Star-shaped mediation in influence games*

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We are interested in analyzing the properties of multi-agent systems [13] where a set of agents have to take a decision among two possible alternatives with the help of the social environment or network of the system itself. The ways in which people influence each other through their interactions in a social network and, in particular, the social rules that can be used for the spread of influence have been proposed in an alternative simple game model [11]. However not all individuals play the same role in the process of taking a decision. In this paper we are interested in formalizing and analyzing the simple game model that results in a *mediation system*. In this scenario we have a social network together with an external participant, *the mediator*. The mediator can interact, in different degrees, with the agents and thus help to reach a decision.

1 Preliminaries

The area of *simple games*, a subfamily of cooperative game theory, provides a formal model for analyzing decision systems [14]. A *simple game* is a set system providing the coalitions that can made an alternative pass. The ways in which people influence each other through their interactions in a social network have received a lot of attention in the last decade with important links to sociology, economics, epidemiology, computer science, and mathematics [1, 9, 6]. Agents face the choice of adopting a specific product or not, choose among competing programs from providers of mobile telephones, having the option to adopt more than one product at an extra cost, etc. Those decisions have to be taken by individuals participating in a social networks. A social network can be represented by a graph where each node is an agent, individual or player, and each edge represents the degree of influence of one agent over another one. Several "motivations" (ideas, trends, fashions, ambitions, rules, etc.) can be initiated by one or more agents and eventually be adopted by the system. The mechanism defining how these motivations are propagated within the network, from the influence of a small set of nodes initially *motivated*, is called the model for *influence spread*. In this subject, motivated by viral marketing and other applications, has been established the *influence maximization problem*

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[5, 12], and the *linear threshold* and *independent cascade models* for influence spread [10] among other ones [2, 4, 1]. In such a setting there is also work done towards analyzing the problem from the point of view of non-cooperative game theory [8].

In a recent paper we proposed a simple game based on a model of spread of influence in a social network, where influence spreads according to the linear threshold model, the so-called *influence games* [11]. Now we extend this model in order to incorporate two different levels of influence spread, the one taking place at the social network and the other one exerted by a *mediator*, again analyzed as a simple game.

2 Definitions and Results

For simple games, we follow definitions and notation from [14]. As usual, given a finite set $N = \{1, \ldots, n\}$ of *players* or *voters*, $\mathcal{P}(N)$ denotes its *power set*. A family of subsets $\mathcal{W} \subseteq \mathcal{P}(N)$ is said *monotonic* when $\forall X \in \mathcal{W}$, if $X \subseteq Z$, then $Z \in \mathcal{W}$.

Definition 2.1 *A* simple game *is a tuple* $(N, \mathcal{W})$ *where* N *is called the* grand coalition*, and* $\mathcal{W}$*, the set of* winning coalitions*, is a monotonic family of subsets of* N*.*

In the context of simple games, subsets not appearing in $\mathcal{W}$ are called *losing coalitions*, and a *minimal winning (maximal losing) coalition* is a winning coalition such that by removing (adding) one player results in a losing (winning) coalition. We use $\mathcal{W}$, $\mathcal{L}$, $\mathcal{W}^m$ and $\mathcal{L}^M$ to denote the sets of winning, losing, minimal winning and maximal losing coalitions. Any of those sets determine uniquely the game and constitute the usual forms of representation for simple games [14]. Before introducing formally the family of influence games we need to define a family of labeled graphs based on the *linear threshold model*. We use standard graph [3] and computational complexity [7] notation. Note that *player* (element of N), *coalition* (subsets of N), *winning* and *losing* for simple games usually mean, respectively, *agent* (vertex of V), *team* (subsets of V), *successful* and *unsuccessful* for influence games.

Definition 2.2 *An* influence graph *is a tuple* $(G; f)$*, where* $G = (V, E)$ *is a labeled and directed graph (without loops) —with* V *its set of vertices and* E *its set of edges— and* $f : V \to \mathbb{N}$ *is a labeling function that quantify how influenceable each player or agent is.*

Given an influence graph $(G; f)$ and an *initial activation* set $X \subseteq V$, the *spread of influence* $F : V \to V$ is a function such that $F(X)$ is the set of agents activated by an iterative process. Initially the vertices in X are activated, i.e., $X \subseteq F(X)$. Let be $\texttt{In}(u) = \{v \,|\, (v, u) \in E\}$, at each step any $u \notin F(X)$ such that $|\texttt{In}(u) \cap F(X)| \geq f(u)$ is added to $F(X)$. The process stops when no additional activation occurs.

Definition 2.3 *An* influence game *is a tuple* $(G; f, q)$*, where* $(G; f)$ *is an influence graph and* $q \geq 0$ *is an integer, the quota. A coalition* $X \subseteq V$ *is winning iff* $|F(X)| \geq q$*.*

In this paper we incorporate another influence layer. On the bottom layer the influence is exerted among the agents and on another layer the relationship of influence between the agents and an external mediator is kept. The mediator can exert influence on some nodes and accept advice from others, thus introducing a modification on the way that influence spreads through the network. We model the society by a set of nodes V where the relation with the mediator can be expressed by three disjoint sets $A, B, C \subseteq V$, where A is formed by the agents that can influence the mediator but are not influenced by him, B contains those agents that influence and can be influenced by the mediator and, finally, C is formed by the agents that can be influenced by the mediator but cannot exert influence. This kind of relationship can be understood by means of a star graph $(V \cup \{c\}, E)$ in which the mediator correspond to an additional vertex $c \notin V$ and where $E = \{(u, c) \mid u \in A \cup B\} \cup \{(c, v) \mid v \in B \cup C\}$.

In the following definition we assume that $A, B, C \subseteq V$ are disjoint.

Definition 2.4 *In a* star influence game $\Gamma(V, A, B, C, k, q)$*, where* $k, q \in \mathbb{N}$*, a coalition* $X \subseteq V$ *is winning iff either (1)* $|X| \geq q$ *or (2)* $|X \cap (A \cup B)| \geq k$ *and* $|X \cup B \cup C| \geq q$*.*

In a star mediation influence game $\Gamma(V, E, f, A, B, C, k, q)$*, where* $((V, E); f)$ *is an influence graph and* $k, q \in \mathbb{N}$*, a coalition* $X \subseteq V$ *is winning iff either (1)* X *is winning in the influence game* $((V, E); f, q)$ *or (2)* $|X \cap (A \cup B)| \geq k$ *and* $|X \cup B \cup C|$ *is winning in the influence game* $((V, E); f, q)$*.*

Theorem 2.5 *Given a star influence game, determine* $\mathcal{W}^m$ *or* $\mathcal{L}^M$ *can be done in incremental-polynomial time.*

Next we also analyze some properties of an influence game, conditions under which the system can reach an alternative, as decision problems from a computational point of view:

IsProper: Determine whether the complement of any winning coalition is losing.
IsStrong: Determine whether the complement of any losing coalition is winning.
IsDecisive: Determine whether a coalition is winning iff its complement is losing.

Moreover, other problems consider properties of an agent i with respect to an influence game:

IsDummy: In any winning coalition X including i, is $X \setminus \{i\}$ also winning?
IsPasser: Is any coalition including i winning?
IsVetoer: Is any coalition not including i losing?
IsDictator: Does the set of winning coalitions coincide with the set of coalitions including i?

Theorem 2.6 *Given a star influence game, the problems* IsProper*,* IsStrong*,* IsDecisive*,* IsDummy*,* IsPasser*,* IsVetoer *and* IsDictator *are in P.*

Theorem 2.7 *Given a star mediation influence game, the problems* IsProper*,* IsStrong*,* IsDecisive *and* IsDummy *are coNP-complete, while* IsPasser*,* IsVetoer *and* IsDictator *are in P.*

The above results are obtained by providing characterizations of the properties in terms of q, k, $|A|$, $|B|$ and $|C|$.

3 Future work

There remain many open problems, in particular to analyze whether different conditions on the influence relationship among the agents and the mediator can lead to characterizations or polynomial time algorithms for the problems considered in this paper or for other problems of interest, like some coming from social choice theory [15]. We have analyzed the results of the superposition of two influence networks in which one of them is restricted to be a star a future line of research is to analyze influence games resulting from the superposition of two or more complex social networks. Another area of interest is to consider weighted influence networks with edge weights or networks with several mediators related to different parts of the network with or without common agents.

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