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Functional Output-Controllability of Time-Invariant Singular Linear Systems

Abstract

In the space of finite-dimensional singular linear continuous-time-invariant systems described in the form 1

$$\left.\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\}$$

where
$E, A \in M = M_n(\mathrm{C})$,
$B \in M_{n\times m}(\mathrm{C})$,
$C \in M_{p\times n}(\mathrm{C})$, functional output-controllability character is considered. A simple test based in the computation of the rank of a certain constant matrix that can be associated to the system is presented.

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María Isabel García-Planas and Sonia Tarragona

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$$\left.\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\} \tag{1}$$

where $E, A \in M = M_n(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$, $C \in M_{p\times n}(\mathbb{C})$, functional output-controllability character is considered. A simple test based in the computation of the rank of a certain constant matrix that can be associated to the system is presented.

1 Introduction

A great many physical problems as for example electrical networks, multibody systems, chemical engineering, Economics, semidiscretized Stokes equations, Convolutional codes among others, use state space representation as (1) for description.

This linear system can be described with a input-output relation called transfer function obtained by applying Laplace transformation to Eq. (1)

$$\left.\begin{aligned} sEX - x(0) &= AX + BU \\ Y &= CX, \end{aligned}\right\},$$

obtaining the following relation

$$H(s)U(s) = C(sE - A)^{-1}x(0) + C(sE - A)^{-1}BU(s). \tag{2}$$

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If the system is relaxed (that is to say if the initial state is $x(0) = 0$), Eq. (2) is reduced to

$$H(s) = C(sE - A)^{-1}B. \tag{3}$$

The controllability concept of a dynamical standard system is largely studied by several authors and under many different points of view, (see [1–3,14] for example). Nevertheless, functional controllability for the output vector of a system has been less treated for the standard case and even less for the singular case, (see [7, 10, 12, 13] for example).

The functional output-controllability generally means, that the system can steer output of dynamical system along the arbitrarily given curve over any interval of time, independently of its state vector. A similar but least essentially restrictive condition is the pointwise output-controllability.

J.L. Domínguez in [6] examine the functional output controllability of a linear system describing a fixed speed wind turbine formed by a squirrel cage generator connected directly to the grid. Working over finite fields, Fragouli and Wessel [9] analyze the minimality among strictly equivalent encoders using the functional output controllability character. The authors use the term output observable instead of functional output controllable, it is the same concept but working in discrete variable.

In this paper functional output-controllability for singular systems is analyzed generalizing the study realized for standard systems and a simple test based on computing the ranks of certain matrices in order to study this property is presented. Notice that, in [11], the authors present a test for the study of functional output-controllability of regular singular systems, which result is therefore a particular case of the one presented in this article where regularizable systems are considered.

2 Preliminaries

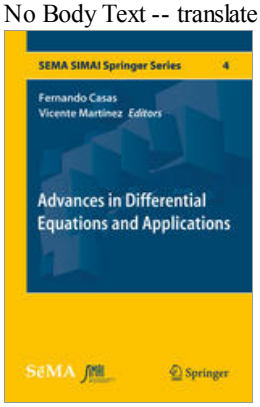
In this paper, it is considered the singular state space system introduced in Eq. (1)

$$\left.\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}\right\},$$

where x is the state vector, y is the output vector, u is the input (or control) vector, $A \in M_n(C)$ is the state matrix, $B \in M_{n\times m}(C)$ is the input matrix and $C \in M_{p\times n}(C)$ is the output matrix.

For simplicity, we will write the systems by quadruples of matrices (E, A, B, C).

In particular we will are interested in systems (called regular) which are those that satisfy the relation $\det(\lambda E + \mu A) \neq 0$ for some $(\lambda, \mu) \in \mathbb{C}^2$, or those systems (called regularizable), which through a feedback proportional and/or derivative and/or an output injection proportional and/or derivative become regular. More



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