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Dynamical sectors for a spinning particle in AdS_3

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We consider the dynamics of a particle of mass M and spin J in AdS_3. The study reveals the presence of different dynamical sectors depending on the relative values of M, J and the AdS_3 radius R. For the subcritical $M^2R^2 - J^2 > 0$ and supercritical $M^2R^2 - J^2 < 0$ cases, it is seen that the equations of motion give the geodesics of AdS_3. For the critical case $M^2R^2 = J^2$ there exist extra gauge transformations which further reduce the physical degrees of freedom, and the motion corresponds to the geodesics of AdS_2. This result should be useful in the holographic interpretation of the entanglement entropy for two-dimensional conformal field theories with gravitational anomalies.

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I. MOTIVATION AND RESULTS

Point particles in spacetime offer a simplified setting to study the interplay between the geometry of spacetime and quantum mechanics. As noted long ago by Deser, Jackiw and 't Hooft, point particles in $2+1$ gravity are naked conical singularities in which the mass is related to the angular deficit. The energy-momentum tensor has Dirac deltas with support at the position of the particles, which induces delta singularities in the curvature at those points. Infinite curvature concentrated at an isolated point corresponds to a conical singularity produced by the removal of a wedge. These conical defects do not affect the local geometry on open sets that do not include the singularities, but they change the global topology [1].

Point particles in AdS_3 are also related to black holes (BH). Since point particles are conical singularities, they are obtained by identification in AdS_3 by a global spacelike Killing vector with a fixed point in a similar way as the $2+1$ black hole is obtained by identifications in the universal covering of AdS_3 [2]. The spacetime geometry of a conical singularity in AdS_3 is identical to the Bañados, Teitelboim, Zanelli (BTZ) geometry, where the mass of the black hole is minus the mass of the point particle, and therefore a point particle of mass M in $2+1$ can be viewed as a black hole of mass $-M$ [3].

The $2+1$ black hole can have angular momentum J and, by the same token, a point particle can be endowed with spin. An important quantity that characterizes both black holes and point particle states is $\kappa \equiv J^2 - M^2R^2$, where R is the anti–de Sitter radius. Nonextremal black holes and spinning point particles correspond to $\kappa < 0$ (subcritical case). The extreme (critical) geometries, $\kappa = 0$, have additional special features, like admitting globally defined Killing spinors [supersymmetric Bogomolnyi-Prasad-Sommerfield (BPS) states]. Figure 1 displays the different sectors of $2+1$ black hole (BH)-particle states in the M-J plane. The AdS_3 geometry, without identifications, is the point $J = 0, MR = -1$; black holes cover region I ($\kappa \leq 0$, $M > 0$); point particles are described by III ($\kappa \leq 0$, $M < 0$). The supercritical regions II ($\kappa > 0$) are unphysical

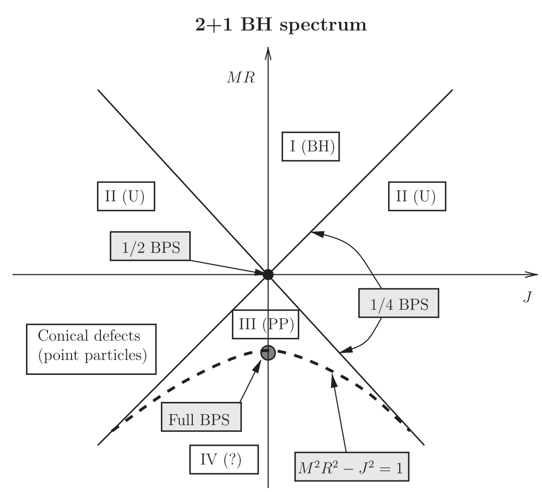

FIG. 1. J-M plot for locally AdS_3 geometries: Physical black holes ($MR \geq |J|$), point particles ($MR \leq -|J|$) and unphysical states ($|M|R \leq |J|$). The critical cases $|M|R = \pm|J|$ correspond to extremal configurations.

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states. States below the hyperbola $\kappa = -1, M < 0$ correspond to angular excesses rather than defects, and also display deltalike curvature singularities at $r = 0$. On the other hand, it is also possible to think of a particle as a probe that does not significantly affect the spacetime geometry around it, a localized perturbation with negligible backreaction. The appropriate setting to describe this would be an action for a relativistic point particle whose trajectories, in the absence of other interactions, are geodesics of the spacetime background. In that case M and J are parameters in the action and κ is an intrinsic feature, while for the black hole they are constants of motion that depend on the initial conditions.

The geodesic equation is a local statement and is therefore the same for a particle on a patch of AdS_3 or around a $2 + 1$ black hole. The only difference would be in the orbits, since they depend on the global properties of the manifold. Hence, the conserved quantities for the different orbits could be the same, e.g. energy and angular momentum, but their specific values would determine the class of geodesics that the particle traces.

Motion of particles in AdS_3 can also be related to the holographic entanglement entropy. As shown in [4], the entanglement entropy in AdS_D is related to minimal $(D-2)$-surfaces in AdS_D. For the case of AdS_3, the minimal surface is the length of the geodesics, and these are the trajectories of particles in AdS_3. The holographic description of the entanglement entropy for conformal field theories in two dimensions with gravitational anomalies [5] has been studied in [6].

Motivated by these observations, we consider the dynamics of a particle of mass M and spin J in AdS_3. The Lagrangian equations of motion reveal the presence of different dynamical sectors depending on M, J and R. For the subcritical ($\kappa < 0$) and supercritical ($\kappa > 0$) cases it is seen that the equations of motion give the corresponding geodesics of AdS_3. For the critical case ($\kappa = 0$) there exists an extra gauge transformation which further reduces the physical degrees of freedom.[1] The orbits correspond to geodesics of AdS_2.

The presence of dynamical sectors appears also in a $(2 + 1)$-dimensional harmonic oscillator system with exotic Newton-Hooke symmetry. The system displays three different phases depending on the values of the parameters [11]. The reduced phase space description reveals a symplectic structure similar to that of Landau problem in the noncommutative plane [12,13]. There is a close relation between the $(2 + 1)$-dimensional exotic Newton-Hooke symmetry and the noncommutative Landau problem [14].[2]

[1]This gauge transformation is the bosonic analog of kappa symmetry [7,8] for superparticles [9,10] that kills half of the degrees of freedom.

[2]The motion of an anyon in an electromagnetic field also has sectors; see e.g. [15].

The nonrelativistic limit of a spinning particle in AdS_3 shows the Newton-Hooke symmetry and gives the $(2 + 1)$-dimensional exotic harmonic oscillator system.

Since the AdS_3 algebra can be written as a sum of two chiral $so(2, 1)$ factors, we construct a Lagrangian for spinning particles in terms of chiral coordinates. The relation between the chiral and nonchiral variables is given in terms of a differential equation that can be solved perturbatively. The equations of motion for the critical case in the chiral formulation describe orbits that are geodesics of AdS_2.

The subcritical case ($\kappa < 0$) includes a special representative, $J = 0$, while $M = 0$ is a representative for the supercritical case ($\kappa > 0$). In each case we explicitly give the coordinate transformation that takes the Lagrangian into one described by the corresponding representative. We expect the points $J = 0$ or $M = 0$ to be described in terms of noncommutative coordinates, in a similar way as for the flat case (see e.g. [16]).

Some of the particle states have a lowest energy (BPS) bound. In particular, the critical sector with $\kappa = 0$, with $M > 0$ saturates a BPS bound and therefore is a candidate to be a supersymmetric configuration, with $1/4$ supersymmetry, if the system is embedded in a supersymmetric model. These particles belong to the BH sector of the J-M plane. The subcritical configurations with $M < 0$ do not have a lowest energy bound but an upper one, and therefore do not seem to correspond to stable BPS states. As shown in [3], the critical states with $M < 0$ also admit globally defined Killing spinors. Nevertheless, a complete correspondence with the BH spectrum should not be expected since we assume the particle as a probe that does not modify the spacetime background.

Summing up, we prove the existence of sectors in the dynamics of a spinning particle in AdS_3. The critical sector has half the physical degrees of freedom due to existence of an extra, kappalike bosonic gauge symmetry. The presence of sectors is in correspondence with the spectrum of BH. Our results shed light on the holographic interpretation of the entanglement entropy for CFT_2 with gravitational anomalies [6], for which the left c_L and right c_R central charges are different, and one will have sectors for this class of theories. The critical sector corresponds to chiral CFTs with only left or right moving sectors, while the sign of $c_L c_R$ determines the subcritical (+) and supercritical (−) sectors. Any particle with spin J in a locally AdS_3 metric will exhibit the same kind of dynamical sectors, but they will be absent, for instance, for local dS_3 geometries. The analysis should also be useful in the study of the motion of anyons around a BTZ black hole.

The paper is organized as follows. Section II presents the Lagrangian, and discusses the dynamical sectors and the equations of motion. Section III introduces the chiral variables and analyzes the dynamical sectors in this description. The relation between chiral and nonchiral variables is established in Sec. IV by means of a set of nonlinear differential equations that can be solved perturbatively.

Section V presents an explicit biparametric transformation, in terms of the chiral variables, that connects Lagrangians with different M, J parameters in the super- and subcritical sectors. Finally, Sec. VI uses the chiral form of the Lagrangian to discuss the existence of BPS energy bounds for several values of the parameters.

II. ADS$_3$ ACTION AND EQUATIONS OF MOTION

The action of a massive spinning particle in AdS$_3$ is constructed from the coordinates of the world line $x^a(\tau)$ and a generic Lorentz transformation $\Phi_a{}^b(\tau)$.[3] To lowest order in derivatives, the action is [16,17]

$$I_0[x,\Phi] = -M\int d\tau(\dot{x}^m e^a{}_m \Phi_a{}^0) - \frac{J}{2}\int d\tau \epsilon_{a'b'}\eta^{cd}\Phi_d{}^{a'}[\dot{\Phi}_c{}^{b'} + \dot{x}^m \omega_{mc}{}^e \Phi_e{}^{b'}], \quad (1)$$

where $e^a{}_m(x)$ and $\omega_m^{ab}(x)$ are the dreibein and spin connection of AdS$_3$. The Lagrangian is given by the Maurer-Cartan (MC) form (nonlinear realization [18]) for the coset $G/H = SO(2,2)/SO(2)$. The AdS$_3$ generators satisfy the so(2,2) algebra

$$[P_a, P_b] = -iR^{-2}M_{ab}, [P_a, M_{cd}] = -i\eta_{a[c}P_{d]},$$
$$[M_{ab}, M_{cd}] = -i\eta_{b[c}M_{ad]} + i\eta_{a[c}M_{bd]}, \quad (2)$$

and the stability group H is generated by M_{12}. We consider a local parametrization of the coset element

$$g = g_0 U \in G/H, \qquad g_0 = e^{iP_0x^0}e^{iP_1x^1}e^{iP_2x^2},$$
$$h = e^{iM_{12}\alpha} \in H, \qquad U = e^{iM_{02}v^1}e^{-iM_{01}v^2}. \quad (3)$$

The Lorentz transformation U can be expressed as

$$\Phi_a{}^b = \begin{pmatrix} \cosh v^1 & 0 & -\sinh v^1 \\ 0 & 1 & 0 \\ -\sinh v^1 & 0 & \cosh v^1 \end{pmatrix}\begin{pmatrix} \cosh v^2 & \sinh v^2 & 0 \\ \sinh v^2 & \cosh v^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The dreibein and spin connection in this parametrization are obtained from the MC form $\Omega_0 = -ig_0^{-1}dg_0 = P_a e^a + \frac{1}{2}M_{ab}\omega^{ab}$,

$$e^0 = \cosh\hat{x}^1\cosh\hat{x}^2 dx^0, \qquad e^1 = \cosh\hat{x}^2 dx^1,$$
$$e^2 = dx^2, \qquad \omega^{02} = \cosh\hat{x}^1\sinh\hat{x}^2 d\hat{x}^0,$$
$$\omega^{12} = \sinh\hat{x}^2 d\hat{x}^1, \qquad \omega^{01} = \sinh\hat{x}^1 d\hat{x}^0, \quad (4)$$

where $\hat{x}^a = x^a/R$. The MC form associated to g is $\Omega = -ig^{-1}dg = P_a L^a + \frac{1}{2}M_{ab}L^{ab}$, where

$$L^a = e^b\Phi_b{}^a, \qquad L^{ab} = \Phi_c{}^a(\eta^{cd}d + \omega^{cd})\Phi_d{}^b. \quad (5)$$

The Lagrangian (1) is constructed from the pullback of the (pseudo) invariant forms L^0 and L^{12} as

$$\mathcal{L}^{\mathrm{non}} = -ML^0 - JL^{12}. \quad (6)$$

The Euler-Lagrange equations of motion can be written as

$$(-MR^2L^{01} + JL^2) = (-MR^2L^{02} - JL^1) = 0,$$
$$(ML^2 - JL^{01}) = (ML^1 + JL^{02}) = 0. \quad (7)$$

If $J^2 \neq M^2R^2$ these equations become $L^{a'} = 0$ and $L^{0a'} = 0$. They relate the spin variables $v^{a'}$ to the coordinates of the world line by

$$\Phi_a{}^0 = \frac{\dot{x}^m e_m{}^b\eta_{ba}}{\sqrt{-g}}, \qquad g \equiv \dot{x}^m\dot{x}^n g_{mn}, \quad (8)$$

and yield also the geodesic equation,

$$\frac{d}{d\tau}\frac{\dot{x}^m}{\sqrt{-g}} + \Gamma^m_{rn}\frac{\dot{x}^r\dot{x}^n}{\sqrt{-g}} = 0 \quad (9)$$

for the metric $g_{mn} = e_m{}^b\eta_{ba}e_n{}^a$. Note that (9) has no contribution from the spin variables $v^{a'}$ because the physical states in configuration space are given by $x^{a'}$ only and the $v^{a'}$ are not independent local degrees of freedom, as can be seen from (8) as well as from the Hamiltonian analysis. In the reduced phase space, the velocity $\dot{x}^m$ is proportional to the momentum, and therefore the coordinates do not exhibit *Zitterbewegung*.

In the critical case $J^2 = M^2R^2$, only two of the equations in (7) are independent and there are new gauge symmetries, besides diffeomorphisms, that reduce the number of degrees of freedom from 4 to 2. The sectors are also present in local AdS$_3$ or warped AdS$_3$, but not for dS$_3$.

III. CHIRAL FORMULATION

The chiral form of (1) is obtained by making use of the isomorphism $SO(2,2) = SO(2,1) \times SO(2,1)$, namely

$$\mathcal{L}^{\mathrm{ch}} = \mu^+L^0_+ + \mu^-L^0_-, \quad (10)$$

with

$$\mu^\pm = -\frac{1}{2}(J \pm MR), \quad (11)$$

and where

$$L^a_\pm = \frac{1}{2}\epsilon^{abc}L_{bc} \pm L^a/R, \quad (12)$$

$$j^\pm_a = \frac{1}{2}\left(-\frac{1}{2}\epsilon_{abc}M^{bc} \pm RP_a\right) \quad (13)$$

[3] The tangent space metric is $\eta_{ab} = \mathrm{diag}(-;++)$. Indices $m, n, \ldots$ refer to the spacetime manifold where the particle moves. $a, b, \ldots = 0, 1, 2$ are tangent space indices, and $a', b', \ldots = 1, 2$.

are the chiral MC forms and generators, related to those of AdS_3.

The sub- and supercritical sectors are defined by the sign of $4\mu^+\mu^- = (J^2 - M^2R^2) = \kappa$, and in the critical sector either $\mu^+ = 0$ or $\mu^- = 0$, which corresponds to an AdS_2 Lagrangian in each case. The Lagrangian (6) was obtained from the coset $SO(2,2)/SO(2)$. In order to construct a Lagrangian in terms of chiral variables we would like to know in which form an element of $SO(2,2)/SO(2)$ can be written as a product of chiral coset elements $SO(2,1)/SO(2)$. To answer this, let us introduce

$$\begin{aligned} g^+ &= (e^{ij_0^+x^0}e^{ij_1^+x^1}e^{ij_2^+x^2}e^{ij_1^+v^1}e^{ij_2^+v^2}) \\ &= e^{ij_0^+X^0}e^{ij_1^+X^1_+}e^{ij_2^+X^2_+}e^{ij_0^+\alpha} \\ &\equiv \tilde{g}^+e^{ij_0^+\alpha}, \\ g^- &= (e^{-ij_0^-x^0}e^{-ij_1^-x^1}e^{-ij_2^-x^2}e^{ij_1^-v^1}e^{ij_2^-v^2}) \\ &= e^{-ij_0^-X^0}e^{-ij_1^-X^1_-}e^{-ij_2^-X^2_-}e^{ij_0^-\alpha} \\ &\equiv \tilde{g}^-e^{ij_0^-\alpha}, \end{aligned} \tag{14}$$

where the terms containing α are compensating elements of the chiral $H = SO(2)$ factors. The elements of $SO(2,2)$ are then given by

$$g^+g^- = \tilde{g}^+\tilde{g}^-h, \tag{15}$$

where

$$h = e^{ij_0^+\alpha}e^{ij_0^-\alpha} = e^{i(j_0^+ + j_0^-)\alpha} \in H, \tag{16}$$

and $(\tilde{g}^+\tilde{g}^-) \in G/H$ is parametrized by five coordinates $(X^0, X^{\pm1}, X^{\pm2})$. Using the expressions for $\tilde{g}^\pm$, the Lagrangian (10) can be written as

$$\begin{aligned} \mathcal{L}^{\mathrm{ch}} &= \mu^+(\cosh X^2_+ \cosh X^1_+ \dot{X}^0 + \sinh X^2_+ \dot{X}^1_+) \\ &\quad + \mu^-(-\cosh X^2_- \cosh X^1_- \dot{X}^0 + \sinh X^2_- \dot{X}^1_-), \end{aligned} \tag{17}$$

and the chiral equations of motion $L^{a'}_\pm = 0$ are

$$\begin{aligned} \dot{X}^1_\pm(\tau) &= \mp \tanh X^2_\pm(\tau) \cosh X^1_\pm(\tau)\dot{X}^0(\tau), \\ \dot{X}^2_\pm(\tau) &= \pm \sinh X^1_\pm(\tau)\dot{X}^0(\tau). \end{aligned} \tag{18}$$

In the critical case $(\mu^- = 0)$, the equations of motion are $L^{a'}_+ = 0$ and give the geodesics in AdS_2, while the $X^{a'}_-$s are gauge degrees of freedom since they are absent from the Lagrangian.

In the nonrelativistic limit $X^{a'}_\pm \to X^{a'}_\pm/\omega, \mu^\pm \to \omega^2\mu^\pm$, the Lagrangian takes the Newton-Hooke form [11] up to a divergent total derivative, and the equations of motion (18) become those of two harmonic oscillators.

IV. NONCHIRAL TO CHIRAL VARIABLES

The relation between chiral and nonchiral coordinates is established by comparing the corresponding expressions for the coset element in the two parametrizations. Introducing an auxiliary parameter t to rescale v^i in the coset expressions (3) and (14), one finds

$$\begin{aligned} &e^{\pm ij_0x^0}e^{\pm ij_1^\pm x^1}e^{\pm ij_2^\pm x^2}e^{itj_1^\pm v^1}e^{itj_2^\pm v^2} \\ &\quad = e^{\pm ij_0^\pm X^0(t)}e^{\pm ij_1^\pm X^{\pm1}(t)}e^{\pm ij_2^\pm X^{\pm2}(t)}e^{ij_0^\pm\alpha(t)}. \end{aligned} \tag{19}$$

Expanding in powers of t corresponds to expansions in v^i, and differentiating with respect to t yields a set of nonlinear differential equations relating the nonchiral variables $(t = 0)$ and chiral variables $(t = 1)$,

$$\begin{aligned} &\pm \cosh X^{\pm2} \cosh X^{\pm1}\partial_t X^0 + \sinh X^{\pm2}\partial_t X^{\pm1} + \partial_t\alpha \\ &\quad = \sinh(tv^2)v^1, \\ &\cos\alpha(\sinh X^{\pm2}\cosh X^{\pm1}\partial_t X^0 \pm \cosh X^{\pm2}\partial_t X^{\pm1}) \\ &\quad - \sin\alpha(-\sinh X^{\pm1}\partial_t X^0 \pm \partial_t X^{\pm2}) = \cosh(tv^2)v^1, \\ &\sin\alpha(\sinh X^{\pm2}\cosh X^{\pm1}\partial_t X^0 \pm \cosh X^{\pm2}\partial_t X^{\pm1}) \\ &\quad + \cos\alpha(-\sinh X^{\pm1}\partial_t X^0 \pm \partial_t X^{\pm2}) = v^2. \end{aligned} \tag{20}$$

These equations can be solved as a series in t with initial conditions $X^0(0) = x^0/R$, $X^{\pm1}(0) = x_1/R$, $X^{\pm2}(0) = x_2/R$ and $\alpha(0) = 0$. To lowest order, the solution is given by

$$X^0(t) = \hat{x}^0 - tv_1 \mathrm{sech} h\hat{x}_1 \sinh\hat{x}_2 + O(t^3), \tag{21}$$

$$\begin{aligned} X^{\pm1}(t) &= \hat{x}_1 \pm tv_1\cosh\hat{x}_2 + O(t^2), \\ X^{\pm2}(t) &= \hat{x}_2 \pm t(v_2 - v_1\tanh\hat{x}_1\sinh\hat{x}_2) + O(t^2), \\ \alpha(t) &= O(t^2). \end{aligned} \tag{22}$$

V. TRANSFORMATION OF LAGRANGIANS

Lagrangians with different values of (M, J) can be related by a biparametric family of point transformations, $(X^{\pm1}, X^{\pm2}) \to (X^{\pm1}(s_\pm), X^{\pm2}(s_\pm))$, given by

$$\begin{aligned} X^{\pm1}(s^\pm) &= \cosh^{-1}\left(e^{s^\pm}\frac{\cosh X^{\pm1}}{\sqrt{1 + (e^{2s^\pm} - 1)\alpha(X^\pm)}}\right) \\ X^{\pm2}(s^\pm) &= \cosh^{-1}\left(\cosh X^{\pm2}\sqrt{1 + (e^{2s^\pm} - 1)\alpha(X^\pm)}\right), \end{aligned} \tag{23}$$

where

$$\alpha(X^\pm) = \frac{\sinh^2 X^{\pm2}}{\cosh^2 X^{\pm2} - \mathrm{sech}^2 X^{\pm1}}.$$

The left and right chiral Lagrangians transform as $L^0_\pm \to e^{s^\pm}L^0_\pm$, up to total derivatives. From the relation between the chiral and nonchiral forms of the Lagragian given by (10) and (11) one can change the coefficients of the L_P and L_J terms of the nonchiral form as

$$\begin{aligned} M \to M' &= e^{s_1}(M\cosh s_2 + J/R \sinh s_2), \\ J \to J' &= e^{s_1}(J\cosh s_2 + MR\sinh s_2), \end{aligned} \tag{24}$$

with $s_\pm = s_1 \pm s_2$, so that $\kappa \to \kappa' = e^{2s_1}\kappa$. In particular, for the subcritical sector, taking $\tanh s_2 = -J/(MR)$ makes $J' = 0$, while in the supercritical case $\tanh s_2 = -MR/J$ yields $M' = 0$. The $M = 0$ and $J = 0$ nonchiral Lagrangians are thus canonical representatives of the super- and subcritical sectors, for which a noncommutative description analogous to the flat case might be expected [16].

VI. BPS ENERGY BOUNDS

The BH spectrum (Fig. 1) shows that the critical regions $|M|R = |J|$ are 1/4 BPS supersymmetric configurations, except for $M = 0 = J$ which is 1/2 BPS. This hints to the existence of BPS energy bounds for the particle moving in AdS_3. Indeed, let us consider the energy associated to the chiral Lagrangian (17) in the static gauge ($\tau = X^0$). When $MR = J$, the energy bound $E \geq -\mu^+ = MR \geq 0$ is saturated by the BPS configuration $X^1_+ = X^2_+ = 0$, but this is not the case when $|J| = -MR$ ($M \leq 0$). Hence, the critical configurations are in correspondence with the $\frac{1}{4}$ BPS supersymmetric states for $M > 0$, but not for $M < 0$.

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