

RIGIDITY OF POISSON LIE GROUP ACTIONS

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ABSTRACT. In this paper we prove that close infinitesimal momentum maps associated to Poisson Lie actions are equivalent under some mild assumptions. We also obtain rigidity theorems for actual momentum maps (when the acting Lie group G is endowed with an arbitrary Poisson structure) combining a rigidity result for canonical Hamiltonian actions ([25]) and a linearization theorem ([13]). These results have applications to quantization of symmetries since these infinitesimal momentum maps appear as the classical limit of quantum momentum maps ([4]).

1. INTRODUCTION

In 1961 Palais proved that close actions of compact Lie groups on compact manifolds can be conjugated by a diffeomorphism [29]. The interest of this rigidity theorem relies on the approximation of actions by nearby ones. As application of this rigidity theorem of Palais we can recover normal form theorems such as the linearization theorem by Bochner [5]. Several generalizations of this result have been obtained in [12] and [23] for the case of symplectic structures and in [25] for the case of Hamiltonian actions of semisimple Lie algebras on Poisson manifolds.

In this paper we generalize a rigidity result from [25] to the context of Poisson Lie groups and pre-Poisson Hamiltonian actions. The main result in [25] establishes that two *close* moment maps $\mu : (M, \Pi) \rightarrow \mathfrak{g}^*$ associated to standard Hamiltonian actions of Lie groups on a Poisson manifold are diffeomorphic when \mathfrak{g} is a compact semisimple Lie algebra and M is a compact manifold.

In this paper we consider the counterpart for compact Poisson Lie groups and pre-Poisson Hamiltonian actions. In this case the actions do not necessarily lift to a canonical moment map (the obstruction being a closed 1-form which is not necessarily exact). The Lie group itself is endowed with a Poisson structure and thus the action of the Lie group on the Poisson manifold (M, Π) does not necessarily preserve the Poisson structure Π on M . When the Poisson structure on the Lie group is the trivial one, we recover a standard Poisson action which is Hamiltonian if the above-mentioned closed 1-form is exact.

As explained in [19] it is necessary to consider this generalization of Hamiltonian actions on Poisson manifolds, in order to take into account the properties of the dressing transformations under *hidden symmetry group* in the

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case of R -matrices. Poisson Lie group actions on Poisson manifolds with non-trivial Poisson structures appear naturally in the study of R -matrices. For these, the notion of momentum mapping for Poisson manifolds coincides with the monodromy matrix of the associated linear system (see [19]). Thus studying rigidity for Poisson Lie group actions can be useful to understand stability of the integrable systems associated to R -matrices.

On the other hand, it is worth mentioning here that the momentum map associated to Poisson Lie group actions represents the semiclassical limit of a quantum Hamiltonian action, as shown in [4]. Thus the study of rigidity properties can be useful to comprehend quantum momentum maps.

There are two main novelties in this paper: First to consider Poisson Lie groups and Poisson Lie group actions instead of standard Poisson actions (for which the associated moment map is called *canonical*) and also to consider infinitesimal momentum maps. Infinitesimal momentum maps are the local counterpart to momentum maps and topology on the acted manifold is an obstruction to its integration to global momentum maps. This is also the case when the Poisson structure on G is not trivial but there are additional obstructions as shown in [11]. In particular the theorems that we prove in this article for infinitesimal momentum maps yield as a corollary a stronger result than the one contained in [25] about rigidity of momentum maps for Poisson structures.

In section 3 rigidity results are considered in the Poisson Lie group setting for actual moment maps. In this case the infinitesimal momentum map actually integrates to a moment map $\mu : M \rightarrow G^*$. When the Lie group G is semisimple and compact, we can indeed prove that close actions are equivalent. The proof uses a global linearization theorem due to Ginzburg and Weinstein [13] and the rigidity result for Hamiltonian actions on Poisson manifolds obtained by Miranda, Monnier and Zung [25].

In section 4 we consider the more general case in which the infinitesimal group action does not integrate to a moment map and prove a rigidity result in case the Lie group is semisimple and compact. The proof uses techniques native to geometrical analysis and an abstract normal form theorem from [25]. This abstract normal form encapsulates a Newton's iterative method used by Moser and Nash to prove the inverse function theorem in infinite dimensions (see for example [18]). Newton's method is used to prove normal forms results by approximating the solution using an iterative process [25]. The solution is then presented as a limit. Due to the loss of differentiability in this process, one needs to use smoothing operators and a deep knowledge of geometric analysis.

The abstract normal form for SCI spaces presented in [25] allows to prove normal forms results (and in particular, linearization and rigidity theorems) without having to plunge into the details of the iterative method. The abstract normal form theorem in [25] has had other applications in the theory of generalized complex manifolds (see [2] and [3]) and a variant of it to normal forms in a neighbourhood of a symplectic leaf of a Poisson manifold [28]. In this paper we provide a new application of this normal form for SCI spaces (the details of all the SCI spaces paraphernalia are included as an appendix so that the reader who is not interested in these details

can skip them without losing the essence of the paper). As in [25] we first prove an infinitesimal rigidity result and then we apply the SCI normal form theorem to conclude rigidity. The normal form theorem that we prove for infinitesimal moment maps requires as additional condition that the image of the homotopy operator of an infinitesimal momentum map –whose existence is guaranteed by the infinitesimal rigidity theorem– is a closed one form; this condition is equivalent to the preservation of the Maurer-Cartan equation by h and it is automatically satisfied in the canonical case. Our theorem can be seen as another reincarnation of Mather’s principle “*infinitesimal stability implies stability*” (see [22] and its sequel).

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2. HAMILTONIAN ACTIONS IN THE POISSON LIE SETTING

In this section we give a brief summary of the notions of Poisson action and momentum map, which generalize the concept of Hamiltonian action to the Poisson Lie context. A Poisson Lie group is defined by a pair (G, π_G) , where G is a Lie group and π_G is a Poisson structure compatible with the multiplication on G . The corresponding infinitesimal object is given by a Lie bialgebra, i.e. the Lie algebra \mathfrak{g} corresponding to the Lie group G , equipped with the 1-cocycle,

$$(1) \quad \delta = d_e \pi_G : \mathfrak{g} \rightarrow \mathfrak{g} \wedge \mathfrak{g}.$$

Drinfel’d’s principle [8] establishes a one-to-one correspondence between the Poisson Lie group (G, π_G) and the Lie bialgebra (\mathfrak{g}, δ) if G is connected and simply connected (for this reason we assume this hypothesis to hold throughout this paper). The 1-cocycle δ also makes \mathfrak{g}^* into a Lie algebra, thus we can define the dual Poisson Lie group G^* as the Lie group associated to the Lie algebra \mathfrak{g}^* .

Definition 2.1. *The action of (G, π_G) on (M, π) is called **Poisson action** if the map $\Phi : G \times M \rightarrow M$ is Poisson, that is*

$$(2) \quad \{f \circ \Phi, g \circ \Phi\}_{G \times M} = \{f, g\}_M \circ \Phi \quad \forall f, g \in C^\infty(M)$$

where the Poisson structure on $G \times M$ is given by $\pi_G \oplus \pi$.

It is important to remark that if G carries the zero Poisson structure $\pi_G = 0$, the action is Poisson if and only if it preserves π . Among the class of Poisson actions, Hamiltonian ones play an important role. In general, when $\pi_G \neq 0$, the structure π is not invariant with respect to the action.

Given an action Φ , its infinitesimal generator is a map which associates a vector field \hat{X} on M to any element $X \in \mathfrak{g}$.

Definition 2.2 (Lu, [20], [21]). *A **momentum map** for the Poisson action $\Phi : G \times M \rightarrow M$ is a map $J : M \rightarrow G^*$ such that*

$$(3) \quad \hat{X} = \pi^\sharp(J^*(\theta_X))$$

where θ_X is the left invariant 1-form on G^* defined by the element $X \in \mathfrak{g} = (T_e G^*)^*$ and J^* is the cotangent lift $T^*G^* \rightarrow T^*M$.

In other words, the momentum map generates \widehat{X} by means of the following construction

$$\mathfrak{g} \xrightarrow{\theta} \Omega^1(G^*) \xrightarrow{\alpha} \Omega^1(M) \xrightarrow{\pi^\sharp} TM$$

where, for $X \in \mathfrak{g}$, $\alpha_X = J^*(\theta_X)$. Notice that the maps θ and π^\sharp are Lie algebra homomorphisms. It is useful to recall that given a Poisson structure π , the anchor map π^\sharp defined as $\pi^\sharp(\alpha) := \pi(\alpha, \cdot)$, defines a skew-symmetric operation $[\cdot, \cdot]_\pi : \Omega^1(M) \times \Omega^1(M) \rightarrow \Omega^1(M)$. The following proposition states the main properties of this operation

Proposition 1. *Let (M, π) be a Poisson manifold. Then there exists a unique bilinear, skew-symmetric operation $[\cdot, \cdot]_\pi : \Omega^1(M) \times \Omega^1(M) \rightarrow \Omega^1(M)$ such that*

$$(4) \quad [df, dg]_\pi = d\{f, g\}, \quad f, g \in C^\infty(M)$$

$$(5) \quad [\alpha, f\beta]_\pi = f[\alpha, \beta]_\pi + (\iota_{\pi^\sharp(\alpha)}f)\beta \quad f \in C^\infty(M), \alpha, \beta \in \Omega^1(M).$$

This operation is given by the general formula

$$[\alpha, \beta]_\pi = \mathcal{L}_{\pi^\sharp(\alpha)}\beta - \mathcal{L}_{\pi^\sharp(\beta)}\alpha - d(\pi(\alpha, \beta)) = \mathcal{L}_{\pi^\sharp(\alpha)}\beta - \iota_{\pi^\sharp(\beta)}d\alpha.$$

*Furthermore, it provides $\Omega^1(M)$ with a Lie algebra structure such that $\pi^\sharp : T^*M \rightarrow TM$ is a Lie algebra homomorphism.*

In general, $J^* : T^*G^* \rightarrow T^*M$ is not a Lie algebra homomorphism; for this reason we introduce the concept of equivariance of momentum map and we recall that a momentum map is said equivariant if and only if it is a Poisson map, i.e.

$$J_*\pi = \pi_{G^*}.$$

Finally, we can say that a **Poisson Hamiltonian action** in this context is a Poisson action induced by an equivariant momentum map. This definition generalizes Hamiltonian actions in the canonical setting. Indeed, we notice that, if the Poisson structure on G is trivial, the dual G^* corresponds to the dual of the Lie algebra \mathfrak{g}^* , the one-form θ_X is the constant one-form X on \mathfrak{g}^* and

$$J^*(\theta_X) = d(H_X)$$

where $H_X(m) = \langle J(m), X \rangle$. Thus, it recovers the usual definition of momentum map for Hamiltonian action in the canonical setting $J : M \rightarrow \mathfrak{g}^*$

$$\widehat{X} = \pi^\sharp(d(H_X)) = \{H_X, \cdot\}$$

Proposition 2. [10] *Given a Poisson Hamiltonian action $\Phi : G \times M \rightarrow M$ with momentum map $J : M \rightarrow G^*$, the forms $\alpha_X = J^*(\theta_X)$ satisfy the following identities*

$$(6) \quad \alpha_{[X, Y]} = [\alpha_X, \alpha_Y]_\pi$$

$$(7) \quad d\alpha_X + \alpha \wedge \alpha \circ \delta(X) = 0$$

The second condition is classically known as *Maurer-Cartan equation*. This observation allows us to introduce a weaker definition of momentum map, in terms of forms. In order to give this new definition we need to recall the notion of Gerstenhaber algebras:

Definition 2.3. A **Gerstenhaber algebra** is a differential graded commutative algebra endowed with a Lie bracket which satisfies the following identities

- $|[a, b]| = |a| + |b| - 1$ (The Lie bracket has degree -1)
- $[a, bc] = [a, b]c + (-1)^{(|a|-1)|b|} [a, c]$ (graded Leibniz identity)

where $|a|$ is the degree of an element a .

Example 2.1. The Poisson structure on G gives its Lie algebra a structure of a Lie bialgebra $(\mathfrak{g}, [\cdot, \cdot], \delta)$ and hence a structure of Gerstenhaber algebra on $\wedge^\bullet \mathfrak{g}$.

Example 2.2. The Poisson bracket on M induces a structure of Lie algebra on $\Omega^\bullet(M)$ with bracket $[\cdot, \cdot]_\pi$; this makes $\Omega^\bullet(M)$ into a Gerstenhaber algebra.

Thus, we can introduce a weaker definition of momentum map, motivated by Proposition 2:

Definition 2.4. Let (M, π) be a Poisson manifold and (G, π_G) a Poisson Lie group. An **infinitesimal momentum map** is a morphism of Gerstenhaber algebras

$$(8) \quad \alpha : (\wedge^\bullet \mathfrak{g}, \delta, [\cdot, \cdot]) \longrightarrow (\Omega^\bullet(M), d_{DR}, [\cdot, \cdot]_\pi).$$

It is worthwhile to mention that, in the Heisenberg case, a concrete study of the conditions under which we can lift the infinitesimal momentum map to the global one gives us the following

Theorem 2.1 (Esposito, Nest [11]). *Let G be a Poisson Lie group acting on a Poisson manifold M with an infinitesimal momentum map α and such that G^* is the Heisenberg group. Let ξ, η, ζ denote the basis of \mathfrak{g} dual to the standard basis x, y, z of \mathfrak{g}^* , with z central and $[x, y] = z$. Then*

$$(9) \quad \pi(\alpha_\xi, \alpha_\eta) = c$$

where c is a constant on M . The form α lifts to a momentum map $J : M \rightarrow G^*$ if and only if $c = 0$. When $c = 0$ the set of momentum maps with given α is one dimensional with free transitive action of \mathbb{R} .

This induces a new definition, that we call pre-Hamiltonian Poisson action as it is somehow weaker than the Poisson Hamiltonian definition given above.

Definition 2.5. A **pre-Hamiltonian Poisson action** is a Poisson action of (G, π_G) on (M, π) induced by an infinitesimal momentum map $\alpha : (\mathfrak{g}, \delta, [\cdot, \cdot]) \rightarrow (\Omega^1(M), d_{DR}, [\cdot, \cdot]_\pi)$.

Clearly this notion is weaker than the Hamiltonian one, as it does not reduce to the canonical one when the Poisson structure on G is trivial. In fact, if $\pi_G = 0$ we have $\delta = 0$ and the Maurer-Cartan equation implies that α_X is a closed form, but in general this form is not exact. If, for example, M is simply connected, α_X is also exact and we can recover the usual definition of momentum map and Hamiltonian system. If M is not simply connected we can get examples in the symplectic realm like rotations on a torus or more sophisticated ones for general Poisson structures.

Example 2.3. Consider the torus \mathbb{T}^2 , with Poisson structure $\pi = \sin \theta_1 \frac{\partial}{\partial \theta_1} \wedge \frac{\partial}{\partial \theta_2}$ where the coordinates on the torus are $\theta_1, \theta_2 \in [0, 2\pi]$. This Poisson structure is symplectic away from the set $Z = \{\theta_1 \in \{0, \pi\}\}$ and the Poisson structure satisfies a transversality condition at the vanishing set. This Poisson structure pertains to a class called *b-Poisson structures* (or *b-symplectic structures*) studied in [16]. The circle action of rotation on the θ_2 coordinate defines a pre-Hamiltonian Poisson action on \mathbb{T}^2 . Indeed it is possible to associate a b-symplectic form to this Poisson structure (see [16]) and work with b-symplectic actions. In this case $\frac{1}{\sin \theta_1} d\theta_1 \wedge d\theta_2$. The circle action of rotation on the θ_2 coordinate is pre-Hamiltonian and the associated one-form is $\frac{1}{\sin \theta_1} d\theta_1$ (see [17] for properties of these actions on b-Poisson manifolds).

In general (when the Poisson structure on the Lie group is not trivial) the problem is harder. The study of the conditions in which an infinitesimal momentum map determines a momentum map in the usual sense can be found in [11]. A concrete example of infinitesimal momentum map has been computed in [4].

3. RIGIDITY FOR HAMILTONIAN ACTIONS

The goal of this section is to prove that two close actions of Poisson Lie group G with arbitrary Poisson structures on G with moment maps $\mu_0 : M \rightarrow G^*$ and $\mu_1 : M \rightarrow G^*$ are equivalent. That is, there exists a diffeomorphism $\phi : M \rightarrow M$ such that $\phi^*(\mu_1) = \mu_0$. We can prove this when the Poisson-Lie group is *semisimple and compact* by combining well-known results of Ginzburg and Weinstein [13] concerning linearization of Poisson-Lie groups with a rigidity theorem for canonical moment maps contained in [25].

Let us start by clarifying we clarify what we mean in this paper by “close” actions.

An action $\rho : G \times M \rightarrow M$ of a Lie group G on a smooth manifold M is a morphism from G to the group of diffeomorphisms $Diff(M)$. As a consequence, we can view this action as an element in $Map(G \times M, M)$ and use the C^k -topology there to refer to close elements¹.

In this paper we can define the topology by using the associated momentum maps, either infinitesimal or not. In the case the infinitesimal moment maps integrate to actual moment maps, we consider the standard C^k -topology in the space of smooth mappings $\mathcal{C}^k(M, G^*)$. If the action is given by an infinitesimal momentum map we can also use the C^k -norm of the infinitesimal momentum map $\alpha : \mathfrak{g} \rightarrow \Omega^1(M)$ and work with α_X , for $X \in \mathfrak{g}$ as mappings $\alpha_X : M \rightarrow TM$.

In order to prove the main result in this section we need to recall the rigidity theorem for Hamiltonian actions on Poisson manifolds contained in [25]:

Theorem 3.1 (Miranda, Monnier, Zung [25]). *Consider a compact Poisson manifold $(M, \{, \})$ and a Hamiltonian action on M given by the momentum map $\lambda : M \rightarrow \mathfrak{g}^*$ where \mathfrak{g} is a semisimple Lie algebra of compact type.*

¹ Observe that two momentum maps $\mu_1 : M \rightarrow \mathfrak{g}^*$ and $\mu_2 : M \rightarrow \mathfrak{g}^*$ are close then the two Hamiltonian actions are close.

There exist a positive integer l and two positive real numbers α and β (with $\beta < 1 < \alpha$) such that, if μ is another momentum map on M with respect to the same Poisson structure and Lie algebra, satisfying

$$(10) \quad \|\lambda - \mu\|_{2l-1} \leq \alpha \quad \text{and} \quad \|\lambda - \mu\|_l \leq \beta$$

then, there exists a diffeomorphism ψ of class C^k , for all $k \geq l$, on M such that $\mu \circ \psi = \lambda$.

Observe that since a Poisson structure on a Poisson Lie group (with Poisson structure π) must vanish at $e \in G$, its linearization at e is well-defined (recall that $d_e \pi_G : \mathfrak{g} \rightarrow \mathfrak{g} \wedge \mathfrak{g}$).

The following theorem says that if G is compact and semisimple, the Poisson structure π_G is linearizable, thus equivalent to $d_e \pi_G$ by diffeomorphisms.

Theorem 3.2 (Ginzburg, Weinstein). *Let G be a compact semisimple Poisson Lie group then the dual Poisson Lie group G^* is globally diffeomorphic to \mathfrak{g}^* with the linear Poisson structure defined as $\{f, g\}_\eta = \langle \eta, [df_\eta, dg_\eta] \rangle$.*

Thus when the Pre-Hamiltonian Poisson action is indeed Hamiltonian the infinitesimal momentum map lifts to a mapping $\mu : M \rightarrow G^*$ then we can apply theorem 3.2 and combine it with theorem 3.1 to obtain rigidity for the action.

More concretely,

Theorem 3.3. *Consider a compact Poisson manifold $(M, \{, \})$ and a Poisson Lie Hamiltonian action on M of a compact semisimple Poisson Lie group G given by the momentum map $\mu_0 : M \rightarrow G^*$.*

There exist a positive integer l and two positive real numbers α and β (with $\beta < 1 < \alpha$) such that, if μ_1 is another momentum map on M with respect to the same Poisson structure and Poisson Lie group, satisfying

$$(11) \quad \|\mu_0 - \mu_1\|_{2l-1} \leq \alpha \quad \text{and} \quad \|\mu_0 - \mu_1\|_l \leq \beta$$

then, there exists a diffeomorphism ψ of class C^k , for all $k \geq l$, on M such that $\mu_1 \circ \psi = \mu_0$.

Proof. Denote by Φ the linearizing Poisson diffeomorphism² $\Phi : G^* \rightarrow \mathfrak{g}^*$ given by theorem 3.2 and consider the compositions $\tilde{\mu}_0 = \Phi \circ \mu_0$ and $\tilde{\mu}_1 = \Phi \circ \mu_1$. The mappings $\tilde{\mu}_0 : M \rightarrow \mathfrak{g}^*$ and $\tilde{\mu}_1 : M \rightarrow \mathfrak{g}^*$ are canonical moment maps and we may consider the infinitesimal Hamiltonian actions of \mathfrak{g} (β_0 and β_1). These actions integrate to infinitesimal standard Lie group actions of the Lie group G which preserve the Poisson structure on M . We may now apply theorem 3.1 to obtain a diffeomorphism $\tilde{\psi}$ such that $\tilde{\mu}_1 \circ \tilde{\psi} = \tilde{\mu}_0$ and therefore $\mu_1 \circ \psi = \mu_0$.

□

3.1. The case of coboundary Poisson Lie groups. We start with the definition of coboundary Lie bialgebra and coboundary Poisson Lie group.

Definition 3.1. *A Lie bialgebra \mathfrak{g} is called a coboundary Lie bialgebra if δ is the coboundary of some element $r \in \mathfrak{g} \wedge \mathfrak{g}$ (that is, $\delta(\xi) = ad_\xi(r)$).*

²The differentiability class can be assumed to be k by the construction in the proof of theorem 3.2

A coboundary Poisson Lie group is a Poisson Lie group with corresponding coboundary Lie bialgebra under Drinfeld's correspondence. The category of coboundary Poisson Lie groups includes the case of quasitriangular Poisson Lie groups.

The formal linearization of quasitriangular Poisson Lie groups was studied by Enriquez, Etingof and Marshall [9]. Recently Alekseev and Meinrenken have generalized this further proving that coboundary Poisson Lie groups are indeed linearizable [1] as the following theorem shows,

Theorem 3.4 (Alekseev, Meinrenken [1]). *For any coboundary Poisson Lie group G , the dual G^* is linearizable at e .*

As we did in the proof of Theorem 3.3, we could try to combine this result with some rigidity for canonical moment maps but such a rigidity result is not proved in the canonical context for non-semisimple Lie groups. We believe that it is possible to adapt the Moser's techniques developed in [1] to give a direct proof of rigidity of moment maps for coboundary Poisson Lie groups but we are not addressing this problem here.

4. RIGIDITY FOR INFINITESIMAL MOMENTUM MAPS

In this section we prove that close infinitesimal momentum maps of Pre-Hamiltonian Poisson actions of Poisson Lie groups are equivalent. In order to do that we first revise the ideas of the proof of rigidity contained in [25].

The main idea in [25] is to approximate a given moment map by an iteration of moment maps.

As explained in [15], a first approach to proving the equivalence of Lie group actions on manifolds would follow the steps below:

In general a Lie group action gives an element in $\mathcal{M} = \text{Hom}(G, \text{Diff}(M))$ and we can consider the additional action,

$$\begin{aligned} \beta : \text{Diff}(M) \times \mathcal{M} &\longmapsto \mathcal{M} \\ (\phi, \alpha) &\longmapsto \phi \circ \alpha \circ \phi^{-1} \end{aligned}$$

Two actions α_0 and α_1 are conjugated if they are on the same orbit under β so, in particular, if β has open orbits the action is rigid.

Observe that

- The tangent space to the orbit of β coincides with 1-coboundaries of the group cohomology with coefficients in $V = \text{Vect}(M)$ and the tangent space to \mathcal{M} are the 1-cocycles.
- The generalized Whitehead lemma implies that for compact G the cohomology group $H^1(G; \text{Vect}(M))$ vanishes. This phenomenon is known as infinitesimal rigidity. In this case the tangent space to the orbit equals the tangent space to \mathcal{M} .
- If \mathcal{M} is a manifold (or tame Fréchet) we can apply the inverse function theorem Nash-Moser to go from the tangent space to the manifold. We can use this fact to prove that β has open manifolds and thus the action is rigid.

In general it is hard to verify the ‘‘tame Fréchet’’ condition but we can apply the method used in the proof of Nash-Moser's theorem (Newton's iterative method). This methods allows to proof several results of type

infinitesimal rigidity implies rigidity. For Hamiltonian actions on Poisson manifold [25] we consider the Chevalley-Eilenberg complex associated to the representation given by the moment map following the next steps:

- (1) Assume that the close moment maps are $\mu_0 : M \rightarrow \mathfrak{g}^*$ and $\mu_1 : M \rightarrow \mathfrak{g}^*$. The difference $\phi = \mu_0 - \mu_1$ defines a 1-cochain of the complex which is a near 1-cocycle.
- (2) We define Φ as the time-1-map of the Hamiltonian vector field $X_{S_t(h(\phi))}$ with h the homotopy operator and S_t is a smoothing operator.
- (3) The Newton iteration is given by,

$$\phi_d = \phi_{X_{S_t(h(\eta_d))}}^1$$

with $\eta_d = (\mu_1 - \mu_0) \circ \phi_{d-1}$. This converges to a Poisson diffeomorphism that conjugates both actions.

Convergence is a *hard* part of the proof. This is why in [25] a strong use of geometric analysis tools is performed in order to check this using the paraphernalia of SCI spaces (see the appendix). In particular the theorem needed to prove convergence is the abstract normal form presented in the first subsection of this section.

In the Poisson Lie group case, we will follow a similar scheme, the difference is that we need to replace a Chevalley-Eilenberg complex which considers the set of smooth functions as a \mathfrak{g} -module by a Chevalley-Eilenberg complex which considers the set of smooth forms as \mathfrak{g} -module. We devote a subsection to defining this complex. The diffeomorphisms considered in each step of the iteration will not be Hamiltonian but Poisson diffeomorphisms of type,

$$\phi_d = \phi_{X_{S_t(h(\eta_d))}}^1$$

with η the difference of two one-forms. In order for this proof to work, we will require that the homotopy operator sends forms α_X satisfying the Maurer-Cartan equation to closed one-forms. We will call actions satisfying this condition *admissible*. Before presenting the proof of the main theorem of this section which holds for Pre-Hamiltonian Poisson actions, we will present a sketch of the proof for Pre-Hamiltonian actions that integrate for Hamiltonian ones (this gives a different proof of Theorem 3.3 given in Section 3) and is included here for sheer pedagogical purposes.

4.1. Preliminaries: An abstract normal form for SCI-spaces. SCI-spaces (where SCI stands for *scaled C^∞ -type*) are a generalization of scaled spaces and tame Fréchet spaces. This analytical apparatus is needed to prove normal form theorems in the most possible general setting which includes neighbourhood of a point, a compact invariant submanifold or a compact manifold. We refer to the appendix for the basic definitions of SCI-spaces, SCI-groups and SCI-actions. It is good to keep in mind the following archetypical example: an example of SCI-spaces is the set of Poisson structures, an example of SCI-group is the group of diffeomorphism (which can be germified, semilocal or global), and in this case an example of SCI-action is the pushforward of a Poisson structure via a diffeomorphism.

The scheme of proof of a normal form theorem in this abstract setting is the following:

- (1) \mathcal{G} (for instance diffeomorphisms) which acts on a set \mathcal{S} (of structures).
- (2) We consider the subset of structures in normal forms \mathcal{N} inside \mathcal{S} .
- (3) The equivalence of an element in \mathcal{S} to a normal form is understood in the following way:

For each element $f \in \mathcal{S}$ there is an element $\phi \in \mathcal{G}$ such that $\phi.f \in \mathcal{N}$.

For practical purposes it is convenient to assume that a \mathcal{S} (in the example above, the set of Poisson structures) is a subset of a linear space \mathcal{T} (in the example above \mathcal{T} would be the set of bivector fields).

The SCI-group \mathcal{G} acts on \mathcal{T} , and the set of normal forms $\mathcal{N} = \mathcal{F} \cap \mathcal{S}$ where \mathcal{F} is a linear subspace of \mathcal{T} .

The following theorem contained in [25] is an abstract normal form theorem for SCI in order to apply it to particular situations, we need to identify the sets \mathcal{S} , \mathcal{F} , \mathcal{T} and the SCI-group \mathcal{G} in each case.

We also need to identify \mathcal{G}_0 a closed subgroup of \mathcal{G} which is not necessarily an SCI-subgroup (for instance, the set of Poisson diffeomorphisms inside the set of diffeomorphisms).

As a consequence the equivalence to the normal form is given by the existence of $\psi \in \mathcal{G}$ (or in a closed subgroup) \mathcal{G}_0 for each $f \in \mathcal{S}$ such that $\psi \cdot f \in \mathcal{N}$.

Theorem 4.1 (Miranda, Monnier, Zung [25]). *Let \mathcal{T} be a SCI-space, \mathcal{F} a SCI-subspace of \mathcal{T} , and \mathcal{S} a subset of \mathcal{T} . Denote $\mathcal{N} = \mathcal{F} \cap \mathcal{S}$. Assume that there is a projection $\pi : \mathcal{T} \rightarrow \mathcal{F}$ (compatible with restriction and inclusion maps) such that for every f in $\mathcal{T}_{k,\rho}$, the element $\zeta(f) = f - \pi(f)$ satisfies*

$$(12) \quad \|\zeta(f)\|_{k,\rho} \leq \|f\|_{k,\rho} \text{Poly}(\|f\|_{[(k+1)/2],\rho})$$

for all $k \in \mathbb{N}$ (or at least for all k sufficiently large), where $[\]$ is the integer part.

Let \mathcal{G} be an SCI-group acting on \mathcal{T} by a linear left SCI-action and let \mathcal{G}^0 be a closed subgroup of \mathcal{G} formed by elements preserving \mathcal{S} .

Let \mathcal{H} be a SCI-space and assume that there exist maps $\mathbb{H} : \mathcal{S} \rightarrow \mathcal{H}$ and $\Phi : \mathcal{H} \rightarrow \mathcal{G}^0$ and an integer $s \in \mathbb{N}$ such that for every $0 < \rho \leq 1$, every f in \mathcal{S} and g in \mathcal{H} , and for all k in \mathbb{N} (or at least for all k sufficiently large) we have the three properties:

$$(13) \quad \|\mathbb{H}(f)\|_{k,\rho} \leq \|\zeta(f)\|_{k+s,\rho} \text{Poly}(\|f\|_{[(k+1)/2]+s,\rho}) \\ + \|f\|_{k+s,\rho} \|\zeta(f)\|_{[(k+1)/2]+s,\rho} \text{Poly}(\|f\|_{[(k+1)/2]+s,\rho}),$$

$$(14) \quad \|\Phi(g) - Id\|_{k,\rho'} \leq \|g\|_{k+s,\rho} \text{Poly}(\|g\|_{[(k+1)/2]+s,\rho})$$

and

$$(15) \quad \|\Phi(g_1) \cdot f - \Phi(g_2) \cdot f\|_{k,\rho'} \leq \|g_1 - g_2\|_{k+s,\rho} \|f\|_{k+s,\rho} \text{Poly}(\|g_1\|_{k+s,\rho}, \|g_2\|_{k+s,\rho}) \\ + \|f\|_{k+s,\rho} \text{Poly}_{(2)}(\|g_1\|_{k+s,\rho}, \|g_2\|_{k+s,\rho})$$

if $\rho' \leq \rho(1 - c\|g\|_{2,\rho})$ in (14) and $\rho' \leq \rho(1 - c\|g_1\|_{2,\rho})$ and $\rho' \leq \rho(1 - c\|g_2\|_{2,\rho})$ in (15).

Finally, for every f in \mathcal{S} denote $\phi_f = Id + \chi_f = \Phi(\mathbb{H}(f)) \in \mathcal{G}^0$ and assume that there is a positive real number δ such that we have the inequality

$$(16) \quad \|\zeta(\phi_f \cdot f)\|_{k,\rho'} \leq \|\zeta(f)\|_{k+s,\rho}^{1+\delta} Q(\|f\|_{k+s,\rho}, \|\chi_f\|_{k+s,\rho}, \|\zeta(f)\|_{k+s,\rho}, \|f\|_{k,\rho})$$

(if $\rho' \leq \rho(1 - c\|\chi_f\|_{1,\rho})$) where Q is a polynomial of four variables and whose degree in the first variable does not depend on k and with positive coefficients.

Then there exist $l \in \mathbb{N}$ and two positive constants α and β with the following property: for all $p \in \mathbb{N} \cup \{\infty\}$, $p \geq l$, and for all $f \in \mathcal{S}_{2p-1,R}$ with $\|f\|_{2l-1,R} < \alpha$ and $\|\zeta(f)\|_{l,R} < \beta$, there exists $\psi \in \mathcal{G}_{p,R/2}^0$ such that $\psi \cdot f \in \mathcal{N}_{p,R/2}$.

Remarks on notation:

- $Poly(\|f\|_{k,r})$ stands for a polynomial term in $\|f\|_{k,r}$ where the polynomial has positive coefficients and does not depend on f (though it may depend on k and on r continuously).
- The notation $Poly_{(p)}(\|f\|_{k,r})$, where p is a strictly positive integer, denotes a polynomial term in $\|f\|_{k,r}$ where the polynomial has positive coefficients and does not depend on f (though it may depend on k and on r continuously) and which contains terms of degree greater or equal to p .

Remark 4.2. It would be possible to relax the SCI-hypotheses in order to prove rigidity for Poisson Lie group actions on compact manifolds. However, thanks to the SCI-scheme we can obtain the local and semilocal statements directly from the rigidity statement for compact manifolds.

4.2. A Chevalley-Eilenberg complex associated to a infinitesimal momentum map.

As in [25], the first step is the infinitesimal rigidity: we construct the Chevalley-Eilenberg cohomology associated to an infinitesimal momentum map. The first cohomology group of the complex, H^1 , can be interpreted as infinitesimal deformations and so, when $H^1 = 0$ under the hypotheses of the Whitehead lemma for Fréchet spaces (see [12]) we obtain infinitesimal rigidity.

In this section we aim to introduce the Chevalley-Eilenberg cohomology associated to an infinitesimal momentum map, as defined above. We show that it is related to the Chevalley-Eilenberg cohomology associated to a Hamiltonian action in the canonical setting.

Let $\alpha : \mathfrak{g} \rightarrow \Omega^1(M) : X \mapsto \alpha_X$ be the infinitesimal momentum map of a Hamiltonian action of (G, π_G) on (M, π) . The Lie algebra \mathfrak{g} defines a representation ρ of \mathfrak{g} on $\Omega^1(M)$ defined, for any $X \in \mathfrak{g}$, as

$$(17) \quad \rho_X(\beta) := [\alpha_X, \beta]_\pi$$

where $[\cdot, \cdot]_\pi$ denotes the Lie bracket on $\Omega^1(M)$ defined in Theorem 1. More precisely, we get

$$(18) \quad \rho_X \rho_Y(\beta) - \rho_Y \rho_X(\beta) = \rho_{[X,Y]}(\beta).$$

This is a direct consequence of properties of the bracket $[\cdot, \cdot]_\pi$ and of α since we have:

$$(19) \quad [\alpha_X, [\alpha_Y, \beta]_\pi]_\pi - [\alpha_Y, [\alpha_X, \beta]_\pi]_\pi = [[\alpha_X, \alpha_Y]_\pi, \beta]_\pi = [\alpha_{[X, Y]}, \beta]_\pi.$$

This proves that ρ defines a Lie algebra representation.

Notice that

$$(20) \quad [\alpha_X, \beta]_\pi = \mathcal{L}_{\pi^\#(\alpha_X)}\beta - \iota_{\pi^\#(\beta)}d\alpha_X = \mathcal{L}_{\pi^\#(\alpha_X)}\beta - \iota_{\pi^\#(\beta)}\alpha \wedge \alpha \circ \delta(X).$$

Thus, we can define the space of cochains as follows: For $q \in \mathbb{N}$, $C^q(\mathfrak{g}, \Omega^1(M)) = \text{Hom}(\wedge^q \mathfrak{g}, \Omega^1(M))$ is the space of alternating q -linear maps from \mathfrak{g} to $\Omega^1(M)$, with the convention $C^0(\mathfrak{g}, \Omega^1(M)) = \Omega^1(M)$. The associated differential is denoted by d_i . Explicitely, we have

$$\Omega^1(M) \xrightarrow{\partial_0} C^1(\mathfrak{g}, \Omega^1(M)) \xrightarrow{\partial_1} C^2(\mathfrak{g}, \Omega^1(M))$$

where

$$(21) \quad \partial_0(\beta)(X) = \rho_X(\beta), \quad \beta \in \Omega^1(M)$$

$$(22)$$

$$\partial_1(\gamma)(X \wedge Y) = \rho_X(\gamma(Y)) - \rho_Y(\gamma(X)) - \gamma([X, Y]), \quad \gamma \in C^1(\mathfrak{g}, \Omega^1(M))$$

with $X, Y \in \mathfrak{g}$. It is well-known that these differentials satisfy $\partial_i \circ \partial_{i-1} = 0$ and we can define the quotients

$$H^i(\mathfrak{g}, \Omega^1(M)) = \ker(\partial_i) / \text{Im}(\partial_{i-1}) \quad \forall i \in \mathbb{N}.$$

Finally, we can see that there exists homotopy operator h_i satisfying

$$\partial_i \circ h_i + h_{i+1} \circ \partial_{i+1} = id_{C^{i+1}(\mathfrak{g}, \Omega^1(M))}$$

for $i = 0, 1$.

$$\Omega^1(M) \begin{array}{c} \xrightarrow{\partial_0} \\ \xleftarrow{h_0} \end{array} C^1(\mathfrak{g}, \Omega^1(M)) \begin{array}{c} \xrightarrow{\partial_1} \\ \xleftarrow{h_1} \end{array} C^2(\mathfrak{g}, \Omega^1(M)).$$

For the Chevalley-Eilenberg complexes used in [25] and [6], certain inequalities are proved for the homotopy operators. These are necessary to control the loss of differentiability in the iterative process. This is somewhat hidden in the abstract normal form theorem in [25] by requiring that the data are SCI spaces. We will need the following lemma (which extends lemma 5.7 in [25]) in order to guarantee that our spaces comply with the SCI requirement.

The trick used in [25] and [6] in order to prove the lemma below is to first use Sobolev metrics and then Sobolev inequalities and then take the real part in order to obtain the desired inequalities. For the Chevalley-Eilenberg complex that we consider in this paper we need those inequalities applied to mappings $\alpha : \wedge^k \mathfrak{g} \rightarrow \Omega^1(M)$ and work with α_X , for $X \in \wedge^k \mathfrak{g}$ as mappings $\alpha_X : M \rightarrow TM$. Since M is compact, Sobolev inequalities holds too. A different way to do this is to consider Sobolev norms in the space of one-forms³ and C^k -topology for the space of one-forms (see for instance [7]) or [14] and adapt the same steps.

³For one-forms on oriented manifolds, we may consider sophistications of the following norm: $\langle \alpha, \beta \rangle = \int_X \alpha \wedge * \beta$ where $*\beta$ stands for the Hodge dual of β .

Lemma 4.1. *In the Chevalley-Eilenberg complex associated to ρ :*

$$\Omega^1(M) \xrightarrow{\partial_0} C^1(\mathfrak{g}, \Omega^1(M)) \xrightarrow{\partial_1} C^2(\mathfrak{g}, \Omega^1(M))$$

there exists a chain of homotopy operators

$$\Omega^1(M) \xrightleftharpoons[h_0]{\partial_0} C^1(\mathfrak{g}, \Omega^1(M)) \xrightleftharpoons[h_1]{\partial_1} C^2(\mathfrak{g}, \Omega^1(M)) .$$

such that

$$\partial_0 \circ h_0 + h_1 \circ \partial_1 = id_{C^1(\mathfrak{g}, \Omega^1(M))}$$

and

$$\partial_1 \circ h_1 + h_2 \circ \partial_2 = id_{C^1(\mathfrak{g}, \Omega^1(M))} .$$

Moreover, for each k , there exists a real constant $C_k > 0$ such that

$$(23) \quad \|h_j(S)\|_{k,r} \leq C_k \|S\|_{k+s,r}, \quad j = 0, 1, 2$$

for all $S \in C^{j+1}(\mathfrak{g}, \Omega^1(M))$

Proof. We apply the same strategy of the proof of lemma 5.7 for compact manifolds in [25] replacing the Sobolev inequalities for smooth function by the analogous for differential forms. A key point in [6] and [25] is that those Sobolev norms are invariant by the action of the Lie group which is linear. The linearity of the action is needed to decompose the Hilbert space into spaces which are invariant.

In our case we can assume that this action is also linear using an appropriate G -equivariant embedding by virtue of Mostow-Palais theorem ([27], [29])⁴.

As it was done in [25], we can check the regularity properties of the homotopy operators with respect to these Sobolev norms and then deduce, as a consequence, regularity properties of the initial norms by looking at the real part. The proof holds step by step by replacing the standard Sobolev inequalities by the ones for differential one-forms.

□

Remark 4.3. *Using the definition of infinitesimal momentum map in terms of Gerstenhaber morphism $\alpha : (\wedge^\bullet \mathfrak{g}, \delta, [,]) \longrightarrow (\Omega^\bullet(M), d_{DR}, [,]_\pi)$ we can immediately generalize the above discussion. In this case α defines the Chevalley-Eilenberg complex $C^q(\wedge^\bullet \mathfrak{g}, \Omega^\bullet(M))$.*

Remark 4.4. *Let us consider the particular case in which the Poisson structure π_G on the Lie group G is trivial. As discussed above, the infinitesimal momentum map associates a closed one-form to each $X \in \mathfrak{g}$. From eq. (20) follows that the Lie algebra representation reduces to*

$$(24) \quad \partial_0(\beta)(X) = [\alpha_X, \beta]_\pi = \mathcal{L}_{\pi^\#(\alpha_X)}\beta$$

⁴Using an orthonormal basis in the vector space E for this action we can define the corresponding Sobolev norms in the ambient spaces provided by the Mostow-Palais embedding theorem. This norm is invariant by the action of G (we can even assume G is a subgroup of the orthogonal group).

Remark 4.5. *If we restrict only to exact forms, it follows immediately from Theorem 1 that*

$$(25) \quad \partial_0(\beta)(X) = [\alpha_X, \beta]_\pi = [dH_X, df]_\pi = d\{H_X, f\}.$$

This means that the infinitesimal momentum map in this case defines a Chevalley-Eilenberg complex $C^q(\mathfrak{g}, \Omega_{ex}^1(M))$, where $\Omega_{ex}^1(M)$ denotes the space of exact one-forms on M . Furthermore, the coboundary operator ∂_0 coincides with the differential of the standard representation δ_0 of \mathfrak{g} on $C^\infty(M)$ discussed in [25]. Similarly, we can see that the homotopy operator \mathfrak{h}_0 coincides with the differential of the homotopy operator h_0 of $C^q(\mathfrak{g}, C^\infty(M))$. In other words, the diagram

$$(26) \quad \begin{array}{ccccc} C^\infty(M) & \xrightleftharpoons[h_0]{\delta_0} & C^1(\mathfrak{g}, C^\infty(M)) & \xrightleftharpoons[h_1]{\delta_1} & C^2(\mathfrak{g}, C^\infty(M)) \\ \downarrow d & & \downarrow d & & \downarrow d \\ \Omega_{ex}^1(M) & \xrightleftharpoons[h_0]{\partial_0} & C^1(\mathfrak{g}, \Omega_{ex}^1(M)) & \xrightleftharpoons[h_1]{\partial_1} & C^2(\mathfrak{g}, \Omega_{ex}^1(M)) \end{array}$$

commutes.

In general, α_X satisfies the Maurer-Cartan equation (7); this implies that our complex reduces to

$$\Omega_{cl}^1(M) \xrightleftharpoons[h_0]{\partial_0} C^1(\mathfrak{g}, \Omega^1(M)) \xrightleftharpoons[h_1]{\partial_1} C^2(\mathfrak{g}, \Omega^1(M)).$$

where $\Omega_{cl}^1(M)$ denotes the space of closed one-forms on M . Notice that we are assuming that $dh_0(\alpha) = h_0(d\alpha)$, thus $dh_0(\alpha) = 0$.

4.3. Rigidity of Hamiltonian actions in the Poisson Lie setting. In this section we state the main theorem of this paper, which prove the rigidity of pre-Hamiltonian Poisson actions. The above discussion shows that the rigidity of Hamiltonian action can be seen as a particular case of the rigidity of pre-Hamiltonian Poisson actions. For this reason we first rewrite Theorem 3.3 in terms of infinitesimal momentum map.

Let $\Phi : G \times M \rightarrow M$ a pre-Hamiltonian Poisson action with infinitesimal momentum map $\alpha : \mathfrak{g} \rightarrow \Omega^1(M)$. Assume that $\pi_G = 0$ and $H^1(M) = 0$ (which guarantees that closed forms are also exact), thus $\alpha_X = dH_X$ for any $X \in \mathfrak{g}$ and it induces the infinitesimal generator of the action by means of the construction

$$(27) \quad \begin{array}{ccc} \mathfrak{g} & \xrightarrow{\quad} & \\ \searrow \alpha & \searrow & \\ & \Omega_{ex}^1(M) & \longrightarrow Ham(M) \end{array}$$

where we denoted by $Ham(M)$ the set of Hamiltonian vector fields on M . In this particular case, it has been proved (see [11]) that the infinitesimal momentum map is generated by a momentum map $J : M \rightarrow G^*$; since $G^* = \mathfrak{g}^*$ it coincides with the canonical momentum map. It is important to recall that, in the canonical setting, giving a equivariant momentum map

$J : M \rightarrow \mathfrak{g}^*$ is equivalent to specify a Lie algebra homomorphism $H : \mathfrak{g} \rightarrow C^\infty(M)$ (called Hamiltonian) making the diagram

$$(28) \quad \begin{array}{ccc} \mathfrak{g} & & \\ & \searrow H & \\ & & C^\infty(M) \longrightarrow Ham(M) \end{array}$$

commute. Since specifying α does not determine the Hamiltonian function in a unique way, we say that the construction (28) is *almost* equivalent to (27). Indeed, given α_X we can reconstruct H_X by solving a cohomological equation, i.e.

$$(29) \quad \alpha_X = dH_X;$$

in other words, α_X determines H_X up to a constant.

In the following we prove the rigidity of the infinitesimal momentum map in the cases in which is equivalent to the canonical momentum map $J : M \rightarrow \mathfrak{g}^*$.

Theorem 4.6. *Consider a pre-Hamiltonian Poisson action of a trivial Poisson Lie group $(G, 0)$ on a compact Poisson manifold (M, π) with $H^1(M) = 0$, given by the construction (27), where \mathfrak{g} is a semisimple Lie algebra of compact type.*

There exist a positive integer l and two positive real numbers a and b (with $b < 1 < a$) such that, if $\tilde{\alpha}$ is another infinitesimal momentum map on M with respect to the same Poisson structure and Lie algebra, satisfying

$$(30) \quad \|\alpha - \tilde{\alpha}\|_{2l-1} \leq a \quad \text{and} \quad \|\alpha - \tilde{\alpha}\|_l \leq b$$

then, there exists a Poisson diffeomorphism $\phi : (M, \pi) \rightarrow (M, \pi)$ of class C^k , for all $k \geq l$, on M such that $\phi^(\tilde{\alpha}_X) = \alpha_X$.*

Proof. This theorem can be also proved applying the affine version of the general norm form theorem 4.1. Let us define the SCI-space \mathcal{T} by the spaces \mathcal{T}_k of C^k -differentiable maps from \mathfrak{g} to $\Omega_{ex}^1(M)$, equipped with the maximal norm. The subset \mathcal{S} is given by the infinitesimal momentum maps (i.e. Lie algebra homomorphisms). The origin of the affine space (see Theorem 4.1) is given by α and $\mathcal{F} = \mathcal{N} = 0$, so that the estimate (12) is obvious.

The SCI-group \mathcal{G} consists of the C^k -differentiable maps from $\Omega_{ex}^1(M)$ to itself, where the action is $\psi \cdot \alpha = \psi \circ \alpha$, with $\psi \in \mathcal{G}$ and $\alpha \in \mathcal{T}$. The closed subgroup \mathcal{G}_0 of \mathcal{G} is given by the Lie algebra homomorphisms on $\Omega_{ex}^1(M)$. The elements of \mathcal{G}_0 preserve \mathcal{S} .

We define the SCI-space \mathcal{H} as the spaces of exact one-forms $\Omega_{ex}^1(M)$, i.e. by the spaces \mathcal{H}_k of differential of C^k -differentiable functions on M . Using the results of Section 4.2, an infinitesimal momentum map α is obviously a 1-cochain in the Chevalley-Eilenberg complex $C^q(\mathfrak{g}, \Omega_{ex}^1(M))$, so we can define the application \mathbb{H} in theorem 4.1 as

$$(31) \quad \mathbb{H} : \mathcal{S} \longrightarrow \mathcal{H} : \alpha \longmapsto \mathfrak{h}_0(\alpha - \tilde{\alpha}).$$

Notice that, as the diagram (26) commutes, we have

$$(32) \quad \mathbb{H}(\alpha_X) = \mathfrak{h}_0(dH_X) = dh_0(H_X)$$

so the map \mathbb{H} is simply given by the differential of Hamiltonian function defined by the canonical momentum map. The homotopy operator h_0 satisfies the inequality (23), so the relation (13) is obvious.

Finally, for every $\alpha \in \mathcal{H}$, we denote by \widehat{X} the Hamiltonian vector field associated to α_X by

$$(33) \quad \widehat{X} = \pi(\alpha_X, \cdot) = \{H_X, \cdot\}$$

Let ψ_t be the flow of the Hamiltonian vector field \widehat{X} and define $\Phi(\alpha) := d\psi_t$. Since the flow ψ_t preserves π , the differential $d\psi_t$ is a Lie algebra homomorphism, thus it is evident that Φ preserves the set of momentum maps \mathcal{S} .

The estimates (14)-(15)-(16) are direct consequences of the Lemmas in Section 5.

Let us now generalize this construction to the case of a generic pre-Hamiltonian Poisson action with corresponding diagram.

$$(34) \quad \begin{array}{ccc} \mathfrak{g} & & \\ & \searrow \alpha & \\ & & \Omega^1(M) \longrightarrow PHam(M) \end{array}$$

The map $\alpha : \mathfrak{g} \rightarrow \Omega^1(M)$ is a Lie algebra homomorphism and it associates to each element X in \mathfrak{g} a generic one-form α_X on M , which satisfies the Maurer-Cartan equation (7).

We will assume that our pre-Hamiltonian Poisson action is **admissible** that is to say, we assume that the homotopy operator sends a Maurer-Cartan form α_X to a closed form.

In this case, using the Chevalley-Eilenberg complex discussed in the previous section, Theorem 4.6 becomes:

Theorem 4.7. *Consider a (connected and simply connected) Poisson Lie group (G, π_G) , a compact Poisson manifold (M, π) and an admissible pre-Hamiltonian Poisson action of (G, π_G) on (M, π) where \mathfrak{g} is a semisimple Lie algebra of compact type.*

There exist a positive integer l and two positive real numbers a and b (with $b < 1 < a$) such that, if $\tilde{\alpha}$ is another infinitesimal momentum map on M with respect to the same Poisson structure and Lie algebra, satisfying

$$(35) \quad \|\alpha - \tilde{\alpha}\|_{2l-1} \leq a \quad \text{and} \quad \|\alpha - \tilde{\alpha}\|_l \leq b$$

then, there exists a Poisson diffeomorphism $\phi : (M, \pi) \rightarrow (M, \pi)$ of class C^k , for all $k \geq l$, on M such that $\phi^(\tilde{\alpha}_X) = \alpha_X$. This Poisson diffeomorphism on M induces a Lie algebra homomorphism $\psi : \mathfrak{g} \rightarrow \Omega^1(M)$ of class C^k , for all $k \geq l$, on M such that $\psi \circ \alpha = \tilde{\alpha}$.*

Proof. This theorem can be proved by applying the same technique as used for Theorem 4.6. In this case, the identification is done as follows:

- The SCI-space \mathcal{T} is defined to be the space \mathcal{T}_k of C^k -differentiable maps from \mathfrak{g} to $\Omega^1(M)$.
- The subset \mathcal{S} is given by the infinitesimal momentum maps (i.e. Lie algebra homomorphisms and Maurer-Cartan forms).

- The origin of the affine space is given by α and $\mathcal{F} = \mathcal{N} = 0$ so that the estimate (12) is obvious.
- The SCI-group \mathcal{G} consists of the C^k -differentiable maps from $\Omega^1(M)$ to itself, where the action is $\psi \cdot \alpha = \psi \circ \alpha$, with $\psi \in \mathcal{G}$ and $\alpha \in \mathcal{T}$.
- The closed subgroup \mathcal{G}_0 of \mathcal{G} is given by the Lie algebra homomorphisms which preserve the Maurer-Cartan equation on $\Omega^1(M)$. The elements of \mathcal{G}_0 preserve \mathcal{S} .
- The SCI-space \mathcal{H} by the space of generic one-forms on M .

A momentum map can be obviously viewed as a 1-cochain in the Chevalley-Eilenberg complex $C^q(\mathfrak{g}, \Omega^1(M))$, thus the image of α by \mathbb{H} is just $\mathfrak{h}_0(\alpha - \tilde{\alpha})$. In this case, α_X is not an exact form so we can not use the commutative diagram (26). Nevertheless, as \mathfrak{h}_0 is the homotopy operator of the Chevalley-Eilenberg complex $C^q(\mathfrak{g}, \Omega^1(M))$, the relation (13) is obvious.

Finally, consider the vector field associated to a generic one-form α

$$\widehat{X} = \pi(h(\alpha_X), \cdot).$$

Since we have assumed that $h(\alpha_X)$ is a closed one-form, its flow ψ_t preserves π and sends an infinitesimal momentum map to an infinitesimal momentum map thus leaving the set of infinitesimal momentum maps \mathcal{S} invariant; Therefore we can define the application $\Phi : \mathcal{H} \rightarrow \mathcal{G}_0$ by $\Phi := \psi_t^*$.

The estimates (14)-(15)-(16) are direct consequences of the Lemmas in Sect.5.

Remark 4.8. *Theorem 4.7 can be easily generalized to the infinitesimal momentum map defined in terms of Gerstenhaber morphism $\alpha : (\wedge^\bullet \mathfrak{g}, \delta, [,]) \longrightarrow (\Omega^\bullet(M), d_{DR}, [,]_\pi)$. In this case the infinitesimal momentum map generates the action by means of the construction*

$$(36) \quad \begin{array}{ccc} (\wedge^\bullet \mathfrak{g}, \delta, [,]) & \xrightarrow{\quad \quad \quad} & (\wedge^\bullet TM, 0, [,]_S) \\ & \searrow \alpha & \\ & & (\Omega^\bullet(M), d_{DR}, [,]_\pi) \end{array} \longrightarrow$$

where the multivector fields $\wedge^\bullet TM$ on M form a Gerstenhaber algebra using the Schouten-Nijenhuis bracket $[,]_S$.

As mentioned in Section 4.2, α defines the Chevalley-Eilenberg complex $C^q(\wedge^\bullet \mathfrak{g}, \Omega^\bullet(M))$ and we can prove the rigidity of α using the technique discussed above.

Remark 4.9. *When the infinitesimal momentum map $\alpha : \mathfrak{g} \rightarrow \Omega^1(M)$ can be integrated to an actual moment map $\mu : M \rightarrow G^*$, then a different proof of theorem 3.3 can be obtained following the steps of this proof. This can be done doing a hands-on manipulation of the integral formulae that give the homotopy operators. Explicit formulae can be obtained dealing with the group cohomology $C^p(G, C^\infty(M))$ (instead of Lie algebra cohomology). In particular, we can use the following integral formula provided by V. Ginzburg [12] for semisimple compact groups,*

$$\mathfrak{h}(c)(g_1, \dots, g_{n-1}) = \int_G \rho_x(g^{-1})c(g, g_1, \dots, g_{n-1})dg.$$

Remark 4.10. *This result also holds for symplectic manifolds but in this case, the proof can be made easier without any need of hard geometric analysis tools. This is because as proved in [24] and [25] when the Lie group G has the trivial Poisson structure, the adaptation of equivariant Moser path method entails rigidity for symplectic (not necessarily) Hamiltonian actions.*

When the Poisson structure on G is not trivial we can still adapt this strategy as we did in the proof of the theorem by post-composing the symplectic diffeomorphism to obtain equivalence of infinitesimal momentum maps.

Observe, in particular, that the technical requirements on two infinitesimal momentum maps being close is relaxed in the symplectic case.

As a corollary of this theorem we obtain a rigidity theorem for Pre-Hamiltonian actions on Poisson manifold (which is more general than the one included in [25] since it applies to Poisson actions that do not integrate to global momentum maps as it is shown in example 2.3).

Corollary 4.11. *Consider a connected and simply connected Lie group G with trivial Poisson structure, a compact Poisson manifold (M, π) and a pre-Hamiltonian Poisson action of G on (M, π) given by the construction (34), where \mathfrak{g} is a semisimple Lie algebra of compact type. There exist a positive integer l and two positive real numbers a and b (with $b < 1 < a$) such that, if $\tilde{\alpha}$ is another infinitesimal momentum map on M with respect to the same Poisson structure and Lie algebra, satisfying*

$$(37) \quad \|\alpha - \tilde{\alpha}\|_{2l-1} \leq a \quad \text{and} \quad \|\alpha - \tilde{\alpha}\|_l \leq b$$

then, there exists a Poisson diffeomorphism $\phi : (M, \pi) \rightarrow (M, \pi)$ of class C^k , for all $k \geq l$, on M such that $\phi^(\tilde{\alpha}_X) = \alpha_X$.*

Remark 4.12. *This corollary can be useful for the study of normal forms and rigidity problems on b-symplectic manifolds extending thus the results of normal forms for toric actions contained in [17] to the non-toric context.*

Remark 4.13. *Since we have used the apparatus of SCI-spaces the analogues of Theorems 4.7 and 4.7 and Corollary 4.11 also hold in the local and semilocal case (neighbourhood of an invariant compact submanifold). Thus, in the same spirit of [25] we also obtain rigidity for pre-Hamiltonian Poisson Lie group actions for actions in a neighbourhood of an invariant compact submanifold (which can be reduced to a single point in the case of fixed points for the action).*

5. TECHNICAL RESULTS

In this section we prove that the identifications given in the proof of theorem 4.7 satisfies the hypothesis of the SCI-setting (refer to the appendix for definitions in the SCI-setting).

5.1. Momentum maps. Consider an infinitesimal momentum map $\alpha : \mathfrak{g} \rightarrow \Omega^1(M)$ with respect to the Poisson structure π . We saw in section 4.2 that we can associate to α a Chevalley-Eilenberg complex $C^\bullet(\mathfrak{g}, \Omega^1(M))$, with differential ∂ , and homotopy operator h . If $\tilde{\alpha}$ is another momentum map with respect to the same Poisson structure then we can see the difference $\alpha - \tilde{\alpha}$ as an 1-cochain in the complex. We then define $\psi_t = Id + \chi_t$ the

flow of the vector field $\widehat{X}_{\mathfrak{h}(\alpha-\tilde{\alpha})}$ with respect to the Poisson structure and $\psi = \psi^1$ the time-1 flow.

Lemma 5.1. *Let $r > 0$ and $0 < \eta < 1$ be two positive numbers. With the notations above, we have the two following properties:*

a) *For any positive integer k we have*

$$(38) \quad \|\partial(\alpha - \tilde{\alpha})\|_{k,r} \leq C \|\alpha - \tilde{\alpha}\|_{k+1,r}^2,$$

where C is a positive constant independent of α and $\tilde{\alpha}$.

b) *There exists a constant $a > 0$ such that if $\|\alpha - \tilde{\alpha}\|_{s+2,r(1+\eta)} < a\eta$, then we have, for any positive integer k :*

$$(39) \quad \|\psi^* \circ \alpha - \tilde{\alpha}\|_{k,r} \leq \|\alpha - \tilde{\alpha}\|_{k+s+2,r(1+\eta)}^2 P(\|\alpha - \tilde{\alpha}\|_{k+s+1,r(1+\eta)})$$

where P is a polynomial with positive coefficients, independent of α and $\tilde{\alpha}$.

Proof.

a) Let us consider a basis $\{X_1, \dots, X_n\}$ of the Lie algebra \mathfrak{g} and the real numbers c_{ij}^p defined by $[X_i, X_j] = \sum_{p=1}^n c_{ij}^p X_p$. In this proof, we adopt for instance the notation α_i , for α_{X_i} . In order to simplify, we denote by $\beta = \alpha - \tilde{\alpha}$. By definition of the differential ∂ , we have:

$$(40) \quad \partial\beta(\xi_i \wedge \xi_j) = [\alpha_i, \beta_j]_\pi - [\alpha_j, \beta_i]_\pi - \beta([\xi_i, \xi_j]).$$

It allows us to write the following equality :

$$(41) \quad [\beta_i, \beta_j]_\pi = [\alpha_i, \alpha_j]_\pi - [\alpha_i, \tilde{\alpha}_j]_\pi - [\tilde{\alpha}_i, \alpha_j]_\pi + [\tilde{\alpha}_i, \tilde{\alpha}_j]_\pi$$

Now, since α and $\tilde{\alpha}$ are infinitesimal momentum maps, we have

$$(42) \quad [\alpha_i, \alpha_j]_\pi = \sum_{p=1}^n c_{ij}^p \alpha_p$$

and also $[\tilde{\alpha}_i, \tilde{\alpha}_j]_\pi = \sum_{p=1}^n c_{ij}^p \tilde{\alpha}_p$.

Therefore, we obtain :

$$(43) \quad \partial\beta(\xi_i \wedge \xi_j) = [\beta_i, \beta_j]_\pi.$$

Finally, we just write the following estimates :

$$(44) \quad \|\partial\beta\|_{k,r} \leq n(n-1) \|\pi\|_{k,r} \|\beta\|_{k+1,r}^2,$$

where π is the Poisson structure considered.

b) Let us consider

$$(45) \quad \psi^*(\alpha_i) - \tilde{\alpha}_i = \psi^*(\alpha)_i - \psi^*(\tilde{\alpha}_i) + \psi^*(\tilde{\alpha}_i) - \tilde{\alpha}_i.$$

Now, we have for each $i \in \{1, \dots, n\}$:

$$(46) \begin{aligned} \psi^*(\tilde{\alpha}_i) - \tilde{\alpha}_i &= \int_0^1 \psi_t^* \mathcal{L}_{\pi^\sharp(\mathfrak{h}(\tilde{\alpha}-\alpha))} \alpha_i dt \\ &= \int_0^1 \psi_t^* [\mathfrak{h}(\tilde{\alpha}-\alpha), \alpha_i]_\pi dt + \int_0^1 \psi_t^* \iota_{\pi^\sharp(\alpha_i)} d\mathfrak{h}(\tilde{\alpha}-\alpha) dt \\ &= \int_0^1 \psi_t^* \partial(\mathfrak{h}(\tilde{\alpha}-\alpha))_i dt + \int_0^1 \psi_t^* \iota_{\pi^\sharp(\alpha_i)} d\mathfrak{h}(\tilde{\alpha}-\alpha) dt. \end{aligned}$$

We can conclude that, if the one-form $\mathfrak{h}(\tilde{\alpha}-\alpha)$ is closed, following the same steps of [25], the claim is proved. It is worth to mention that

in this construction the Maurer-Cartan identity plays a fundamental role; without this compatibility between the structures of the Lie bialgebra and the de Rham complex, the rigidity could not have been proved.

6. APPENDIX: BASIC DEFINITIONS OF SCI SPACES

In this appendix we give the basic definitions of SCI-spaces. This appendix closely follows [25] and [26].

Definition 6.1 (SCI-spaces). *An SCI-space \mathcal{H} is a collection of Banach spaces $(\mathcal{H}_{k,\rho}, \|\cdot\|_{k,\rho})$ with $0 < \rho \leq 1$ and $k \in \mathbb{Z}_+ = \{0, 1, 2, \dots\}$ (ρ is called the radius parameter, k is called the smoothness parameter; we say that $f \in \mathcal{H}$ if $f \in \mathcal{H}_{k,\rho}$ for some k and ρ , and in that case we say that f is k -smooth and defined in radius ρ) which satisfies the following properties:*

- If $k < k'$, then for any $0 < \rho \leq 1$, $\mathcal{H}_{k',\rho}$ is a linear subspace of $\mathcal{H}_{k,\rho}$: $\mathcal{H}_{k',\rho} \subset \mathcal{H}_{k,\rho}$.
- If $0 < \rho' < \rho \leq 1$, then for each $k \in \mathbb{Z}_+$, there is a given linear map, called the projection map, or radius restriction map,

$$\pi_{\rho,\rho'} : \mathcal{H}_{k,\rho} \rightarrow \mathcal{H}_{k,\rho'}.$$

These projections don't depend on k and satisfy the natural commutativity condition $\pi_{\rho,\rho''} = \pi_{\rho,\rho'} \circ \pi_{\rho',\rho''}$. If $f \in \mathcal{H}_{k,\rho}$ and $\rho' < \rho$, then by abuse of language we will still denote by f its projection to $\mathcal{H}_{k,\rho'}$ (when this notation does not lead to confusions).

- For any f in \mathcal{H} we have

$$(47) \quad \|f\|_{k,\rho} \geq \|f\|_{k',\rho'} \quad \forall k \geq k', \rho \geq \rho'.$$

In the above inequality, if f is not in $\mathcal{H}_{k,\rho}$ then we put $\|f\|_{k,\rho} = +\infty$, and if f is in $\mathcal{H}_{k,\rho}$ then the right hand side means the norm of the projection of f to $\mathcal{H}_{k',\rho'}$, of course.

- There is a smoothing operator for each ρ , which depends continuously on ρ . More precisely, for each $0 < \rho \leq 1$ and each $t > 1$ there is a linear map, called the smoothing operator,

$$(48) \quad S_\rho(t) : \mathcal{H}_{0,\rho} \longrightarrow \mathcal{H}_{\infty,\rho} = \bigcap_{k=0}^{\infty} \mathcal{H}_{k,\rho}$$

which satisfies the following inequalities: for any $p, q \in \mathbb{Z}_+$, $p \geq q$ we have

$$(49) \quad \|S_\rho(t)f\|_{p,\rho} \leq C_{\rho,p,q} t^{p-q} \|f\|_{q,\rho}$$

$$(50) \quad \|f - S_\rho(t)f\|_{q,\rho} \leq C_{\rho,p,q} t^{q-p} \|f\|_{p,\rho}$$

where $C_{\rho,p,q}$ is a positive constant (which does not depend on f nor on t) and which is continuous with respect to ρ .

As explained in [25] the properties (49) and (50) of the smoothing operator imply the *interpolation inequality*:

For any positive integers p, q and r with $p \geq q \geq r$ we have

$$(51) \quad (\|f\|_{q,\rho})^{p-r} \leq C_{p,q,r} (\|f\|_{r,\rho})^{p-q} (\|f\|_{p,\rho})^{q-r},$$

where $C_{p,q,r}$ is a positive constant which is continuous with respect to ρ and does not depend on f .

Definition 6.2. An SCI-subspace of an SCI-space \mathcal{H} is a collection \mathcal{V} of subspaces $\mathcal{V}_{k,\rho}$ of $\mathcal{H}_{k,\rho}$, which themselves form an SCI-space (under the induced norms, induced smoothing operators, induced inclusion and radius restriction operators from \mathcal{H} - it is understood that these structural operators preserve \mathcal{V}).

A subset of an SCI-space \mathcal{H} , is a collection \mathcal{F} of subsets $\mathcal{F}_{k,\rho}$ of $\mathcal{H}_{k,\rho}$, invariant under the inclusion and radius restriction maps of \mathcal{H} .

Definition 6.3. We will say that there is a linear left SCI-action of an SCI-group \mathcal{G} on an SCI-space \mathcal{H} if there is a positive integer γ (and a positive constant c) such that, for each $\phi = Id + \chi \in \mathcal{G}_{k,\rho}$ and $f \in \mathcal{H}_{k,\rho'}$ with $\rho' = (1 - c\|\chi\|_{1,\rho})\rho$, the element $\phi \cdot f$ (the image of the action of ϕ on f) is well-defined in $\mathcal{H}_{k,\rho'}$, the usual axioms of a left group action modulo appropriate restrictions of radii (so we have scaled action laws) are satisfied, and the following inequalities expressing some continuity conditions are also satisfied:

i) For each k there are polynomials Q and R (which depend on k) such that

$$(52) \quad \|(Id + \chi) \cdot f\|_{2k-1,\rho'} \leq \|f\|_{2k-1,\rho} (1 + \|\chi\|_{k+\gamma,\rho} Q(\|\chi\|_{k+\gamma,\rho})) \\ + \|\chi\|_{2k-1+\gamma,\rho} \|f\|_{k,\rho} R(\|\chi\|_{k+\gamma,\rho})$$

ii) There is a polynomial function T of 2 variables such that

$$(53) \quad \|(\phi + \chi) \cdot f - \phi \cdot f\|_{k,\rho'} \leq \|\chi\|_{k+\gamma,\rho} \|f\|_{k+\gamma,\rho} T(\|\phi - Id\|_{k+\gamma,\rho}, \|\chi\|_{k+\gamma,\rho})$$

In the above inequalities, ρ' is related to ρ by a formula of the type $\rho' = (1 - c(\|\chi\|_{1,\rho} + \|\phi - Id\|_{1,\rho}))\rho$. ($\phi = Id$ in the first two inequalities).

Note that a consequence of the property i) is the following inequality, where P is a polynomial function depending on k :

$$(54) \quad \|(Id + \chi) \cdot f\|_{k,\rho'} \leq \|f\|_{k,\rho} (1 + \|\chi\|_{k+\gamma,\rho} P(\|\chi\|_{k+\gamma,\rho})) .$$

REFERENCES

- [1] A. Alekseev and E. Meinrenken. Linearization of Poisson Lie group structures. *preprint*, arxiv:11312.1223v1, 2013.
- [2] M. Bailey. Local classification of generalized complex structures. *J. Differential Geom.*, 95(1):1–37, 2013.
- [3] M. Bailey and M. Gualtieri. Local analytic geometry of generalized complex structures. *preprint*, 2014.
- [4] P. Bieliavsky, C. Esposito, and R. Nest. Quantization of Poisson-Hamiltonian systems. in preparation.
- [5] S. Bochner. Compact groups of differentiable transformations. *Ann. of Math. (2)*, 46:372–381, 1945.
- [6] J. F. Conn. Normal forms for smooth Poisson structures. *Ann. of Math. (2)*, 121(3):565–593, 1985.
- [7] J. Dodziuk. Sobolev spaces of differential forms and de Rham-Hodge isomorphism. *J. Differential Geom.*, 16(1):63–73, 1981.
- [8] V. G. Drinfel'd. Hamiltonian structures on Lie groups, Lie bialgebras and the geometric meaning of classical Yang-Baxter equations. *Dokl. Akad. Nauk SSSR*, 268(2):285–287, 1983.

- [9] B. Enriquez, P. Etingof, and I. Marshall. Comparison of Poisson structures and Poisson-Lie dynamical r -matrices. *Int. Math. Res. Not.*, (36):2183–2198, 2005.
- [10] C. Esposito. *On the classical and quantum momentum map*. PhD thesis, University of Copenhagen, 2012.
- [11] C. Esposito and R. Nest. Uniqueness of the Momentum map. Preprint, <http://arxiv.org/abs/1208.1486>, 2012.
- [12] V. L. Ginzburg. Momentum mappings and Poisson cohomology. *International Journal of Mathematics*, 7(3):329–358, 1996.
- [13] V. L. Ginzburg and A. Weinstein. Lie-Poisson structure on some Poisson Lie groups. *J. Amer. Math. Soc.*, 5(2):445–453, 1992.
- [14] V. Gol'dstein and M. Troyanov. Sobolev inequalities for differential forms and $L_{q,p}$ -cohomology. *J. Geom. Anal.*, 16(4):597–631, 2006.
- [15] V. Guillemin, V. Ginzburg, and Y. Karshon. *Moment maps, cobordisms, and Hamiltonian group actions*, volume 98 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2002. Appendix J by Maxim Braverman.
- [16] V. Guillemin, E. Miranda, and A. R. Pires. Symplectic and Poisson geometry on b -manifolds. *Adv. Math.*, 264:864–896, 2014.
- [17] V. Guillemin, E. Miranda, A. R. Pires, and G. Scott. Toric actions on b -symplectic manifolds. *Int Math Res Notices*, 2014.
- [18] R. S. Hamilton. The inverse function theorem of Nash and Moser. *Bull. Amer. Math. Soc. (N.S.)*, 7(1):65–222, 1982.
- [19] Y. Kosmann-Schwarzbach. Lie bialgebras, Poisson Lie groups and dressing transformations. In *Integrability of nonlinear systems (Pondicherry, 1996)*, volume 495 of *Lecture Notes in Phys.*, pages 104–170. Springer, Berlin.
- [20] J-H. Lu. *Multiplicative and Affine Poisson structure on Lie groups*. PhD thesis, University of California (Berkeley), 1990.
- [21] J-H. Lu. Momentum mappings and reduction of Poisson actions. In *Symplectic geometry, groupoids, and integrable systems*, pages 209–226. Math. Sci. Res. Inst. Publ., Berkeley, California, 1991.
- [22] J. N. Mather. Stability of C^∞ mappings. I. The division theorem. *Ann. of Math. (2)*, 87:89–104, 1968.
- [23] E. Miranda. Some rigidity results for symplectic and Poisson group actions. In *XV International Workshop on Geometry and Physics*, volume 11 of *Publ. R. Soc. Mat. Esp.*, pages 177–183. R. Soc. Mat. Esp., Madrid, 2007.
- [24] E. Miranda. Some rigidity results for symplectic and Poisson group actions. In *XV International Workshop on Geometry and Physics*, volume 11 of *Publ. R. Soc. Mat. Esp.*, pages 177–183. R. Soc. Mat. Esp., Madrid, 2007.
- [25] E. Miranda, N. Tien Zung, and P. Monnier. Rigidity of Hamiltonian actions on poisson manifolds. *Advances in mathematics*, 229:1136–1179, 2012.
- [26] P. Monnier and N. T. Zung. Levi decomposition for smooth Poisson structures. *J. Differential Geom.*, 68(2):347–395, 2004.
- [27] G. D. Mostow. Equivariant embeddings in Euclidean space. *Ann. of Math. (2)*, 65:432–446, 1957.
- [28] I. Mărcuț. Rigidity around Poisson submanifolds. *Acta Math.*, 213(1):137–198, 2014.
- [29] R. S. Palais. Equivalence of nearby differentiable actions of a compact group. *Bull. Amer. Math. Soc.*, 67:362–364, 1961.

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