

Measuring the translational and rotational velocities of particles in helical motion using structured light

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Abstract: We measure the rotational and translational velocity components of particles moving in helical motion under a Laguerre-Gaussian mode illumination. The moving particle reflects light that acquires an additional frequency shift proportional to the velocity of rotation in the transverse plane, on top of the usual frequency shift due to the longitudinal motion. We determined both the translational and rotational velocities of the particles by switching between two modes: by illuminating with a Gaussian beam, we can isolate the longitudinal frequency shift; and by using a Laguerre-Gaussian mode, the frequency shift due to the rotation can be determined. Our technique can be used to characterize the motility of microorganisms with a full three-dimensional movement.

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OCIS codes: (280.3340) Laser Doppler velocimetry; (000.2170) Equipment and techniques; (280.4788) Optical sensing and sensors; (120.3180) Interferometry.

References and links

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1. Introduction

The search for reliable methods to detect the velocity of micro- and nano-particles, and microorganisms, in a three-dimensional (3D) motion is challenging and is continually being addressed [1–3]. Certainly, the ability of quantifying the full velocity of a particle’s movement opens new possibilities. For example, it can unveil some of the most intriguing biomechanical causes and ecological consequences of particle movement such as in helical swimming [4, 5]. Nearly all aquatic microorganisms smaller than 0.5 mm long, exhibit helical swimming paths either in search for food, to move toward appropriate temperature or pH, or to escape from predators. These paths are inherently three-dimensional [6]. This is for instance the case of spermatozoa when traveling towards the ovum. This specific characteristics of the movement should be taken into account when creating fertilization models. [7, 8] .

To characterize 3D motion, researchers usually extend two-dimensional (2D) measurement schemes, commonly based on standard optical systems such as video cameras and microscopes [9]. Other researchers employ numerical analysis [10, 11] or digital holography with extensive numerical computations [12]. Also, standard techniques based on the classical non-relativistic Doppler effect have been widely applied to determine velocity components perpendicular to the direction of propagation of the illuminating light beam, relying upon measurements of the longitudinal component for a large set of directions [13]. Unfortunately, many of these schemes require the use of multiple laser sources pointing at different directions, or alternatively a single laser source with fast switching of its pointing direction.

In 2011, Belmonte and Torres put forward a novel method to measure directly transverse velocity components using structured light as illumination source [14]. In that scheme, the illumination beam that is pointing in a fixed direction, and that is wide enough in the transverse plane to cover all possible locations of interest of the moving particle, can take on different spatial phase profiles that are tailored to adapt to the target’s motion. In this way the measurement of the transverse velocity is enormously simplified. Two recent experiments have used structured light beams to measure the angular velocity of targets rotating perpendicularly to the direction of illumination, something that cannot be determined with commonly used Gaussian beams [15, 16]. In [15], the angular velocity of a micron-size rotating particle was measured using a Laguerre-Gauss (LG_0^ℓ) mode, a beam that contains an azimuthally varying phase gradient. The angular velocity of a rotating macroscopic body, on the other hand, was measured using a superposition of two LG_0^ℓ modes with opposite winding number ℓ in [16]. However, both experiments restricted their attention to a 2D motion. We note that this technique is different from

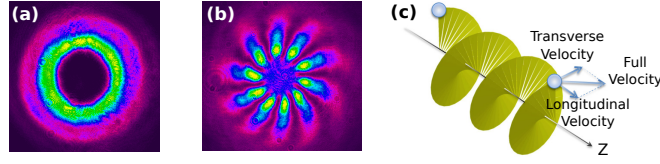


Fig. 1. (a) Intensity profile of the LG_0^{10} beam illuminating the Digital Micromirror Device (DMD). (b) Interference of the LG_0^{10} beam with the reference beam. The 10 lobes observed are due to the phase profile $\Phi = 10\phi$. (c) Schematic representation of the helical trajectory followed by particles. Z is the propagation axis of the beam.

other techniques that use the frequency shift induced by rotating objects, such as the rotational frequency shift experienced when a beam is rotated at certain angular velocity [17, 18].

Here we demonstrate that this technique can also be used to detect all velocity components in a full 3D helical motion using LG_0^ℓ modes, which has an azimuthally varying phase profile. This is a two-step technique wherein we first illuminate the target with a Gaussian mode to determine its translation velocity. Then we change the illumination to an LG_0^ℓ mode to obtain the velocity of rotation. For the case when the direction of translation is known, it is possible to determine the sense of rotation by simply reversing the sign of the mode index ℓ of the LG_0^ℓ mode. Conversely, if we know the sense of rotation, we can compute the direction of translation, again by reversing the sign of ℓ . Even though for convenience we implement the technique in two steps, one can envision illuminating the target with the two beams simultaneously.

2. Theoretical framework

In the classical non-relativistic scheme, light reflected from a moving target is frequency shifted proportional to the target's velocity as $\Delta f_{\parallel} = 2|\mathbf{v}|\cos(\theta)/\lambda$, where λ is the wavelength of light and θ is the angle between the velocity \mathbf{v} of the target and the direction of propagation of the light beam. Under structured light illumination, the phase along the transverse plane is no longer constant. Therefore in the presence of a transverse velocity component, the frequency shift will have an additional term and will given by [14],

$$\Delta f = \Delta f_{\parallel} + \Delta f_{\perp} = \frac{1}{2\pi}(2kv_z + \nabla_{\perp}\Phi \cdot \mathbf{v}_{\perp}), \quad (1)$$

where v_z is the velocity of the target along the line of sight, \mathbf{v}_{\perp} is the velocity in the transverse plane, $k = 2\pi/\lambda$ is the wave vector and $\nabla_{\perp}\Phi$ is the transverse phase gradient.

Helical motion is a combination of a translation along the line of sight and a rotation in the transverse plane [Fig. 1(c)]. Therefore, the use of an azimuthally varying phase $\Phi = \ell\phi$, present in an LG beam [Figs. 1(a) and 1(b)], simplifies the determination of the angular velocity of rotation. Here, ϕ is the azimuthal angle and ℓ is the number of 2π phase jumps of the light beam as one goes around ϕ . The second term of Eq. (1) becomes $\nabla_{\perp}\Phi \cdot \mathbf{v}_{\perp} = \ell\Omega$, so that

$$\Delta f = \frac{1}{2\pi}(2kv_z + \ell\Omega). \quad (2)$$

Two aspects of this equation should be highlighted. First, if we illuminate with a Gaussian mode ($\ell = 0$), as most radar Doppler systems do, $\Delta f = \Delta f_{\parallel} = kv_z/\pi$ and the translational velocity can be determined. Second, Δf depends on the relative signs of v_z , ℓ and Ω . It will acquire maximum value when v_z and $\ell\Omega$ have the same signs and a minimum value when they have opposite signs. Moreover, the larger the value of ℓ , the larger is the frequency shift observed.

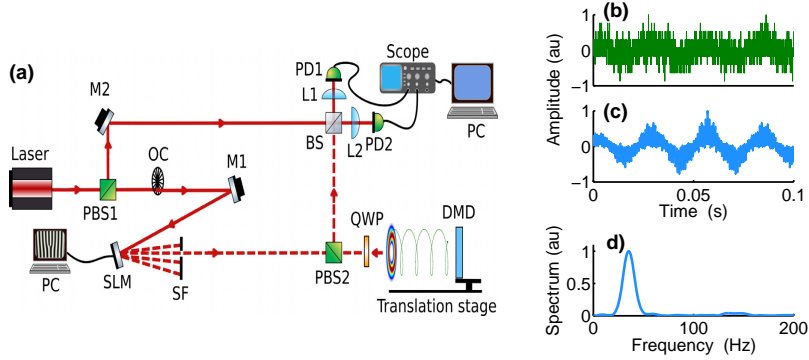


Fig. 2. (a) Experimental setup. (b) Raw signal after balanced detection. (c) Autocorrelation function of the signal. (d) Power spectral density. We only show the spectrum above the chopping frequency. PBS: polarizing beam splitter; M: mirror; L: lens; PD: photodetector; SLM: Spatial Light Modulator; SF: spatial filter; OC: optical chopper; QWP: Quarter-Wave Plate; DMD: Digital Micromirror Device. See text for details.

To generate the helical motion of particles, we employed a Digital micromirror device (DMD). A cluster of 512 diamond-shape squares ($10 \times 10 \mu\text{m}$) randomly distributed within an area with a diameter of 3 mm, was set to rotation as a solid body in a similar way to [15]. The DMD was attached to a translation stage (BP1M2-150 from Thorlabs, with a maximum displacement of 150 mm), aligned perpendicular to the plane of rotation of the DMD. The axis of the illumination beam is aligned to coincide with the axis of the helical trajectory.

3. Experimental setup

We extract the Doppler frequency shift imparted by the moving particles via an interferometric technique using the modified Mach-Zehnder interferometer shown in Fig. 2(a). A 15 mW continuous wave He-Ne laser ($\lambda = 632.8 \text{ nm}$) is spatially cleaned and expanded to a diameter of 5 mm. This beam is split into two (*signal* and *reference*) using a polarizing beam splitter (PBS1). A mirror (M1) redirects the signal beam to a Spatial Light Modulator (SLM) that imprints the beam with the structured phase, as shown in Fig. 1. The first diffracted order of the fork-like hologram encoded into the SLM is used to illuminate the target while the rest of the diffracted orders are spatially filtered (SF). A second polarizing beam splitter (PBS2) in combination with a quarter-wave plate (QWP) collects light reflected from the target back into the interferometer. The light reflected interferes with the reference signal using a beam splitter (BS).

A balanced detection is implemented with two photodetectors (PD1 and PD2), connected to an oscilloscope (TDS2012 from Tektronix). To eliminate most of the low-frequency noise, an optical chopper (OC) placed in the path of the signal beam shifts our detected frequency from Hz to kHz. Figure 2(b) shows a typical signal resulting from the difference of the signals detected from output ports PD1 and PD2. An autocorrelation process allows us to improve the signal-to-noise ratio significantly by finding periodic patterns obscured by noise [Fig. 2(c)]. The signal is Fourier transformed to find the fundamental frequency content. Hamming windowing and zero padding are applied to smooth the Fourier spectrum, shown in Fig. 2(d).

Due to the interferometric nature of the experimental set up, we can measure only $|\Delta f|$. However, since the frequency shift given by Eq. (2) is dependent on the relative signs of v_z , ℓ , and Ω , we can determine the absolute magnitudes of $|\Delta f_{\parallel}|$ and $|\Delta f_{\perp}|$, and consequently $|v_z|$, $|\Omega|$ and the relative sign between v_z and Ω . The frequency shifts always fulfill $|\Delta f| > |\Delta f_{\parallel}|$ for v_z and $\ell\Omega$ showing the same sign, while $|\Delta f| < |\Delta f_{\parallel}|$ for v_z and $\ell\Omega$ showing opposite signs.

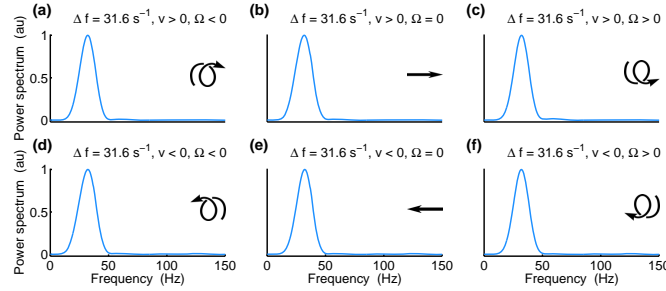


Fig. 3. Frequency shift measured under illumination with a Gaussian mode ($\ell = 0$) for any direction of translation ($v_z > 0$ and $v_z < 0$), and any sense of rotation ($\Omega > 0$ and ($\Omega < 0$)). For the sake of comparison, the case with $\Omega = 0$ is also shown.

Therefore, if $|\Delta f|$ is larger for $\ell > 0$ than for $\ell < 0$, v_z and Ω have the same sign, while if it is smaller, v_z and Ω have opposite signs. The important point is that the sign of ℓ is a free parameter imposed on the illumination beam chosen in the experiment. The experimental procedure to measure $|v_z|$, $|\Omega|$ and the sign of $v_z \cdot \Omega$ is thus to choose a beam with $\ell = 0$ to obtain $|\Delta f_{\parallel}|$, use afterward a *LG* beam with the sign of the index ℓ so that it maximizes $|\Delta f|$, and obtain $|\Delta f_{\perp}| = |\Delta f| - |\Delta f_{\parallel}|$. Furthermore, if $\ell > 0$, we have $v_z \cdot \Omega > 0$, and $v_z \cdot \Omega < 0$ otherwise. Previous knowledge of the sign of v_z , for instance knowing that the particle or microorganism under study advances in a fluid stream, allows to determine the sign of Ω .

4. Experimental results

In our experiments, we first illuminate the helically moving particles with a Gaussian beam. Here, $\Delta f = \Delta f_{\parallel}$ since the particles see a constant phase along the transverse plane. Figure 3 shows the experimental results. We do all possible combinations: $v_z > 0$ (forward, moving away from the illuminating source) with negative [Fig. 3(a)] and positive [Fig. 3(c)] rotations, as well as $v_z < 0$ (backwards, moving towards the illuminating source) with negative [Fig. 3(d)] and positive [Fig. 3(f)] rotation. Positive (negative) rotation refers to counterclockwise (clockwise) rotation for an observer looking towards the illumination source. For the sake of comparison, we also show the spectra when particles translate without any rotation for both $v_z > 0$ and $v_z < 0$ [Figs. 3(b) and 3(e), respectively]. The frequency shift measured, $|\Delta f_{\parallel}| = 31.6 \text{ s}^{-1}$, is the same in all cases, regardless of the relative signs of v_z and ℓ . From this information, computing the translational velocity is straight forward from Eq. (2). The velocity we obtained is $|v_z| = \pm 10 \mu\text{m/s}$, exactly the same as the velocity we programmed our rail to move. This leaves only the rotational velocity unknown.

Figures 4 and 5 show experimental results when the moving particles are illuminated by an LG_0^{ℓ} mode with $\ell = \pm 10$. With any value of ℓ we would obtain similar results. The presence of some lower-intensity secondary peaks is attributed to the remaining low-frequency noise still present in the detection system. For $\ell = -10$ (Fig. 4) and $v_z > 0$, Δf is larger if the particles rotate with $\Omega < 0$ [Fig. 4(a)], while it is smaller if the particles rotate in the opposite direction ($\Omega > 0$) [Fig. 4(c)]. The frequency shifts measured are $\Delta f = 83.7 \text{ s}^{-1}$ and $\Delta f = 20.5 \text{ s}^{-1}$ for $\Omega < 0$ and $\Omega > 0$, respectively. By accounting these and using the procedure described above, we can compute Δf_{\perp} due to the rotation of the particles according to Eq. (2). In all cases, $|\Delta f_{\perp}| = 52.1 \text{ s}^{-1}$, yielding an angular velocity of $|\Omega| = 32.7 \text{ s}^{-1}$. Conversely, when the particles move backwards ($v_z < 0$), Δf is larger for $\Omega > 0$ [Fig. 4(f)], while it is smaller for $\Omega < 0$ [Fig. 4(d)]. On the contrary, if we illuminate with LG_0^{10} , we observe a larger Δf when either $v_z > 0$ and $\Omega > 0$ [Fig. 5(c)] or $v_z < 0$ and $\Omega < 0$ [Fig. 5(d)]. We also show the spectra when the

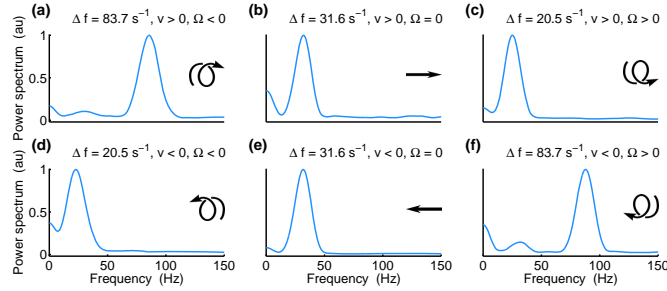


Fig. 4. Frequency shift measured under illumination with a LG_0^{-10} mode for $v_z > 0$ and $v_z < 0$, and for $\Omega > 0$ and $\Omega < 0$.

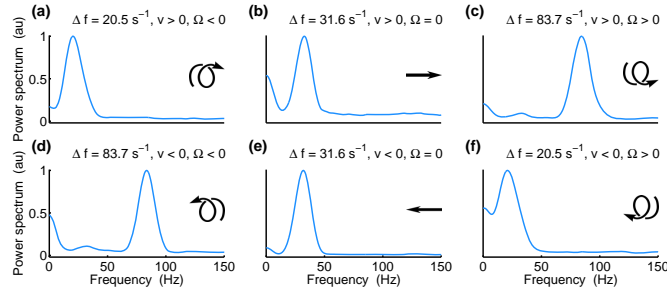


Fig. 5. Frequency shift measured under illumination with a LG_0^{10} mode for $v_z > 0$ and $v_z < 0$, and for $\Omega > 0$ and $\Omega < 0$.

particles move forward [Figs. 4(b) and 5(b)] or backward [Figs. 4(e) and 5(e)] without rotation ($\Omega = 0$).

The translational and rotational velocities used in our experiment are within the range of velocities common to biological specimens. Human sperm for example, have helical rotation speeds from ~ 18 to 120 s^{-1} and linear speeds of approximately 20 to $100 \mu\text{m/s}$ [19].

5. Conclusions

We have measured the full velocity of particles moving in a helical motion by using the frequency shift generated by reflecting particles moving in a structured light beam, i.e., a beam with an engineered transverse phase profile. The technique requires choosing the appropriate spatial mode of the illumination beam, which depends on the particular type of movement under investigation. First, we determine the longitudinal velocity component by illuminating with a Gaussian mode and afterwards, we use an LG_0^l mode to obtain the angular velocity of the rotational motion.

The technique can be generalized to other types of motion by choosing appropriately the transverse phase profile of the illuminating beam. Our technique can also be implemented in other frequency bands. For example, the microwave band is of special interest for detecting transverse velocities of macroscopic objects.

Acknowledgments

This work was supported by project FIS2010-14831, program SEVERO OCHOA, and Fundacio Privada Cellex Barcelona.