STEPWISE ADVANCING STRATEGY FOR THE SIMULATION OF FATIGUE PROBLEMS

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Abstract. A time advance strategy for cyclic loading will be presented, applied to the fatigue formulation first proposed by [1]. The coupling of both formulations provides a comprehensive approach to simulate high cycle fatigue problems accurately and with an important computational cost reduction. The capabilities of the proposed procedure are shown in a numerical example.

1 INTRODUCTION

Fatigue is a phenomenon generally understood as an alteration in material properties leading to failure under cyclic loads below its elastic threshold. Habitually, the number of cycles required for the total fracture of the mechanical part was found to be in the range of 10^4 – 10^7 cycles or more.

However, it has been observed that when either the entire load or some cycles of it, induce stresses superior to the elastic limit of the material, an alteration of material properties still occurs, not only due to the inelastic nature of the load but also due to its cyclic application. In this case the number of cycles required for rupture is found to be drastically reduced and is generally below 10^3 -10^4 cycles. Based on these observations, fatigue processes can be differentiated into high cycle, low cycle and ultra-low-cycle fatigue, as further inquiry into mechanical behaviour showed fundamental differences between them.

Regarding the high cycle fatigue phenomenon, it has been documented that the type of fracture involved at macroscale level is a brittle type. Therefore, high – cycle fatigue (HCF) does not introduce macroscopic plastic strain. When looking at the same phenomena from the microscale, however, it can be seen that a large part of the material's internal energy is spent in a rearrangement of its internal structure to accommodate better the elastic cyclical load, followed by the gliding of the interatomic planes phase. Therefore, metal grains suffer plastic slip and non-linear behaviour [2], and these irreversible processes are responsible for crack initiation under cyclic loading.

The model hereby proposed is based on the classical continuum damage formulation made sensible to fatigue effects by incorporating the number of cycles as a new internal variable.

High cycle fatigue experiments habitually exhibit a fatigue life in the order of millions or dozens of millions of loading cycles. If a single loading cycle is described by n loading steps, then the number of loading steps required to complete a HCF analysis would be in the order of 10^7 x n. Furthermore, if the mechanical piece has a complex geometry and a high level of discretization is required at finite element level, then at each of the 10^7 x n load steps a large number of constitutive operations need to be computed for each integration point.

The above serve as a clear example of why time-advance strategies are of the utmost importance in HCF simulations. The strategy presented in this paper is based on the model defined in [1], utilizing, however, the stages of the algorithm in a wider display of situations, as will be presented later on.

2 DAMAGE MODEL

2.1 Mechanical formulation

The free Helmholtz energy is formulated in the reference configuration for elastic Green strains, $E_{ij} = E_{ij}^e$, as [3][4]

$$\Psi = \Psi(E_{ij}, d) = (1-d)\,\frac{1}{2m^0}\left(E_{ij}\,\boldsymbol{C}_{ijkl}^0 E_{kl}\right) \tag{1}$$

Considering the second thermodynamic law (Clausius-Duhem inequality – [5] [6] [7]), the mechanical dissipation can be obtained as [3]

$$\Xi = -\frac{\partial \Psi}{\partial d}\,\dot{d} \geq 0 \tag{2}$$

The accomplishment of this dissipation condition (Equation 2) demands that the expression of the stress should be defined as (Coleman method; see [7])

$$S_{ij} = m^0\,\frac{\partial \Psi}{\partial E_{ij}} = (1-d)\boldsymbol{C}_{ijkl}^0 E_{kl} \tag{3a}$$

Also, from the last expressions, the secant constitutive tensor can be obtained as:

$$\boldsymbol{C}_{ijkl}^s(d) = \frac{\partial S_{ij}}{\partial E_{ij}} = m^0\,\frac{\partial^2 \Psi}{\partial E_{ij}\partial E_{kl}} = (1-d)\boldsymbol{C}_{ijkl}^0 \tag{3b}$$

where m^0 is the material density, $E_{ij} = E_{ij}^e$ is the total strain tensors, $d^{ini} \leq d \leq 1$ is the internal damage variable enclosed between its initial value d^{ini} and its maximum value 1, $\boldsymbol{C}_{ijkl}^0$ and $\boldsymbol{C}_{ijkl}^s$ are the original and secant constitutive tensors and S_{ij} is the stress tensor for a single material point.

2.2 Threshold damage function oriented to fatigue analysis. Macroscopic approach

The effects caused by applying an increasing number of loading cycles are taken into account by means of a proposed $f_{red}(N, S_{max}, R)$ function. This function is introduced in the above formulation in the expression of the discontinuity threshold proposed by [7], [8] and [9], $F^D(S_{ij}, d)$. The number of cycles can then be incorporated as a state value.

This enables the classical constitutive damage formulation to account for fatigue phenomena by translating the accumulation of number of cycles into a readjustment and/or movement of the damage threshold function.

The non-lineal behaviour caused by fatigue is introduced in this procedure implicitly, by incorporating a fatigue state variable that is irreversible and depends on the number of cycles, the amplitude and the maximum value of the stresses in the material, and on the factor of reversion of the load. This state variable affects the residual strength of the material by modifying the damage threshold either on the strength threshold (left term) or on the equivalent stress function (right term)[1].

$$F^D(S_{ij}, d, N) = f^D(S_{ij}) - \underbrace{\overline{K}^D(S_{ij}, d) \cdot f_{red}(N, S_{max}, R)}_{K^D(S_{ij}, d, N)} = 0 \tag{4}$$

$$F^D(S_{ij}, d, N) = \underbrace{\left(\frac{f^D(S_{ij})}{f_{red}(N, S_{max}, R)}\right)}_{f^{D\prime}(S_{ij}, N, R)} - \overline{K}^D(S_{ij}, d) = 0$$

In Equation (4), N is the current number of cycles, $R = \frac{S_{min}}{S_{max}}$ is the stress reversion factor, S_{max} the maximum applied stress (see Figure 2) and $f_{red}(N, S_{max}, R)$ is the reduction function influenced by the number of the cycles N. Furthermore, in the above, $f^{D\prime} = f^D / f_N$, is the equivalent stress function in the undamaged space, $K^D(S_{ij}, d, N)$ is the damage strength threshold, and $d = \int_0^t \dot{d}\, dt$ the damage internal variable. The evolution of the damage variable is defined as,

$$\dot{d} = \dot{\mu} \frac{\partial F^D}{\partial f^D} \tag{5}$$

being μ the consistency damage factor, which is equivalent to the consistency plastic factor defined in [3]. Consequently, for the isotropic damage case,

$$\dot{d} = \frac{\dot{\mu}}{f_{red}} \tag{6}$$

The reduction function $f_{red}(N, S_{max}, R)$ makes the damage model dependent on the phenomenon of fatigue.

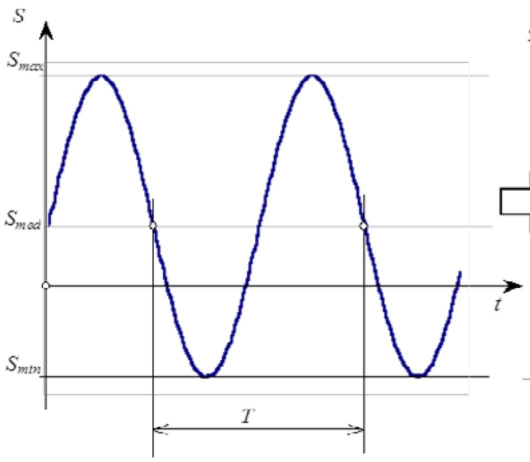

Figure 1a: Stress evolution at a single point

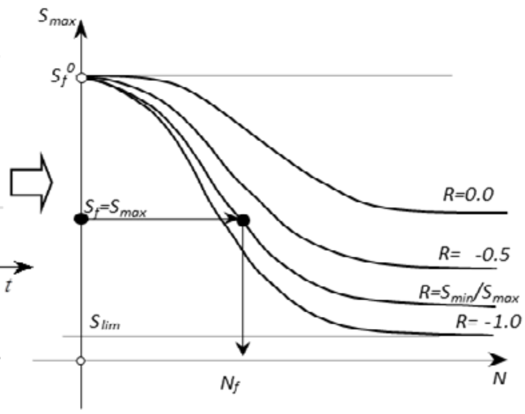

Figure 1b: S-N (Wöhler's) Curves

2.3 Function of residual strength reduction for fatigue – Wöhler curve definition

Wöhler or "Stress-Num. of cycles" (*S-N*) curves are experimentally obtained by subjecting identical smooth specimens to cyclic harmonic stresses and establishing their life span measured in number of cycles. The curves depend on the level of the maximum applied stress and the ratio between the lowest and the highest stresses ($R=S_{min}/S_{max}$). Usually, *S-N* curves are obtained for fully reversed stress ($R=S_{min}/S_{max}=-1$) by rotating bending fatigue tests.

S-N curves are, therefore, fatigue life estimators for a material point with a fixed maximum stress and a given ratio R. If, after a number of cycles lower than the cycles to failure, the cyclic load stops, a change in the material's elastic threshold is expected due to accumulation of fatigue cycles. Furthermore, if the number of cycles exceeds *Nf*, being *Nf* the fatigue life as resulting from Figure 2, the material will fail with the consequent reduction of strength and stiffness. The change in strength is quantified by the strength reduction function $f_{red}(N, S_{max}, R)$.

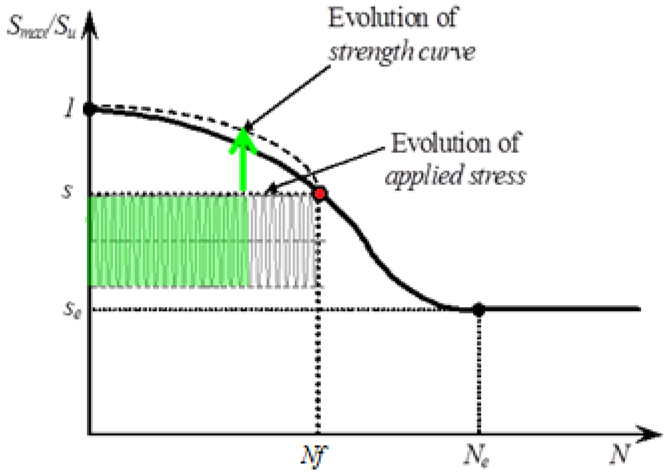

Figure 2: Evolution of the residual strength with the applied load and number of cycles

In the case of a cyclic load with constant maximum value and reversion factor throughout the entire life of a material, the S-N curve is sufficient for determining fatigue life. However, when dealing with different load interactions the main focus resides on the residual strength curve. The curve quantifies the loss of strength in the material as the number of cycles accumulates and as load characteristics change.

All fatigue numerical simulations are based on experimentally obtained Wöhler curves. These curves are described in an analytical form with the help of material parameters. Their expression, as well as the analytical definition of the strength reduction function, is connected to the experimental curve and, therefore, subjected to change if the material changes.

The analytical expression of the curves used in this paper can be found in [1]. Different analytical definitions can also be found in [10], [11] and [12].

3 STEPWISE TIME ADVANCEMENT STRATEGY

3.1 Introduction

The stepwise time-advancing strategy proposed in this paper is based on the formulation presented in [1] and consists of two different stages. The first one is defined by time-advance being conducted by small time increments with the consequent load variation following a cyclic path. The second stage is characterized by time-advance being done with large increments of number of cycles.

3.2 Load-tracking stage

The first stage is characterized by the load being applied in small increments. The purpose of this stage is to determine and save the characteristics of the cyclical load. After having detected the maximum and the minimum stress, at each integration point, the reversion factor is computed, $R=S_{min}/S_{max}$. Several more cycles are then described by small increments. After each one of them, a stabilization norm quantifying the sum of the normalized variation of the reversion factor, compared to its previous cycle value, is evaluated as shown below in equation 7.

$$\eta = \sum_{GP} \left\| \frac{R_{GP}^{i+1} - R_{GP}^{i}}{R_{GP}^{i+1}} \right\| \rightarrow 0 \tag{7}$$

When this norm is below a given tolerance it can be said that the reversion factor has a stable value throughout the solid.

This stage is necessary at the beginning of each different cyclical load in order to determine the parameters that define the cyclic behavior at each Gauss point of the structure (R and Smax). In case of modifying the cyclic load, a new activation of this stage is necessary in order to recalculate these parameters. The flow chart for this stage is presented on the left side of Figure 3.

In this stage the variable is the level of the load.

3.3 Large increments tracking stage

After the stress parameters, R and Smax, stabilize throughout the solid there is no need to keep applying small increments as there will be no change in the stress state unless either the elastic threshold is reached or the applied cyclical load changes. Therefore, the load level can be maintained at its maximum value and large number of cycles increments can be applied.

In this stage the variable is not the level of the load, kept constant, but the number of cycles.

A flow chart is presented below in Figure 3 for each of the two stages. The algorithm for the large increments stage is presented on the right side of the figure. The pass from one stage to the other is indicated by red arrows.

A key point of the above formulation is the strength reduction function, $f_{red}(N, S_{max}, R)$. Its expression depends on the ratio between the maximum stress and the elastic threshold and on the fatigue life given by the Wöhler curve, as can be seen in [1]. These two parameters are determined at the beginning of the analysis at each Gauss point and they are constant unless internal forces change.

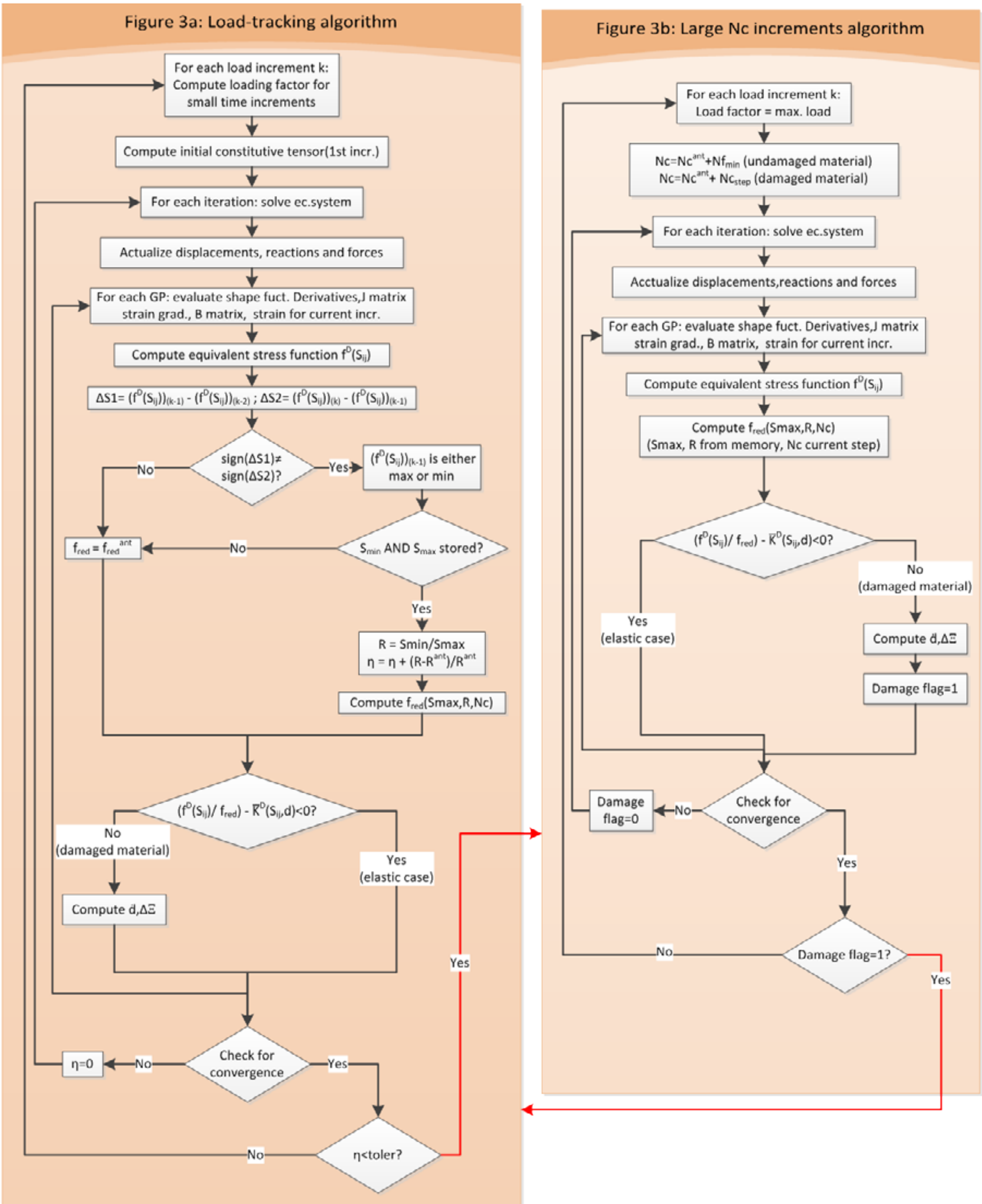

Figure 3: Flow chart for the two stages of the time-advance algorithm

This intrinsic step can predict the exact number of cycles at which damage initiates. After evaluating the Wöhler's *Nf* (see Figure 2) corresponding to each stress level at the beginning of the analysis, a search is made to find the minimum fatigue life throughout the solid. The resulting number of cycles is considered to be the first step of the large increments stage ensuring that the entire span of number of cycles before the damage process initiates is done in one step. The nonlinear processes occurring past the point damage initiates in the first Gauss point will be simulated with a user-defined N_c step.

3.4 Automatic load-tracking stage activation

The above mentioned strategy has the following implications:

When applying a single cyclical load, time advance will be done by passing once thru the load-tracking stage and then advancing by number of cycles increments both before and after reaching the elastic threshold. However, when inside the constitutive model a Gauss point surpasses its elastic threshold, the internal forces of the structure are modified in order to achieve a new equilibrium configuration.

This situation leads to a variation of the reversion factor and, therefore, of the stress state at integration point level. At this point, the load-tracking stage it automatically activated. Furthermore, it will be activated at each step where damage accumulates ($\dot{d} > 0$).

In the case of applying different cyclic loads, damage can appear either due to fatigue or due to a new load applied that leads to stress values over the elastic threshold. In both cases the model will jump automatically from large increments tracking stage to load-tracking stage.

4 NUMERICAL EXAMPLES

4.1 Test case geometry and material

In order to validate the proposed constitutive model a test case analysis of one linear hexahedral element with 8 integration points was performed. Geometry dimensions were 1x1x1cm. The material used has the following characteristics: Young modulus = $2.01\,x\,10^5\,MPa$; Poisson ratio = 0.1 ; Static elastic threshold is $S_u = 838.9\,MPa$ and the material fracture energy has a value of $G_t = G_c = 10\ kN/m$. The damage model used has exponential softening and a Von Mises failure surface.

The element has one of its faces subjected to a cyclical displacement while the opposite face has boundary conditions that fix its longitudinal displacement, allowing transversal expansion and contraction.

One of the model's particularities is the strength alteration occurring previous to the damage initiation moment. The progressive loss of resistance leading to the initiation of damage is represented in the strength reduction curve. In order for it to be clearly differentiated from the Wöhler curve, a direct jump to the point where damage initiates was not done. Rather, an approximation of the damage initiation point was made by choosing a suitable number of cycles as time step.

Below, two different cases are presented. In the first case a cyclical load with a reversion factor of 0.3, minimum displacement of 0.0000114m and maximum displacement of 0.000038m is applied. The load applied in the second case has a null reversion factor, a maximum displacement of 0.000035m and a null minimum displacement. The number of cycles adopted as a step for the large increments stage in the first case is 10^6 cycles. The second case was calculated with a step of 10^5 cycles.

4.2 HCF1 load case

In the following table the stresses generated at integration point level by the imposed maximum and minimum displacements are displayed as well as the fatigue life resulting from the FEM model.

Table 1: Characterization of load HCF1

		(normalized with threshold limit)			
Case code	Reversion factor	Max. Stress PG	Min. Stress PG	Med. Stress PG	Nc at which damage initiates
hcf1	0,3	0,91	0,273	0,59	4,90E+06

The stresses induced by the cyclic displacement applied lead to a fatigue life, according to the material Wöhler curve, of 4,9 x 10^6 cycles. This number of cycles marks the beginning of the nonlinear process and, therefore, of the energy dissipation in the volume associated to the integration point presented.

Below, in Figure 4, strength, Wöhler stress and damage evolution are presented. It can be seen that, while the residual strength curve is above the Wohler fatigue life curve, there is no stress alteration or damage accumulation. The material is considered to be in an elastic stage. However, once the residual strength curve intersects the Wöhler fatigue life curve at exactly the maximum stress level induced in the material volume analysed, damage accumulation starts.

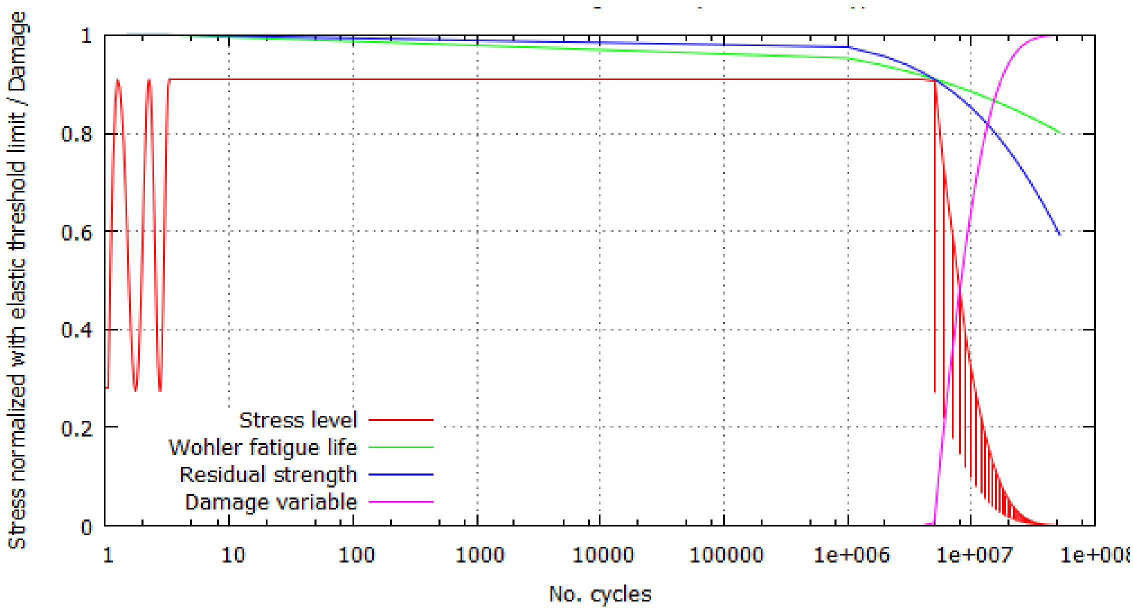

Figure 4: Parameters of interest for the fatigue analysis under load HCF1

From that point forward, after each large increment where $\dot{d} > 0$, the load-tracking stage is automatically activated so that damage evolution can be monitored from cycle to cycle. If, after describing several cycles with small increments, the stress state throughout the solid has stabilized, a new large increment will be applied. The process thus automatically repeats until the material reaches a state of complete degradation.

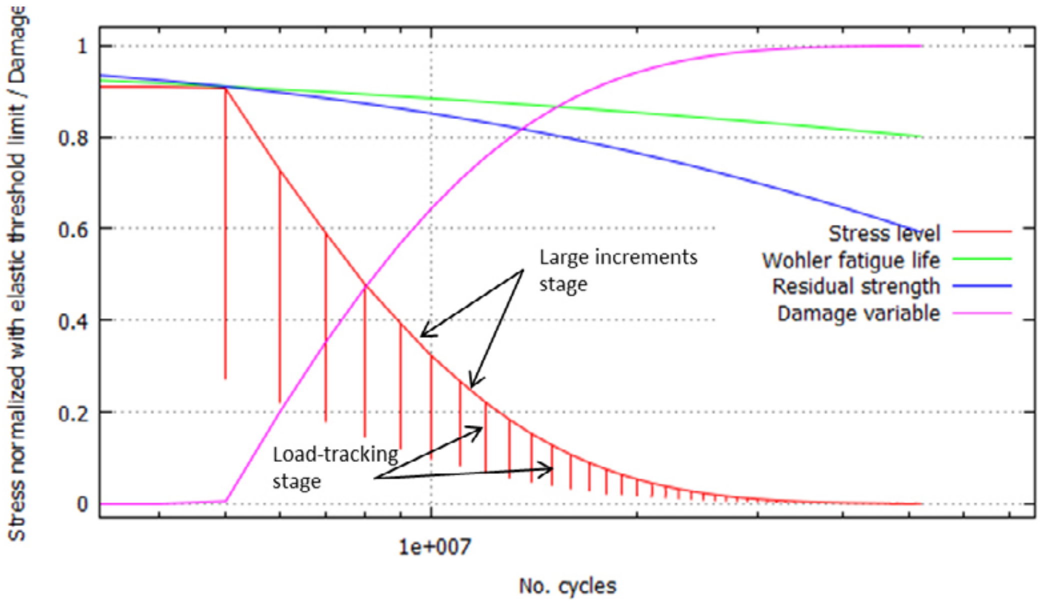

Figure 5: Parameters of interest for the fatigue analysis under load HCF1 in the nonlinear zone

The process can be observed with more detail in Figure 5 above, where a zoom on the variables' evolution in the nonlinear zone is presented. Load-tracking and large increments tracking stages are indicated.

Both Figure 4 and Figure 5 have a logarithmical scale along the horizontal axis.

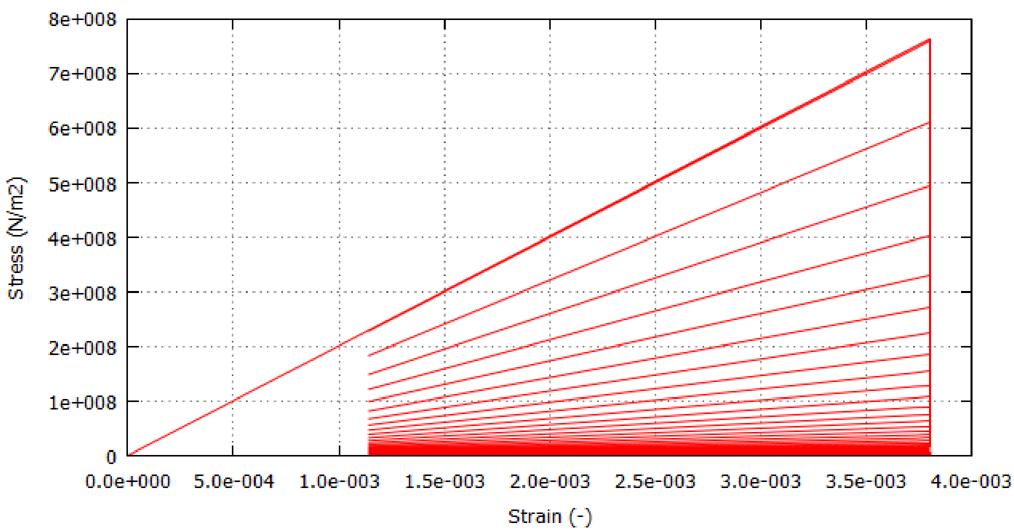

Figure 6: Stress-Strain at integration point for load HCF1

In Figure 6, the effects of material degradation are shown in the stress-strain curve. The large increments tracking stage, where the displacement is maintained at its maximum value, is represented by the vertical lines descending from the point of maximum stress. The stress interval represented by each descent quantifies the stress softening caused by a single, large number of cycles, interval. Each of these stress-softening intervals is followed by a few unloading (until minimum displacement) - loading cycles. These mark the load-tracking stage where a clear change in material stiffness is visible.

It can be seen that, as the material progressively suffers loss of stiffness, for the same large step there is less stress softening.

4.3 HCF3 load case

In the following table, same as for the previous case, the stresses generated at integration point level by the imposed maximum and minimum displacements are displayed as well as the fatigue life resulting from the FEM model.

Table 2: Characterization of load HCF3

		(normalized with threshold limit)			
Case code	Reversion factor	Max. Stress PG	Min. Stress PG	Med. Stress PG	Nc at which damage initiates
Hcf3	0	0,839	0	0,42	3,46E+06

Compared to the previous case, the maximum stress induced in the material is smaller, going from 0.91 of the elastic threshold limit to approximately 0.84. This can lead to believe that, since the material is subjected to lower stresses, fatigue life increases. However, the fatigue life resulting from applying load HCF3 is smaller than that corresponding to load HCF1.

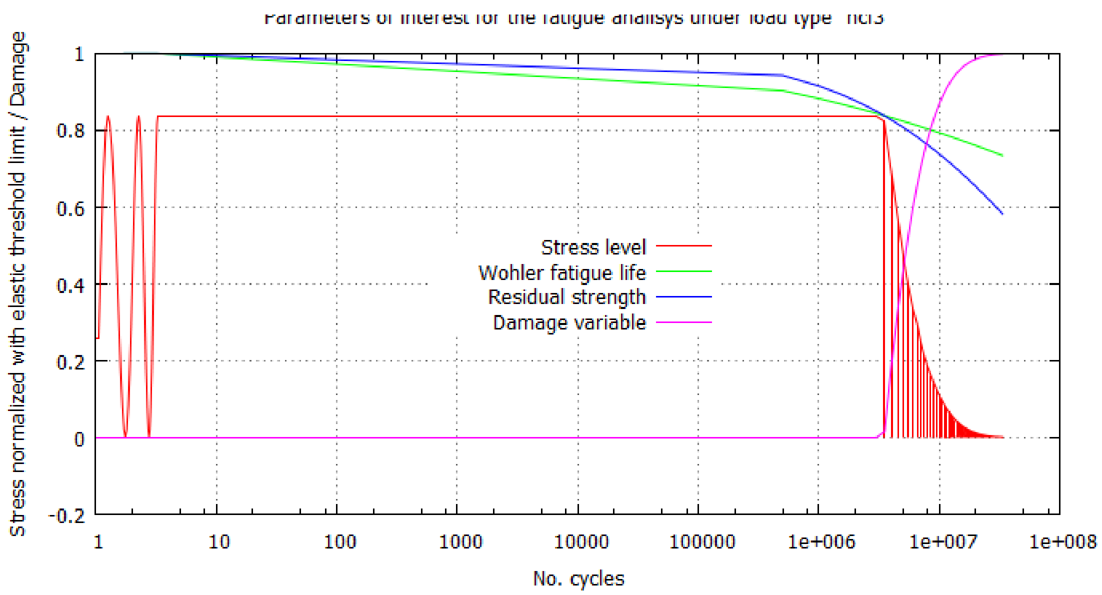

Figure 7: Parameters of interest for the fatigue analysis under load HCF3

This is due to the fact that load HCF3 has a different stress reversion factor (0 compared to 0.3 for HCF1) and that the material used is more vulnerable to a smaller reversion factor. Thus, the first effect (smaller maximum stress leads to larger fatigue life) is countered by the second one (smaller reversion factor leads to shorter fatigue life).

The applied displacement for load case HCF3 induces a maximum stress of 0,839 of the elastic limit and unloads to 0, leading to a fatigue life of 3,46 x 10^6 cycles, as can be seen in Figure 7. Since by using a step of 10^6 cycles, as for the first case, the damage initiation point was not sufficiently well approximated, a 5 x 10^5 cycles step was used. A logarithmical scale has been used for the horizontal axis.

Figure 8 shows the stress – strain curve where the stiffness reduction can be seen at each automatic unloading made after having detected that $\dot{d} > 0$.

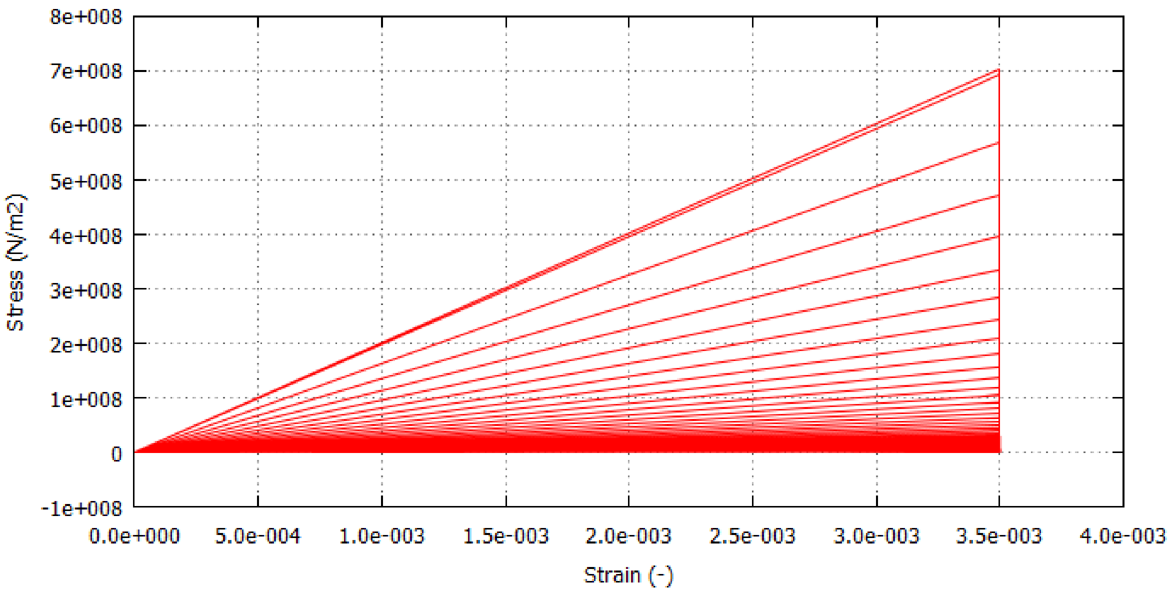

Figure 8: Stress-Strain at integration point for load HCF3

5 CONCLUSIONS

This paper has presented a fatigue formulation that takes into account the effects caused by the accumulation of number of cycles of loading by both an alteration in the strength and in the stiffness of the material. First, material strength is reduced until it reaches the induced maximum stress level. From that point on, energy dissipation in the mesoscale is done by means of stiffness reduction.

The cyclical load is taken into consideration by means of two parameters: maximum generated stress and stress reversion factor. Both parameters have a direct influence on the onset of damage and on the strength reduction. This allows a quantification of the effects induced by different cyclical loads and discrimination between different load-applying orders.

A stepwise time-advance strategy has been proposed in order to save computational time and improve convergence in a number of cases, such as load combinations and nonlinear material behaviour.

The procedure divides the load in two different loading processes: load tracking and large increments tracking stages. The jump between the two loading schemes is made automatically, depending on the mechanical response of the structure. The capabilities of the algorithm have been proven with two numerical examples.

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