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CONTACT CHARACTERIZATION BETWEEN A CAM AND A CROWNED ROLLER FOLLOWER

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ABSTRACT
In the higher pairs of a mechanism, like the one existing between a cam and roller follower, it is common to localize the maximum deformations, the maximum contact pressures and that appears the surface fatigue failure due to the relative motion between the cam and the roller. The formulations that characterized the contact are based in Hertz elastic contact theory. In this paper the contact between a cam and a crowned roller is studied applying the formulation for elliptical point contact proposed by Johnson. The contact ellipse obtained experimentally is presented to validate the elliptical point contact. The comparison of the width of the axes of the contact ellipse obtained experimentally and by the formulation presents the same level of errors than more sophisticated experimental methods.
Keywords: *crowned roller, cam mechanism, elliptical point contact, Hertz elastic contact theory*

1. INTRODUCTION
In the contact between a cam and a roller follower, it is common to find the largest deformation, the maximum contact pressure and the surface fatigue failure appears due to the relative motion between solids linked. Formulations to characterize the contact between a cam and a roller follower mostly are based on the Hertz elastic contact theory. The geometry of the cam and the roller determines a priori the applied formulation. When the roller follower is cylindrical, the line contact formulation [1] can be used and when the roller is crowned the elliptical point contact can be used [2]. The usage of a crowned roller follower rather than a cylindrical roller follower is recommended to avoid unexpected increases in the contact pressures due to manufacturing errors [3].
This paper studies the contact case between a cam and a roller crowned follower by means of the formulation proposed by Johnson [2]. To validate the suitability of the formulation for elliptical point contact proposed by Johnson, experimental measurements of the width of the axes of the contact ellipse are made in a designed test bed.

2. ELLIPTICAL POINT CONTACT FORMULATION FOR A CAM ROLLER CROWNED FOLLOWER MECHANISM
When a contact force *F* is applied between a cam and a crowned roller follower, an elliptical contact region like the one in Figure 1a is generated in the plane t-t' tangent to both solid surfaces. In the elliptical point contact formulation, the geometry for each solid is locally defined with two principal curvature radii. For the crowned roller follower, the maximum principal curvature radius $R_{\mathrm{II}}^{\max}$ is the crowned radius r_{crowned} and the minimum principal curvature radius $R_{\mathrm{II}}^{\min}$ is the nominal radius of the roller *r*. For the cam, the maximum principal curvature radius $R_{\mathrm{I}}^{\max}$ is infinite due to it is cylindrical and the minimum principal curvature radius $R_{\mathrm{I}}^{\min}$ is the local curvature radius of the profile of the cam $r_{\mathrm{c}}(\theta)$. In the frontal plane of design of the mechanism that contains the contact

point J and it is perpendicular to the axis of rotation of the cam, there are the two minimum principal curvature radii as can be seen in Figure 1b. In the plane n-n', perpendicular to the frontal plane of design, there are the two maximum principal curvature radii as can be seen in Figure 1c.

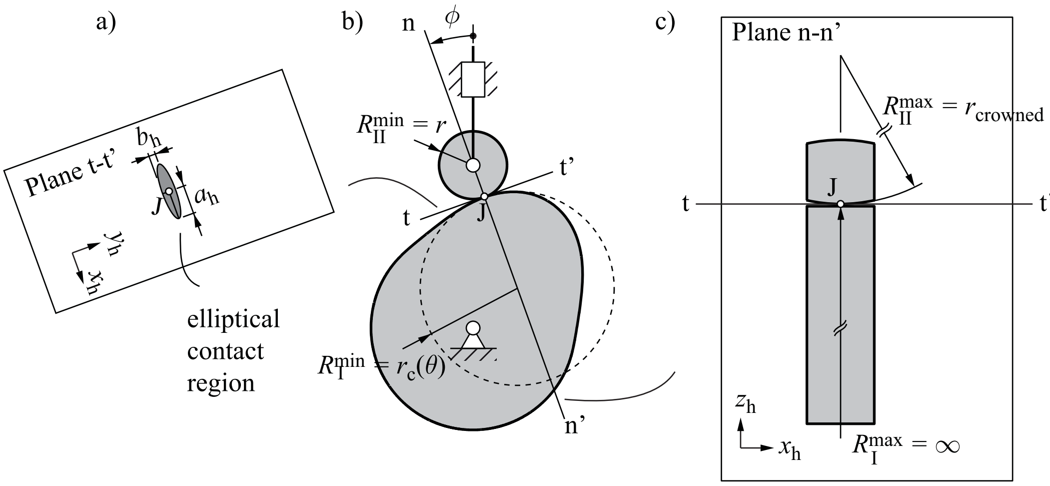

Figure 1 Cam crowned roller follower mechanism: a) elliptical contact region. Principal curvature radii: b) minimum, c) maximum

The formulation for elliptical point contact proposed by Johnson [2] to calculate the semi-width of the axes a_h and b_h of the contact ellipse involves Eq. (1) to Eq. (7). Where A, B and R_e are variables defined by Johnson, γ is the angle between the planes that contains the maximum principal curvature radius $-R_{II}^{max}$, R_I^{max}–, $K(e)$ is the complete elliptic integral of the first kind, $E(e)$ is the complete elliptic integral of the second kind, e is a ratio between the axes of the contact ellipse, F_{e1} is a tabulated parameter, E^* is an equivalent Young Modulus, ν_I is the Poisson ratio of the cam and ν_{II} is the Poisson ratio for the roller.

$$\left\{\begin{aligned} A+B &= \frac{1}{2}\left(\frac{1}{R_I^{max}}+\frac{1}{R_I^{min}}+\frac{1}{R_{II}^{max}}+\frac{1}{R_{II}^{min}}\right) \\ |B-A| &= \frac{1}{2}\sqrt{\left(\frac{1}{R_I^{min}}-\frac{1}{R_I^{max}}\right)^2+\left(\frac{1}{R_{II}^{min}}-\frac{1}{R_{II}^{max}}\right)^2+2\left(\frac{1}{R_I^{min}}-\frac{1}{R_I^{max}}\right)\left(\frac{1}{R_{II}^{min}}-\frac{1}{R_{II}^{max}}\right)\cos 2\gamma} \end{aligned}\right\} \tag{1}$$

$$R_e = \frac{1}{2\sqrt{AB}} \tag{2}$$

$$\frac{B}{A} = \frac{(a_h/b_h)^2 E(e) - K(e)}{K(e)-E(e)} \tag{3}$$

$$\begin{aligned} e &= \sqrt{1-(b_h/a_h)^2} \\ K(e) &= \int_0^{\pi/2}\left(1-e\sin^2\psi\right)^{-1/2}\mathrm{d}\psi \\ E(e) &= \int_0^{\pi/2}\left(1-e\sin^2\psi\right)^{1/2}\mathrm{d}\psi \end{aligned} \tag{4}$$

$$F_{e1}=\left(\frac{4}{\pi e^2}\right)^{1/3}\left(\frac{b_h}{a_h}\right)^{1/2}\left(\left(\left(\frac{a_h}{b_h}\right)^2 E(e)-K(e)\right)\left(K(e)-E(e)\right)\right)^{1/6} \quad (5)$$

$$E^*=\left(\frac{1-\nu_I^2}{E_I}+\frac{1-\nu_{II}^2}{E_{II}}\right)^{-1} \quad (6)$$

$$a_h b_h=\left(\frac{3FR_e}{4E^*}\right)^{2/3} F_{e1}^2 \quad (7)$$

3. TEST BENCH TO MEASURE THE AXES OF THE CONTACT ELLIPSE

In Figure 2 it is presented the designed test bed to measure the axes $2a_h$ and $2b_h$ of the contact ellipse. The test bed consists of a fixed column, a crowned roller IKO NURT20R, a follower, a grinded cylinder and a worm gearbox. The follower and the fixed column are joined with a prismatic pair and the follower has a platform to add weights and apply the desired contact force *F*.

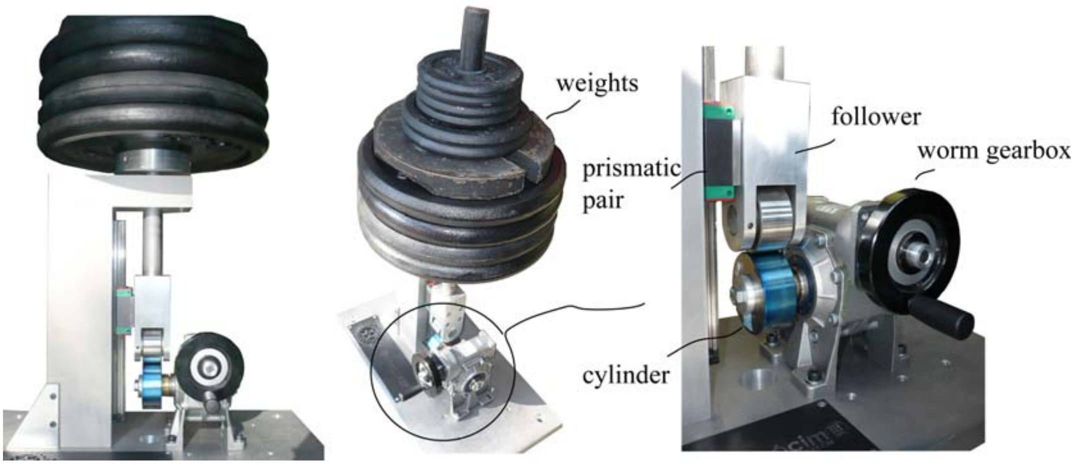

Figure 2 Designed test bed

The analyzed contact case is between a crowned roller and grinded cylinder. A cam locally in each contact point is approximated to a cylinder with the radius the local curvature radius $r_c(\theta)$ of the profile as it is seen in Figure 1b. In Table 1 are presented the surface roughness, geometrical and material properties of the crowned roller and the grinded cylinder.

Cylinder			Crowned roller IKO NURT 20R			Solid properties shared		
R_a [µm]	R_I^{max} [mm]	R_I^{min} [mm]	R_a [µm]	R_{II}^{max} [mm]	R_{II}^{min} [mm]	γ [º]	$E_I = E_{II}$ [GPa]	$\nu_I = \nu_{II}$
0,11	∞	30	0,34	500	23,5	0	206,8	0,28

Table 1 Surface Roughness, geometrical and material properties

To measure the axes of the contact ellipse the surface of the cylinder is painted with a Prussian blue ink which is used into mechanical adjustment operations. When the crowned roller and the cylinder are in contact, an area is marked on the surface of the cylinder. The measure of the axes of the contact ellipse is done with an electronic microscope and a digital camera Leica DC 295. The camera realizes photograph of the surface and with the information of the photograph, the software Leica Application Suite determines the lengths and we measure the axis $2a_h$ and $2b_h$ .In Figure 3 it is presented three contact areas measured in different regions of the grinded cylinder, when the contact force *F* is

790,8 N. In the three photographs it is seen how the marked area it is very similar to the theoretical contact ellipse. Thought the good surface roughness properties there are regions where the cylinder and the roller have not touch. The perturbation done by the surface finish is reduced if the contact force is increased.

Figure 3 Experimental contact ellipse when F=790,8 N

In Table 2 are presented the width of the axes of the contact ellipse obtained with the mean of the three experimental measures, obtained analytical with Johnson formulation and their differences. It is observed that there is just a 0,9% of difference in the measure of the major axis $2a_h$ and there is a 29% of difference in the measure of the minor axis $2b_h$. The difference in the measure of the minor axis are high due to the precision of measure method is of 0,1 mm, at the same order of magnitude than the width of the minor axis. The precision of the method increases if the contact force F increases. However it must be said that the obtained differences are in the same order than more sophisticated methods [4].

Experimental		Analytical		Differences	
$2a_h$ [mm]	$2b_h$ [mm]	$2a_h$ [mm]	$2b_h$ [mm]	$2a_h$ [%]	$2b_h$ [%]
4,21	0,53	4,25	0,41	0,9	29

Table 2 Width of the axes of the contact ellipse: experimental, analytical and differences

4. CONCLUSIONS

In this paper the formulation for elliptical point contact proposed by Johnson is applied in the contact between a cam and crowned roller follower. To validate the suitability of this formulation to this particular case a test bed is designed to measure the width of the axes of the contact ellipse. The results predicted by the formulation and the measured ones presents difference in the same order than more sophisticated method, so a cheap method of validation is proposed. Further investigation can be done to validate the method by the methodology of designed experiments. Better results are expected if the contact force is increased.

6. REFERENCES

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