Estimation of Survival Functions Subject to Order Restrictions

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OUTLINE

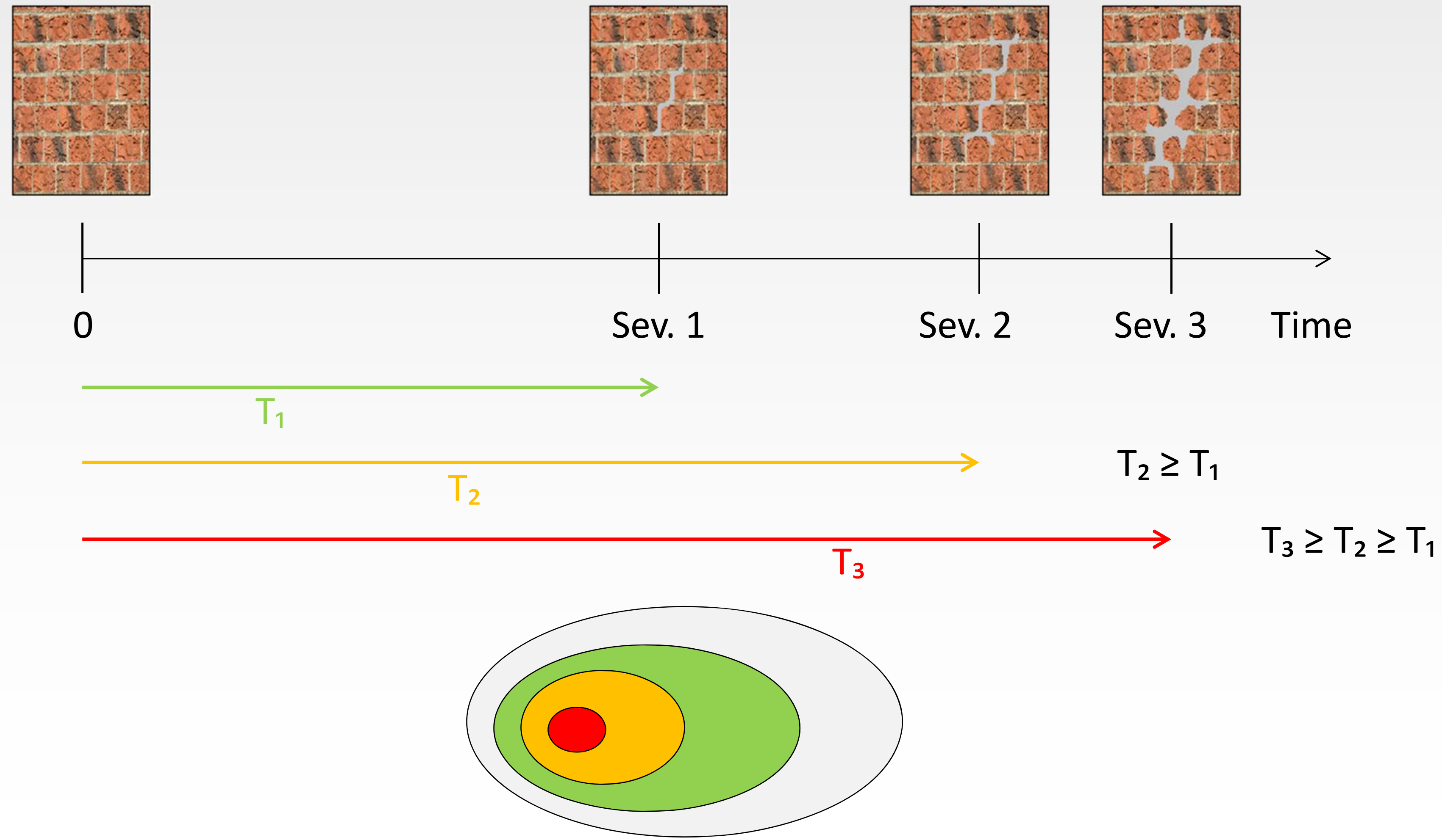
1. Introduction
2. Methodology
3. The AMPL language
4. Estimation of the durability with order restrictions
5. Simulation of one practical case
6. Conclusions and ongoing research

1.1 BACKGROUND

DURABILITY ANALISIS IN THE BUILDING STOCK

- Development of the methodology (*Martín, 2004*)
- Implementation in S-PLUS (*Liébana and Molons, 2005*)
- Application in Hospitalet de Llobregat (*Barriuso and Estupiñà, 2006*)
- General proposal of durability estimators (*Gibert and Royano, 2010*)

1.1 BACKGROUND

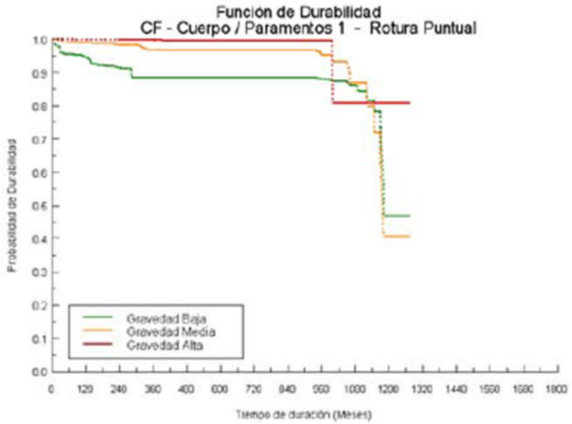


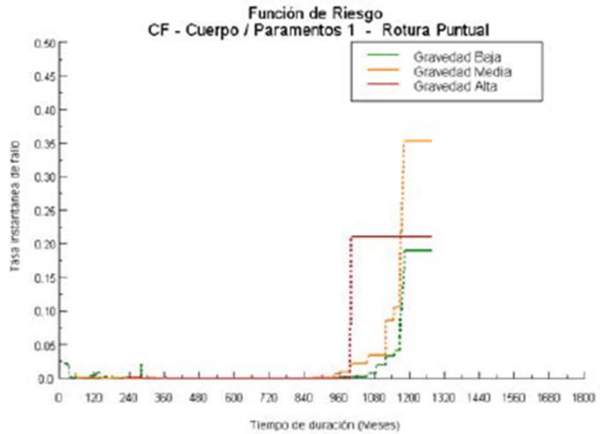
1.1 BACKGROUND

Laboratori d'Edificació de l'EPSEB

ESTUDIO POR GRAVEDAD DE LA EVOLUCIÓN DE LA PRESENCIA DE LESIONES EN FACHADAS PRINCIPALES DE HOSPITALET DE LLOBREGAT

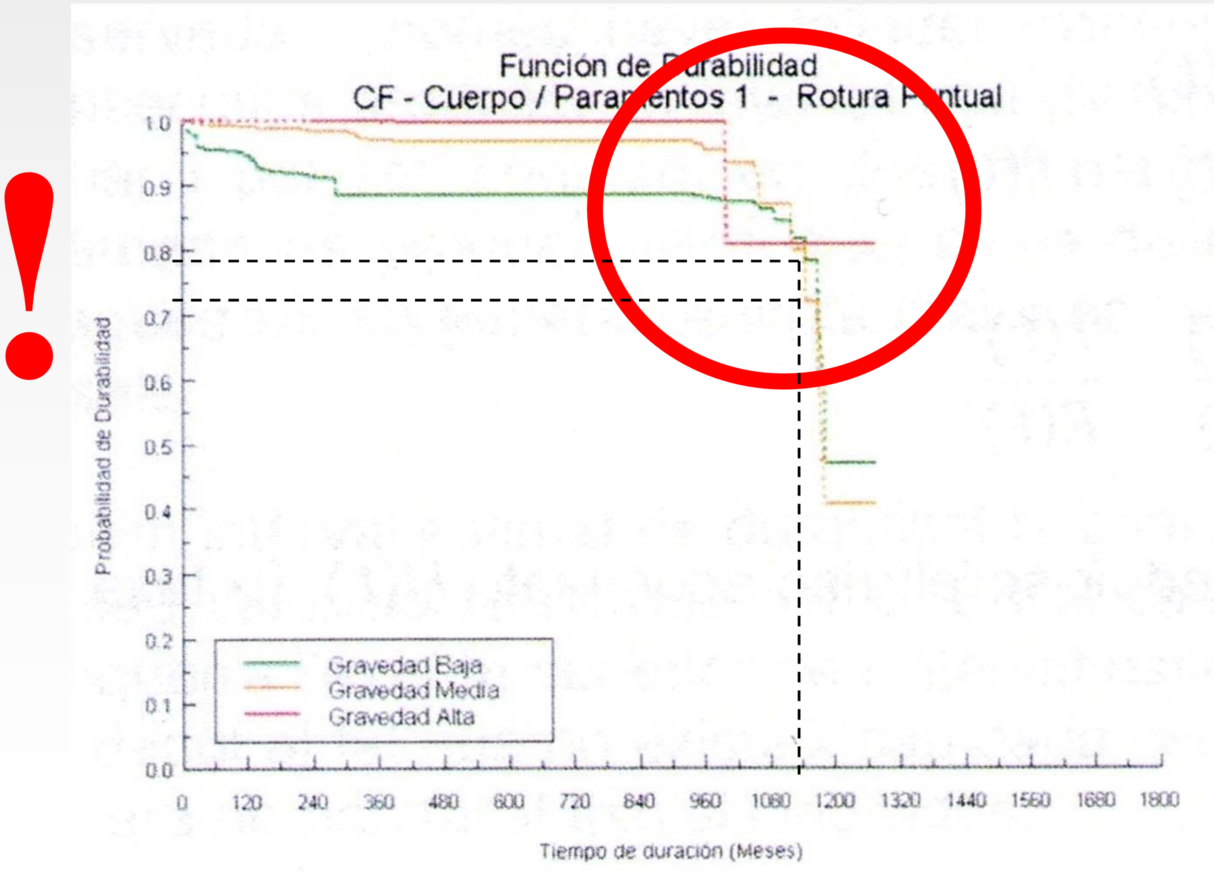
CÓDIGO DEL ESTIMADOR:
0022



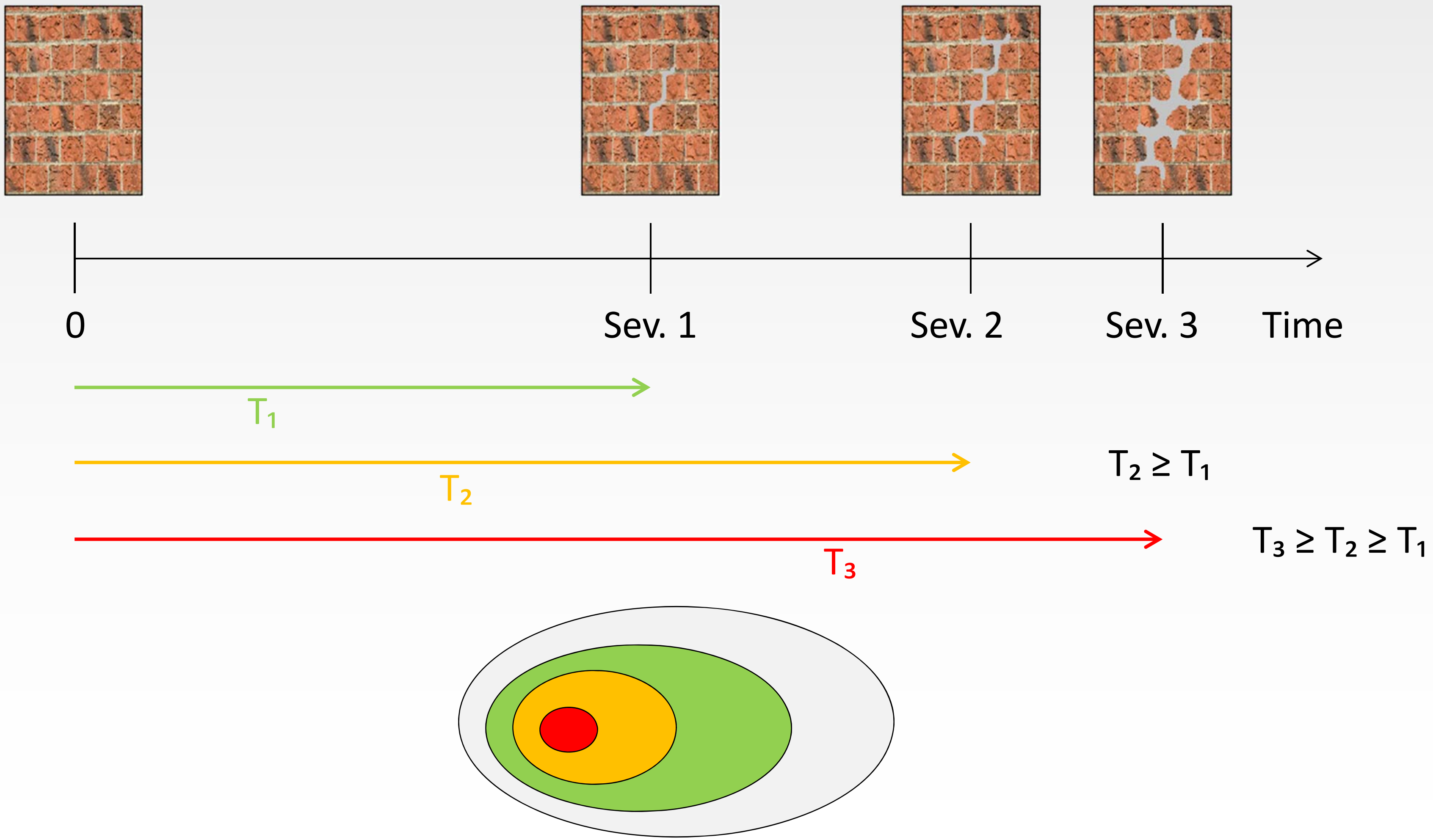


NUM. A RIESGO	% DE CENSURADOS		ESTIMACIÓN % LESIONES						TIEMPO EN AÑOS HASTA UN % ACUMULADO DE LESIONES				%ACUMULADO DE LESIONES EN UN TIEMPO EN AÑOS				UMBRAL DE RIESGO	AÑOS CORTE CON UMBRAL	
			INICIAL			FINAL													
	IZQ	DCHA	AÑOS	%	RIESGO	AÑOS	%	RIESGO	10%	25%	50%	75%	10	25	50	100		t1	t2
10414	10.9	89.1	1.4	2.2	0.023	106.1	53.1	0.189	23.5	97.4	98.6	--	5.3	11.4	11.6	53.1	0.01	2.7	90.6
10414	2.8	97.2	4.6	0.8	0.008	106.1	59.5	0.354	88.4	95.7	98.1	--	1.0	1.7	3.2	59.5	0.01	83.3	83.3
10414	0.3	99.7	19.7	0.1	0.001	106.1	19.3	0.210	83.3	--	--	--	0.1	0.2	0.3	19.3	0.01	83.3	83.3

1.2 GOAL


!
Función de Durabilidad
CF - Cuerpo / Paramentos 1 - Rotura Puntual
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Probabilidad de Durabilidad
Gravedad Baja
Gravedad Media
Gravedad Alta
0 120 240 360 480 600 720 840 960 1080 1200 1320 1440 1560 1680 1800
Tiempo de duración (Meses)

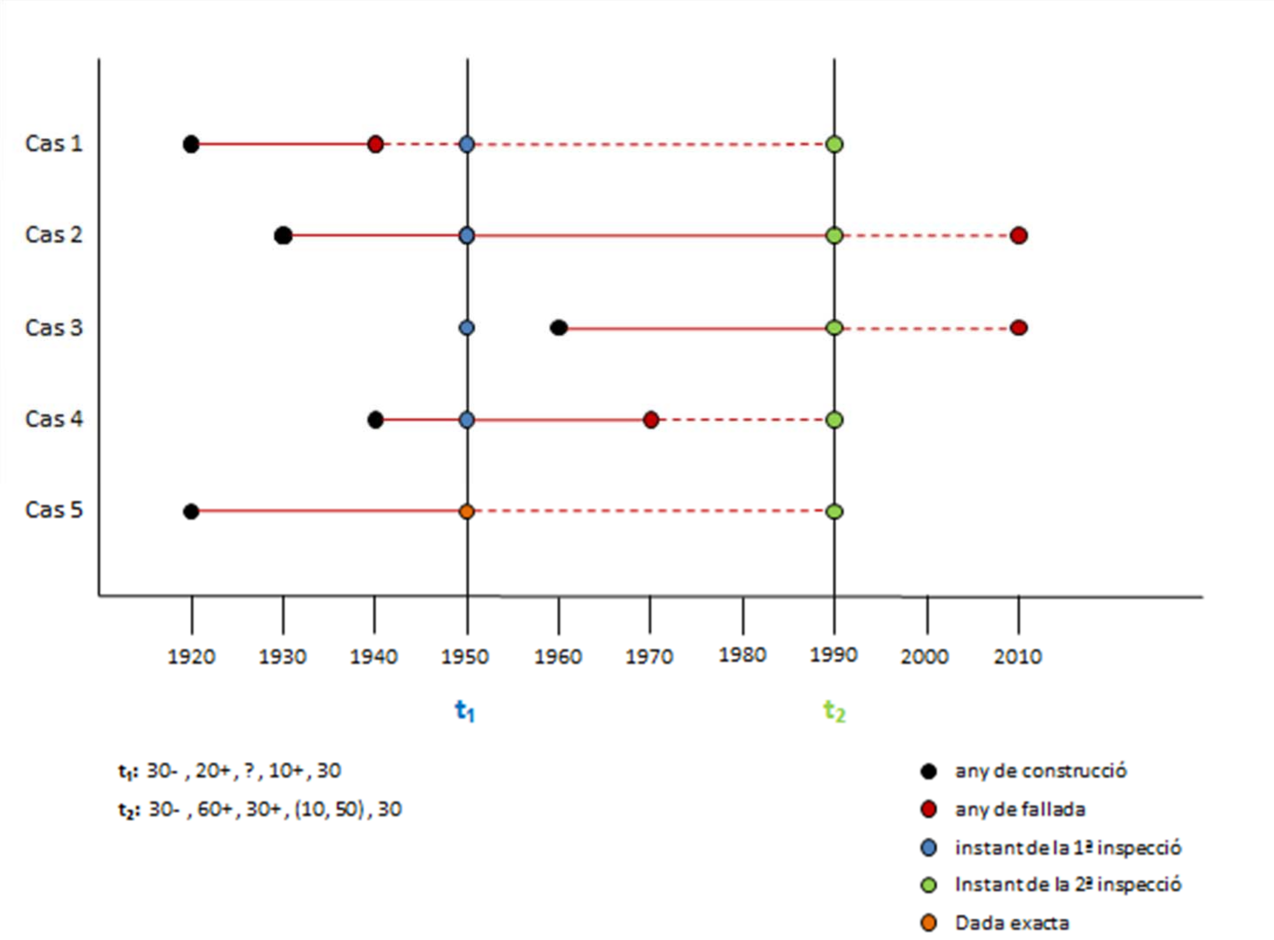
1.2 GOAL



1.2 GOAL

- Solve the problem of inconsistency
- Having a tool to estimate the durability for successive events

2.1 CENSORED DATA



2.1 CENSORED DATA

- Left-censored data : $(0, C_i)$ or C_i^-
- Right-censored data: (C_i, ∞) or C_i^+
- Interval-censored data: (C_{i1}, C_{i2})
- Exact data: (C_i, C_i)

2.2 TURNBULL's ESTIMATOR

- KAPLAN and MEIER estimator for right-censored data (*Kaplan and Meier, 1958*)
- TURNBULL's estimator (*Turnbull, 1976*) maximizes the non-parametric likelihood function for the data $(L_i, R_i) \quad i = 1, \ldots, n$

$$L = \underbrace{\prod_{i \in O} (F(o_i) - F(o_i^-))}_{\text{Full observed data}} \underbrace{\prod_{i \in R} (1 - F(r_i))}_{\text{Right-censored data}} \underbrace{\prod_{i \in L} F(l_i)}_{\text{Left-censored data}} \underbrace{\prod_{i \in I} (F(r_i) - F(l_i))}_{\text{Interval-censored data}}$$

2.2 TURNBULL's ESTIMATOR

Turnbull (1976) demonstrates that the optimal solution gives only positive probability mass, ω_j, in *m* disjoint intervals $(q_j, p_j]$

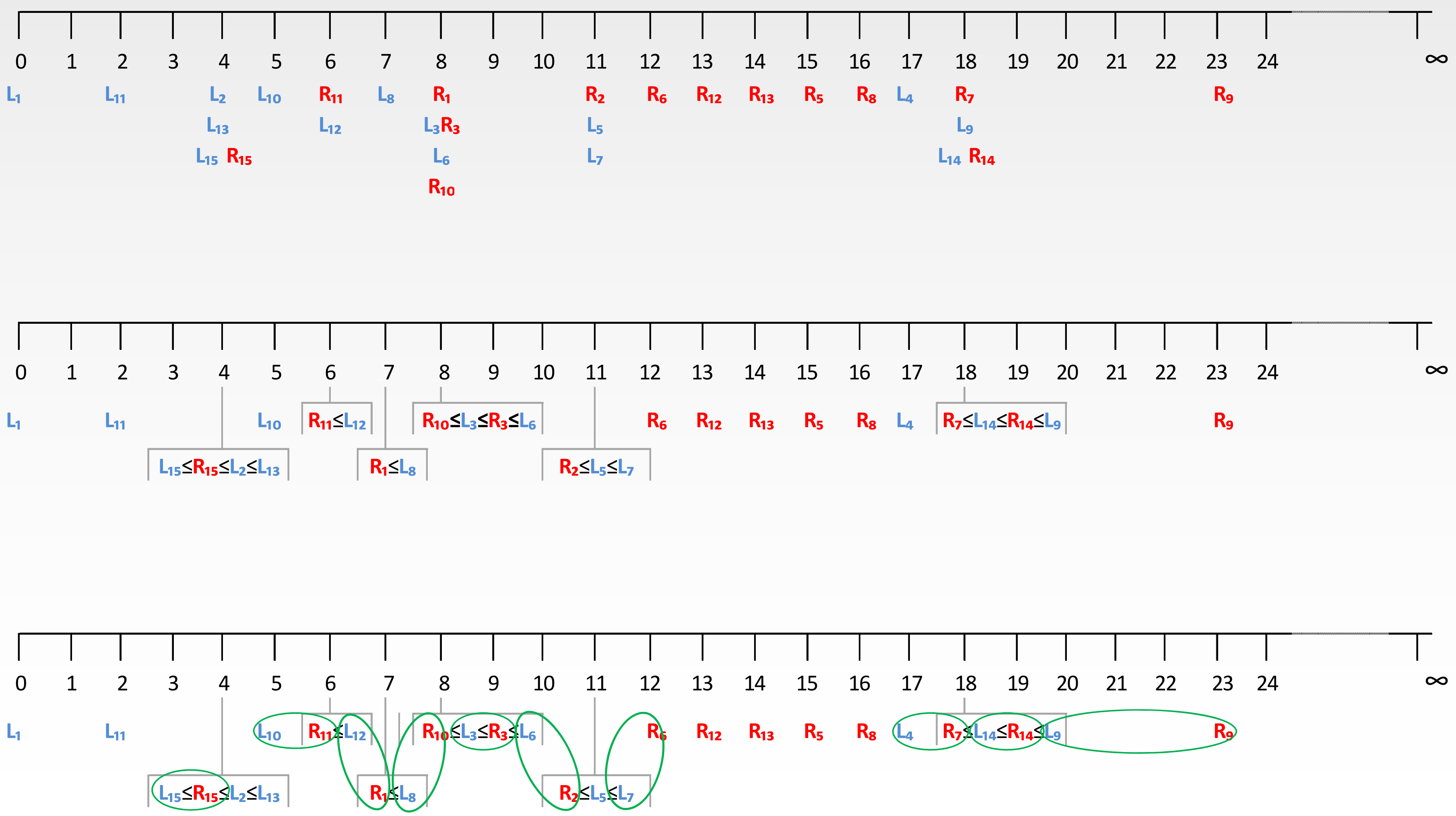
- ω_j is the probability of failure for j-th Turnbull interval.
- $\sum_{j=1}^{m} \omega_j = 1$

and the problem is reduced to maximize the likelihood function

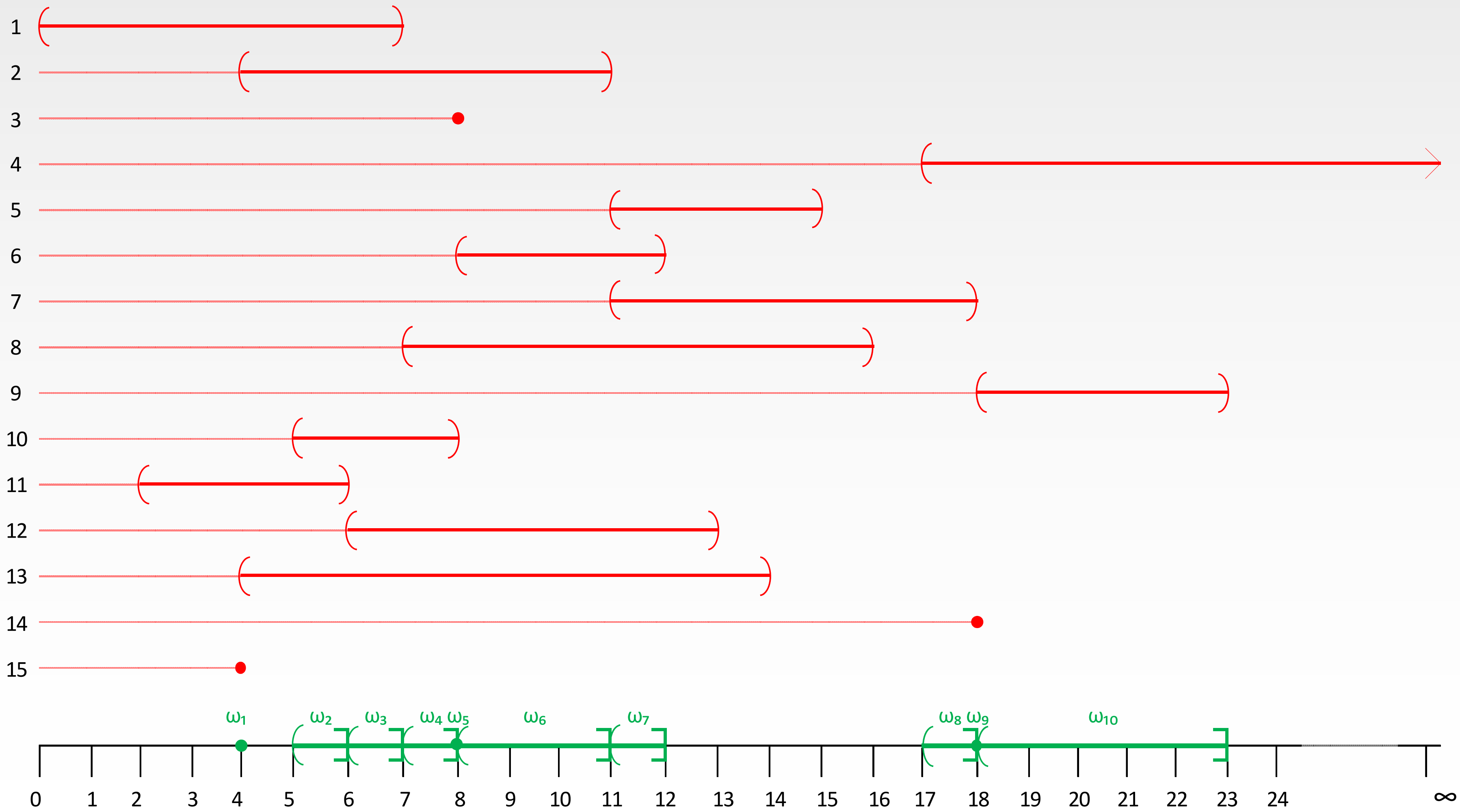
$$L(\omega_1, \omega_2, ..., \omega_m) = \prod_{i=1}^{n} (\sum_{j=1}^{m} \alpha_{ij} \omega_j)$$

where $\alpha_{ij} = I((q_j, p_j) \subset (L_i, R_i))$

2.2 TURNBULL's ESTIMATOR



2.2 TURNBULL's ESTIMATOR



2.2 TURNBULL's ESTIMATOR

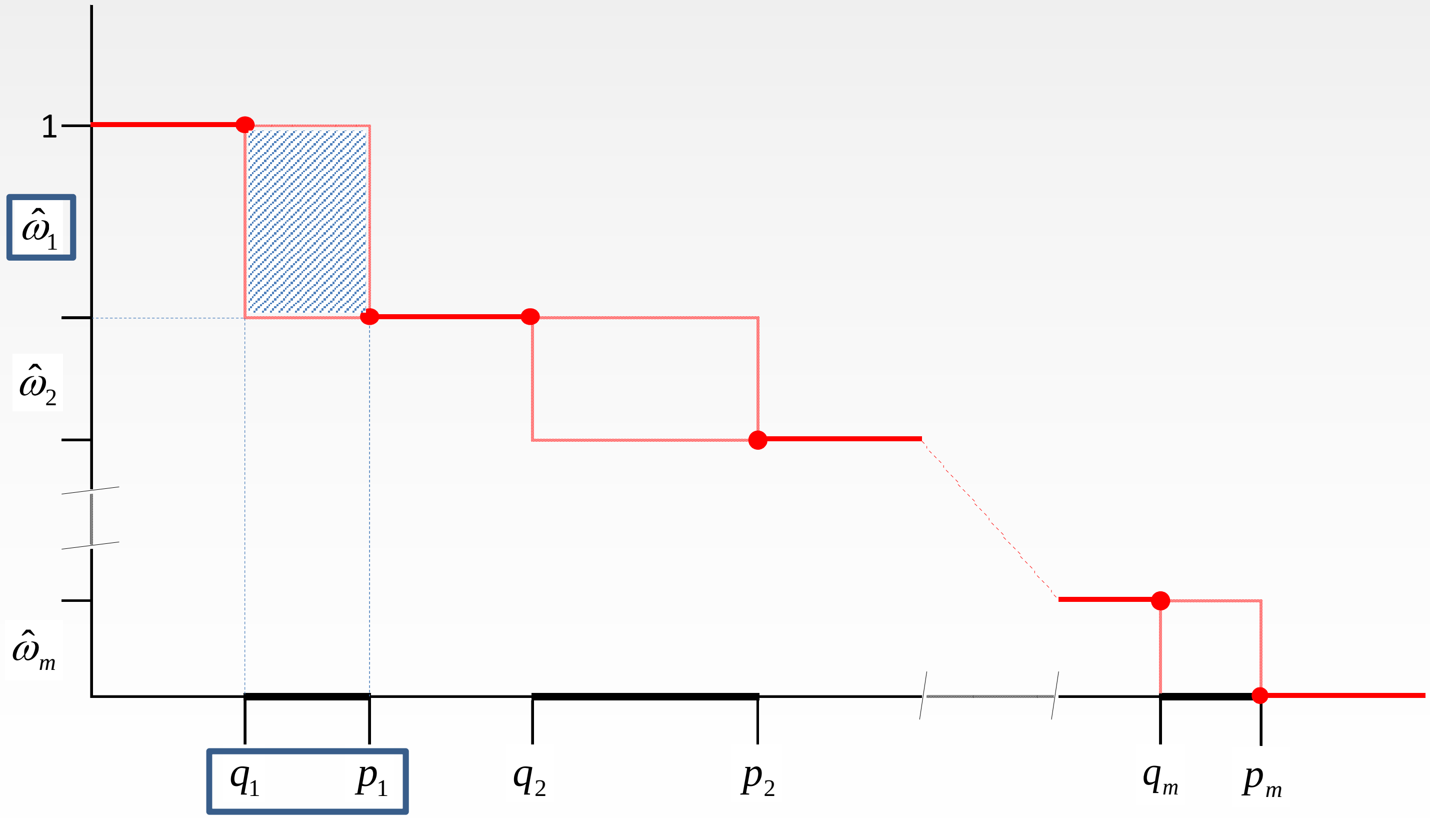
$$L\ (\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6,\omega_7,\omega_8,\omega_9,\omega_{10}) = \prod_{i=1}^{15}[S(l_j)-S(r_j)] = \prod_{i=1}^{15}(\sum_{j=1}^{10}\alpha_{ij}\omega_j) =$$

$$= (\omega_1 + \omega_2 + \omega_3)\cdot(\omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6)\cdot(\omega_5)\cdot(\omega_8 + \omega_9 + \omega_{10})\cdot(\omega_7)\cdot(\omega_6 + \varpi_7)\cdot$$

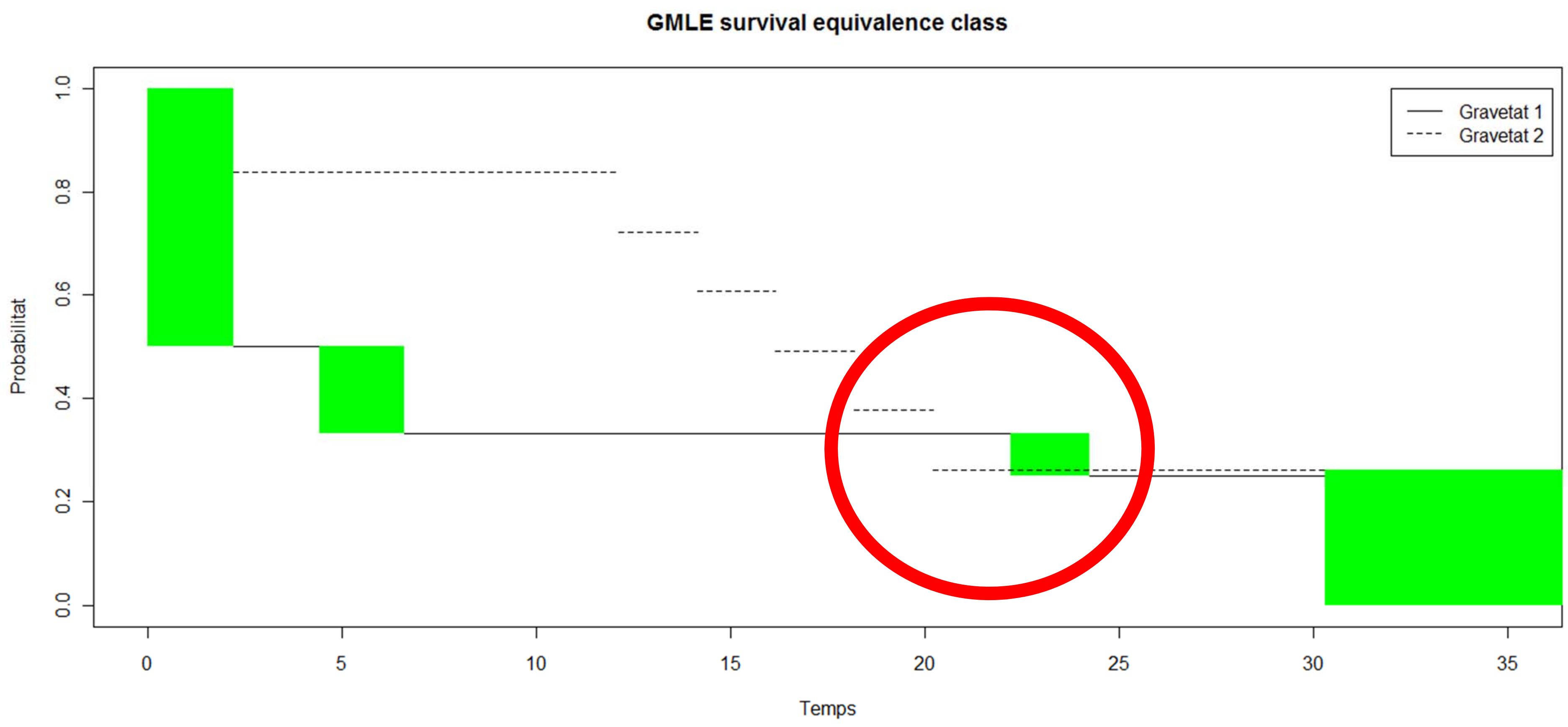
$$\cdot(\omega_7 + \omega_8)\cdot(\omega_4 + \omega_5 + \omega_6 + \omega_7)\cdot(\omega_{10})\cdot(\omega_2 + \omega_3 + \omega_4)\cdot(\omega_1 + \omega_2)3(\omega_3 + \omega_4 + \omega_5 +$$

$$+ \omega_6 + \omega_7)\cdot(\omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7)\cdot(\omega_9)\cdot(\omega_{10})$$

2.2 TURNBULL's ESTIMATOR



2.3 THE R SOFTWARE



3.1 THE AMPL LANGUAGE

- Modeling language to solve optimization problems (*Fourer et al., 1990*)

  - Enter programming

  - Linear programming

  - Non-linear programming

- *Available solvers* on the NEOS server (*Czyzyk, Mesnier and More, 1998)*

  - At a local level
  - Through its direct access

  } Number of variables and constraints

3.2 THE SNOPT SOLVER

- Solve problems with nonlinear constraints.
- At the local level it is limited to 300 variables and/or constraints.
- It requires three files:

  - The Model file

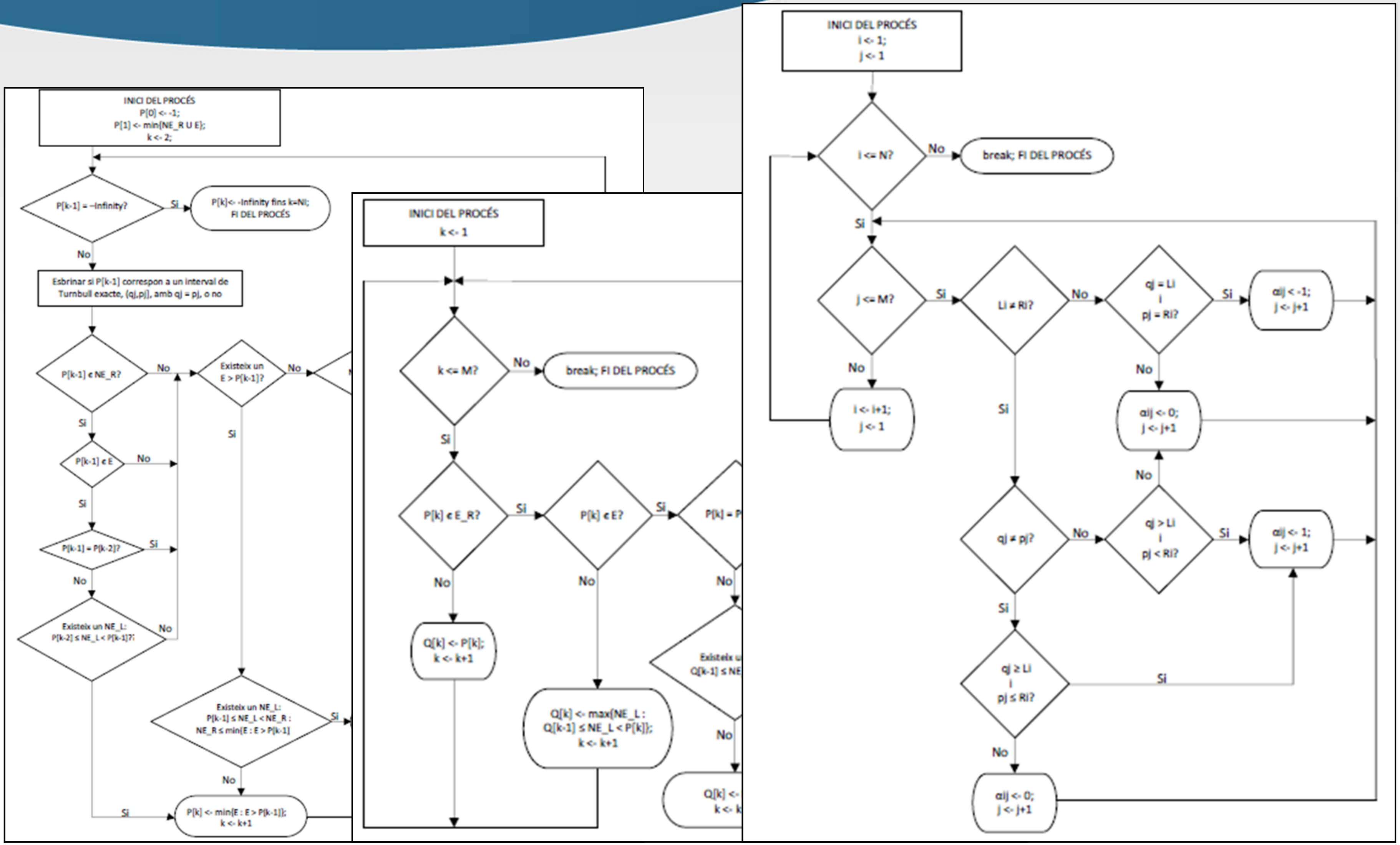
  - The Data file

  - The Run file

3.2 THE SNOPT SOLVER

- Langohr and Gómez (2005) implemented in AMPL, using NEOS, a survival estimator for interval-censored data in a fully parametric approach.
- In the non-parametric framework, and as a first step, we propose the implementation of the Turnbull estimator in AMPL, what we call the TEA algorithm.

3.3 THE TEA METHOD



3.3 THE TEA METHOD

```
########################################
## DEFINITION OF SETS and PARAMETERS ##
########################################

param N;                                        #number of observations
param datmat{1..N,1..2};                        #data matrix

param zl{i in 1..N} := datmat[i,1];             #left endpoints
param zr{i in 1..N} := datmat[i,2];             #right endpoints

#Set of ends of exact observations
set E ordered := setof {i in 1..N: zl[i]=zr[i]} (zl[i]);
param ME := card(E);
param e {t in 1..ME} := member(t,E);

#Set of left endpoints of not exact observations
set NE_L ordered := setof {i in 1..N: zl[i]<>zr[i]} (zl[i]);
param ML := card(NE_L);
param nel {p in 1..ML} := member(p,NE_L);

#Set of right endpoints of not exact observations
set NE_R ordered := setof {i in 1..N: zl[i]<>zr[i]} (zr[i]);
param MR := card(NE_R);
param ner {q in 1..MR} := member(q,NE_R);


##########################
## TURNBULL INTERVALS ##
##########################

#maximum possible number of Turnbull intervals
param NI := ME + min(ML,MR);

## RIGHT ENDPOINTS OF TURNBULL INTERVALS ##

param tr0 {k in 0..NI} :=
if (k=0) then -1 else ( if (k=1)
then (min(min{q in 1..MR}ner[q], min{t in 1..ME}e[t]))
else
{
  if (tr0[k-1]<>-Infinity) then
  {
    if (tr0[k-1] in NE_R) then
    {
      if (tr0[k-1] in E) then
      {
        if (tr0[k-1]=tr0[k-2]) then    #it is an E
        {
          if (exists {t in 1..ME} (e[t]>tr0[k-1])) then
          {
          #the next is a NE_R or an E bigger
            if (exists {q in 1..MR, p in 1..ML} (nel[p]>=tr0[k-1] and
              nel[p]<ner[q] and ner[q]<=(min{t in 1..ME : e[t]>tr0[k-1]}e[t])))
            then (min{q in 1..MR, p in 1..ML : nel[p]>=tr0[k-1] and nel[p]<ner[q]
              and ner[q]<=(min{t in 1..ME : e[t]>tr0[k-1]}e[t])}ner[q])
            else (min{t in 1..ME : e[t]>tr0[k-1]}e[t])
          }
          else    #the next is a NE_R
          {
            if (exists {q in 1..MR, p in 1..ML} (nel[p]>=tr0[k-1] and
              ner[q]>nel[p]))
            then (min{q in 1..MR, p in 1..ML : nel[p]>=tr0[k-1] and
              ner[q]>nel[p]} ner[q])
```

The MODEL file:

- Definition and initialization parameters.
- Computation of the Turnbull intervals.
- Computation of the matrix contributions.
- Definition of the variables.
- Definition of the objective function.
- Definition of the constraints.

3.3 THE TEA METHOD

```
param N := 15;

param datmat:
      1        2:=
1     0        2.20
2     0        8.80
3     0        18.18
4     0        24.24
5     0        28.28
6     0        6.60
7     0        10.10
8     0        14.14
9     0        20.20
10    0        26.26
11    4.40     999
12    12.12    999
13    16.16    999
14    22.22    999
15    30.30    999;
```

The DATA file:

- Definition of the number of data *N*.
- Definition of the matrix data *datmat*.

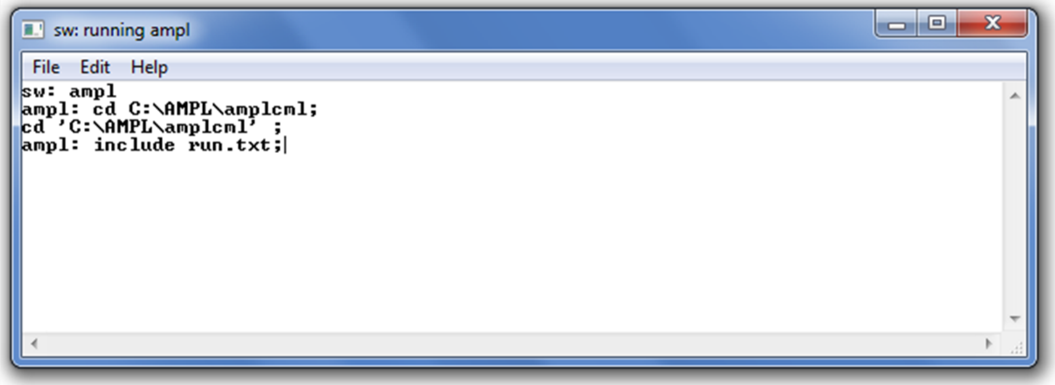
3.3 THE TEA METHOD

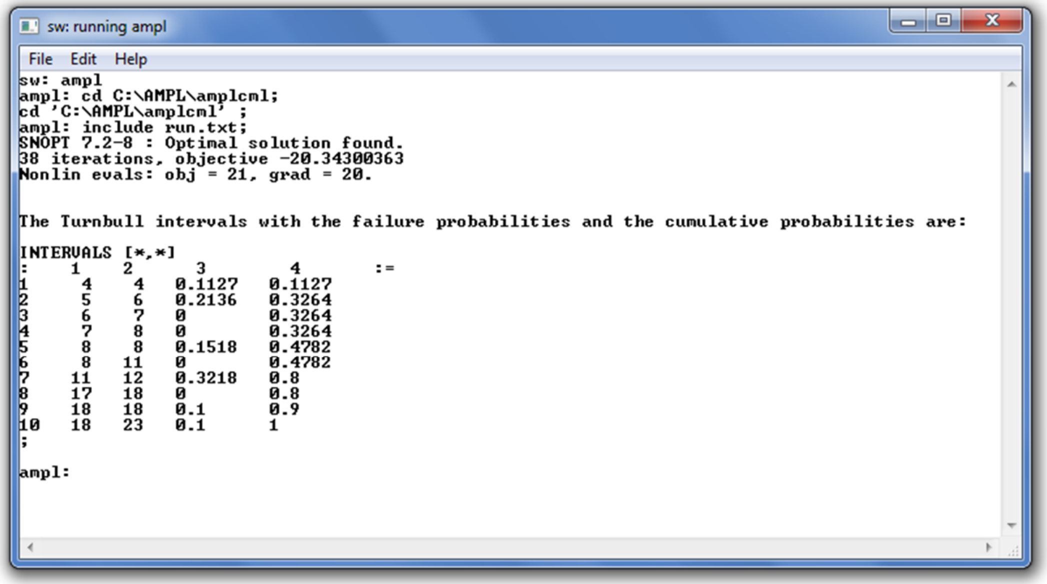
```
####################################
## INVOKING MODEL AND DATE FILE ##
####################################
reset;
model name_model_file.txt;              #UPDATE the code with the corrent filename
data name_data_file.txt;                #UPDATE the code with the corrent filename


######################
## CHOOSING SOLVER ##
######################
option solver snopt;


################################
## INITIATION OF MAXIMIZATION ##
################################

let num_run := 1;
repeat{
  let {j in 1..M} w[j] := Uniform(0,1);
  let sum_one := sum{j in 1..M} w[j];
  let {j in 1..M} w[j] := w[j]/sum_one;
  solve;
  if solve_result =='solved' then break;
  let num_run := num_run+1;
  if num_run == 56 then break;
}

# pass variable w to parameter ww
let {i in 1..M} ww[i] := w[i];


###################
## OUTPUT DESIGN ##
###################

printf "\n\n The Turnbull intervals with the failure probabilities and the cumulative probabilities are:\n\n";
display INTERVALS;
```

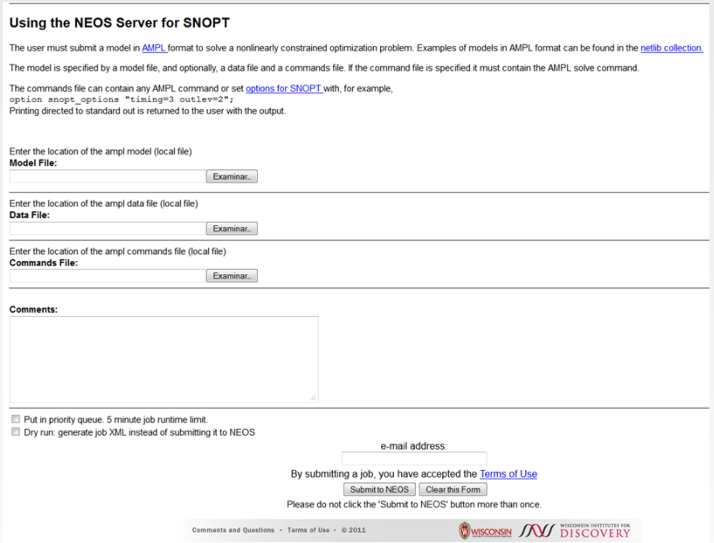
The RUN file:

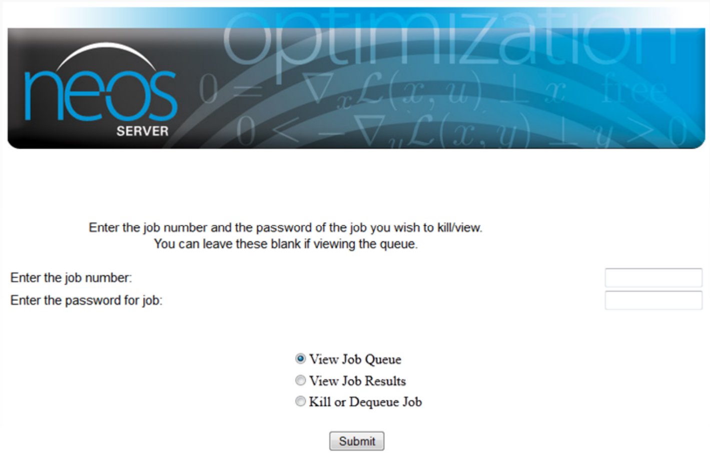
- Calls to model and data files.
- Choice of the solver.
- Initialitzation of the maximization.
- Print of the results.

3.3 THE TEA METHOD









3.3 THE TEA METHOD

Number job and password.

Date to start and finish running

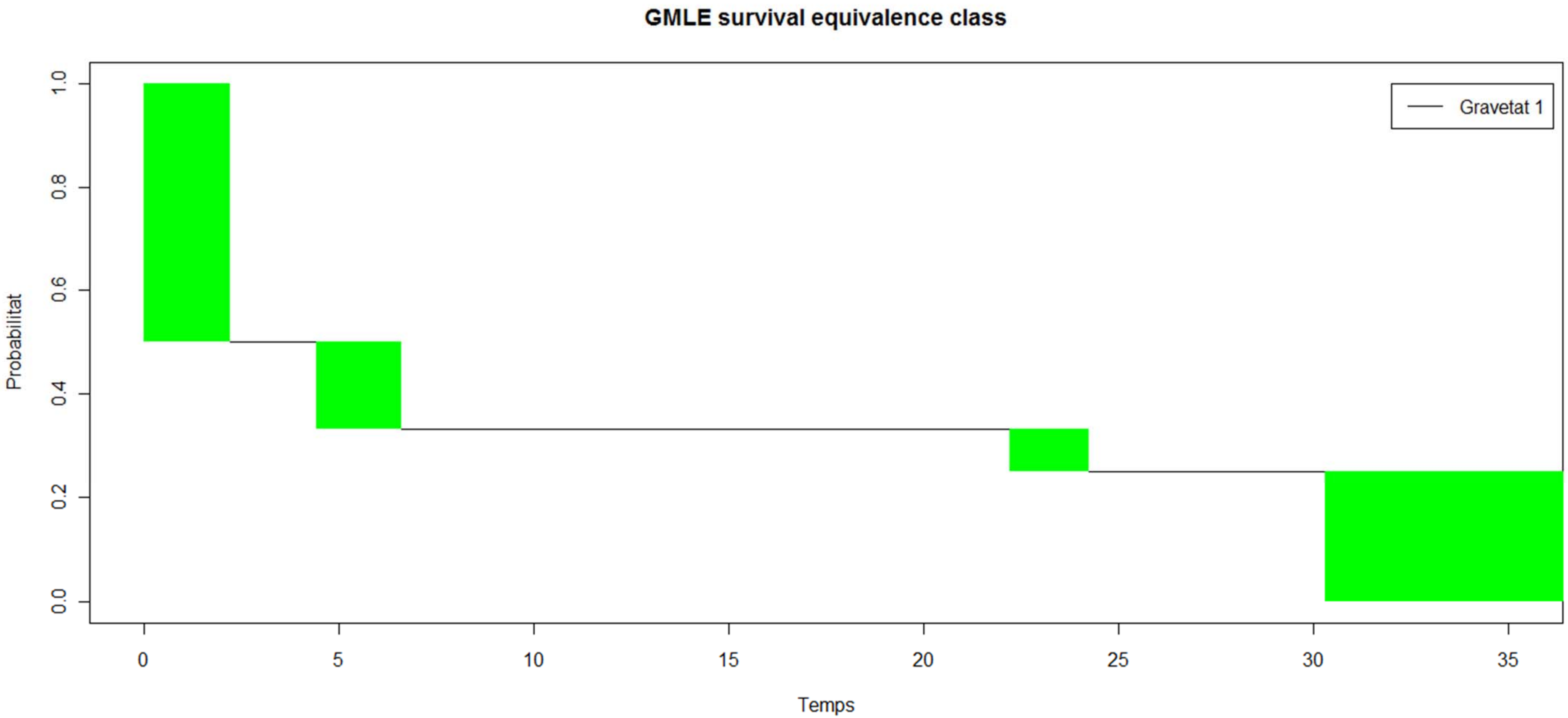
Checking the existence of three files.

Number of variables and restrictions of problem to optimize.

RESULTS

```
*************************************************************
        NEOS Server Version 5.0
        Job#     : 449470
        Password : bahutCEg
        Solver   : nco:SNOPT:AMPL
        Start    : 2013-01-10 10:29:53
        End      : 2013-01-10 10:30:49
        Host     : neos-1.chtc.wisc.edu

        Disclaimer:
        This information is provided without any express or
        implied warranty. In particular, there is no warranty
        of any kind concerning the fitness of this
        information for any particular purpose.
*************************************************************
Job 449470 sent to neos-1.chtc.wisc.edu
password: bahutCEg
---------- Begin Solver Output -----------
Executing /opt/neos/Drivers/snopt-ampl/snopt-driver.py at time:2013-01-10
10:29:53.157442
File exists
You are using the solver snopt.
Executing AMPL.
processing data.
processing commands.

6 variables, all nonlinear
1 constraint, all linear; 6 nonzeros
1 equality constraint
1 nonlinear objective; 6 nonzeros.
SNOPT 7.2-10 : Optimal solution found.
24 iterations, objective -9.364262454
Nonlin evals: obj = 19, grad = 18.

The Turnbull intervals with the failure probabilities and the cumulative
probabilities are:

INT_TURNBULL [*,*]
:     1        2       3        4        :=
1     0        2.2     0.5      0.5
2     4.4      6.6     0.1667   0.6667
3    12.12    14.14    0        0.6667
4    16.16    18.18    0        0.6667
5    22.22    24.24    0.0833   0.75
6    30.3    999       0.25     1
;
```

3.3 THE TEA METHOD



4.1 THE TEAR ALGORITHM

Is the identification process of the Turnbull's intervals the same when the maximization problem includes order restrictions?

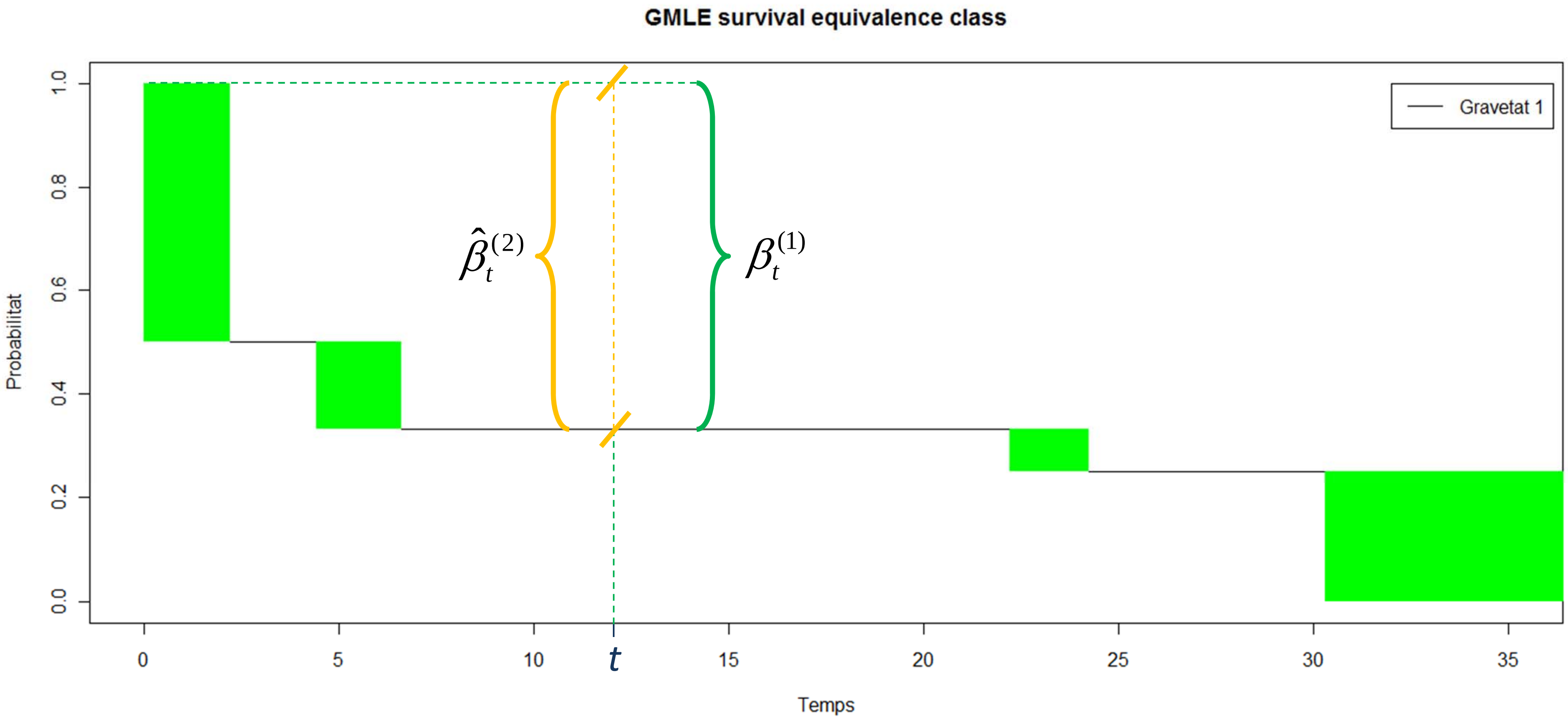
?

4.1 THE TEAR ALGORITHM

4.1 THE TEAR ALGORITHM

- We programmed again the TEA method
  - Calculating all possible intervals.
  - Adding order restrictions.

4.1 THE TEAR ALGORITHM



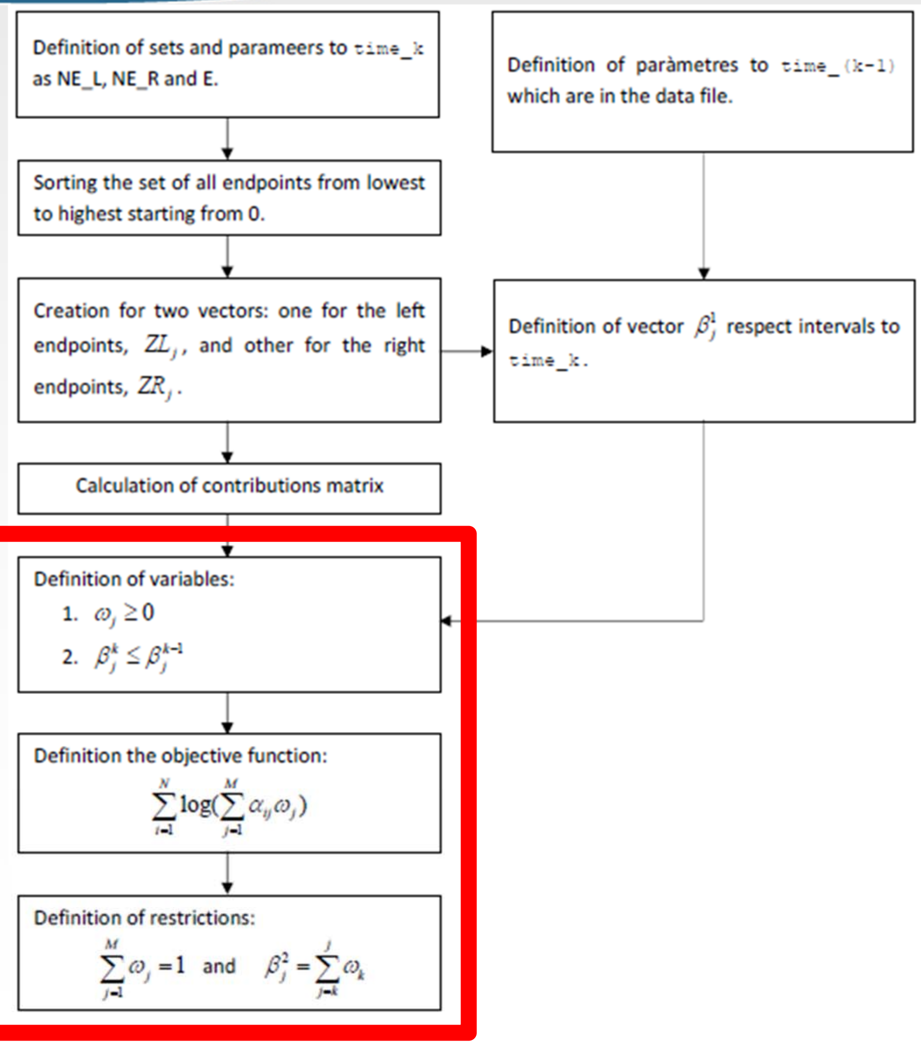
4.1 THE TEAR ALGORITHM

- Formally the optimization problem is :

$$
\begin{cases}
\max\limits_{\omega_1,\ldots,\omega_m} \prod\limits_{i=1}^{n} \left(\sum\limits_{j=1}^{m} \alpha_{ij}\,\omega_j\right) \\
s.t. \\
\quad \omega_j \geq 0 \qquad j = 1,\ldots, m \\
\quad \sum\limits_{j=1}^{m} \omega_j = 1 \\
\quad \beta_k^{(2)} = \sum\limits_{j=1}^{k} \omega_j \leq \beta_k^{(1)} \qquad k = 1,\ldots, m
\end{cases}
$$

where $\alpha_{ij} = I((q_j, p_j) \subset (L_i, R_i))$ and $(\beta_1^{(1)},\ldots,\beta_m^{(1)})$ is a known vector of probabilities

4.1 THE TEAR ALGORITHM

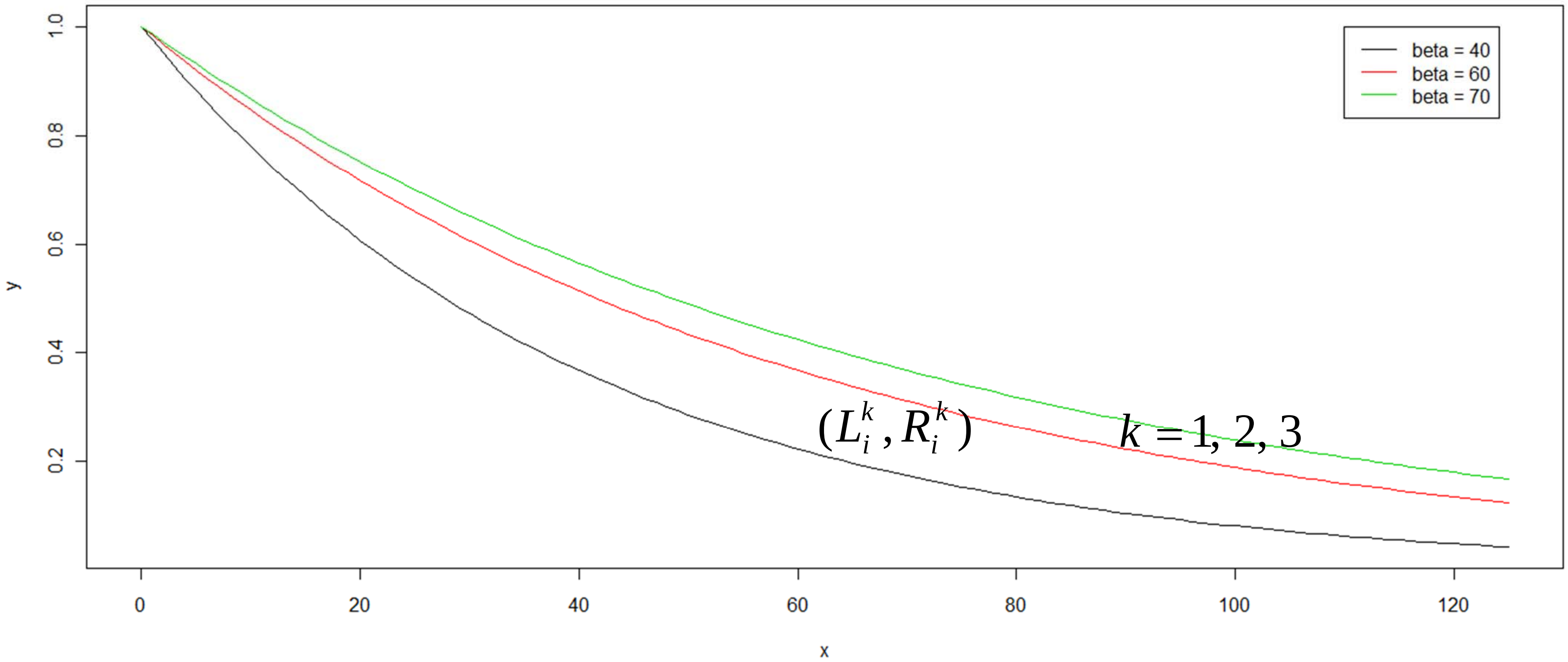


4.1 THE TEAR ALGORITHM

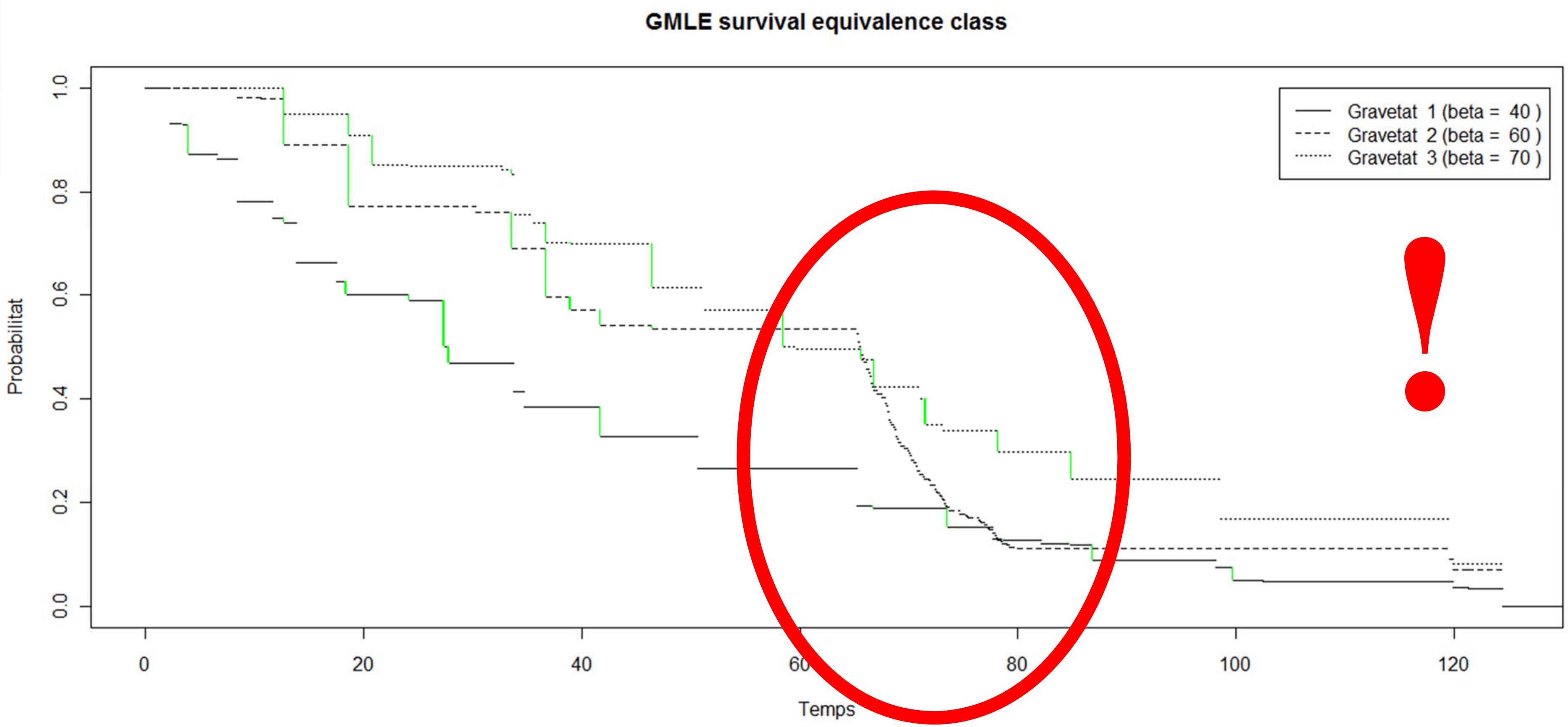
```
####################################################
## VARIABLES, OBJECTIVE FUNCTION and CONSTRAINTS ##
####################################################

param num_run;
param sum_ome;

## VARIABLES ##
var w {j in 1..M} >=0;
var beta2{j in 1..M} <= beta1[j];

## OBJECTIVE FUNCTION ##
maximize logLikelihood1: sum{i in 1..N}log(sum{j in 1..M}gamma[i,j]*w[j]);

## CONSTRAINTS ##
subject to sum_w: sum {j in 1..M} w[j]=1;
subject to cum_w{j in 1..M}: beta2[j]=abs(sum{k in 1..j}w[k]);
```

4.1 THE TEAR ALGORITHM

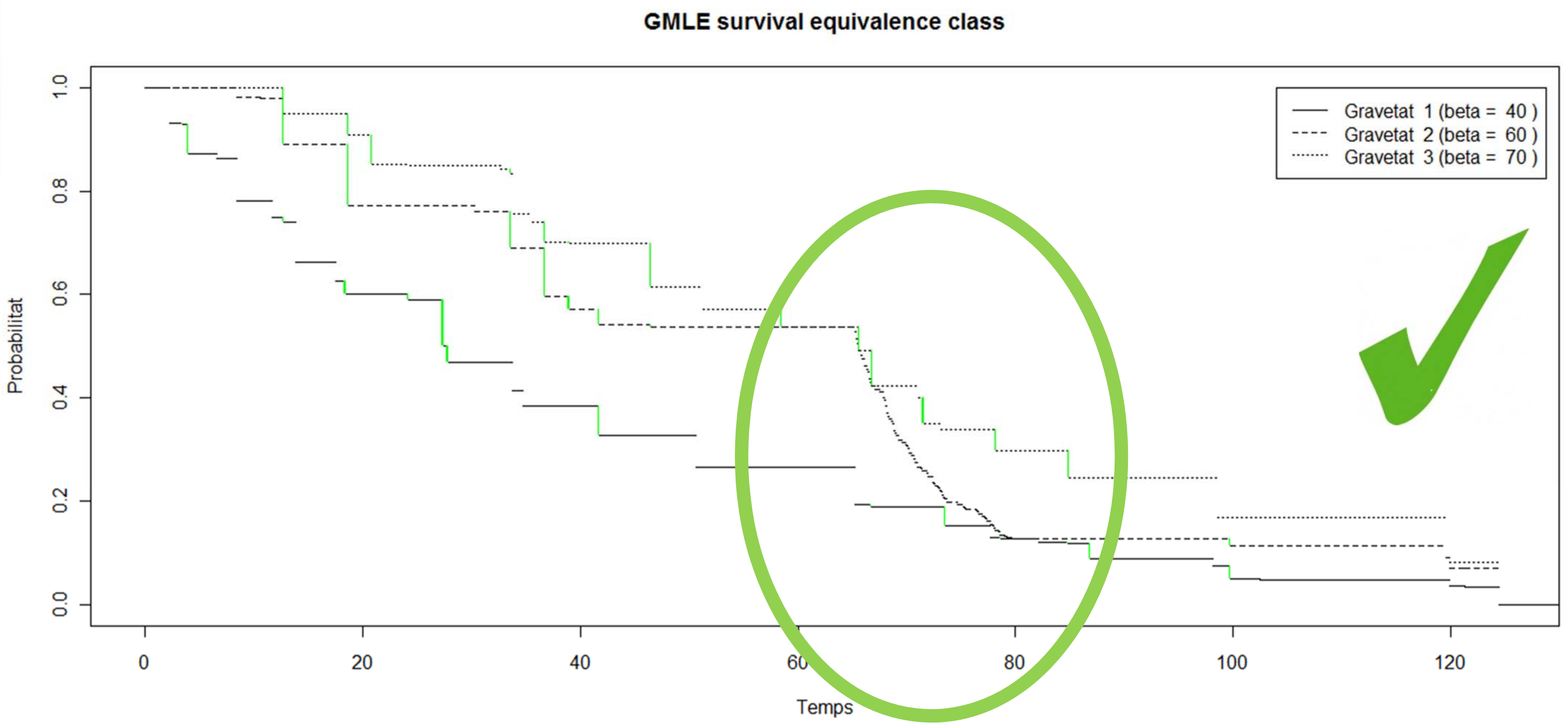
5.1 DATA SIMULATION



5.2 MODIFIED SIMULATED DATA



5.3 ESTIMATION OF THE DURABILITY



6.1 CONCLUSIONS

- Implementation of the Turnbull algorithm in AMPL language.

- Implementation of TEAR algorithm to estimate durability curves with order restrictions.

6.2 FUTURE RESEARCH

- To prove theoretically the extension of the Turnbull estimator for successive survival functions.
- To develop the implementation of a library in R to estimate the Turnbull estimator with order restrictions.
- To improve the implementation for TEA and TEAR in AMPL language.

Estimation of Survival Functions Subject to Order Restrictions

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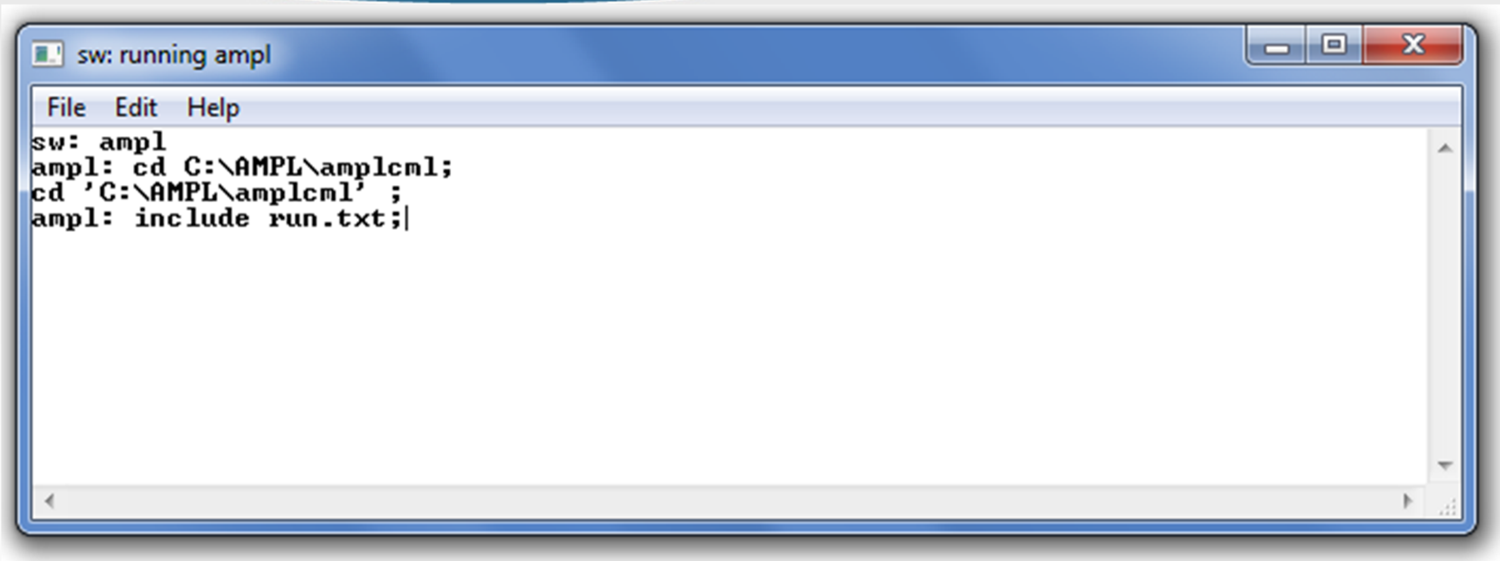
Institut d'Estadística i Matemàtica Aplicada a l'Edificació

UPC-BarcelonaTECH

Thank you for your attention !


sw: running ampl
File Edit Help
sw: ampl
ampl: cd C:\AMPL\amplcml;
cd 'C:\AMPL\amplcml' ;
ampl: include run.txt;

sw: running ampl

File Edit Help

```
sw: ampl
ampl: cd C:\AMPL\amplcml;
cd 'C:\AMPL\amplcml' ;
ampl: include run.txt;
SNOPT 7.2-8 : Optimal solution found.
38 iterations, objective -20.34300363
Nonlin evals: obj = 21, grad = 20.


The Turnbull intervals with the failure probabilities and the cumulative probabilities are:

INTERVALS [*,*]
:     1    2      3        4       :=
1     4    4    0.1127   0.1127
2     5    6    0.2136   0.3264
3     6    7    0        0.3264
4     7    8    0        0.3264
5     8    8    0.1518   0.4782
6     8   11    0        0.4782
7    11   12    0.3218   0.8
8    17   18    0        0.8
9    18   18    0.1      0.9
10   18   23    0.1      1
;

ampl:
```

Using the NEOS Server for SNOPT

The user must submit a model in AMPL format to solve a nonlinearly constrained optimization problem. Examples of models in AMPL format can be found in the netlib collection.

The model is specified by a model file, and optionally, a data file and a commands file. If the command file is specified it must contain the AMPL solve command.

The commands file can contain any AMPL command or set options for SNOPT with, for example,

```
option snopt_options "timing=3 outlev=2";
```

Printing directed to standard out is returned to the user with the output.

Enter the location of the ampl model (local file)

Model File:

Examinar...

Enter the location of the ampl data file (local file)

Data File:

Examinar...

Enter the location of the ampl commands file (local file)

Commands File:

Examinar...

Comments:

☐ Put in priority queue. 5 minute job runtime limit.

☐ Dry run: generate job XML instead of submitting it to NEOS

e-mail address:

By submitting a job, you have accepted the Terms of Use

Submit to NEOS | Clear this Form

Please do not click the 'Submit to NEOS' button more than once.

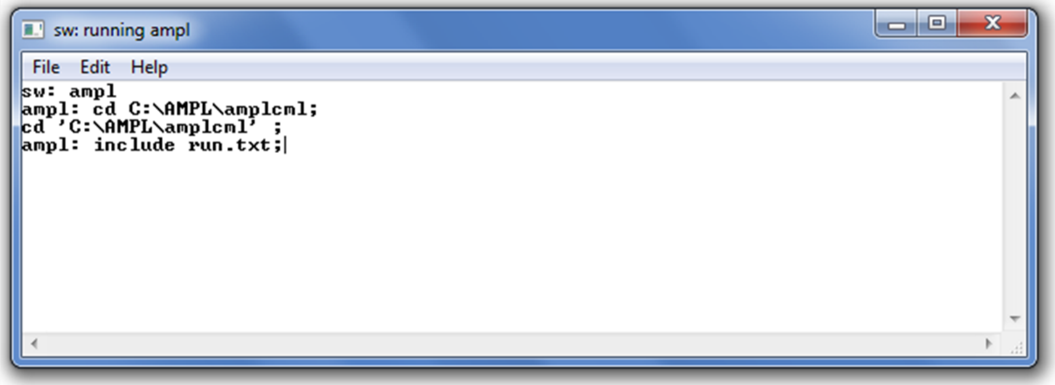
Enter the job number and the password of the job you wish to kill/view.
You can leave these blank if viewing the queue.

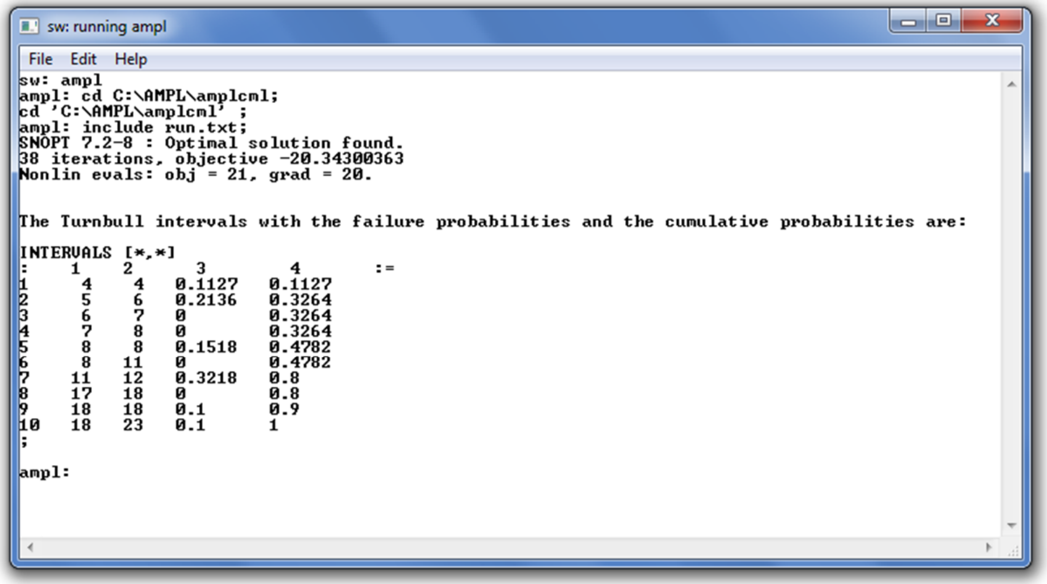
Enter the job number:

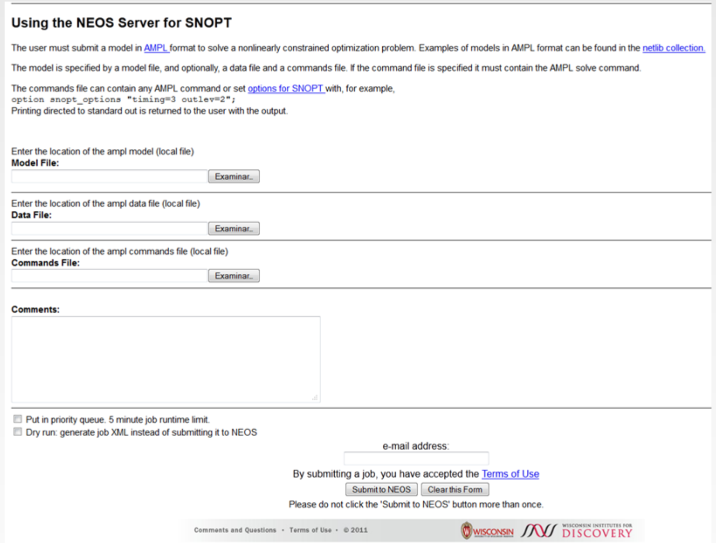
Enter the password for job:

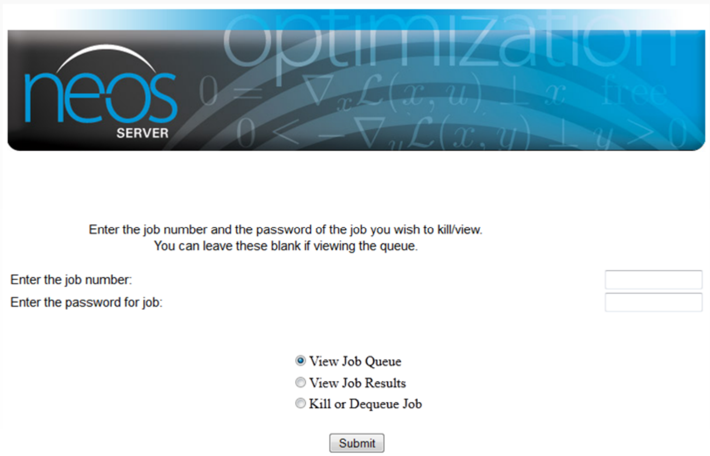
- View Job Queue
- View Job Results
- Kill or Dequeue Job

Submit


sw: running ampl
File Edit Help
sw: ampl
ampl: cd C:\AMPL\amplcml;
cd 'C:\AMPL\amplcml' ;
ampl: include run.txt;


sw: running ampl
File Edit Help
sw: ampl
ampl: cd C:\AMPL\amplcml;
cd 'C:\AMPL\amplcml' ;
ampl: include run.txt;
SNOPT 7.2-8 : Optimal solution found.
38 iterations, objective -20.34300363
Nonlin evals: obj = 21, grad = 20.
The Turnbull intervals with the failure probabilities and the cumulative probabilities are:
INTERVALS [*,*]
: 1 2 3 4 :=
1 4 4 0.1127 0.1127
2 5 6 0.2136 0.3264
3 6 7 0 0.3264
4 7 8 0 0.3264
5 8 8 0.1518 0.4782
6 8 11 0 0.4782
7 11 12 0.3218 0.8
8 17 18 0 0.8
9 18 18 0.1 0.9
10 18 23 0.1 1
;
ampl:


Using the NEOS Server for SNOPT
The user must submit a model in AMPL format to solve a nonlinearly constrained optimization problem. Examples of models in AMPL format can be found in the netlib collection.
The model is specified by a model file, and optionally, a data file and a commands file. If the command file is specified it must contain the AMPL solve command.
The commands file can contain any AMPL command or set options for SNOPT with, for example,
option snopt_options "timing=3 outlev=2";
Printing directed to standard out is returned to the user with the output.
Enter the location of the ampl model (local file)
Model File:
Examinar..
Enter the location of the ampl data file (local file)
Data File:
Examinar..
Enter the location of the ampl commands file (local file)
Commands File:
Examinar..
Comments:
Put in priority queue. 5 minute job runtime limit.
Dry run: generate job XML instead of submitting it to NEOS
e-mail address:
By submitting a job, you have accepted the Terms of Use
Submit to NEOS
Clear this Form
Please do not click the 'Submit to NEOS' button more than once.
Comments and Questions • Terms of Use • © 2011
WISCONSIN
WISCONSIN INSTITUTES FOR DISCOVERY


optimization
neos
SERVER
Enter the job number and the password of the job you wish to kill/view.
You can leave these blank if viewing the queue.
Enter the job number:
Enter the password for job:
View Job Queue
View Job Results
Kill or Dequeue Job
Submit

Using the NEOS Server for SNOPT

The user must submit a model in AMPL format to solve a nonlinearly constrained optimization problem. Examples of models in AMPL format can be found in the netlib collection.

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```
option snopt_options "timing=3 outlev=2";
```

Printing directed to standard out is returned to the user with the output.

Enter the location of the ampl model (local file)
Model File:
Examinar...

Enter the location of the ampl data file (local file)
Data File:
Examinar...

Enter the location of the ampl commands file (local file)
Commands File:
Examinar...

Comments:

☐ Put in priority queue. 5 minute job runtime limit.
☐ Dry run: generate job XML instead of submitting it to NEOS

e-mail address:

By submitting a job, you have accepted the Terms of Use

Submit to NEOS | Clear this Form

Please do not click the 'Submit to NEOS' button more than once.

neos SERVER optimization

Enter the job number and the password of the job you wish to kill/view.
You can leave these blank if viewing the queue.

Enter the job number:

Enter the password for job:

◉ View Job Queue
○ View Job Results
○ Kill or Dequeue Job

Submit