MODELLING OF A CLAMPED-PINNED PIPELINE CONVEYING FLUID FOR VIBRATIONAL STABILITY ANALYSIS

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Abstract

Recent developments in materials and cost reduction have led the study of the vibrational stability of pipelines conveying fluid to be an important issue. Nowadays, this analysis is done both by means of simulation with specialized softwares and by laboratory testing of the preferred materials. The former usually requires of complex modelling of the pipeline and the internal fluid to determine if the material will ensure vibrational stability; and in the latter case, each time there is a mistake on the material selection is necessary to restart all the process making this option expensive. In this paper, the classical mathematical description of the dynamic behavior of a clamped-pinned pipeline conveying fluid is presented. Then, they are approximated to a Hamiltonian system through Garlekin's method being modelled as a simple linear system. The system stability has been studied by means of the eigenvalues of the linear system. From this analysis, characteristic expressions dependent on material constants has been developed as inequalities, which ensures the stability of the material if it matches all expressions. This new model provides a simplified dynamical approximation of the pipeline conveying fluid depending on material and fluid constants that is useful to determine if it is stable or not. It is worth to determine that the model dynamics does not correspond with the real, but the global behaviour is well represented. Finally, some simulations of specific materials have been use to validate the results obtained from the Hamiltonian model and a more complex model done with finite element software.

Key words

Stability, eigenvalues analysis, pipe conveying fluid, material selection, Garlekin's method, Hamiltonian systems.

1 Introduction

In the last decades, the dynamics and stability of pipes conveying fluid has been studied thoroughly with various techniques of analysis, considering different end conditions and different models of the fluid-conveying pipeline (see for example [Gregory and Païdoussis, 1966; Kameswara and Simha, 2008; Kerboua and Lakis, 2008; Kuiper and Metrikine, 2004; Mediano, 2011; Misra, Wong, Païdoussis, 2001; Païdoussis, 1998; Païdoussis and Issid, 1974; Païdoussis, Tian, Misra, 1993; Stoicuta et al., 2010]. These authors analyze stability of pinned-pinned, clamped-clamped and cantilevered fluid-conveying pipes, even in the presence of a tensile force and a harmonically perturbed flow field.

The Interest in the problem of vibrational stability not only is in modeling pipes but in other fields as for example analyzing the stability of fixed speed wind turbine (see [Dominguez-Garcia]) or analyzing vibrations of a thin stretched string, with an alternating electric current passing through, in a nonuniform magnetic field, (see [López Reyes and Kurmyshev]).

It is well known that the dynamical behavior of pipes of a finite length depends strongly on the type of boundary. The type of supports considered (fixed, one end fixed, etc.) and their position (horizontal, vertical) must be distinguished.

The dynamics of the system can be described by a partial differential equation [Seyranian and Mailybaev, 2003; Thompson, 1982]

$$a_4\frac{\partial^4 y}{\partial x^4} + a_3\frac{\partial^2 y}{\partial x^2} + a_2\frac{\partial^2 y}{\partial x \partial t} + a_1\frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

with boundary conditions at ends of a clamped-pinned pipe. We find approximate solution of this equation using Galerkin's method obtaining as a result a linear gyroscopic system possessing the properties of linear

Hamiltonian systems. Then, the eigenvalues of this linear Hamiltonian system gives information about stability: a stable Hamiltonian system is characterized by pure imaginary eigenvalues. It is known that the stability of a linear Hamiltonian system is not asymptotic, nevertheless the study provides the necessary stability condition for the original non-linear system.

Different qualitative analysis of multiparameter linear systems as well bifurcation theory of eigenvalues can be found in [Seyranian and Mailybaev, 2003; Galin, 1982; Garcia-Planas, 2008; Garcia-Planas and Tarragona, 2012; Mediano, 2011; Mediano and Garcia-Planas, 2011; Mediano and Garcia-Planas, 2014].

The aim of the paper is by means of linear Hamiltonian system to model the clamped-pinned pipeline problem and to analyze the structural stability of the proposed model. This paper refers to a one end fixed horizontal pipeline.

The paper is structured as follows. Section 2 presents a mathematical statement of the problem. Section 3 is devoted to analyze the stability of linear gyroscopic system obtained in subsection 2.1. Section 4 presents and a simulation of the dynamic system using ANSYS for some different materials used in real cases, such as PVC, Polyethylene and Concrete, in order to validate the results obtained analytically.

2 Preliminaries

The system under consideration is a straight, tight and of finite length pipeline, passing through it a fluid. The following assumptions are taken into account in the analysis of the system:

i) Are ignored the effects of gravity, the coefficient damping material, the shear strain and rotational inertia
ii) The pipeline is considered horizontal
iii) The pipe is inextensible
iv) The lateral movement of $y(x,t)$ is small, and with large length wave compared with the diameter of the pipe, so that theory Euler-Bernoulli is applicable for the description of vibration bending of the pipe.
v) It ignores the velocity distribution in the cross section of pipe.

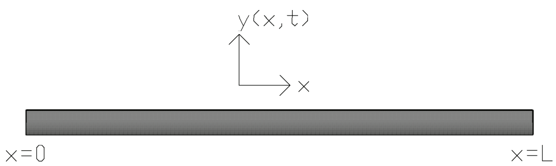

Figure 1. Pipeline

The equation for a single span prestressed pipeline where the fluid is transported is a function of the distance x and time t and is based on the beam theory, [Abid-Al-Sahib, Jameel, Abdlateef, 2010; Morand and Ohayon, 1995]:

$$EI\frac{\partial^4 y}{\partial x^4} + m_p\frac{\partial^2 y}{\partial t^2} = f_{int}(x,t) \tag{2}$$

where EI is the bending stiffness of the pipe (Nm^2), m_p is the pipe mass per unit length ($\frac{kg}{m}$) and f_{int} is an inside force acting on the pipe.

The internal fluid flow is approximated as a plug flow, so all points of the fluid have the same velocity U relative to the pipe. This is a reasonable approximation for a turbulent flow profile. Because of that the inside force can be written as:

$$f_{int} = -m_f\frac{d^2y}{dt^2}\bigg|_{x=Ut} \tag{3}$$

where m_f is the fluid mass per unit length ($\frac{kg}{m}$) and U is the fluid velocity ($\frac{m}{s}$).

Total acceleration can be decomposed into local acceleration, Coriolis and centrifugal.

$$\begin{aligned} m_f\frac{d^2y}{dt^2}\bigg|_{x=Ut} &= m_f\left[\frac{d}{dt}\left(\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x}\frac{dx}{dt}\right)\bigg|_{x=Ut}\right] = \\ &= m_f\left[\frac{d}{dt}\left(\frac{\partial y}{\partial t} + U\frac{\partial y}{\partial x}\right)\bigg|_{x=Ut}\right] = \\ &= m_f\left[\frac{\partial^2 y}{\partial t^2} + 2U\frac{\partial^2 y}{\partial x \partial t} + U^2\frac{\partial^2 y}{\partial x^2}\right] \end{aligned} \tag{4}$$

The internal fluid causes an hydrostatic pressure on the pipe wall.

$$T = -A_iP_i \tag{5}$$

where A_i is the internal cross sectional area of the pipe (measured in m^2) and P_i is the hydrostatic pressure inside the pipe (measured in Pa).

Finally if by considering that the total acceleration is equal to the composition of local, coriolis and centrifugal acceleration. The resulting equation describing the oscillations of the pipe ([Mediano, 2011]):

$$\begin{aligned} &EI\frac{\partial^4 y}{\partial x^4} + (m_fU^2 - T)\frac{\partial^2 y}{\partial x^2} + 2m_fU\frac{\partial^2 y}{\partial x \partial t} + \\ &(m_p + m_f)\frac{\partial^2 y}{\partial t^2} = 0 \end{aligned} \tag{6}$$

2.1 Linear Hamiltonian System

This subsection is devoted to prove that equation (6) can be written as a linear dynamic system.

Proposition 2.1. *The equation*

$$EI\frac{\partial^4 y}{\partial x^4} + (m_f U^2 - T)\frac{\partial^2 y}{\partial x^2} + 2m_f U\frac{\partial^2 y}{\partial x \partial t} + (m_p + m_f)\frac{\partial^2 y}{\partial t^2} = 0$$

can be written as

$$\dot{X} = AX$$

for some specific Hamiltonian matrix.

To find approximate solution to equation (6), the method used is the Galerkin's method with two coordinate function, that is to say, taking $n = 2$ with respect $\left\{\sin\frac{i\pi}{L}x\right\}_{i=1,2,...}$ basis defined over a open set $\Omega \subset \mathbb{R}^n$ and the scalar product $< f, g >= \int_0^L fg$ the approximate solution is:

$$y(x,t) = q_1(t)\text{sen}\,\frac{\pi}{L}x + q_2(t)\text{sen}\,\frac{2\pi}{L}x$$

Replacing the solution in the equation (6), it is obtained that:

$$\begin{aligned}
&EIq_1(t)\left(\frac{\pi^4}{L^4}\text{sen}\,\frac{\pi}{L}x + q_2(t)\frac{16\pi^4}{L^4}\text{sen}\,\frac{2\pi}{L}x\right) + \\
&(m_f U^2 - T)\left(-q_1(t)\frac{\pi^2}{L^2}\text{sen}\,\frac{\pi}{L}x - q_2(t)\frac{4\pi^2}{L^2}\text{sen}\,\frac{2\pi}{L}x\right) + \\
&2m_f U\left(\dot{q}_1(t)\frac{\pi}{L}\cos\frac{\pi}{L}x + \dot{q}_2(t)\frac{2\pi}{L}\cos\frac{2\pi}{L}x\right) + \\
&(m_p + m_f)\left(\ddot{q}_1(t)\text{sen}\,\frac{\pi}{L}x + \ddot{q}_2(t)\text{sen}\,\frac{2\pi}{L}x\right) = 0
\end{aligned} \tag{7}$$

Making the scalar product by sen $\frac{\pi}{L}\xi$ and sen $\frac{2\pi}{L}\xi$, respectively, it can be obtain:

$$\begin{aligned}
&\frac{L}{2}(m_p + m_f)\ddot{q}_1(t) - \frac{8}{3}m_f U\dot{q}_2(t) + \\
&\left(EI\frac{\pi^4}{2L^3} - \frac{(m_f U^2 - T)\pi^2}{2L}\right)q_1(t) = 0 \\
&\frac{L}{2}(m_p + m_f)\ddot{q}_2(t) - \frac{8}{3}m_f U\dot{q}_1(t) + \\
&\left(EI\frac{8\pi^4}{L^3} - (m_f U^2 - T)\frac{4\pi^2}{L^2}\right)q_2(t) = 0
\end{aligned} \tag{8}$$

The previous equation system can be written as matrix form like:

$$M\ddot{q} + B\dot{q} + Cq = 0$$

that corresponds to gyroscopic lineal system:

$$\ddot{x} + G\dot{x} + Kx = 0 \tag{9}$$

with $M^{-1/2}q = x$ (we write the variable as x if confusion it is not possible),

$$G = M^{-1/2}BM^{-1/2} = \frac{16m_f}{L(m_f + m_p)}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$K = M^{-1/2}CM^{-1/2} = \frac{2}{L(m_f + m_p)}\begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$$

and

$$K_1 = EI\frac{\pi^4}{2L^3} - \frac{(m_f U^2 - T)\pi^2}{2L}$$

$$K_2 = EI\frac{8\pi^4}{L^3} - (m_f U^2 - T)\frac{4\pi^2}{L^2}$$

Introducing the vector:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} + Gx/2 \end{pmatrix}$$

and calculating the derivatives of x and y it can be found $\dot{x} = y - Gx/2$, $\dot{y} = \ddot{x} + G\dot{x}/2$ and considering that $\ddot{x} = -G\dot{x} - Kx$ and linearizing the system a linear Hamiltonian equation is obtained:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -G/2 & I_2 \\ G^2/4 - K & -G/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}.$$

It is easy to prove that the matrix A is Hamiltonian because QA is symmetrical, where Q is the antisymmetrical matrix:

$$Q = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

So, the linear gyroscopic system obtained in (9) possess the properties of linear Hamiltonian systems.

In order to simplify, the following parameters considered

$$\begin{aligned} \Lambda &= \frac{EI\pi^4}{L^3} \\ \delta &= (m_f U^2 - T)\frac{\pi^2}{L} \\ \beta &= \frac{1}{L(m_f + m_p)} \end{aligned} \tag{10}$$

and the matrices G and K are written as:

$$\begin{aligned} G &= 16 m_f \beta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ K &= 2\beta \begin{pmatrix} \frac{1}{2}\Lambda - \frac{1}{2}\delta & 0 \\ 0 & 8\Lambda - \frac{4}{L}\delta \end{pmatrix} \end{aligned} \tag{11}$$

Therefore, matrix A is:

$$A = \begin{pmatrix} 0 & a & 1 & 0 \\ -a & 0 & 0 & 1 \\ b & 0 & 0 & a \\ 0 & c & -a & 0 \end{pmatrix} \tag{12}$$

where:

$$\begin{aligned} a &= 8 m_f \beta \\ b &= -64 m_f^2 \beta^2 - \beta\Lambda + \beta\delta \\ c &= -64 m_f^2 \beta^2 - 16\beta\Lambda + \frac{8}{L}\beta\delta. \end{aligned} \tag{13}$$

Removing the variable change it is known that:

$$\begin{aligned} a &= \frac{8 m_f}{L(m_f + m_p)} \\ b &= \frac{-64 m_f^2}{L^2(m_f + m_p)^2} - \frac{EI\pi^4}{L^4(m_f + m_p)} + \\ &\quad \frac{(m_f U^2 + A_i P_i)\pi^2}{L^2(m_f + m_p)} \\ c &= \frac{-64 m_f^2}{L^2(m_f + m_p)^2} - \frac{16 EI\pi^4}{L^4(m_f + m_p)} + \\ &\quad \frac{8(m_f U^2 + A_i P_i)\pi^2}{L^3(m_f + m_p)}. \end{aligned}$$

So, the pipeline is modeled as a Hamiltonian system.

3 Stability

In this section the stability properties of linear dynamic systems representing the pipeline is studied. Also, a detailed explanation of the effect of the stabilization in terms of the bifurcation theory of eigenvalues is presented.

Concretely, this section is devoted to prove that the space of stability can be described in terms of parameters of the matrix A ([Mediano and Garcia-Planas, 2011; Mediano and Garcia-Planas, 2014]) and to prove that it is an open set whose boundary consists of algebraic varieties of smaller dimension

Theorem 3.1. *let (a, b, c) be the parameters of the matrix A in the hamiltonian equation. The set of points (a, b, c) of stability is determined by*

$$\left. \begin{array}{l} 2a^2 - b - c > 0 \\ (a^2 + c)(a^2 + b) > 0 \\ (2a^2 - b - c)^2 > 4(a^2 + c)(a^2 + b) \end{array} \right\}. \tag{14}$$

A stable hamiltonian system is characterized by eigenvalues lying on the imaginary axis. The characteristic equation of the matrix is:

$$\lambda^4 + (2a^2 - b - c)\lambda^2 + (a^2 + c)(a^2 + b) = 0 \tag{15}$$

and the eigenvalues

$$\lambda = \pm\sqrt{\frac{-2a^2 + b + c \pm \sqrt{-8a^2 b - 8a^2 c - 2bc + b^2 + c^2}}{2}} \tag{16}$$

that in terms of Λ, δ, β are

$$\lambda = \pm\sqrt{\frac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}} \tag{17}$$

with

$$
\begin{aligned}
\lambda_1 &= -256m_f^2\beta^2 - 17\beta\Lambda + \left(1 + \frac{8}{L}\right)\beta\delta \\
\lambda_2 &= 65536m_f^4\beta^2 + 8704m_f^2\beta\Lambda - \\
&\left(512 + \frac{4096}{L}\right)m_f^2\beta\delta + 225\Lambda^2 + \\
&\left(1 + \frac{64}{L^2} - \frac{16}{L}\right)\delta^2 + \left(30 - \frac{240}{L}\right)\Lambda\delta
\end{aligned}
\tag{18}
$$

As we says, the system is stable in Lyapunov's sense, if the eigenvalues lie on the imaginary axe and they are smple or semi-simple.

Taking into account that the values in the system are know only approximately, the matrix A in the system can be considered as a family of matrices depending on parameters a, b, c in a neighborhood of a fixed point p_0, that permit us to study the stability border. The point p_0, in which correspond only simple pure imaginary eigenvalues, is always an interior point of the stability domain, while the points on the boundary of the stability domain are characterized by the existence of multiple pure imaginary or zero eigenvalues, (when the other eigenvalues are simple and pure imaginary).

Stability conditions requires that the roots obtained in (17), $\lambda^2 = \dfrac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}$ are real and negative. Imposing these conditions the stability zone in the parameter space can be determined.

It can be observed that the points $p = (a, b, c)$ such that

$$
\left.\begin{aligned} 2a^2 - b - c = 0 \\ (a^2 + c)(a^2 + b) = 0 \end{aligned}\right\}, \tag{19}
$$

the characteristic polynomial is λ^4,

The set (19) corresponds to the union of parameterized curves $\varphi(\alpha) = (\alpha, 3\alpha^2, -\alpha^2)$ and $\varphi(\alpha) = (\alpha, -\alpha^2, 3\alpha^2)$. In the intersection can be found the most degenerate case, with respect the algebraic structure of the system as it can be seen below.

If $a \neq 0$ the Jordan form of A is

$$
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

A Jordan basis transforming the matrix in its reduced form is

$$
S_1 = \begin{pmatrix} 1 & 0 & b - a^2 & 0 \\ 0 & -a & 0 & -a(a^2 + b) \\ 0 & b & 0 & c(a^2 - b) \\ 0 & 0 & -a(b + c) & 0 \end{pmatrix}
$$

if $a^2 + b \neq 0$, and

$$
S_2 = \begin{pmatrix} 0 & a & 0 & a(a^2 + c) \\ 1 & 0 & c - a^2 & 0 \\ 0 & 0 & a(b + c) & 0 \\ 0 & c & 0 & -b(a^2 + c) \end{pmatrix}
$$

if $a^2 + c \neq 0$. (Observe that $a^2 + b$ and $a^2 + c$ can not be zero simultaneously because $a \neq 0$).

If $a = 0$ the Jordan form of A is

$$
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}
$$

In this case, a Jordan basis is

$$
S = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}
$$

At the points (19) the system have singularities of the type 0^4 and the more degenerate $0^2 0^2$ on the stability boundary. In both cases eigenvalues lie in imaginary axis but they are not semisimple.

Near of these singularities it is possible to find the lest degenerate matrices, for example

$$
A(\varepsilon) = \begin{pmatrix} 0 & \varepsilon & 1 & 0 \\ -\varepsilon & 0 & 0 & 1 \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & -\varepsilon & 0 \end{pmatrix}
$$

where the eigenvalues are $\pm\varepsilon i$, and the stable case:

$$
\widetilde{A} = \begin{pmatrix} 0 & -0.1 & 1 & 0 \\ 0.1 & 0 & 0 & 1 \\ -0.0001 & 0 & 0 & -0.1 \\ 0 & -0.0001 & 0.1 & 0 \end{pmatrix}
$$

where the eigenvalues are $0 + 0.1100i$, $0 - 0.1100i$, $0 + 0.0900i$, $0 - 0.0900i$.

Following the analysis of eigenvalues, also it is obtained the eigenvalue 0 at the points such that (a, b, c)

$$
\left.\begin{aligned} (a^2 + c)(a^2 + b) = 0 \\ 2a^2 - b - c \neq 0 \end{aligned}\right\} \tag{20}
$$

At the points $(a, b, -a^2)$ there are two possibilities depending on b if it is equal or not to $-a^2$

For $b \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{-3a^2+b} & 0 \\ 0 & 0 & 0 & -\sqrt{-3a^2+b} \end{pmatrix}$$

For $b = -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2ai & 0 \\ 0 & 0 & 0 & -2ai \end{pmatrix}.$$

This case correspond to a stability point because of all eigenvalues are semisimple and lie in the imaginary axe. It is important to note that in this case the reduced form is not structurally stable (a small perturbation makes that the double eigenvalue bifurcates into two distinct eigenvalues or into a double nonderogatory eigenvalue of type 0^2.

By symmetry, at the points $(a, -a^2, c)$ there are two cases depending on c be equal or not to $-a^2$

For $c \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{-3a^2+c} & 0 \\ 0 & 0 & 0 & -\sqrt{-3a^2+c} \end{pmatrix}$$

For $c = -a^2$ the Jordan form coincides with the case $b = -a^2$.

Analogously, the case $c = a^2$ is out of the stability space

For the case $b \neq -a^2$ and $c \neq -a^2$ the system have singularities of the type 0^2 in the boundary of stability.

It remains to study the case that no eigenvalue is zero

The roots of $\mu^2+(2a^2-b-c)\mu+(a^2+c)(a^2+b) = 0$, are real and negative when

$$\left.\begin{array}{l} 2a^2-b-c>0 \\ (a^2+c)(a^2+b)>0 \\ (2a^2-b-c)^2 \geq 4(a^2+c)(a^2+b) \end{array}\right\} \quad (21)$$

In the case $(2a^2-b-c)^2 = 4(a^2+c)(a^2+b)$ the eigenvalues are $\lambda = \pm i\sqrt{2a^2-b-c} = \pm i\omega$ double.

It is easy to observe that rank $(A - (\pm i\omega)I) = 3$ so the Jordan form is

$$\begin{pmatrix} i\omega & 1 & 0 & 0 \\ 0 & i\omega & 0 & 0 \\ 0 & 0 & -i\omega & 1 \\ 0 & 0 & 0 & -i\omega \end{pmatrix}$$

At the points (a, b, c) with $2a^2-b-c > 0$, $(a^2+c)(a^2+b) > 0$ and $(2a^2-b-c)^2 = 4(a^2+c)(a^2+b)$ the system have singularities of the type $\pm i\omega^2$.

The last case

$$\left.\begin{array}{l} 2a^2-b-c>0 \\ (a^2+c)(a^2+b)>0 \\ (2a^2-b-c)^2 > 4(a^2+c)(a^2+b) \end{array}\right\} \quad (22)$$

determined the stability points (a, b, c) remaining within the area bounded by the above singularities.

4 Stability of the Pipes: Case Study

Taking as a constant parameters $L = 1000mm$, $I = 2,185 \cdot 10^6$, $A_i = 2500\pi$ due to the geometry of the pipe and $m_f = 2,5\pi \cdot 10^{-6}\dfrac{Tn}{mm}$ assuming the fluid is water. It is also supposed that the study is applied to the inside wall of the pipe so U at these points are zero.

Therefore the values a, b y c are:

$$a = \frac{2\pi \cdot 10^{-8}}{(2,5\pi \cdot 10^{-6} + m_p)}$$
$$b = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6} + m_p)^2} - \frac{2,185 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6} + m_p)} + \frac{2,5 \cdot 10^{-3}P_i\pi^3}{(2,5\pi \cdot 10^{-6} + m_p)}$$
$$c = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6} + m_p)^2} - \frac{34,96 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6} + m_p)} + \frac{2 \cdot 10^{-5}P_i\pi^3}{(2,5\pi \cdot 10^{-6} + m_p)}$$

That permit to obtain the following relation depending only on m_p, E and P_i:

$$\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi^2 E - 2.52\pi P_i > 0$$
$$15.27752 \cdot 10^{-4}\pi^2 E^2 + P_i^2 - 17.48874 \cdot 10^{-1}\pi E P_i > 0$$
$$\left(\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi E - 2.52 P_i\right)^2 >$$
$$4\pi^2(76.3877 \cdot 10^{-6}\pi^2 E^2 - 87.4437 \cdot 10^{-3}\pi E P_i +$$
$$5 \cdot 10^{-2}P_i^2). \quad (23)$$

This study is done to show the stability of pipes with different materials assuming in all of them that the fluid transported is water and causes a constant pressure on its walls of 4 bar. The geometrical conditions of the pipe are the inside diameter equal to 50 mm and the thickness of the pipe which is 6 mm. The materials chosen are PVC, Polyethylene and Concrete.

The values of E and m_p of the PVC pipe are:

$$E = 30,581\frac{N}{mm^2}$$

$$m_p = 2,76 \cdot 10^{-6} \frac{Tn}{mm}$$

Applying the inequalities (23) it is found that the solution is unstable.

The values of E and m_p of the PE pipe are:

$$E = 9,174 \frac{N}{mm^2}$$

$$m_p = 1,91 \cdot 10^{-6} \frac{Tn}{mm}$$

Applying the inequalities (23) it is found that the solution is unstable.

The values of E and m_p of the Concrete pipe are:

$$E = 221,203 \frac{N}{mm^2}$$

$$m_p = 4,40 \cdot 10^{-6} \frac{Tn}{mm}$$

Applying the inequalities (23) it is found that the solution is stable.

So, the eigenvalues obtained for this materials are

Material	Eigenvalues			
PVC	48.042	48.042	$98.936i$	$98.936i$
PE	-54.549	54.549	$56.341i$	$56.341i$
Concrete	$0.362i$	$0.362i$	$2.479i$	$2.479i$

Figure 2 shows the distribution of these values in the complex plane.

Observe that the case of PVC pipe is the furthest away from stability zone.

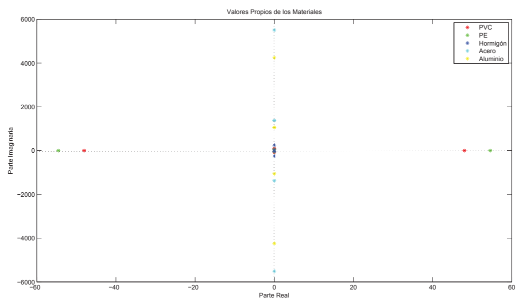

Figure 2. Values of the different pipes

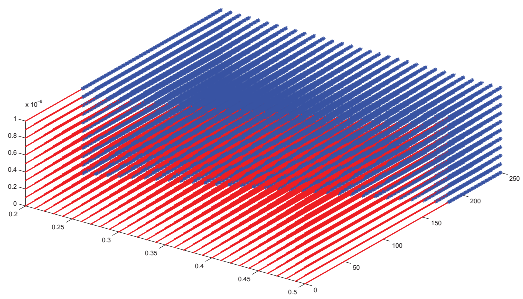

Figure 3. Representation for values of E less than 250 N/mm^2

To perform the analysis we have seen that we had to find the Jordan reduced form of the matrix for each particular case because as it is well known the obtention of its continuous invariants is not stable when they are not simple and that these values are inevitable for small disturbances. One way to address these kind of problems is analyzing families of elements dependent on parameters, defined in a neighborhood of the given element. In this approach versal deformations can be constructed. A versal deformation represents the most general family possessing (in some sense) the properties of all deformations of the element ([**?**]).

Let us consider the Lie algebra $sp(2k, \mathbb{C}) = \{A \in M_4(\mathbb{C}) \mid QA + A^tQ = 0\}$ with $Q = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}$ of the Hamiltonian matrices under the action α of Lie group $G = \{P \in Gl(n; \mathbb{C}) \mid P^tQP = Q\}$ defined in the form $\alpha(P, A) = P^{-1}AP$. A miniversal deformation is given $A + T_A\mathcal{O}(A)^\perp$ where $\mathcal{O}(A)$ is the orbit of A with respect the action and $\perp$ is the orthogonal with respect the standard scalar product. It is easy to compute $T\mathcal{O}(A)^\perp = \{X \mid X^tA - AX^t = 0\}$.

Although the stability analysis for the Hamiltonian matrix have been made without having a miniversal deformation, this method can be used for higher-order matrix symplectic dependent on an arbitrary number of parameters.

In fact, if it is considered the miniversal deformation in a neighborhood of a most degenerate case we obtain the following four parameter familiy of Hamiltonian matrices

$$A(\mathbf{p}) = \begin{pmatrix} 0 & -p_1 & 1 & 0 \\ p_1 & 0 & 0 & 1 \\ p_2 & p_3 & 0 & -p_1 \\ p_3 & p_4 & p_1 & 0 \end{pmatrix}$$

that restricted to the particular setup of systems describing the pipe, is $p_3 = 0$ and $A(\mathbf{p})$ coincides with the matrix A in (12).

4.1 Simulation for Some Specific Materials

In this section the equation (6) is solved and the structural stabilities found in the previous section with the stability of the solution is compared. Moreover, vibration characteristics of a pipe conveying fluid is calculated using a Finite Element package called ANSYS.

To determinate the vibration characteristics modal analysis have been used, with this analysis you find natural frequencies and mode shapes which are important parameters in the design of a structure for dynamic studies.

The simulation of the problem varies depending on the boundary conditions. On the left side of the pipe is a rigid support, while the side right lateral displacement can not but have a time proportional to the angle of rotation of the pipe. So, the boundary conditions at ends of a clamped-pinned pipe are given as:

$$y(0,t)=0, \qquad y(L,t)=0$$
$$\frac{\partial^2 y(0,t)}{\partial t}=0, \quad EI\frac{\partial^2 y(L,t)}{\partial x^2}=-K_{rs}\frac{\partial y(L,t)}{\partial x} \tag{24}$$

where K_{rs} is the stiffness of the rotational spring at the right end.

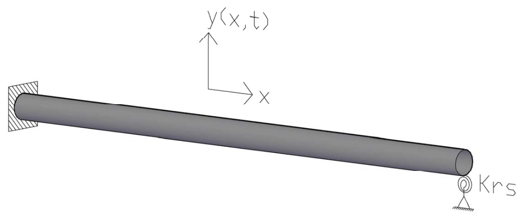

Figure 4. Boundary conditions

The following table lists the values of natural frequency and displacement in both cases:

	First shape		Second shape	
	f	dx	f	dx
Concrete	0	0.034258	0.023795	0.022697
PE	19.435	36.609	19.668	38.279
PVC	9.806	50.513	24.043	31.546

Remark that the frequencies obtained solving linear Hamiltonian system does not coincides with frequencies that can be obtained solving the second order differential equation (6) with initial conditions (23), in fact with Hamiltonian equation is studied structural stability giving information about the qualitative changes that can be in the behavior of systems when the systems are known only approximately.

In the following pictures it is shown the performance of the first and the second shapes and the natural frequencies of them.

As seen in picture 3, 4 and 5 the lowest natural frequency is the concrete pipe (it is not write on the picture

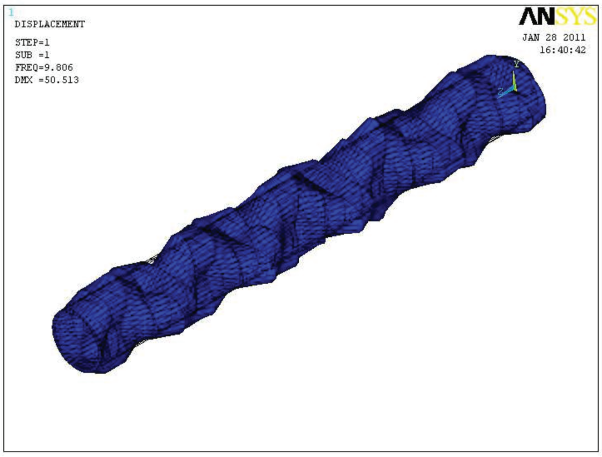

Figure 5. First shape of PVC

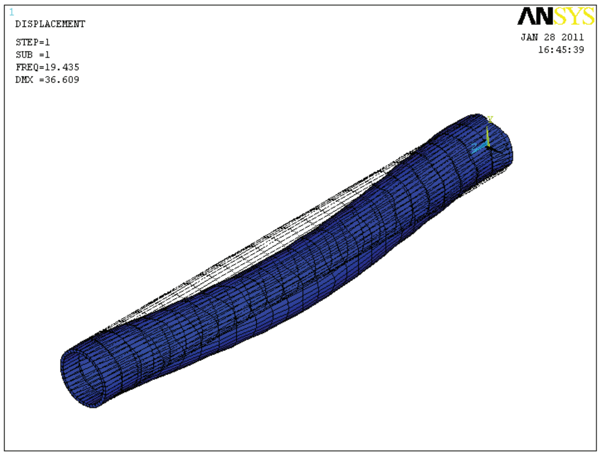

Figure 6. First shape of Polyethylene

because is zero) and the biggest one is the Polyethylene pipe ($19,435$ Hz) but the greater displacement of x axis is the PVC pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete pipe is stable.

As seen in picture 6, 7 and 8 the lowest natural frequency is the concrete pipe (0,0238 Hz) and the biggest one is the PVC pipe ($24,043$ Hz) but the greater displacement of x axis is the PE pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete pipe is stable.

5 Conclusion

In this paper the classic non-linear dynamic model for a pipe conveying fluid have presented. Moreover, a linearization method have been done by approximation of the non-linear system to the linear gyroscopic system (Garlekin's method) providing a Hamiltonian system. From the linear system, the stability of the pipeline has been analyzed in a general form by means of the first Lyapunov's methods. The stability generalization of

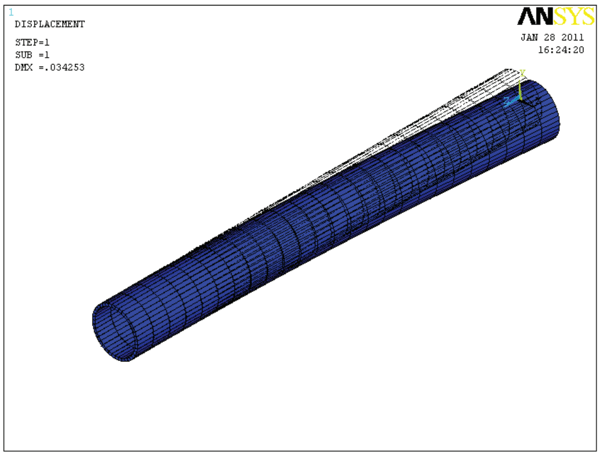

Figure 7. First shape of Concrete

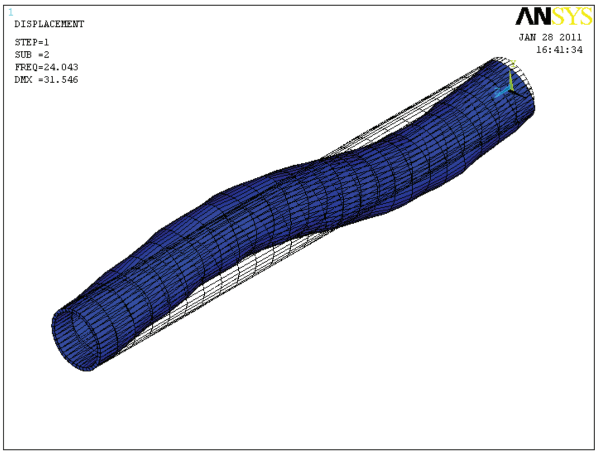

Figure 8. Second shape of PVC

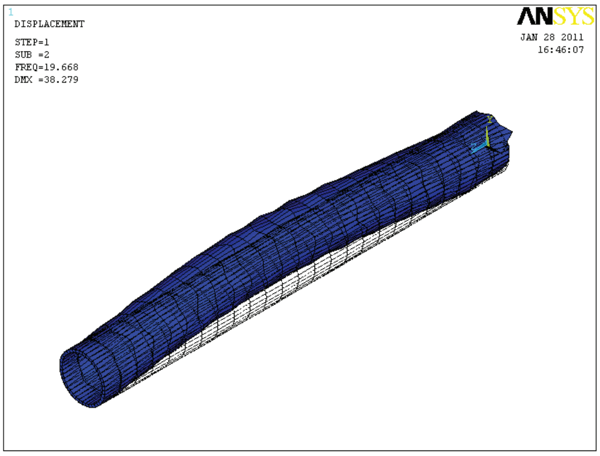

Figure 9. Second shape of Polyethylene

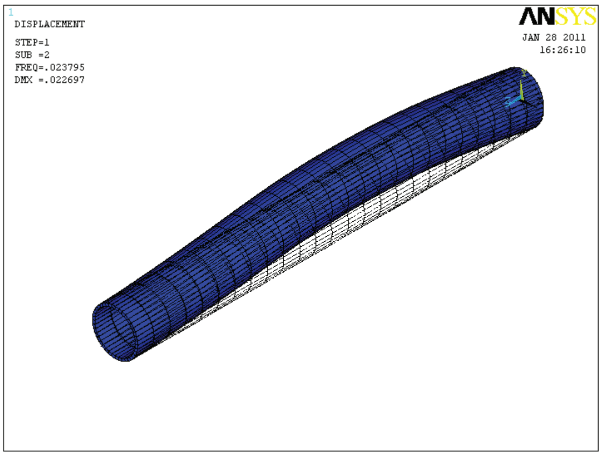

Figure 10. Second shape of Concrete

the system have been done obtaining the stability limits as function of the material and fluid parameters.

It have been shown that the dynamics and stability of pipes conveying fluid not only depends on the boundary conditions but it is also strongly important the material of the pipe and the pressure produced by the fluid.

The proposed Hamiltonian model of a pipeline conveying fluid provides a simple case to evaluate if the selected materials will remain stables.

In this paper the calculations of the proposed model and the simulation of a complex model of typical materials for a pipe used in public works have been compared to verify the results obtained.

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