

MULTI-RATE DECENTRALIZED MPC STRATEGY FOR DRINKING WATER NETWORKS: APPLICATION TO THE BARCELONA CASE STUDY

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In this paper, a multi-rate decentralised model predictive control (MDMPC) approach is proposed for drinking water networks (DWN). The upper temporal layer works with a daily time scale and is in charge of achieving the global objectives, i.e., optimal selection of the water sources and the path to the reservoirs. On the other hand, the lower temporal layer is in charge of manipulating the set-point of the actuators to satisfy the local objectives, i.e., the minimisation of the energy needed for pumping water to the reservoirs. Once obtained the system decomposition into subsystems, some bidirectional flows of information between the MPC controllers that control the subsystems appear. Hence, the proposed DMPC strategy copes with them, resulting in a hierarchical-like scheme. Results obtained when used selected simulation scenarios over a partitioned model of the Barcelona DWN show the effectiveness of the control strategy in terms of system modularity, reduced computational burden and, at the same time, the admissible loss of performance in contrast to a centralised MPC (CMPC) strategy.

INTRODUCTION

Drinking Water Networks (DWNs) are large-scale multisource / multi-node flow systems which must be reliable and resilient while being subjected to constraints and continuously varying conditions with both deterministic and probabilistic nature. Optimal management of these systems is a complex task and has become an increasingly environmental and socio-economic research subject worldwide, with special attention to efficient handling of energetic and natural resources in dense urban areas, such as Barcelona city.

Model-based Predictive Control (MPC) has been proved to be one of the advanced control techniques, widely accepted for the operational control of water systems; see Brdys and Ulanicki [1], Marinaki and Papageorgiou [2], and Overloop [3], among others. The main reason is due to once obtained the network dynamical model, the MPC design just consists in expressing the desired performance specifications through different control objectives and constraints on system variables (e.g., minima/maxima of selected process variables and/or their rates of change), which are necessary to ensure process safety and asset health. The rest of the MPC design is automatic: the given model, constraints, and weights define an optimal control problem over a finite time horizon in the future (for this reason the approach is said *predictive*).

Nevertheless, the main hurdle for MPC control, as any other control technique, when applied to large-scale networks in a centralised way, is the non-scalability. The reason is that a huge control model is needed, being difficult to maintain/update and which needs to be rebuilt on every change in the system configuration as for example, when some part of the system should be stopped because of maintenance actions or malfunctions. Subsequently, a model change would require re-tuning the centralised controller. It is obvious that the cost of setting up and maintaining the monolithic solution of the control problem is prohibitive. A way of circumventing these issues might be by looking into *decentralised* MPC (DMPC), where networked local MPC controllers are in charge of the control of part of the entire system.

The main contribution of this paper consists in presenting the application of a hierarchical temporal multi-layer DMPC approach to the Barcelona drinking water network (DWN). The aim is to show that this approach reduces the computational burden with respect to the centralised counterpart and reduces the level of suboptimality with respect to a pure hierarchical DMPC approach presented in Ocampo-Martinez *et al.* [4]. Moreover, important features such as the system modularity are presented in a decentralised scheme. The advantage of the hierarchical-like DMPC approach is the simplicity of its implementation given the absence of negotiations among controllers. To apply the proposed DMPC approach, the network is decomposed into subsystems using a novel automatic decomposition algorithm reported in Ocampo-Martinez *et al.* [4], which is based on graph partitioning.

CASE STUDY DESCRIPTION

The DWN of Barcelona is used as the case study of this paper. This network is managed by Aguas de Barcelona S.A (AGBAR), which not only supplies drinking water to Barcelona city but also to the metropolitan area. The sources of water are the rivers Ter and Llobregat. Currently, there are four potabilisation plants: Abrera and Sant Joan Despí plants, which extract water from river Llobregat; Cardedeu plant, which extracts water from river Ter; and Besòs plant, which treats the underground flows from the aquifer of Besòs river. There are also several underground sources (wells) that may provide water through pumping procedures. These sources should currently provide a flow of around 12 m³/s. The Barcelona drinking water network is currently comprised of 67 tanks and 121 actuators, which are divided in 46 pumps and 75 valves (see Figure 1). Among the pumps, five of them draw water from the underground sources, whereas the others are used to carry the water to those places with different elevation (higher city sectors). Moreover, the network has 88 main sectors of consume and 16 water nodes. Both the demand episode and the calibration setup of the network are provided by AGBAR. The current AGBAR control centre has a tele-control system for the network management. The Barcelona water network is also comprised of more than 98 remote stations, which manages in real time about 450 elements: flow meters, pumping stations, valves, chloride dosing instruments, etc.

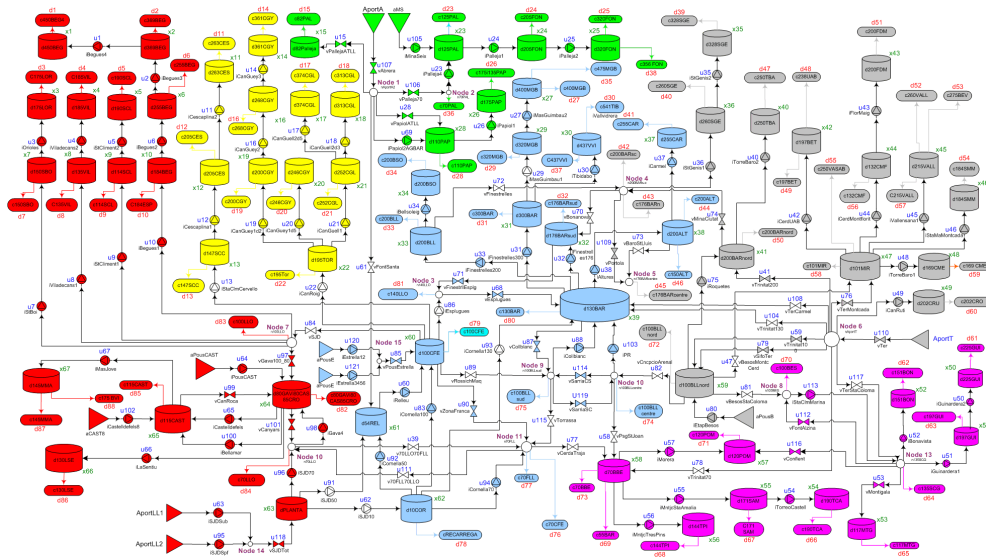


Figure 1: Barcelona DWN

The Barcelona DWN is structurally organised in two layers. The upper layer, named as transport network, links the water treatment plants with the reservoirs distributed all over the city. The lower layer, named distribution network is sectorised in subnetworks. Each subnetwork links a reservoir with each consumer. This paper is focused on the transport network. Thus, each subnetwork of the distribution network is modelled as a demand sector. The demand of each sector is characterised by a demand pattern, which can be predicted by using a time-series model reported in Quevedo *et al.* [5]. The control system of the transport network is also organised in three layers (see Figure 2). The upper layer determines the general operational objectives and working framework. The medium layer is in charge of the global control of the network, establishing the set-points of the regulatory controllers at the lower layer. Regulatory controllers are of PID type, while the supervisory layer controller is of MPC type. Regulatory controllers hide the network non-linear behaviour to the supervisory controller. This fact allows the MPC supervisory controller to use a linear control-oriented model.

DWN CONTROL PROBLEM

DWN Linear Discrete-Time Model

The control-oriented model of a water transport system is a simplified but representative model of the dynamic behavior, which allows predicting the effect of control actions on the entire network. Modeling principles of DWNs have been widely reported in the literature; see Brdys and Ulanicki [1]. Considering the aforementioned references, the model of a DWN in discrete-time state space may be written as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_p\mathbf{d}(k), \quad (1a)$$

$$\mathbf{E}_1\mathbf{u}(k) + \mathbf{E}_2\mathbf{d}(k) = 0, \quad (1b)$$

$$\mathbf{x}^{min} \leq \mathbf{x}(k) \leq \mathbf{x}^{max}, \quad (1c)$$

$$\mathbf{u}^{min} \leq \mathbf{u}(k) \leq \mathbf{u}^{max}, \quad (1d)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the measurable state vector of water stock levels in m^3 corresponding to the n buffer tanks at time $k \in \mathbb{Z}^+$, subject to physical constraints \mathbf{x}^{min} and \mathbf{x}^{max} ; $\mathbf{u} \in \mathbb{R}^m$ is the vector of manipulated flows in m^3/s through the m actuators subject to operational constraints \mathbf{u}^{min} and \mathbf{u}^{max} ; $\mathbf{d} \in \mathbb{R}^p$ corresponds to the vector of the p water demands (sectors of consume) in m^3/s ; \mathbf{A} , \mathbf{B} and \mathbf{B}_p are state-space system matrices of suitable dimensions; and \mathbf{E}_1 and \mathbf{E}_2 are matrices of suitable dimensions dictated by the network topology.

DWN Operational Control

The main goal of the operational control of water transport networks is to satisfy the demands at consumer sectors, but optimizing at the same time, management policies expressed as a multi-objective control problem. Hence, MPC is a suitable technique to control a DWN because its capability to deal efficiently with multivariable dynamic constrained systems and predict the proper actions to achieve the optimal performance according to a user defined cost function. Specifically, in DWN the interest is to minimize the following cost function (Ocampo-Martinez *et al.* [4]):

$$\min_{\Delta\mathbf{u}, \boldsymbol{\varepsilon}} \left[\sum_{i=0}^{H_u-1} f_1(k+i) + \sum_{i=1}^{H_p} f_2(k+i) + \sum_{i=0}^{H_u-1} f_3(k+i) \right], \quad (2)$$

where H_p and H_u are the prediction and control horizons, respectively; index k represents the current time instant while index i represents the predicted time along the horizons; $f_1(k) = \|(\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2(k))\mathbf{u}(k)\Delta t\|_{\mathbf{W}_e}^1$ minimizes the economic cost of network operation taking into account water production cost ($\boldsymbol{\alpha}_1$) and water pumping electric cost ($\boldsymbol{\alpha}_2$); $f_2(k) = \|\boldsymbol{\varepsilon}(k)\|_{\mathbf{W}_x}^2$ is a performance index which penalizes the amount of volume $\boldsymbol{\varepsilon}$ that goes down from a safety volume value; $f_3(k) = \|\Delta\mathbf{u}(k)\|_{\mathbf{W}_u}^2$ minimizes control signal variations to extend actuators life and assure a smooth operation; \mathbf{W}_e , \mathbf{W}_x , and \mathbf{W}_u are diagonal weighting matrices included to prioritize the objectives.

With the above information, the MPC design follows a systematic procedure as in Maciejowski [6], which generates the control input signals to the plant by combining a prediction model and a receding-horizon control strategy. The cost function (2) subject to (1) represents the desired system performance over a future horizon. Once the minimization is performed, only the first computed control action is implemented and the system operates with this constant input until the next sampling instant. Then, the optimization is solved again with new feedback measurements to compensate for unmeasured disturbances and model inaccuracies. This scheme is repeated at each future sampling period.

MULTI-RATE DMPC APPROACH

In the proposed multi-rate DMPC the *Global/Supervisory Control Level* in Figure 2 is divided in two control sub-layers characterized by different behaviours and time scales:

- *Daily Centralized MPC Control*: This centralized optimization operates at daily time scale to coordinate the controllers working at hourly scale.
- *Hourly Decentralized MPC Control*: Each decentralized controller is in charge of a subsystem and operates at hourly time scale. Controllers are coordinated through a hierarchical control scheme that determines a sequence of information distribution among the controllers, where top-down communication is available from upper to lower level of the hierarchy (Figure 3).

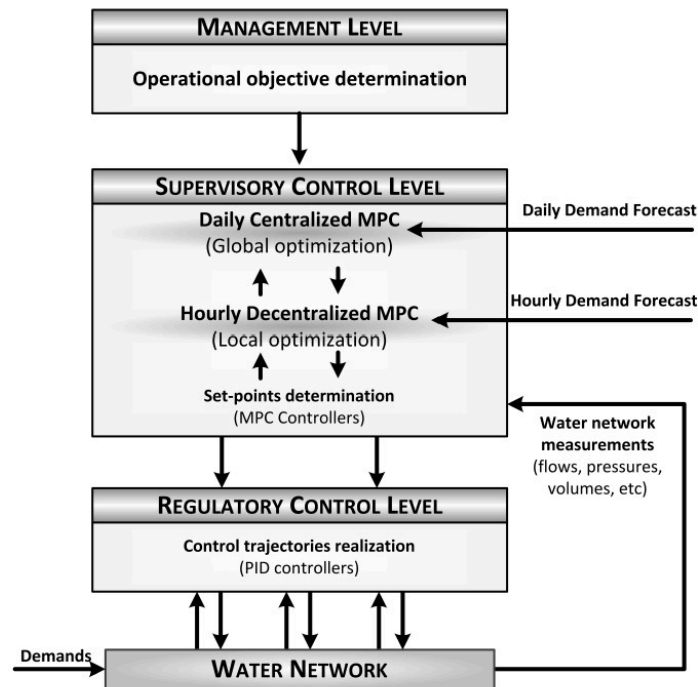


Figure 2: Control levels in the Barcelona Water Transport Network

The motivation for this hierarchical multi-layer DMPC approach comes from the results obtained just using a hierarchical DMPC approach from Ocampo-Martinez *et al.* [4]. Analysing these results it was noticed an increment of the total costs of operation when using the hierarchical DMPC strategy with respect to a CMPC strategy. This loss of performance is due to the DMPC strategy does not take into account in a proper way the water costs related to external water sources since it is a global objective. On the other hand, DMPC controllers are mainly focused on the reduction of pumping costs (local objective) within each subsystem. By contrast, the information of water costs is properly

managed for the CMPC controller by optimising it but at the price of moving more water inside the network. This leads to an increment in the electric costs (the water transportation cost) when CMPC controller is used. Therefore, in order to enforce the global objective, the economical unitary cost of the shared variables that act as sources is calculated by the daily optimization in order to fulfil the global objective. The daily optimization determines this price by finding the optimal paths from all water sources taking into account the flow capacity and unitary cost in each point of the network.

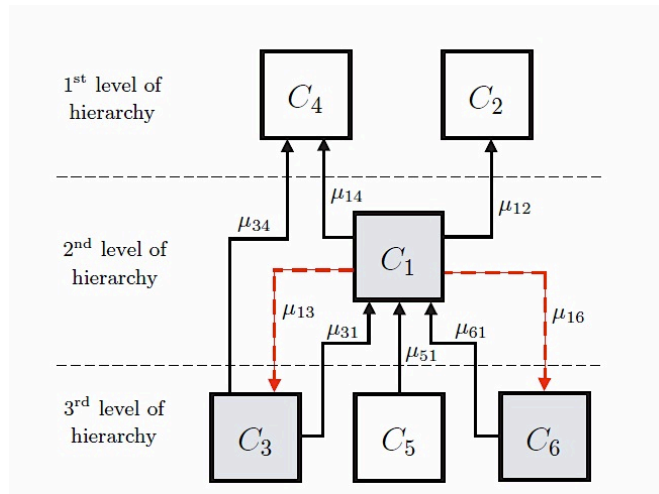


Figure 3: Hierarchy of controllers operating at hourly level

RESULTS

This section presents the results of the application of the proposed hierarchical multi-layer DMPC approach using the partitioned Barcelona water network obtained in Ocampo-Martinez *et al.* [4]. This partitioning method provides a partition consisting of a set of non-overlapping subgraphs whose number of vertices is as similar as possible and the number of interconnecting edges between them is minimal. To achieve this goal, the proposed algorithm applied a set of graph theory procedures based on identifying the highly connected subgraphs with balanced number of internal and external connections. Figure 1 shows, in different colours, the obtained subsystems of Barcelona water network (for more details see [4]). The results obtained by using the proposed multi-layer DMPC strategy are compared with those obtained employing a CMPC approach and DMPC strategy without the multi-layer scheme proposed in Ocampo-Martinez *et al.* [4]. The results are presented for 72 hours (July 24 and July 27 of 2007). The weights of the cost function are $W_e = 100$, $W_x = 10$, and $W_u = 0.05$. The tuning of these parameters has been chosen in a way that highest priority objective is the economic cost, which should be minimized while

maintaining a similar rate of the safety volume and control action smoothness terms. The hourly layer uses the same control and prediction horizons $H_p = H_u = 24$.

Table 1 summarizes the obtained control results in terms of economic cost. For each MPC approach the water, electric and total cost are presented. Taking into account the optimization results of water cost, Hierarchical DMPC has a higher cost of about 230% than the CMPC approach, while the Hierarchical Multilayer and DMPC has a higher cost of about 5%. The graphical results are shown in Figure 4. If the electric cost is considered, Hierarchical DMPC has a lower cost of about 60% compared with the CMPC approach, while the Hierarchical Multilayer and DMPC has a lower cost of about 3%. Figure 5 presents the evolution electric cost along the four days. Finally, comparing the total costs, the Hierarchical Multilayer DMPC presents a similar result than the CMPC and the total cost of Hierarchical DMPC approach is higher of about 30%. According to the results of Table 1 the Hierarchical Multilayer and DMPC approach presents similar results to the CMPC.

Table 1. Performance Comparisons

Index	CMPC	DMPC	ML-DMPC
Water Cost	93.01	205.55	97.11
Electric Cost	90.31	34.58	87.53
Total Cost	183.33	240.13	184.65

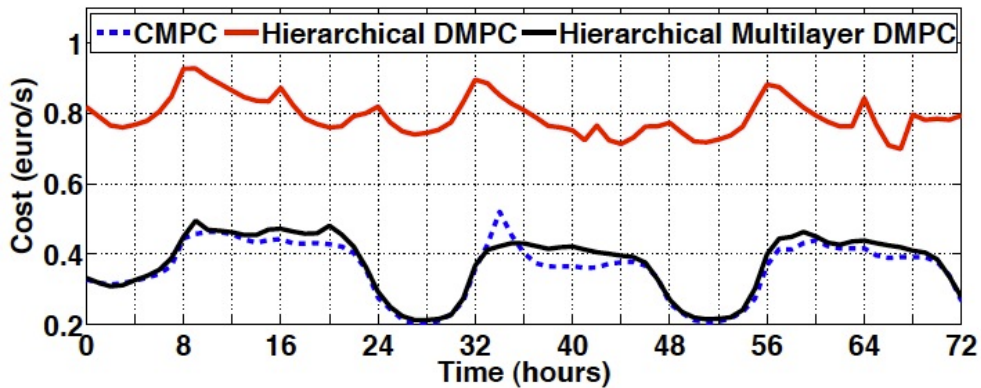


Figure 4: Water cost of the different MPC controllers

CONCLUSIONS

This paper has presented the application of a temporal multi-layer DMPC approach to the Barcelona DWN. To apply this control strategy, the network is partitioned into subsystems by using an automatic decomposition algorithm based on graph partitioning. The

hierarchical-like DMPC scheme should be considered since the resultant DWN decomposition determines a set of subsystems that cannot be grouped in different levels such that the flow of information is unidirectional from clusters at higher hierarchical levels towards clusters at lower levels. This fact implies that a pure hierarchical DMPC approach cannot be considered. A comparison with a CMPC approach shows that the level of sub-optimality in economic costs is acceptable considering the resultant reduction in computational burden. As future research, the proposed hierarchical-like approach, which addresses the loops between levels in a heuristic way, should be further investigated in order to evaluate the introduced degree of suboptimality as well as how feasibility and stability features are preserved.

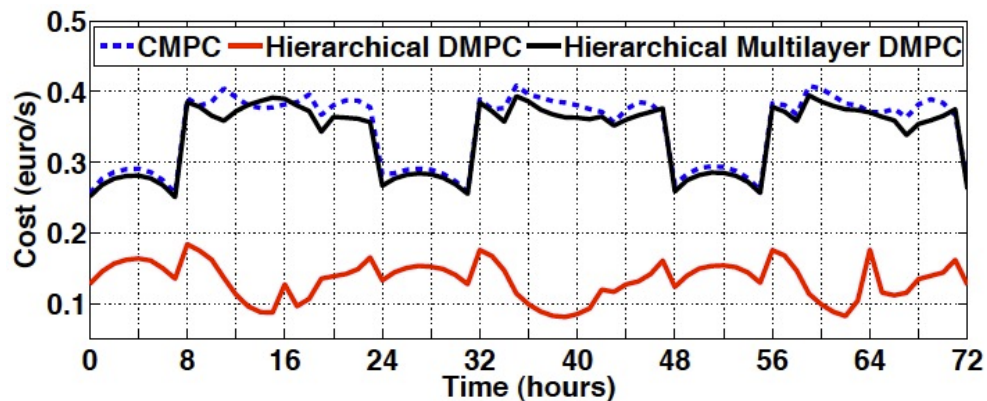


Figure 5: Electric cost of the different MPC controllers

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