# Stochastic optimal generation bid to electricity markets with emission risk constraints. \*\*

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#### **Abstract**

There are many factors that influence the day-ahead market bidding strategies of a generation company (GenCo) in the current energy market framework. Environmental policy issues have become more and more important for fossil-fuelled power plants and they have to be considered in their management, giving rise to emission limitations. This work allows investigating the influence of the emission reduction plan, and the incorporation of the derivatives medium-term commitments in the optimal generation bidding strategy to the day-ahead electricity market. Two different technologies have been considered: the coal thermal units, high-emission technology, and the combined cycle gas turbine units, low-emission technology. The Iberian Electricity Market (MIBEL) and the Spanish National Emission Reduction Plan (NERP) defines the environmental framework to deal with by the day-ahead market bidding strategies. To address emission limitations, some of the standard risk management methodologies developed for financial markets, such as Value-at-Risk (VaR) and Conditional Valueat-Risk (CVaR), have been extended giving rise to the new concept of Conditional Emission at Risk (CEaR). This study offers to electricity generation utilities a mathematical model to determinate the individual optimal generation bid to the wholesale electricity market, for each one of their generation units that maximizes the long-run profits of the utility abiding by the Iberian Electricity Market rules, as well as the environmental restrictions set by the Spanish National Emissions Reduction Plan. The economic implications for a GenCo of including the environmental restrictions of this National Plan are analyzed, and the effect of the NERP in the expected profits and optimal generation bid are analyzed.

*Keywords:* OR in Energy, Stochastic Programming, Risk Management, Electricity market, Emission reduction

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#### 1. Introduction

Due to the constantly increasing pollution of the earth's atmosphere, in recent years emission control has become a matter of paramount importance. Nowadays world energy systems are highly dependent on fossil fuels (such as coal, oil and gas formed from the organic remains of prehistoric plants and animals). The share of fossil fuels in the world's energy production is more than 85% and in electricity generation more than 60% [1]. Although they provide a reliable and affordable source of energy, the use of fossil-fuelled power plants harm the global ecosystem by emitting into the atmosphere noxious gases and toxic substances, causing the greenhouse effect, which is thought to be responsible for climate change.

The EU sets limits for emissions of pollutants from large combustion plants through the so called National Emission Reduction Plan (NERP) (Directive 2001/80/EC [2]). The impact of the NERP on the power industry appears to be very significant and there is a real need for power companies to find new strategies to optimally integrate these new emission limits in their energy's market bid strategy. In this regard, NERP has to be necessarily and explicitly considered in the elaboration of the generation units' optimal sale bid to the wholesale electricity market, and this is the main commitment of this work.

## 1.1. The Iberian Electricity Market (MIBEL)

An important factor to determine the efficiency of electricity markets is the specific market structure and trading rules, such as regulations, applied in each specific market. The Iberian Electricity Market (MIBEL) is the result of a joint initiative of the Governments of Portugal and Spain to integrate their markets. The generation companies have to make daily bids to sell its electricity through the wholesale market, while distribution companies perform an energy demand. This market is organized by the Iberian Market Operator of Energy (OMIE, by its Spanish initials) who has to match supply with demand in real time. Nowadays the day-ahead market (DAM) (short-term mechanism) is the market where the most important part of the electricity demand is negotiated (78% in the case of the MIBEL), explaining why finding the optimal bid to the DAM is of utmost significance in the daily operation of any GenCo. However DAM is not only the main physical energy market of the MIBEL, in terms of the amount of traded energy, but also the mechanism thought which other energy products, as bilateral and futures contracts (medium term mechanisms), are integrated into the energy production system. Current electricity markets are organized not only around the short-term spot energy market but also around a variety of mid-term physical and financial products, as future and bilateral contracts, that each generation unit has to integrate in the sale bid submitted to the market operator following the specific rules of each national electricity market.

The DAM of day D consists of a series of twenty-four hourly auctions which are cleared simultaneously between 10:00h and 10:30h of the previous day (D-1). Selling and buying agents must submit their sale/purchase bids to each auction before 10:00h of D-1. Both sale and purchase bids are composed of up to 24 price-energy pairs with non-increasing price values, and each agent is unaware of the bids of the other agents. The clearing price  $\lambda_t^D$  of each hourly auction for time t is determined by the intersection

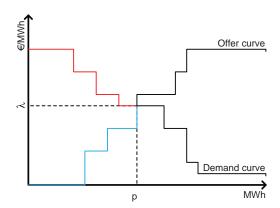


Figure 1: Market clearing for a certain hour: intersection of the aggregated offer and demand curves.

of the aggregated offer and demand curves: Fig. 1 . All the sale/purchase bids with a lower/greater bid price are matched and will be remunerated at the same clearing price  $\lambda_t^D$ , whichever the original bid price.

Bilateral contracts (BC) are agreements between a GenCo and a qualified consumer to provide a given amount of electrical energy at a stipulated price along with a delivering period. The agreements terms, namely: the energy, the price and the delivered period, are negotiated several days before the DAM, and the energy that is destined to the BC cannot be included in the DAM. Moreover, accordingly to the MIBEL rules, the DAM bid of each unit must include the whole available energy not allocated to the BC. This fact makes the optimal sale bid and the optimal BC's dispatching mutually dependents, coupling both problems. From the point of view of the GenCo, a BC represents a scheduled load curve to be delivered, chargeable at a fixed price, that has to be optimally dispatched among the GenCo's units.

A future contract (FC) is an exchange-traded derivative that represents agreements to buy/sell some underlying asset in the future at a specified price [3]. The DAM's operator demands every GenCo to commit the quantity designed to each FC through the DAM bidding of a given sets of generation units. This commitment is done through a sale offer with a bid price of 0€/MWh, the so called *price acceptance offer*. All price acceptant offers will be matched (i.e. accepted) in the clearing process, i.e., the energy shall be produced and will be remunerated at the DAM spot price.

## 1.2. EU National Emission Reduction Plan (NERP)

The EU envisages a Community Strategy to combat acidification within the EU. One of the main objectives of the strategy is "not to exceed, at any time, critical loads and levels" of certain acidifying pollutants such as  $SO_2$  and  $NO_x$ , so that both, people and ecosystems are protected effectively against the risks of air pollution. The EU's NERP directive (Directive 2001/80/EC [2]) applies to combustion plants (technical apparatus in which fuels are oxidized in order to use the heat thus generated) with a

rated thermal input equal to or greater than 50 MW, irrespective of the type of fuel used (solid, liquid or gaseous). This directive limits the amount of sulphur dioxide (SO<sub>2</sub>), nitrogen oxides (NO<sub>x</sub>) emitted form large combustion plants each year. Following this commitments, the Spanish public administration launched in 2004 the Spanish National Emissions Reduction Plan (NERP, Real Decreto 430/2004 [4]). The Spanish NERP imposes, for the period 2008-15, a global reduction of 81% of SO<sub>2</sub> and 15% of NO<sub>x</sub> emissions, compared to emissions in 2001.

### 1.3. Generation Units

In the wholesale electricity market there are a variety of units available for generating electricity, and each has its own characteristics. This work considers a GenCo with a set of coal thermal units (high emission technology) and combined cycle gas turbines (CCGT) generation units (low emission technology).

Thermal generation units have been part of the energy story for nearly fifty years. Thermal energy is generated by burning coal, natural gas, oil, or the combustion of diesel. In thermal power plants electricity is generated by burning fossil fuels such as coal, natural gas, or petroleum (oil). Fuel (coal for this study) and compressed air are mixed in a combustion chamber and ignited. This combustion produces heat that is used to heat water, which turns into steam, and spins a steam turbine rotor which drives an electrical generator to produce electricity. Unfortunately, complete conversion of fuel into energy is no possible. Consequently, fuel gas and cooling water from combustion of the fossil fuels are discharged into the air.

The combined cycle gas turbine units [5] represent a combination of combustion and steam turbines within a power plant. The CCGT plants employ more than one thermodynamic cycle. Water is heated and turns into steam, the heat captured from the combustion turbine exhaust gas, is used in the heat recovery steam generator and spins a steam turbine which consequently drives an electrical generator to produce electricity. This additional electricity improves the efficiency of electricity generation [6]. Currently, in Europe most of the new generating unit installations are CCGT units. They are between 20 and 30 % more efficient than thermal power plants, and can reach up to 60 % of efficiency. They are fast response units, which can be a quite decisive feature in facing fluctuations in the load and generation of the whole electricity production system. Moreover they are less climate-damaging as they don't produce  $SO_2$  emissions at all, and the  $NO_x$  emissions of a CCGT plant are negligible in comparison with that of thermal units.

## 1.4. Literature review

The most important part of the published works concerning the relation between the energy production and the pollutant emissions have been devoted to study the impact of  $CO_2$  emissions trading in the power industry, specially through mid-term ([7, 8, 9]) but also short-term ([10]) models. Although the NERP modifies substantially the shape of the optimal bid strategy of an electricity producer quite few attention has been given in the bibliography to the optimal generation bid strategies under  $SO_2$  and  $NO_x$  emission limits. Most of the research production in relation with  $SO_2$  and  $NO_x$  has been long-term studies of different aspects of the impact of the  $SO_2$  and  $NO_x$  emissions reduction

in the wholesale electricity production system ([11, 12, 13]). Among the few papers that studies the optimal generation under emission limits, [14] develops a load dispatch model to minimize the  $NO_x$  emissions taking the fuel cost and stochastic wind power availability as constraints, disregarding the electricity market. The model in [15] considers a deterministic unit commitment of both thermal and combined cycle units that minimizes the generation (fuel) costs satisfying simple bounds to the  $SO_2$  and  $NO_x$  emissions. A quite common approach in several recent papers to the handling of emission limits are multiobjective optimization techniques where both the profit and emissions are minimized [16, 17], sometimes with additional emissions limit constraints [18]. Despite the interest of all these studies it is worth mentioning that none of them can be considered as optimal-bid models, as their formulation doesn't incorporate the bid rules of the electricity market, limiting the influence of the electricity market to the expression of the total profits either through deterministic forecasted electricity prices [16, 17, 18] or spot price scenarios [19].

### 1.5. Contribution

This work presents a new stochastic programming model to cope with the optimal generation bid to the next day auctions of the MIBEL day-ahead market (DAM) taking into account the SO<sub>2</sub> and NO<sub>x</sub> emission limits of the Spanish NERP. We consider a price taker GenCo with a set of thermal coal and CCGT generation units subject to  $SO_2$  and  $NO_x$  emissions limits. The objective is to find the generation scheduling and sales bid of each one of the generators that maximize the expected value of the net profit of a Genco including the start-up, shut-down and generation costs together with the incomes from the day-ahead market, futures and bilateral contracts. Several characteristics distinguish this paper from the previous works in this area. Contrary to other studies, our model consider the ex-ante negotiated Futures Contracts (FC) and Bilateral Contracts (BC) of the GenCo, that are integrated in the optimal bidding strategy according to the MIBEL directives and provides the optimal generation bid for each one of the generation units assuming the optimal offer curve model developed in [20, 21]. Moreover, the day-ahead electricity market bid with futures and bilateral contracts model (DAMB-FBC) proposed in [21] has been improved in the present work through the formulation of the CCGTs unit commitment modelization introduced in [22] together with the explicit consideration of the NERP emissions limits through a new specific measure of risk called Conditional Emission-at-Risk (CEaR), which is one of the most important contributions of this paper. The resulting model for the optimal day-head market bid with emission risk (DAMB-ER) has been validated with real data from generation units operating in the MIBEL and with real prices from the Spanish day-ahead market and used to study the impact of the Spanish NERP directives onto the optimal generation bid and expected utilities' profit. The results can be easily extrapolated to any country with similar NERP regulation.

The remainder of this paper is organized as follows. Section 2 develops the proposed day-ahead market bid model with emission risk (DAMB-ER). In section 3 a case study with real data of the MIBEL is solved and analyzed. Finally section 4 presents the conclusions of the work.

## 2. Emission Risk Constrained Model for the Optimal Electricity Generation Bid

As a consequence of the deregulation of the countrywide energy production system through the settlement of liberalized electricity markets, the price of electricity has become a significant risk factor because it is unknown at the moment when generation companies have to take operational decisions. This means that the market price has to be considered as a random variable whose realization is only known once the market has been cleared. Stochastic programming [23] provides a powerful and well established methodology to tackle with this uncertainty incorporating in a single mathematical optimization model the available statistical information on the relevant random variables. In this section we begin with a brief description of the so called day-ahead market bid model with futures and bilateral contracts (DAMB-FBC), a twostage stochastic optimization model recently developed in [21] and [22] which allows a GenCo to optimally decide the unit commitment of its generation units, the economic dispatch of the bilateral and futures contracts, and the optimal generation bid to the day-ahead market. Model (DAMB-FBC) is then extended in the second part of the section to cope with the Spanish NERP through a new emission risk measure called Conditional Emission at Risk.

## 2.1. Base model: optimal day-ahead market bid with futures and bilateral contracts

The basic day-ahead market bid model with futures and bilateral contracts (DAMB-FBC) considered in this work was developed in [21] and subsequently extended in [22] to include combined cycle gas turbines (CCGT)units. Lets consider a price-taker GenCo owning a set of thermal generation units  $\mathcal{I}$  and a set CC units that bid to the  $t \in \mathcal{T} = \{1, 2, ..., 24\}$  hourly auctions of the DAM. Each one of the different operation modes of the CCGT units described in section 1.3 is considered as an pseudo-unit, being  $\mathcal{P}$  the set of pseudo-units of all the CCGT units considered (see [22] for more details). As a consequence, the total set of generation units considered by the model is  $\mathcal{U} = \mathcal{I} \cup \mathcal{P}$ , and the parameters for the generation unit  $i \in \mathcal{U}$  are:

- $c_i^b, c_i^l, c_i^q$ : constant, linear and quadratic coefficients of the generation costs function ( $[\in]$ ,  $[\in]/MWh]$  and  $[\in]/MWh^2]$  respectively).
- $\overline{P}_i, P_i$ : upper and lower bounds on the energy generation: [MWh].
- $c_i^{on}, c_i^{off}$ : start-up and shut-down costs [ $\in$ ].
- $t_i^{on}, t_i^{off}$ : minimum operation and minimum idle time [h].
- $st_i^0$ : Number of hours the unit has been on  $(st_i^0 > 0)$  or off  $(st_i^0 < 0)$  previous to the first time period.

The parameters defining a base load physical futures contract  $j \in \mathcal{F}$  are:

- $\mathcal{I}_j \in \mathcal{U}$ : the set of generation units allowed to cover the FC j: .
- $L_j^F$ : the amount of energy [MWh] to be procured each interval of the delivery period by the set  $\mathcal{I}_j$  of generation units to cover contract j.

•  $\lambda_i^F$ : the price of contract  $j \in MWh$ ].

And the parameters defining a base load bilateral contract  $k \in \mathcal{B}$  are:

- $L_{tk}^B$ : the amount of energy [MWh] to be procured during hour t of the delivery period by the set of available generation units to cover the BC k.
- $\lambda_k^B$ : the price of the contract  $k \in MWh$ ].

The random variable  $\lambda^D_t$ , the clearing price of the  $t^{th}$  hourly auction of the DAM, is represented in the two-stage stochastic model by a set of scenarios  $s \in \mathcal{S}$ , each one with its associated clearing price for each DAM auction  $\lambda^{D,s}_t$ ,  $t \in \mathcal{T}$  and the corresponding probability  $P^s$  [24].

The first-stage (here and now) variables of model (DAMB-FBC) are, for every time period  $t \in \mathcal{T}$  and generation unit  $i \in \mathcal{U}$ :

- $u_{ti} \in \{0,1\}$ : the unit commitment binary variables, expressing the on-off operating status of the  $i^{th}$  unit.
- $c_{ti}^u, c_{ti}^d$ : the start-up/shut-down costs variables [ $\in$ ].
- $q_{ti}$ : the price acceptant offer bid [MWh].
- $f_{tij}$ : the scheduled energy for futures contract  $j \in \mathcal{F}$  [MWh].
- $b_{ti}$ : the scheduled energy for bilateral contract [MWh].

Finally, the second stage (wait and see) variables are, for each  $t \in \mathcal{T}$ , generation unit  $i \in \mathcal{U}$  under scenario  $s \in \mathcal{S}$ .:

- $g_{ti}^s$ : the total generation under scenario s [MWh].
- $p_{ti}^s$ : the matched energy in the day-ahead market under scenario s [MWh].

Considering the parameters and variables described below, the (DAMB-FBC) model is defined in [21] as:

$$(DAMB-FBC) \begin{cases} \max h(u,c^u,c^d,g,p,b,f) \\ \text{s.t.:} \\ \sum_{i \in \mathcal{I}_j} f_{tij} = L_j^F \\ \sum_{i \in \mathcal{I}_j} f_{tij} = L_{tk}^F \\ \sum_{i \in \mathcal{I}_j} f_{tij} \\ \sum$$

where the interpretation of the different set of constraints is as follows:

- (1a)-(1d): guarantees the coverage of both the physical futures and bilateral contracts obligations.
- (2a)-(2d): incorporates in the model the MIBEL's rules through which the energies  $L_j^F$  and  $L_{tk}^B$  of the futures and bilateral contracts are integrated in the dayahead market bid of a generation unit. The first rule is that if generator  $i \in \mathcal{U}$  contributes with  $f_{tij}$  MWh at period t to the coverage of the FC j, then the energy  $f_{tij}$  must be offered to the pool for free embedded into the price acceptance sale bid (2a). The second rule establishes that if generator  $i \in \mathcal{U}$  contributes with  $b_{ti}$  MWh at period t to the coverage of any of the BCs, then only the remaining production capacity  $\overline{P}_i b_{ti}$  can be bid to the DAM (constraints (2b) and (2c)).
- (3a),(3b): defines the total generation level of a given unit  $i, g_{ti}^s$ , as the addition of the allocated energy to the BC, plus the matched energy in the DAM, and restricts the total generation output to be  $g_{ti}^s \in \{0\} \cup \underline{P_i}, \overline{P_i}$ .
- (4): restricts the unit commitment variables (those related with the on-off state of each generation unit) to belong to the feasible unit commitment polyhedron  $P_{UC}$ . This feasible polyhedron contains all the values of the binary unit commitment variables u that satisfies the minimum operation and minimum idle time  $t^{on}$  and

 $t^{off}$ , the initial state  $s^0$ , and defines conveniently the value of the start-up/shut-down costs variables  $c^u$ ,  $c^d$ . The linear constraints that define this polyhedron are quite intricate and has been omitted here for the sake of clarity. The detailed formulation of these constraints can be found in [22].

Finally, the objective function h of the model accounts for the expected value of the total profit obtained by the GenCo, and is represented by the following expression:

$$\begin{split} h(u,c^u,c^d,g,p,b,f) = & E_{\lambda^D}\left[profit\right] = \\ = & |\mathcal{T}|\left[\sum_{k\in\mathcal{F}}\lambda_k^FL_k^F\right] + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{B}}\lambda_{tj}^{^{BC}}L_{tj}^{^{BC}} \end{split} \tag{5a}$$

$$-\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left[c_{ti}^{u}+c_{ti}^{d}+c_{i}^{b}u_{ti}\right] \tag{5b}$$

$$-\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \left[ c^u_{t\mathcal{P}_c(1)} + c^u_{t\mathcal{P}_c(2)} + \sum_{i \in \mathcal{P}_c} c^b_i u_{ti} \right]$$
 (5c)

$$+ \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{U}} \sum_{s \in S} P^{s} \left[ \lambda_{t}^{\scriptscriptstyle D,s} p_{ti}^{s} - (c_{i}^{l} g_{ti}^{s} + c_{i}^{q} (g_{ti}^{s})^{2}) \right] \tag{5d}$$

where:

- (5a): corresponds to the incomes of the FCs and the BCs, where  $\lambda_k^F$ ,  $L_k^F$  and  $\lambda_{tj}^{BC}$ ,  $L_{tj}^{BC}$  are the prices and energies of FCs and BCs respectively. Both the energy and price of these contracts has been fixed long before the moment when the bid to the day-ahead market is being decided, and then this is a known constant term in our objective function.
- (5b): accounts for the on/off fixed cost of the unit commitment of the thermal units. This term is independent of the realization of the random variable  $\lambda_t^D$ .  $c_i^b$  are the constant coefficients of the generation costs ( $\in$ ).
- (5c): CC's start-up and fixed generation costs. Only start-up costs are associated to the PU, and no cost is associated to the transition from state 2 to state 1. This term does not depend on the realization of the random variable  $\lambda^D$ .
- (5d): represents the expected value of the benefits from the day-ahead market, where  $P^s$  is the probability of scenario s. The term between brackets corresponds to the expression of the quadratic generation costs with respect to the total generation of the unit,  $g_{ti}^s$ .

## 2.2. Conditional Emission at Risk (CEaR)

The Spanish National Emission Reduction Plan imposes limits  $\overline{SO_2}$  and  $\overline{NO_x}$  [kg/day] to the joint emission of the thermal units (CCGT units are excluded). Of course these limitations could be included in the model (DAMB-FBC) by simply imposing an emission limit at every scenario s through the following set of constraints [22]:

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \le \overline{SO_2}$$
  $s \in \mathcal{S}$  (6)

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{T}} e_i^{NO_x} g_{ti}^s \le \overline{NO_x} \qquad s \in \mathcal{S}$$
 (7)

where, as usual, the emissions of a thermal unit are assumed to be linear w.r.t. the total generation  $g_{ii}^s$  with emission coefficients  $e_i^{SO_2}$  and  $e_i^{NO_x}$  [kg/MWh] that depend on the generation technology. Actually, this formulation would be a simple stochastic extension of the deterministic approach adopted by the few previous works that incorporate explicitly the emission limits [15, 18] by imposing an upper bound to the accumulated emissions along the complete optimization horizon. This approach, although valid is quite restrictive as it forces the optimal bid to abide by the NERP rules even in the most extremes (less likely) scenarios.

The risk management ideas developed for the financial markets offers a new and more flexible approach to address the  $SO_2$  and  $NO_x$  emission constraints. Risk management is the theory about how to handle risks. Classical risk management methods in portfolio optimization theory, include *Value-at-Risk* (VaR) and *Conditional Value-at-Risk* (CVaR) [25, 26]. By analogy to the CVaR concept developed to monitor losses within pre-specified tolerances, a new concept, the Conditional Emission-at-Risk (CEaR) is proposed in this work as a tool to measure and control the risk of violating the NERP emission limits. To this end, we start by formulating the following constraints:

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \le M^{SO_2} y^s \qquad s \in \mathcal{S}$$
 (8)

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \ge M^{SO_2} (y^s - 1) \qquad s \in \mathcal{S}$$
 (9)

$$\sum_{s \in S} P^s y^s \le \gamma \tag{10}$$

The first two equations (8) and (9) conveniently classify the scenarios in which the  $SO_2$  emission exceed the limit.  $y^s$ ,  $s \in \mathcal{S}$  is a binary variable that takes value 1 if the emissions are higher than  $\overline{SO_2}$  and 0 otherwise, and parameter  $M^{SO_2}$  is an upper bound of the emission violation, that is:

$$-M^{SO_2} \leq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s - \overline{SO_2} \leq M^{SO_2}$$

Of course, a trivial valid value for  $M^{SO_2}$  could be  $|\mathcal{T}|\sum_i e_i^{SO_2}\overline{P}_i$ . Equation (10), in turn, limits the joint probability of those scenarios exceeding the upper bound  $\overline{SO_2}$ . Thus, instead of imposing an emission limit at every individual scenario s, this approach allows exceeding the limit, with a given probability not greater than  $\gamma$ . It is worth mentioning that when  $\gamma=0$  constraints (8)- (10) are equivalent to constraints (6), that is, to impose that no scenario can exceed the emission limit. Furthermore,

taking  $\gamma=1$  is equivalent to not to impose any limit at all (and then to revert to the base model (DAMB-FBC)).

The above three constraints: (8) - (10) are the base to the development of a CVaR-like model to limit the average amount by which the emissions can exceed the limit. We will develop, by analogy with the CVaR function, the so-called Conditional Emissionat-Risk (CEaR) in order to establish a new measure of risk associated with the expected value of  $\overline{SO_2}$  violation. To this end, let's define first for every scenario s, a new set of variables  $eS^s$  whose value will be equal to the value of the  $SO_2$  emissions  $(\sum\limits_{t\in\mathcal{T}}\sum\limits_{i\in\mathcal{I}}e_i^{SO_2}g_{ti}^s)$  if the emission of scenario s exceed the limit (that is, if  $y^s=1$ ), or 0 if the emission of scenario s is below the limit (that is, whenever  $y^s=0$ ):

( — — aa

$$eS^{s} = \begin{cases} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_{i}^{SO_{2}} g_{ti}^{s} & \text{if } y^{s} = 1\\ 0 & \text{if } y^{s} = 0 \end{cases}$$

Eq. (11) - (13) below express variables  $eS^s$  in a way amenable to the optimization model:

$$eS^s - \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \le M^{SO_2} (1 - y^s) \qquad s \in \mathcal{S}$$
 (11)

$$eS^s - \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{SO_2} g_{ti}^s \ge -M^{SO_2} (1 - y^s) \qquad s \in \mathcal{S}$$
 (12)

$$eS^s \le M^{SO_2} y^s \qquad \qquad s \in \mathcal{S} \tag{13}$$

Let consider now a solution g,y satisfying Eq. (8) - (13). Then, for any given probability level  $\gamma$  and emission limit  $\overline{SO_2}$ , the conditional emission at risk  $CEaR_{\gamma}^{\overline{SO_2}}$  associated to g,y is defined as the expectation of the  $SO_2$  emissions for those scenarios exceeding  $\overline{SO_2}$ :

$$CEaR_{\gamma}^{\overline{SO_2}} = \frac{1}{\sum\limits_{s \in S} P^s y^s} \sum\limits_{s \in S} P^s eS^s$$

Then, the following risk-limiting constraint allows to control the amount by which the expectation of the violating emissions  $CEaR_{\gamma}^{\overline{SO_2}}$  surpasses the limit  $\overline{SO_2}$ :

$$CEaR_{\gamma}^{\overline{SO_2}} \le (1+\beta)\overline{SO_2}$$

where the parameter  $\beta \geq 0$  (usually < 1), the violation factor, represents the maximum permitted violation as a fraction of the emission limit  $\overline{SO_2}$ . The last inequality ensures that, in case that emissions are above the limit, the expected violation will be, on the average, less than a fraction  $\beta$  of  $\overline{SO_2}$ . Note that when  $\beta=0$  no scenario can exceed the emission limit, irrespective of the value of  $\gamma$ . In order to incorporate the last two equations in a mathematical programming model it is convenient to combine them in the following single linear inequality:

$$\sum_{s \in S} P^s e S^s \le (1+\beta) \overline{SO_2} \sum_{s \in S} P^s y^s \tag{14}$$

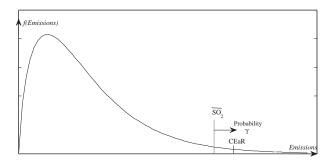


Figure 2: Graphical representation of CEaR concept.

It is worth mentioning that the value  $\overline{SO_2}$  imposed by the NERP plays the role of the VaR level in the classical CVaR definition. The definition of  $CEaR_{\gamma}^{\overline{SO_2}}$  is illustrated graphically in Fig. 2 where f(Emissions) represents the probability density function of the  $SO_2$  emissions.

Similarly to what has been done in the case of  $SO_2$  emissions, it is possible to formulate the  $NO_x$  CEaR's risk constraints through the following set of constraints:

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \le M^{NO_x} z^s$$
  $s \in \mathcal{S}$  (15)

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s - \overline{NO_x} \ge M^{NO_x} (z^s - 1)$$
  $s \in \mathcal{S}$  (16)

$$\sum_{s \in S} P^s z^s \le \gamma \tag{17}$$

$$eN^s - \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s \le M^{NO_x} (1 - z^s)$$
  $s \in \mathcal{S}$  (18)

$$eN^s - \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} e_i^{NO_x} g_{ti}^s \ge -M^{NO_x} (1 - z^s)$$
  $s \in \mathcal{S}$  (19)

$$eN^s \le M^{NO_x} z^s \qquad \qquad s \in \mathcal{S} \tag{20}$$

$$\sum_{s \in S} P^s e N^s \le (1+\beta) \overline{NO_x} \sum_{s \in S} P^s z^s \tag{21}$$

The set of variables  $z^s$  and  $eN^s$ , and parameter  $M^{NO_x}$  are defined analogously to  $y^s$ ,  $eS^s$  and  $M^{SO_2}$  respectively.

## 2.3. Optimal day-ahead market bid model with emission risk constraints

We are now ready to extend the basic optimal day-ahead market bid model presented in section 2.1 with the Conditional Emission at Risk formulation of the national emission reduction plan developed in section 2.2. The resulting optimal day-ahead market bid model with emission risk (DAMB-ER) $_{\gamma,\beta}$  can be expressed through the following mathematical optimization problem:

$$(\text{DAMB-ER})_{\gamma,\beta} \left\{ \begin{array}{ll} \max & h(u,c^u,c^d,g,p,b,f) \\ \text{s.t.} : & \\ & \text{Eq. } (1a) - (1d) & \text{FC and BC} \\ & \text{Eq. } (2a) - (2d) & \text{Day-ahead market} \\ & \text{Eq. } (3a) - (3b) & \text{Total generation} \\ & \text{Eq. } (4) & \text{Unit commitment} \\ & \text{Eq. } (8) - (14) & CEaR_{\gamma}^{\overline{SO_2}} \leq (1+\beta)\overline{SO_2} \\ & \text{Eq. } (15) - (21) & CEaR_{\gamma}^{\overline{NO_x}} \leq (1+\beta)\overline{NO_x} \end{array} \right.$$

Problem (DAMB-ER) $_{\gamma,\beta}$  is a mixed, continuous-binary concave quadratic maximization problem with linear constraints with a well defined global optimal solution.

## 3. Case Study

The model (DAMB-ER) $_{\gamma,\beta}$  developed in the previous section depends parametrically on the confidence probability  $\gamma$  and the violation factor  $\beta$ . The different combinations of values of  $\gamma$  and  $\beta$  between this two extreme cases provides information that will be used in this section to asses the impact of the emission limits onto the optimal generation bid and the expected profits. Model (DAMB-ER) $_{\gamma,\beta}$  has been implemented with the AMPL modeling language [27] and solved with CPLEX 12.4 [28] (mipgap=0.01) over a Fuji RX200 S6 (2 x CPUs Intel Xeon X5680 six core - 12 threads 3.33 GHz, 64Gb RAM), taking advantage of the multithreading capabilities of CPLEX (threads=20).

#### 3.1. Data set

This study uses the same set of 50 scenarios generated in [29] for the random dayahead market spot prices  $\lambda^D$  as the result of the application of a scenario reduction algorithm [30] to the complete set of historic data available from June 2007 to May 2010 available at the website of the Independent Iberian Market Operator OMIE [31]. The generation units of this study correspond to four thermal units and two combined cycle units currently operating in the MIBEL, and its technical characteristics are shown in Table 1 and 2. Table 3 shows the number, energy and price of the bilateral and future contracts. All data related with  $SO_2$  and  $NO_x$  can be obtained from Table 4. The emission limits  $\overline{SO_2}$  and  $\overline{NO_x}$  derive from the National Emission Reduction Plan [4]. The  $SO_2$  and  $NO_x$  emissions rates shown in Table 4 correspond to the values published by the Intergovernmental Panel on Climate Change Emission [32] for coal thermal units. Further details can be obtained from [33].

## 3.2. Impact of the NERP in the expected profits: parameterized efficient frontier

As mentioned earlier, the risk constrained model (DAMB-ER) $_{\gamma,\beta}$  defines a family of problems parameterized by the risk factors  $\gamma$  and  $\beta$  that can be used to asses the dependence of expected returns with the risk level. The *efficient frontier* (Fig. 3) defines the maximum expected profit that can be achieved by a GenCo for a given risk level,

Table 1: Operational Characteristics of the Thermal Units

$\mathcal{I}$	$c_i^b \in$	$c_i^l$ $\in$ /MWh	$c_i^q$ $\in$ /MWh <sup>2</sup>				ı	$c_i^{off} \in$	$t_i^{on}, t_i^{o\!f\!f}$ hr
1	159.24	42.55	0.016	160.0	350.0	+3	435.09	435.09	3
2	901.70	59.38	0.038	250.0	563.2	+1	1307.70	1307.70	3
3	344.68	30.41	0.038	160.0	370.7	-1	462.07	462.07	3
4	322.04	60.04	0.032	160.0	364.1	+1	682.04	682.04	3

Table 2: Operational Characteristics of the Combined Cycle Units

CCGT	$\mathcal{P}$	ı	$c_i^l$ $\in$ /MWh	$c_i^q \in /MWh^2$				· ·	$t_i^{on}$ hr	· ·
1	5	151.08	50.37	0.023	160.0	350.0	-2	803.75	2	3
1	6	224.21	32.50	0.035	250.0	563.2	-2	412.80	2	3
2	7	163.11	55.58	0.019	90.0	350.0	-3	320.50	2	3
2	8	245.32	31.10	0.022	220.0	700.0	-3	510.83	2	3

Table 3: Characteristics of Futures and Bilateral Contracts

j	$L^{{\scriptscriptstyle BC}}_{j,t=124}$ MW	$\lambda_{j,t=124}^{\scriptscriptstyle BC}$ $\in$ /MWh	$L^{\scriptscriptstyle FC}_{j,t=124} \\ {\rm MW}$	$\lambda_{j,t=124}^{FC}$ $\in$ /MWh
2	164	43,35	120	45,6
	50	43,35	120	46,1
	150	43,35	120	51,2

Table 4: Daily emission limits data and thermal unit's emissions data

$\overline{SO_2}$ kg/day	$\overline{NO_x}$ kg/day	$\begin{array}{c} e_i^{SO_2} \\ \text{kg/MWh} \end{array}$	$e_i^{NO_x}$ kg/MWh
3.900	11.460	0.7848	1.368

defined in our model by the two parameters  $\gamma$  and  $\beta$  (actually, the efficient frontier is an "efficient surface" in our case). It can be observed in Fig. 3 that as  $\gamma$  increases, emissions may exceed the limit with greater probability and consequently the expected profit increases. Moreover, for any given confidence probability  $\gamma$  if the average percentage at which emissions exceeds the limit is increased (i.e. if  $\beta$  increases), then the expected value of the profits increases accordingly. There are two extreme cases in Fig. 3

- The bottommost, flat, curve associated to  $\gamma = 0$  that corresponds to the most restrictive optimization problem (DAMB-ER)<sub>0,\beta</sub> where no scenario is allowed to violate the limit (being therefore irrelevant the value of \beta). It is worth mentioning that this case is equivalent to the base model (DAMB-FBC) plus the emission constraints (6)-(7).
- The topmost curve associated to  $\gamma=1$  that corresponds to the less restrictive optimization problem (DAMB-ER)<sub>1,\beta</sub> where any scenario is allowed to violate the emission limit by an amount not greater, on the average, than a fraction  $\beta$  of the maximum emission. The optimal solution of the limiting case  $\beta \to \infty$ , or  $\beta=1$  in practice, (DABM-ER)<sub>1,1</sub>, coincides with the base model (DAMB-FBC).

The economical information provided by the parameterized efficient frontier in Fig. 3 is an example of how model (DAMB-ER) $_{\gamma,\beta}$  can be used by GenCo as a tool to asses several decisions related with the electricity generation under NERP regulations. An example of this would be to determine the convenience of the installation of a SO<sub>2</sub> and NO<sub>x</sub> capture technology with a given capacity and fault probability  $\gamma$ , by comparing the cost of the installation with the increase in the expected profits between the zero-risk case (DAMB-ER)<sub>0,0</sub> and the case (DAMB-ER) $_{\gamma,\beta}$  with a violation  $\beta$  representing the capacity of the capture device.

## 3.3. Impact of the NERP in the optimal generation bid

The purpose of this section is to study in detail the effect of the risk constraints in the optimal generation bid. The study will be based on the comparison of the optimal solution of the original problem (DAMB-FBC), where no emission limits is considered, with the optimal solution of problem (DAMB-ER)<sub>0.3,0.15</sub>, as a representative element of the parameterized family (DAMB-ER) $_{\gamma,\beta}$ . The optimal solution of problem (DAMB-ER)<sub>0.3,0.15</sub> is such that the expected matched generation in the day-ahead market will violate the NERP limits with a probability of 0.3, with an expected violation lesser than a 15% of the emissions limits. The dimensions and execution time of both problems are indicated in Table 5

Table 6 depicts the expected value of the  $SO_2$  and  $NO_x$  emissions, at the optimal solution of the two cases. The reduction of the  $SO_2$  and  $NO_x$  emissions is 36.4% and 51.6% respectively, with a decrease in the expected profits of just a 2.9%. It is interesting to mention that the expected  $SO_2$  emission at the optimal solution of model (DAMB-ER)<sub>0.3,0.15</sub> (3.903 kg/day) is slightly greater than the NERP limit (3.900 kg/day), which doesn't represent any inconsistency as the considered confident probability,  $\gamma = 0.3$ , is greater than zero.

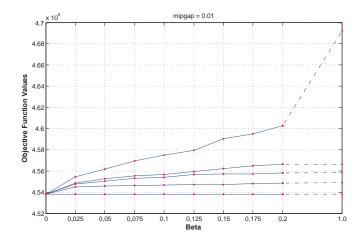


Figure 3: Parameterized efficient CEaR frontier of the problem  $(DAMB-ER)_{\gamma,\beta}$ , showing the change in the value of the expected profits as a function of parameters  $\gamma$  and  $\beta$ . The different curves corresponds, from bottom to top, to the values  $\gamma \in \{0, 0.1, 0.3, 0.5, 1\}$ . Over each one of these curves, the dots denotes the computed optimal expected value of the problems  $(DAMB-ER)_{\gamma,\beta}$  for  $\beta \in \{0, 0.025, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 1.0\}$ .

Table 5: Characteristics of the optimization problems

	Variab	les		
Cases	Continuous	Binary	Constraints	Exec. time
(DAMB-FBC) (DAMB-ER) <sub>0.3,0.15</sub>	20.160 20.260	200 300	49.458 49.962	360 sec. 48 min.

Table 6: Expected daily emissions and profit variation

	$E[SO_2]$ kg/day	$E[NO_x]$ kg/day	$E_{\lambda^D} \left[ profit \right] \\ \in$
(DAMB-FBC)	6.139	14.665	469.597
$(DAMB-ER)_{0.3,0.15}$	3.903	7.104	455.757
Variation	-36.4%	-51.6%	-2.9%

Table 7: Total expected energy production

	(DAMB-FBC) MWh	(DAMB-ER) <sub>0.3,0.15</sub> MWh	Difference MWh	
Thermal 1	3301.4	448.0	-2853.4	(-86%)
Thermal 2	500.0	500.0	0.0	(0%)
Thermal 3	5030.2	3701.4	-1328.8	(-26%)
Thermal 4	320.0	320.0	0.0	(0%)
Total thermal	9151.5	4969	-4182.1	(-46%)
CCGT 1	4973.3	5617.8	644.4	(+13%)
CCGT 2	6829.4	8766.7	1937.3	(+28%)
Total CCGT	11802.7	14384.4	2581.7	(+22%)
Total thermal+CCGT	20954.3	19353.9	-1600.4	(-8%)

The effect of the NERP regulation on the aggregated daily expected energy generation of each unit can be observed in Table 7. The results show that under NERP regulation:

- The total expected production is reduced by 1600.4MWh (-8%), with a reduction of 4182.1MWh (-46%) of the thermal units' production and an increase of 2581.7MWh (+22%) of the CCGTs' production.
- The reduction in the thermal units is concentrated in thermal unit 1, which is switched off as soon as possible, with a decrease of 2853.4MWh (-86%) and thermal unit 3, which is kept in operation, but decreasing by 1328MWh (-26%) its expected production.
- The generation of the CCGT units increases by 644.4MWh for unit 1 (+13%) and 1937MWh for unit 2 (+22%).

Fig. 4 shows the impact of the emission risk constraints over the individual unit commitment of each generation unit, together with the optimal dispatch of the bilateral and future contracts among each generation unit for the (DAMB-FBC) problem (left column) and (DAMB-ER)<sub>0.3,0.15</sub> (right column). The blue area corresponds to the energy allocated to the bilateral contracts (variable  $b_{ti}$ ); the green area is the energy of the price acceptance bid  $q_{ti}$  that includes the energy allocated to the futures contracts  $f_{tij}$ . Finally, the yellow area is, for each generation i and period t, the expected value of the matched energy in the day-ahead market  $\sum_{s \in \mathcal{S}} P^s p_{ti}^s$ . Comparing the generation

profiles in Fig. 4 it is clear how the NERP regulation is affecting the unit commitment: all coal thermal generators (high-emission units) are shut-down early, except thermal unit 3, which is maintained in operation to satisfy future contract 3 (in the absence of future contracts, thermal unit 3 would have been kept shut-down all day long).

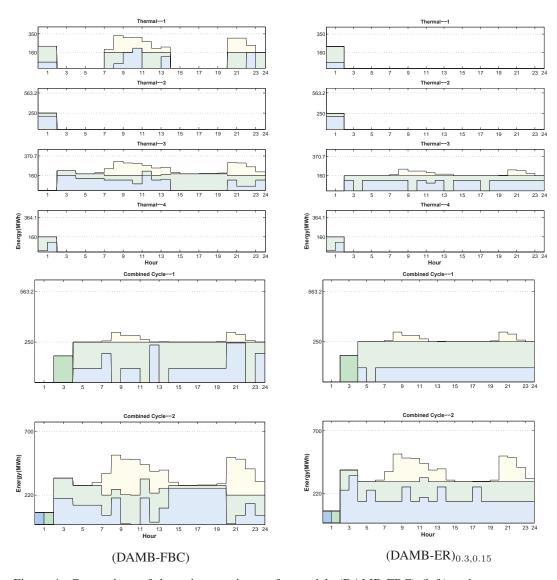


Figure 4: Comparison of the unit commitment for models (DAMB-FBC) (left) and (DAMB-ER) $_{0.3,0.15}$  (Right):  $b_{ti}$  (scheduled energy for bilateral contract, blue),  $q_{ti}$  (price acceptant bid, green). In yellow the expected value of matched energy. For the CC units, dark colors are for pseudo-unit 1 and light colors for pseudo-unit 2.

#### 4. Conclusions

Generation companies have to decide the daily generation bid to be submitted to the day-ahead electricity market, where, in Spain, a total of more than 30 millions of Euros are negotiated daily. The GenCo's optimal generation bid is aimed at both maximizing the expected profit and abiding by the different National Emission Reduction Plan of each country. The new competitive and environmentally constrained electricity supply industry requires new mathematical and computing tools to ensure both competitiveness with other generating companies in the electricity market and secondly, environmental protection by limiting damaging emission into the atmosphere. Trying to advance in that direction this work proposes a new two-stage stochastic programming model to cope with the optimal generation bid to the day-ahead electricity market of a GenCo operating a pool of thermal and combined cycle generation units and a given set of futures and bilateral contracts to be settled next day. The model takes into account the MIBEL market rules and the SO<sub>2</sub> and NO<sub>x</sub> emission limits of the current Spanish NERP regulation through a new measure of risk called Conditional Emissionat-Risk (CEaR). CEaR allows the formulation of a family of models (DAMB-ER) $_{\gamma,\beta}$ parameterized by the emission risk level defined by  $\gamma$  and  $\beta$  which gives a flexible tool to asses a wide range of decisions related with the electricity generation under NERP regulations. The computational experiments performed with real data of the Spanish wholesale electricity market provides the optimal dispatch of each individual thermal and CCGT unit among the different energy contracts an the day-ahead market. The numerical results show that, for a given representative risk level, the  $SO_2$  and  $NO_x$  NERP obligations can be met by reducing the expected total energy production by 8%, with a 3% decrease in the expected profits. This reduction of the total energy production is unevenly distributed among the generation technologies, with a 46% decrease of the thermal production against a 22% increase of the CCGT generation, confirming the central role of the CCGT technology in an environmental friendly energy production system.

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