Effects in fuel consumption of assigning RTAs into 4D trajectory optimisation upon departures

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ABSTRACT
4D trajectory optimisation has showed good potential to reduce environmental impact in aviation. However, a recurrent problematic is the loss in air traffic capacity that these pose, usually overcome with speed and time advisories. This paper aims at the quantification in terms of fuel consumption of implementing suboptimal trajectories to preserve capacity. Via an own developed optimisation framework, we deliver results on how imposing a non-optimal RTA to a trajectory increases the fuel burned. We show how advancing a metering fix in an example departure trajectory translates to an increase of up 15Kg of fuel burned. Similarly, postponing it 50s, will burn around 23Kg more. Also, imposing a level off phase (due to incoming traffic) will typically consume around 25Kg more. Different scenarios and situations are studied for the fairest comparison.

Author Keywords
trajectory optimisation; 4D trajectories; non-linear programming; optimal control; fuel; performance models; BADA; RTA; metering fix;

ACM Classification Keywords
G.1.6; G.1.7

INTRODUCTION
The improvement of air transport efficiency (in terms of economic and environmental impact) is one of the major drivers for research and development in the SESAR and NextGen programmes. New technologies and procedures for future ATM and on-board systems and operations are being investigated and proposed. Initiatives such as Continuous Climb Departures (CCD), Continuous Cruise Climb (CCC), and Continuous Descent Approaches (CDA) propose good fuel reduction in specific phases of the flight. However, such operations usually come with a negative impact in air traffic capacity, given the vast typologies of aircraft and hence diversity in vertical and speed profiles.

Uncertainty in Top of Descent (TOD) for example has been studied in [1]. Solutions to this issue usually come with the

acceptance of sub-optimal trajectories given dynamic speed requests [2], multiple flight path angle (FPA) phases [3], requested time of arrivals in a specific point (RTA) [4], etc. For example, several research has been done in the integration of CDA in dense TMAs [5,6]. The Oceanic Tailored Arrivals program, currently in place in San Francisco airport [4], is another relevant example. These arrivals are supported by the Efficient Descent Advisor (EDA) developed by NASA-AMES, which is able to compute conflict-free optimal descent trajectories and satisfy a given arrival fix metering [7].

Even if this research is indeed very promising and is setting the foundations for future applications, we are still far to fulfil SESAR objectives in terms of significantly improving the trajectory efficiency in terminal airspace. Additionally, in order to preserve air traffic capacity, it is usually assumed that a sub-optimal solution is implemented. This paper aims at the quantification in terms of fuel consumption of such sub-optimal trajectories via an own developed optimisation framework.

OPTIMISATION FRAMEWORK
Trajectory modelling and optimisation has been a subject widely researched in the last decades. Analytically, this optimisation problem can be formally written as a continuous optimal control problem and extensive research on its resolution can be found in the literature. However, realistic trajectories are hardly impossible to solve analytically and a wide variety of numerical solutions have arisen. One of the most relevant ones involves the direct transcription of the problem, leading to a Non-Linear Programming (NLP) problem with a finite set of decision variables [8]. This approach will set-up the basic theoretical background for the research proposed in this paper.

Equations of Motion
In this paper we have taken a Point-Mass representation of the aircraft, where forces apply at its Centre Of Gravity (COG). For the initial assessment proposed in this paper, a winds calm situation, in a flat non-rotating earth has been assumed. The equations of motion are written as follows:

$$\dot{v} = \frac{1}{m}(T - D - m\,g\,\sin\gamma) \tag{1}$$

$$\dot{\gamma} = \frac{g}{v}(n_z \cos\phi - \cos\gamma) \tag{2}$$

$$\dot{\psi} = \frac{g\,\sin\phi}{v\,\cos\gamma}n_z \tag{3}$$

$$\dot{x} = v \cos \gamma \cos \psi \qquad (4)$$
$$\dot{y} = v \cos \gamma \sin \psi \qquad (5)$$
$$\dot{h} = v \sin \gamma \qquad (6)$$

where x, y and h are the spatial location of the aircraft, v is the velocity, γ the flight path angle, ψ the heading and ϕ the bank angle. The load factor (n_z) is defined as the relation between the aerodynamic lift force and the aircraft weight as follows:

$$n_z = \frac{L}{m \, g} \qquad (7)$$

To the calculation of the aerodynamic and propulsive forces, we use BADA performance models [9]:

$$C_L = \frac{2mg}{\rho v^2 S \cos\phi} \qquad (8)$$
$$C_D = C_{D0} + C_{D2}C_L{}^2 \qquad (9)$$
$$D = \frac{1}{2}\rho S v^2 C_D \qquad (10)$$
$$T = C_{Tc1}\left(1 - \frac{h}{C_{Tc2}} + C_{Tc3}h^2\right) \qquad (11)$$

where C_{Tc1}, C_{Tc2} and C_{Tc3} are climb coefficients specified in the BADA tables; C_{D0} is the parasitic drag coefficient; C_{D2} is the induced drag coefficient; and ρ is the air density at altitude assuming ISA atmosphere as calculated in BADA. To model the throttle position we define μ as a percentage multiplying T.

Additionally, BADA defines the fuel flow as follows:

$$ff(v,T) = \eta \, T \qquad (12)$$

where the thrust specific fuel consumption is modelled as:

$$\eta = C_{f1}\left(1 - \frac{v}{C_{f2}}\right) \qquad (13)$$

being the coefficients C_{f1} and C_{f2} also defined in BADA.

Problem Formulation
Optimal control problems are usually nonlinear and generally do not have analytic solutions and it is required to employ numerical methods to solve them [8, 10]. The approach is to convert the infinite-dimensional original problem into a finite-dimensional optimisation by iteratively applying three fundamental steps [8]: collocation (namely Euler, Trapezoidal and Pseudospectral are the most used [11]), solving the NLP (with NLP Solvers such as SNOPT or IPOPT) and re-dimensioning the problem (packages such as for instance GPOPS[1], SOCS[2] or PSOPT[3] will automatically iterate the three steps).

The optimisation framework developed in this paper uses GPOPS, which implements a *pseudospectral* collocation method. It handles multiphase optimal control problems and can automatically resize the number of collocation points in

each iteration (i.e. it does not rely on the number of points given in the guess). GPOPS is developed in MATLAB and is open source and free for research purposes.

Using the Equations of Motion described above, we have formulated an optimal control problem, the solution to which minimises the fuel consumption as:

$$J(\mathrm{t}) = \int ff(v,T)\,\mathrm{d}t \qquad (14)$$

The state ($\boldsymbol{x}$) and control ($\boldsymbol{u}$) vectors of the problem are defined as follows:

$$\boldsymbol{x} = [v\ \gamma\ \psi x\ y\ h]; \qquad \boldsymbol{u} = [n_z\ \phi\ \mu]$$

The following table depicts the constraints considered in the optimisation problem:

Constraint	Definition	
Maximum altitude	$h \le h_{MAX}$	(15)
Minimum operation airspeed	$v \ge v_{MIN}$	(16)
Maximum operation airspeed	$v \le v_{MAX}$	(17)
No deceleration allowed	$\dot{v} \ge 0$	(18)
No descent allowed	$\dot{h} \ge 0$	(19)
Procedure Design Gradient (PDG)	$\dfrac{h}{s} \ge 3.3\%\ s$	(20)
Load factor	$0.85 \le n_z \le 1.15$	(21)
Bank angle	$-25 \le \phi \le 25$	(22)

Table 1. Constraints in the optimal control problem

Many of these are operational constraints, either to stay within the flight envelope or comply with ATM constraints such as ground obstacle avoidance (PDG). Since BADA defines v_{MIN} and v_{MAX} on CAS speeds, we do the conversion from TAS to ensure we stay within the limits. Additionally, the box constraints on n_z and ϕ where defined following usual civil aviation standards. More information on optimal control formulation techniques used in this research can be found in [12] and [13].

Generating the optimal trajectory
BADA defines different flight phases for a departing trajectory, with specific performance values to each phase. In it, there is a first phase where the aircraft must be at TOGA thrust climbing up without the possibility of turning or making changes in the aerodynamic configuration. In many studies, this phase is not contemplated given the low degrees of freedom in it due to operational constraints. After that, the following phases are defined by the time of aerodynamic changes. BADA defines a first flap retraction (from TO to IC) and a second flap retraction into clean configuration (IC to CL). Since flap retraction will change aircraft performance, each phase has different aerodynamic drag coefficients among other particularities. In our framework, these change with defined speed steps [14].

To take into consideration the changes in aerodynamic

configurations, we use continuous and twice differentiable *switching functions*. This method has the negative impact that it adds complexity (non-linearities) to the model (hence greater calculation times and convergence difficulties), and the minor side effect of having a transition effect around the switching value. Both issues are directly related, since the less steep is the function (and thus smoother for the NLP Solver) the bigger is the transition effect, and vice versa. Hence, a trade-off must be sought [13].

With this, we are able to compute a full trajectory from a set of initial conditions to a set of final conditions, including one or more RTA in waypoints along the route using a multi-phase optimal control problem.

SCENARIOS

For the sake of this paper, we have envisaged a set of scenarios that cover different operational constraints: from close-to-current operations to fully optimal scenarios. To this end, we have defined two baseline scenarios that will be compared to N futuristic trajectories with RTA. A first optimal departure scenario (A) has been defined from ground to cruise altitude (FL360 in the example) without traffic constraints. The second scenario (B) tries to closely reproduce a current ATCO conflict resolution with a step climb. In it, we specify a level-off segment at 10000ft from an along track distance of 20Nm to 30Nm (i.e. the aircraft reaches a point where it is told to maintain altitude up to the point where it is cleared to climb again). The optimal vertical profile of the two scenarios (A and B) is seen in Figure 1.
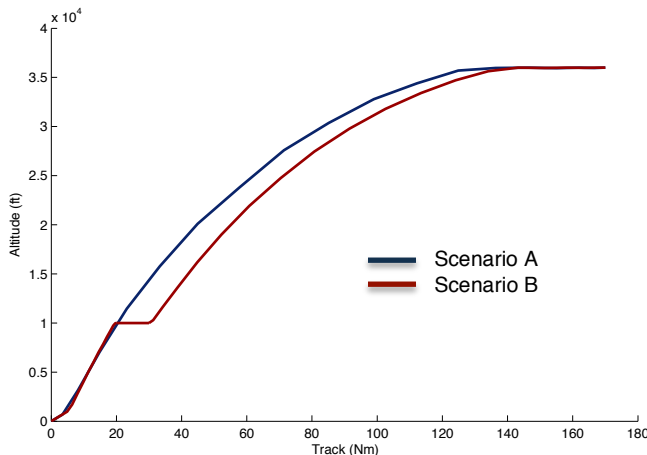

Figure 1. Optimal vertical profile for scenarios A and B

It is important to notice that in order to define the correct reference conditions for all scenarios, we have defined the same ending conditions for all in position, speed and altitude. Therefore we are able to fairly compare the burned fuel and the time spent in the flight. Besides, these ending conditions are defined taking into account that the aircraft must end with enough energy to continue with the subsequent phases of the flight.

Once the reference scenarios have been defined, we are able to create the following subset of scenarios in the direction

of the objectives of the current paper. In a futuristic scenario, with the capability to cope with 4D trajectories we want to quantify the impact in fuel that assigning 4D metering fixes (RTA) due to traffic constraints will pose as compared to the optimal (A) and current operations (B).

The optimal trajectory (A) cannot be flown due to a potential loss of separation with other traffic. For this reason, the ATCO has taken a conservative approach leveling-off the aircraft at 10000ft, before entering in conflict (B). In a 4D futuristic scenario, we could envisage that better separation assurance techniques could be in place, such as giving an RTA to the conflicting point (P) that prevents the conflict. Thus, the separation of the two aircraft will be sought by making one (or both) of them arrive to P at an earlier or later time.

Even if assuming a static geometry of the conflict is naïve, it is out of the scope of the paper to account for dynamic geometries due to uncertainties in the conflicting trajectories. Nevertheless, this allows us to isolate one trajectory and quantify its increase in fuel with the different assigned RTA.

For the following scenario (C) we make the assumption that P is found at 25Nm north. In the reference scenario A, we find out at what altitude (12227ft) and time (271s) the trajectory reaches P. On the other hand, we calculate the fastest possible time to reach it (presumably a bit faster than the optimal reference). Then, starting from the fastest feasible time, we define an RTA and iteratively move it back in time with 10s time steps. Each RTA will give different fuel consumptions and altitudes at P (see Figure 2). These should be further studied in order to detect if the conflict is avoided and if they provide more optimal results.
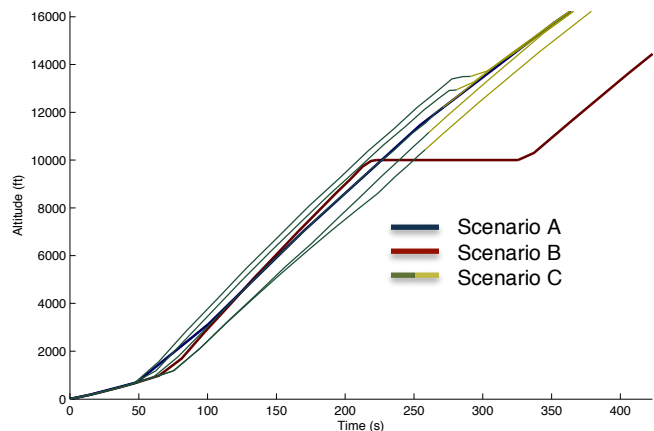

Figure 2. Optimal vertical profile for scenario C as compared to A and B

RESULTS

The fuel burned for scenario B has been quantified with an increase of 25,85Kg of fuel burned, as compared to the optimal reference trajectory A.

Results for trajectories in scenario C are depicted in the chart in Figure 3. Starting from the fastest feasible

trajectory, -13 seconds earlier than the optimal (calculated with the same optimization framework, changing only the objective function to time), to +50 seconds. It relates the fuel burned, the total duration of the flight and the altitude when reaching P.
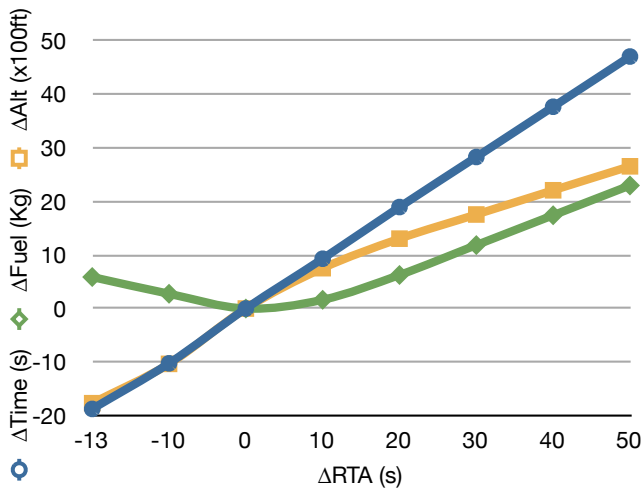

Figure 3. Results for trajectories in Scenario C

In the previous figure, zero denotes the optimal trajectory (as calculated for scenario A) and the positive and negative values are the increase or decrease in fuel or time in comparison to the latter. In this case, if a conflict can be avoided reaching the given point P at time -13, the fuel increase will be of 5,87Kg, almost 20Kg less than B. Likewise, delaying the RTA 50s, the increase in fuel will be 22,93Kg (little less than B), reaching the point at a higher altitude (2650ft higher), though as it can be seen, the total duration of the departing trajectory will be about 47s longer.

CONCLUSION

This paper presents a generic optimisation framework that has been used to quantify the effect of different RTA in a departing trajectory. The results can be useful in the process of choosing a sub-optimal trajectory in respect with separation assurance, time pressure and fuel reduction.

In this paper we have experimented with different scenarios that quantify the effect in fuel consumption that sub-optimal departing trajectories pose. To this end, we have isolated one trajectory on the conflict to be able to study the increase in fuel. Effectively, these results should be used within a collaborative conflict resolution to find the optimal RTA for all conflicting trajectories.

Finally, this framework proposes a generic approach to this issue. Further research will have to be applied to specific real-life examples in order to assess the potential in minimising fuel.

ACKNOWLEDGMENTS

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