The Hilbert Monopole Revisited

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***Abstract*— A comparison between the resonant performance of the third iteration Hilbert monopole and a spiral shaped one of the same size and resonant frequency is presented. Quality factor, bandwidth, efficiency and radiation patterns are investigated. Although the Hilbert monopole total wire length is longer, it shows nearly the same radiation efficiency yet a lower quality factor. Hence the higher compression efficiency of the spiral shaped monopole does not provide any advantage in the antenna electromagnetic performance. Differences in radiation pattern are also observed. It is demonstrated that antenna resonance performance depends on its geometry, and not only on its size or wire length.**

I. Introduction

The suitability of fractal geometries in the design of small wire resonant antennas has been broadly investigated. Small antennas like the Koch and the Hilbert monopole are described in the literature [1-3]. It has been demonstrated [2] that the Hilbert monopole becomes electrically smaller as the fractal iteration increases, being possible to reduce, in terms of resonance frequency, the electrical size of a classical λ/4 monopole up to factors of 11.

Extensive research on the behaviour of antennas with geometries based on the Hilbert curve has been developed [2-8]. The antenna performance properties of the Hilbert monopole have been compared with other geometries [9-11]. Total wire length, resonant frequency, quality factor, 2:1 SWR bandwidth and efficiency are explored. In these comparative studies, there is no mention about the radiation pattern properties which may vary from one geometry to another.

The resonant performance of the third iteration Hilbert monopole and the one of a spiral-shaped monopole is compared in [9]. Both antennas show the same resonant frequency and occupy the same planar area. Total wire length, resonant frequency, resonant resistance, radiation resistance, quality factor, 2:1 SWR bandwidth and efficiency are considered.

In the present paper the comparison of the resonant performance of the third iteration Hilbert monopole and the one of a spiral shaped monopole is revisited. A fully comparison is performed, exploring some antenna parameters that have not been considered before, such as for instance the radiation pattern, together with an additional discussion on bandwidth and quality factor. It will be demonstrated that the geometry of the antenna affects to its resonant performance.

This paper is divided as follows. Section II describes the geometries of the third iteration Hilbert and the spiral shaped monopoles and also investigates their compression efficiency. The compression efficiency is defined as the ratio between the total wire length of the antenna and the wire length that would have a straight vertical monopole resonating at the same frequency [2]. This parameter is related to the coupling effects associated to the antenna geometry. Section III shows the performance of the antennas. Discussion on quality factor, bandwidth, efficiency and radiation pattern is presented. Finally, Section IV contains the main conclusions drawn from this research.

II. Antenna Description

The third iteration Hilbert monopole and the spiral-shaped monopole have a planar area of 156.8x156.8 mm^2 and antenna height is 227 mm in both cases (Fig.1). Both antennas are made of copper and they are placed over and infinite groundplane. The width of the copper strip is 1 mm and its thickness is 0.035 mm. The value of copper conductivity is set to $5.8x10^7$ $S·m^{-1}$. The spiral shaped monopole wire length is selected so that its first resonance frequency matches the corresponding resonance of the third iteration Hilbert monopole.

Performance properties of these antenna designs are calculated with Zeland IE3D simulation software based on MoM.

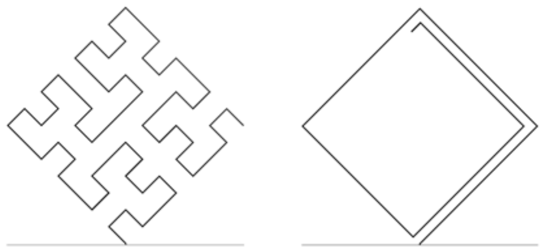

Fig. 1. Third iteration Hilbert monopole and spiral shaped monopole

Both antennas present the same first resonant frequency: 84.5 MHz. The Hilbert monopole shows lower compression efficiency, it needs 54% more wire to achieve the same resonant frequency than the spiral shaped monopole (Table I). The question is how this may impact the electromagnetic performance of the antenna.

TABLE I
COMPRESSION EFFICIENCY COMPARED TO A λ/4 STRAIGHT MONOPOLE

Antenna	Resonant Frequency (MHz)	Wire Length (mm)	Compression Efficiency (%)
Hilbert	84.5	1141.2	62.9
Spiral	84.5	916.8	96.8

At the resonant frequency, current vectors in the spiral shaped monopole are aligned in the same direction in the regions closely spaced (Fig.2), while the Hilbert monopole shows closely spaced regions with opposite current vectors reducing its effective length [12-13], which has been related to the fact that the spiral shaped monopole achieves higher compression efficiency than the Hilbert monopole.

Based on this; it seems that the Hilbert antenna would be less efficient since it needs more wire to resonate at the same frequency than the spiral-shaped antenna. Therefore the impact on the efficiency is analyzed. Moreover, bandwidth, quality factor and radiation pattern are also compared for both cases in the next section.

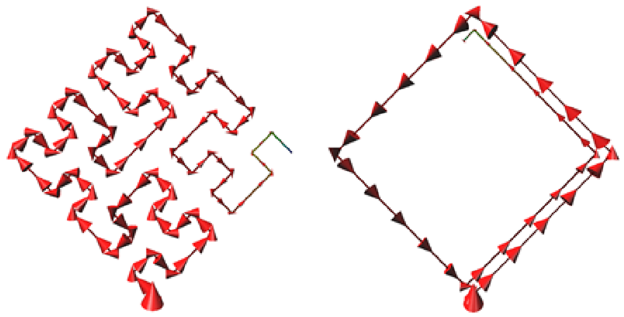

Fig. 2. Current vector distribution of the antennas at the resonance frequency of f=84.5 MHz

III. Antenna Performance

A. *Input impedance, quality factor and bandwidth*

The antenna quality factor is calculated using the following equation [14]

$$Q(\omega) \approx Q_Z(\omega) = \frac{\omega}{2R(\omega)} \cdot \sqrt{\frac{dR(\omega)}{d\omega} + \left(\frac{dX(\omega)}{d\omega} + \frac{|X(\omega)|}{\omega}\right)^2} \qquad (1)$$

where R and X are the antenna's resistance and reactance respectively and ω is de radian frequency equal to $2\pi f$, being f the operation frequency in Hz.

The quality factor is significant lower for the Hilbert monopole, which should be related to a more efficient use of the available volume inside de radiansphere (Fig.3).

The antenna VSWR bandwidth and the quality factor are related as given in [14]

$$Bw(\omega) \approx \frac{2\sqrt{\beta}}{Q\ (\omega)}, with \sqrt{\beta} = \frac{s-1}{2\sqrt{s}} \qquad (2)$$

where s is the VSWR limit established to compute the bandwidth. In this paper a VSWR=2 limit is used. This definition of the VSWR bandwidth assumes that the characteristic impedance of the transmission line connecting the antenna and the source is equal to the antenna's resistance at the tuned frequency.

Since both antennas feature a series RLC circuit around the resonant frequency, eq. (2) may be used to calculate the bandwidth from the quality factor (Fig.4).

The antennas' VSWR, is calculated under the condition that the characteristic impedance of the transmission line equals the antennas' respective resonant resistance (Fig 5). The Hilbert monopole shows larger bandwidth than the spiral shaped monopole. At the resonant frequency, 84.5 MHz, the Hilbert monopole presents higher resonant resistance, lower quality factor and larger 2:1 VSWR bandwidth than the spiral shaped monopole (Table II). Specifically, the Hilbert monopole shows a 2:1 VSWR bandwidth 1.6 times larger than the spiral antenna one.

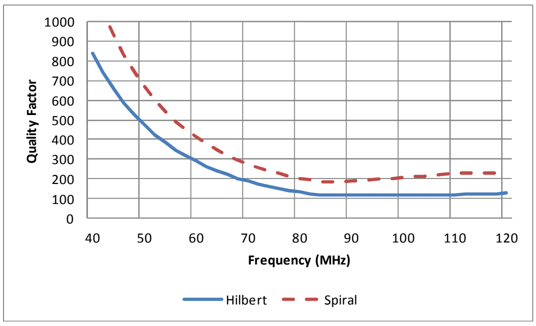

Fig. 3. Quality factor of the antennas

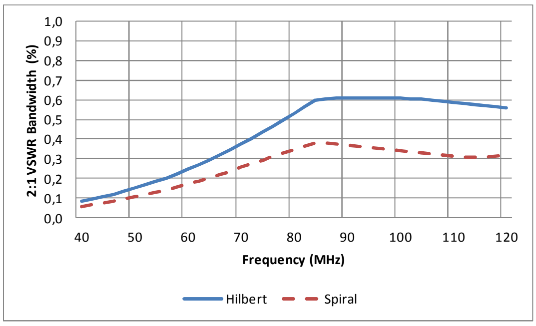

Fig. 4. Matched 2:1 VSWR bandwidth of the antennas

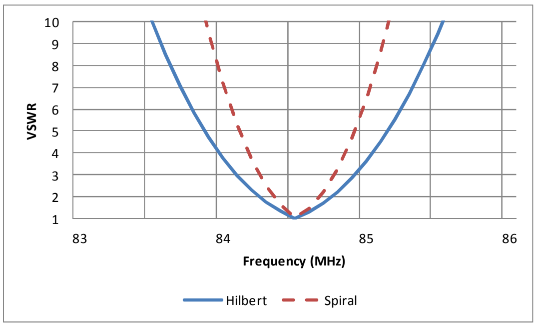

Fig. 5. VSWR of the antennas calculated under the condition that the characteristic impedance of the transmission line equals the antennas' respective resonant resistance.

TABLE II
Input Resistance, Quality Factor and Bandwidth at Resonant Frequency (84.5 MHz)

Antenna	Resonant Resistance (Ohms)	Quality Factor	2:1 VSWR Bandwidth (%)
Hilbert	3.3	118.40	0.60
Spiral	2.7	188.71	0.37

B. Radiation and antenna efficiency

The antenna radiation efficiency is defined as the ratio between the radiated power and the power delivered to the antenna. It can be written as

$$\eta_r = \frac{R_r}{R_r + R_L} \tag{3}$$

where R_r is the radiation resistance and R_L is the losses resistance, used to represent the conduction-dielectric losses. This parameter is used to take into account losses within the structure of the antenna. Antenna radiation efficiency is very similar in both cases (Fig.6). At the resonant frequency, 84.5 MHz, the Hilbert monopole radiation efficiency is 72.6% and the spiral-shaped monopole one is 71.0%.

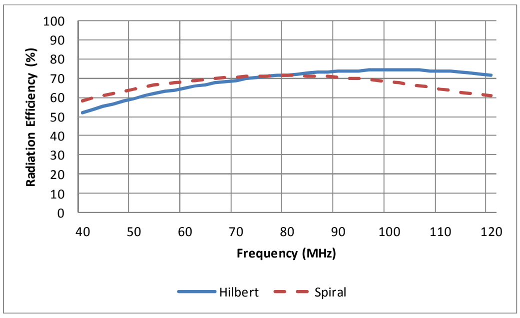

Fig. 6. Radiation Efficiency of the antennas

Antenna efficiency is used to take into account not only losses within the antenna structure but also losses due to the mismatch between the antenna and the transmission line that connect it to the source. It is related to radiation efficiency by

$$\eta_a = \eta_r \cdot \left(1 - |S_{11}|^2\right) \tag{4}$$

Antenna efficiency is calculated near to resonance frequency, considering that the transmission line characteristic impedance equals the antennas' respective resonance resistance (Fig. 7). The maximum antenna efficiency value is obtained at the resonance frequency and it's almost the same for both antennas, since they show almost the same radiation efficiency value and both are perfectly matched at resonance frequency. Moving away from the resonant frequency the antenna efficiency decreases quickly for the spiral shaped monopole due to its narrower bandwidth. At frequencies 1% away from the resonant frequency the Hilbert monopole antenna efficiency is approximately 3dB higher.

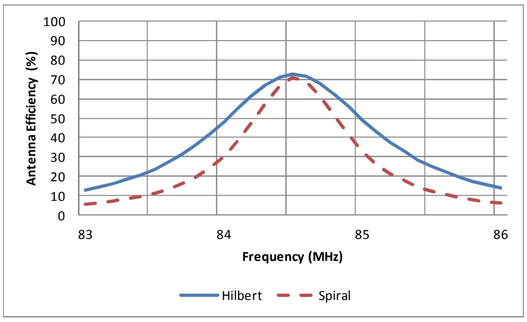

Fig. 7. Antenna Efficiency of the antennas

C. Radiation Pattern

The radiation pattern of the Hilbert monopole at the resonant frequency presents a null at zenith which means that the Hilbert antenna really behaves as a monopole type antenna (Fig.8). However, the spiral shaped monopole shows a different radiation pattern, which does not present a null in the zenith direction. Directivity in the zenith direction is -14.8 dBi for the Hilbert monopole and -3.0 dBi for the spiral shaped. This makes the Hilbert antenna suitable for signal reception where most of the electromagnetic energy comes from the horizon such as the case of broadcast applications.

In the Hilbert monopole the net current vector, resulting from the sum of all current vectors along the antenna path (Fig. 2), shows a predominant component perpendicular to the ground plane, justifying its radiation pattern similar to that of a straight monopole.

By contrast, the net current vector in the spiral shaped monopole (Fig. 2) shows both a component parallel to the ground plane and another perpendicular to it, and thus its radiation pattern is different from the one of the Hilbert monopole and clearly different from that of a straight monopole. The fact that the Q of the Hilbert monopole is lower than that of the spiral monopole suggests that the length of the individual segments of a small antenna are not as relevant as other characteristics such as the spatial arrangement of those segments and the overall antenna geometry. Radiation is obtained through the addition of the individual contributions of such small current elements and despite such a contribution is reduced with an increased fractal iteration, the number of them and overall length increases faster than the reduction of the segment's length. Again, each individual geometry makes a different use of such features (length of segments, overall antenna length, spatial arrangement and use of volume) and the question on how such features relate to the performance of a small antenna still requires further investigation.

On the other hand, the higher Q of the spiral monopole suggests that a higher reactive energy is stored in the surroundings of the antenna compared to the Hilbert monopole. This might be related to the magnetic field stored inside the spiral turns, analogously to the behavior of coil inductors that tipically feature such a spiral shape.

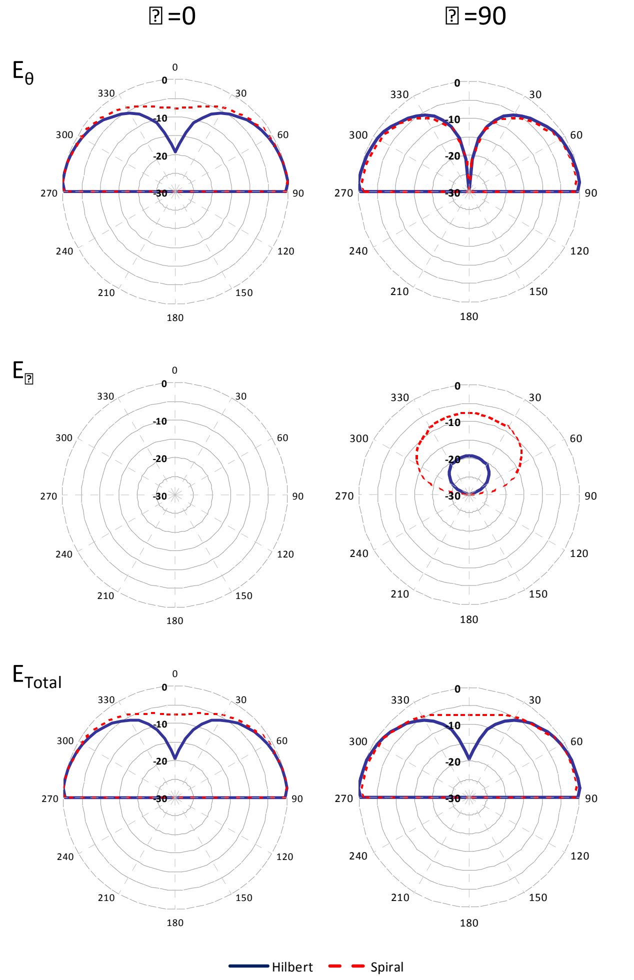

Fig. 8. Main cuts of the radiation pattern of the antennas at the resonance frequency, f=84.5 MHz

IV. Conclusion

A fully comparison of the resonant behaviour of the third iteration of the Hilbert monopole and a spiral shaped monopole is performed in this paper.

The spiral shaped monopole shows better compression efficiency, requiring less wire than the Hilbert monopole to resonate at a certain frequency. However, this fact provides no advantage in the electromagnetic performance of the antenna. In fact the Hilbert monopole shows an improved performance in terms of antenna miniaturization since it presents nearly the same radiation efficiency and lower quality factor and therefore a larger VSWR bandwidth, which is a critical parameter to consider in a small antenna.

It has been shown that both antenna radiation patterns are different. The Hilbert monopole really behaves as a monopole type antenna, showing a null in the zenith direction, while the spiral shaped monopole does not.

It has been observed significant differences in the quality factor and the radiation pattern of the antennas, showing that geometry, and not only size or wire length, plays a role in the performance of a small antenna.

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