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Method for Forming a Magnetic Field of Arbitrary Shape from an Established Template

Josep M. Torrents[1], Pablo Juan-García[2], Miguel Á. Sánchez-Moragues[3]

[1]Technical University of Catalonia (UPC), Jordi Girona 1-3, Building C4, 08034 Barcelona, SPAIN, phone: 34-934054430, fax: 34-934016756, e-mail: josep.m.torrents@upc.edu
[2]Technical University of Catalonia (former PhD), e-mail: pablo.juan@upc.edu
[3]Technical University of Catalonia (former student), Spain, e-mail: sanchez.moragues@gmail.com

Abstract-A strategy to design a particular form of magnetic field along an axis from a coaxial inductor is described. The procedure is illustrated with the design of a constant magnetic field inside a cubic inductor. It validates the goodness of the method by comparing the results with all possible combinations of inductance within a limited set of solutions. Keywords: Magnetic field, magnetic sensor, Helmholtz coils.

I. Introduction

In some applications related to magnetic field, which range from industrial sensors to clinical diagnostic tools, there is a need to create either a constant magnetic field [1]–[2], a constant gradient magnetic field [3] or an arbitrary form of magnetic field [4].
Often, the starting point is the Helmholtz coils. Later, other authors such as [1] and [2] have inserted more parallel inductors wound on the same axis. Moreover, [5] proposed a system of arbitrary magnetic field design; although its mechanization could be complex.
We are interested in having a systematic and easy to perform procedure to aid the design of a particular magnetic field formed by concentric spirals. This communication presents a system of magnetic field design based on a convolution of two arbitrary functions. One (spatial) function is the magnetic field produced by each of the inductors in a certain position. The other function is the weight for each of the inductances which, defined by a template, form the desired magnetic field. Finally, the goodness of the method is verified through the design and measurement of inductances which hold a constant magnetic field along their axis.

II. Theory

For the spatial variable x, the function *y(x)* is defined as the template shape or desired magnetic field. Contributes to this function the magnetic field *b(x),* that produces each evenly spaced inductance M weighted by the number of turns represented by the function of weights α. Thus,

$$y(x) = \sum_{i=1}^{M} \alpha_i \cdot b(x - x_i) \qquad (1)$$

For each of the positions (from 1 to M) that discretise the space, each one contributes to the magnetic field inductance depending on the distance to the x position considered. Thus, we can represent the matrix form.

$$\begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix} = \begin{pmatrix} b(x_1 - x_1) & \cdots & b(x_1 - x_M) \\ \vdots & \ddots & \vdots \\ b(x_M - x_1) & \cdots & b(x_M - x_M) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_M \end{pmatrix} \qquad (2)$$

Or even we can represent the compact matrix form:

$$Y = B\mathrm{A} \qquad (3)$$

If B is a square matrix and nonzero determinant, then it is solved by inverting the matrix B. If the determinant is zero or the matrix B is not square, then it is solved from the pseudoinverse:

$$\mathrm{A} = pinv(B) \cdot Y \qquad (4)$$

The optimal weight vector will contain real numbers. The weights must be integers, as they represent the number of turns of each inductor. Negative weights, although possible, complicate the realization of the winding, and increase parasitic effects.

Limiting the set of feasible weights means that the analytical solution is not ideal. A posteriori, the solution obtained by applying the method is optimized by the Branch&Bound technique [6]. In this case, an optimal or sub-optimal way can be found depending on the initial value proposed. To compare the results of the methods discussed a solution was also sought by trial-and-error technique. Table 1 compares theoretical advantages and disadvantages of the three strategies. The two final solutions are similar but not necessarily identical.

Table 1. Comparison of the three methods

	Advantages	**Disadvantages**
Analytical method	Low computational Cost	High Error (due to feasible weights)
Branch&Bound	Moderate Error	Variable computational Time
Trial-and-error	Lowest Error	High computational Time

The method of trial-and-error calculates the magnetic field due to a combination of inductors, which is compared afterwards with that proposed by the template of desired magnetic field. If the difference is less than that the previous “good” combination of inductances then, it is taken as the new “good” combination result. If not, it is discarded. In any case, all combinations must be tried to cover all possible permutations. To avoid the calculation being too long, the maximum of symmetries and constraints are taken into account from the observation of the analytical method.

However, the method of trial-and-error may be too long and unwieldy (most often impracticable). It may be too long because the number of combinations to test grows exponentially with M (M representing the number of equidistant inductances). If M is very large (e.g. 100), the computing power necessary to solve the problem may be beyond the available means. Unwieldy because if it is iterated for a certain template reaching a result of weights and any parameter from the M inductances is changed, such as shape or size, it must be iterated again.

Graphically, the flowchart of Figure 1 shows the design procedure to form the magnetic field. As mentioned, we define the magnetic field b(x) that produce each of the wire loops and sets the desired template of magnetic field. Then, it is determined the number M of evenly spaced inductors. After defining these parameters, the method to apply is decided: Either the truncated analytical solution for the physical limitations of achievable inductance and optimized by the Branch&Bound strategy, or alternatively, through the trial-and-error: As mentioned, we first define all the possible combinations of solutions that are tested sequentially. If a proposed solution minimizes the error on the desired value of the magnetic field given by the template, it is chosen and stored. Thus, when the method has tried all the possible solutions, the remaining storage is the optimal solution of the trial-and-error method. Except for differences in truncation, both methods reach a similar solution.

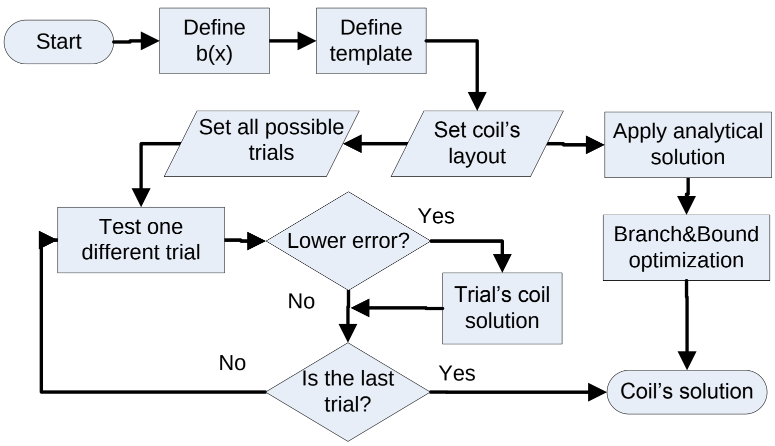

Figure 1. Flowchart of the design process of coils forming the magnetic field

The solution of both methods is similar but not necessarily identical. As a general rule, the method of trial-and-error reaches the best solution among all possible ones (because the method tests all of them), while the analytical method, limited by the physical realization of the inductance and optimized by the Branch&Bound strategy can reach both an optimal or a suboptimal solution.

III. Material and Methods

The iterative computation strategies Branch&Bound and trial-and-error testing and matrix inversion are programmed using Matlab 7.4 and executed by a computer Intel Core 2 Duo 2.13 GHz clock speed with 2 GB of RAM and OS Windows XP Professional SP3.

To determine if the analytical solution converges to the method of trial-and-error, two prismatic 6 cm side inductors are mounted as shown in Figure 2. It is set a target to achieve a magnetic field as uniform as possible on its axis, so the region of interest is set in the interval [-3 cm, +3 cm] from the center of the inductor. If M is equal to 13, each set of coils is almost 5 mm wide. An inductor is wound according to the strategy of the analytical method and the other inductor according to the method of trial-and-error.

Figure 2. Prismatic inductors of 6 cm side. Designed with analytical method (left) and trial-and-error (right).

To verify the coincidence of the designs with the numerical and analytical estimations, a measurement system is set, by transfer function mode (AB mode) of a low frequency impedance analyzer HP 4192A, as shown in Figure 3. The oscillator analyzer generates a sine wave of known frequency and voltage. This sine wave is injected into a splitter. The one half that serves as a reference signal is injected into an input channel of the analyzer, the other half excites the inductance to be characterized. On the same axis, a small inductance is mounted as the sensing element. The signal induced on the small inductance is injected into the other input channel of the analyzer. This inductance sensor is manually moved along its axis at intervals of 5 mm. The attenuation is measured and compared between points on the axis of displacement. The data is acquired through the GPIB-USB connection of PC and using the data acquisition program LabView.

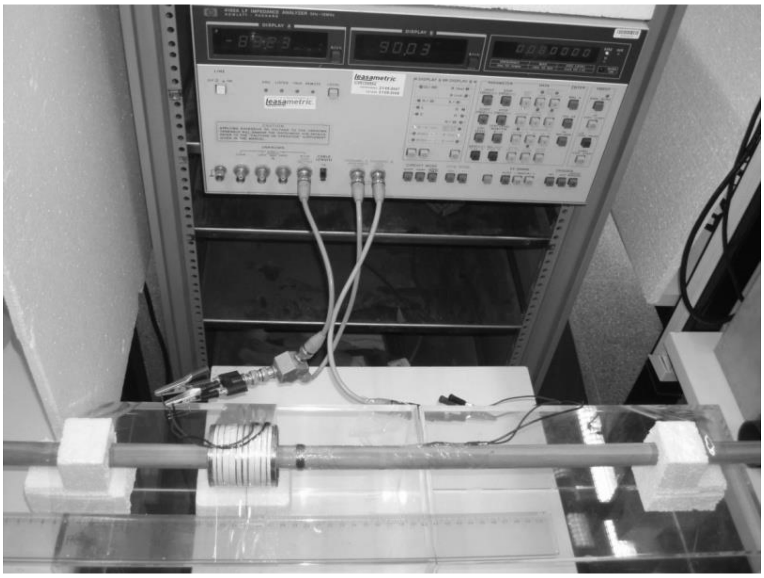

Figure 3. Measurement of a cylindrical inductance at 80 Hz and a significant attenuation (83 dB) measuring the magnetic field at 2 cm off-axis inductance.

The sensitivity and dynamic range of this measure performed with an oscilloscope would be insufficient. HP4192A performs coherent demodulation to obtain sensitivity and dynamic range high enough. For example, the magnetic field on the axis of the inductance shown in Figure 3 at 2 cm off the central part is attenuated over 80 dB at a frequency of 80 Hz.

IV. Results and Discussion

Table 2 shows the results for the design of a prismatic 6 cm side inductor with M=13 cm using the analytical and Trial-and-error method. Such a coarse approach (with steps of 5 mm) led to a calculation of more than 16 hours. Figure 2 shows only winding at the ends (16 turns) and center (9 turns). This result is consistent with the weight vector from Table 2.

Table 2. Results for prismatic 6 cm side inductor designed with two different methods

	Analytical method	**Trial-and-error method**
Vector natural weights (positive integers)	{16, 0, 0, 0, 0, 0, 9, 0, 0, 0, 0, 0, 16}	{40, 0, 0, 0, 0, 1, 12, 1, 0, 0, 0, 0, 40}
Theoretical Error	8.7%	5.6%
Empirical Error	8.5%	5.9%
RMS Error	0.2%	0.3%
Runtime	0.64 s	59448 s ≈ 16.5 hours

The theoretical error is defined as the maximum difference between the value of the design (template) and that obtained from the application of analytical method, evaluated in the range of interest from -3 cm to 3 cm. The empirical error is defined also as the maximum difference between the template value and the measured and also in the same interval of interest between -3 cm to 3 cm. Then, the theoretical error shows entirely the limitations of the method, whilst the empirical error is also influenced by other error sources, like physical asymmetries or parasitic effects. Finally, the RMS error (or effective) was defined as the mean square error from the value obtained by applying the method (analytic case) and measured in the range between -10 cm to 10 cm. These errors are defined as indices of quality of both the difference between the values entered by the template against the theoretical values achievable by the application of the method as well as the difference between the application of the method and the measured values.

Figure 4 shows the results of the measurements in the inductance mounted according to the analytical method with weights of natural numbers (positive integer) loops. They are measured between ranges of -10 cm to 10 cm, being 0 cm the centre of the inductance. The quantification of the error between theoretical and measured values is shown in Table 2, while Figure 4 shows visually the matching between theoretical and measured values. In this case, the maximum difference of 8.7% of the theoretical error corresponds to the ends of the inductance. The maximum difference of 8.5% of the empirical error also corresponds to the ends of the inductance. Measures closely follow the theoretical value so the RMS error is low. The small ripple of the measures to be observed outside the ends of the inductance may be due to the dispersion of measured values for a small field and due to the fact that the 5 mm displacement sensor coil in each measure was carried out manually. It is noted that in the central part of the interior of the inductance the field is constant, consistent with analytical predictions.

Table 2 also shows in the right the results for the designed prismatic 6 cm side inductor with M=13 using the method of trial-and-error. In this method, it makes no sense to assign real numbers to weights as they are limited directly to positive integers. Consistent with the weight vector in Table 2, figure 2 shows only the ends winding (40 turns) and central (12 turns and a small 1 turn "pedestal" on both sides).

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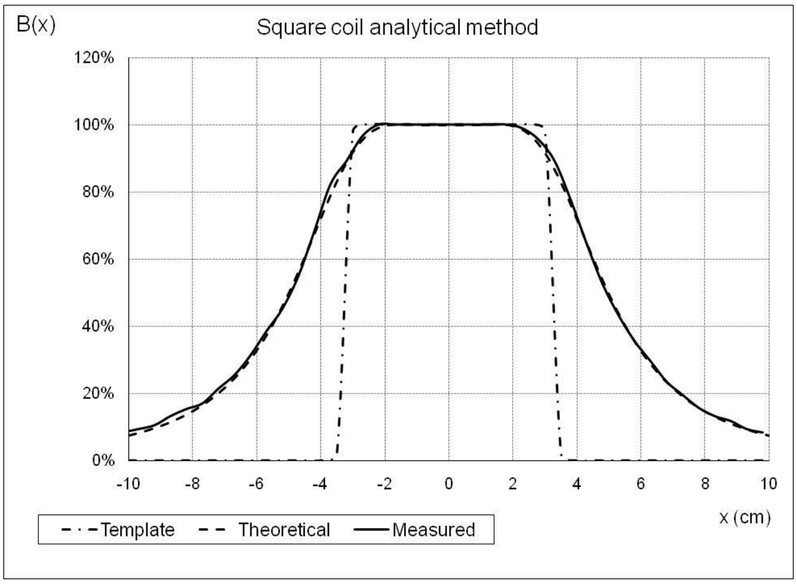

Figure 4. Comparison between theoretical values and measurements of magnetic field on the axis of the 6 cm prismatic inductance designed by analytical method according to natural (positive integers) weights.

Figure 5 shows the results of the measurements in the inductance mounted by the method trial-and-error with weights of natural numbers (positive integer) loops. As in the previous case, it is measured in the range of -10 cm to 10 cm being 0 cm the centre of the inductance. The quantification of the error between theoretical and measured values is shown in Table 2 while Figure 5 shows the coincidence visually. In this case, the maximum difference of 5.6% of the theoretical error is both at the centre of the inductor (which is clearly seen in Figure 5) and at the ends of the inductor (although visually it appears to be more difficult to see in Figure 5). The magnetic field does not decrease so abruptly as the proposed template but this trial-and-error solution improved this aspect at the ends of the inductance (5.6 % is less than 8.7 %) to increase the cost of error in the central area. The maximum error of 5.9% is consistent with the empirical value of 5.6% of the theoretical error. Similarly, the RMS error is small because of the great similarity between the theoretical value of the trial-and-error and the measured one. It should be emphasized that the method of trial-and-error reaches absolute best results, although visually could seem worse than the analytical method.

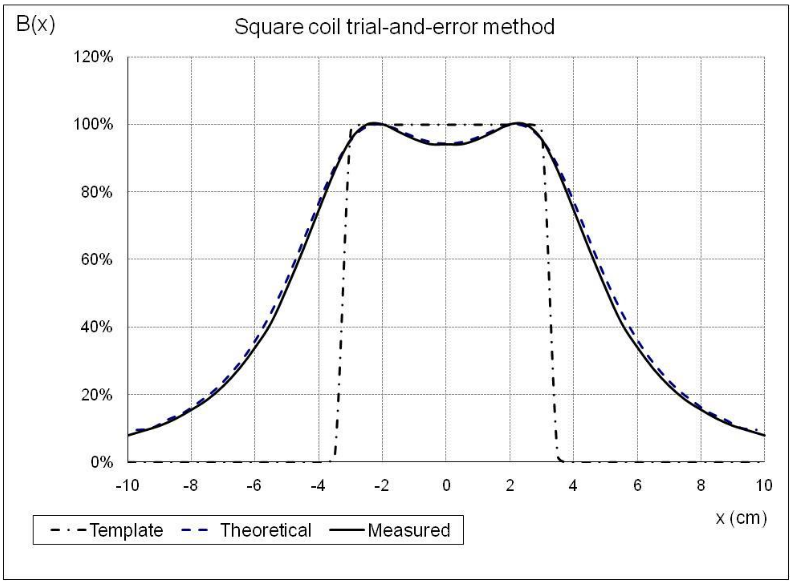

Figure 5. Comparison between theoretical values and measurements of magnetic field on the axis of the prismatic 6 cm side inductor designed with trial-and-error method.

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Eventually, both methods reliably reproduce the magnetic field proposed by the template. The magnetic field produced by the analytical method is solved with a smaller investment of time but with a more inaccurate result in reference to the requested one (at least in some points), constant magnetic field in this case. Anyway, as already noted, if the inductor had a greater length (or an M number slightly higher), the method of trial-and-error would be impracticable by the exponential growth in the consumption of computing time.
The decay out of the constant field bands cannot be as abrupt as requested by the template because the magnetic field produced by coils decays as the cube of the distance. It should be wound reversely to try to eliminate the "tails" of the magnetic field, which increases the cost and makes the winding more difficult.

V. Conclusion

It has been validated a strategy for designing a template shape of magnetic field along an axis from the convolution of the magnetic field function (spatial) produced by each of the inductors (in a certain position) and weight function of each of the inductances.
The magnetic field of each inductance has been convolved with the weight function for the template. In this particular case, the template was constant. The similarity between theoretical and measured results has been quantified.
Finally, we have compared the magnetic fields produced by two inductors, the strategy designed by convolution which reaches a suboptimal result and the one designed by trial-and-error that always comes to the optimal result.

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