AAR-based decomposition algorithm for non-linear convex optimisation

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The analysis of the bearing capacity of structures with a rigid-plastic behaviour can be achieved resorting to computational limit analysis. Recent techniques [3],[4] have allowed scientists and engineers to determine upper and lower bounds of the load factor under which the structure will collapse. Despite the attractiveness of these results, their application to practical examples is still hampered by the size of the resulting optimisation process.

We here propose a method for decomposing a class of convex nonlinear programmes which are encountered in limit analysis. These problems have second-order conic memberships constraints and a single complicating variable in the objective function. The method is based on finding the distance between the feasible sets of the decomposed problems, and updating the global optimal value according to the value of this distance. The latter is found by exploiting the method of Averaged Alternating Reflections (AAR), which is here adapted to the optimisation problem at hand. The method is specially suited for non-linear problems, and as our numerical results show, its convergence is independent of the number of variables of each sub-domain. We have tested the method with problems that have more than 10000 variables.

1 Method of Averaged Alternating Reflections (AAR)

1.1 Preliminary definitions

Definition 1 *The set of fixed points of an operator $T : X \to X$ is denoted by Fix T, i.e.*

$$Fix\ T = \{x \in X | T(x) = x\}.$$

Definition 2 *We define the so-called Averaged Alternating Reflections (AAR) operator, denoted by T and given by,*

$$T = \frac{R_W R_Z + I}{2}, \tag{1}$$

where $R_W = 2P_W - I$, $R_Z = 2P_Z - I$ and P_W, P_Z are projectors on two non-empty closed convex sets W and Z.

We now recall the known convergence results for the method of Averaged Alternating Reflections (AAR).

Fact 1 *(AAR method). Consider the following successive approximation method: Take $\boldsymbol{t}_0 \in \Re^n$, and set*

$$\boldsymbol{t}_n = T^n(\boldsymbol{t}_0) = T(\boldsymbol{t}_{n-1}),\ n = 1, 2, \dots \tag{2}$$

where T is defined in (1), and W, Z are nonempty closed convex subsets of $\Re^n$. Then the following results hold

$$(i)\ \mathit{Fix}\ T \neq \emptyset \iff (T^n(\boldsymbol{t}_0))_{n\in N} \text{ converges to some point in } \mathit{Fix}\ T.$$
$$(ii)\ \mathit{Fix}\ T = \emptyset \iff \|T^n(\boldsymbol{t}_0)\| \to \infty,\ \text{when } n \to \infty.$$
$$(iii)\ \frac{\|\boldsymbol{t}_n\|}{n} \to \inf \|W - Z\|.$$

(i) and (ii) are demonstrated in [1, 5, 2] and (iv) is demonstrated in [6].

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