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CAPTURING SMALL ASTEROIDS INTO SUN-EARTH LAGRANGIAN POINTS FOR MINING PURPOSES

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Abstract

The aim of this paper is to study the capture of small Near Earth Objects (NEOs) into the Sun-Earth L_2 using low-thrust propulsion for mining or science purposes. As it is well known, the vicinity of these points is inside a net of dynamical channels suitable for the transport in the Earth-Moon neighborhood, so different final destinations from here could be easily considered. Asteroids with very small mass and not representing a potential hazard are analyzed. An initial pruning of asteroids is made, considering NEOs with stellar magnitude bigger than 28, which are the smallest available, and NEOs close to the Earth orbit with semi-major axis between 0.85-1.15. Due to the difficult determination of their physical properties, two methods to estimate the asteroid masses are conducted. A procedure to find the low-thrust optimization trajectories has been implemented. The initial seed is obtained integrating forward the equations of motion plus its conjugated equations expressed in cartesian coordinates and applying the Pontryagins maximum principle to obtain the optimal control with a switching function for the thrust. To refine the trajectory a 4 order Runge-Kutta shooting method has been used. The objective function in this study is the fuel consumption. Finally, the capable asteroids to get captured by a low-thrust engine have been listed indicating the main parameters.

1 Introduction

Asteroid mining[1] will play a key role in providing the future resources for the exploration of the Solar System. A rough spectral taxonomy of asteroid types separates them in three types: C-type (carbonaceous), S-type (stony) and M-type (metallic). Type C asteroids comprise more than 70 % of all asteroids. Eventhough Near Earth Asteroids (NEAs) are potentially the most hazardous objects in space, they are the objects that could be easier to exploit for their raw materials. The current paper targets the transfer of asteroids to the Sun-Earth L_2 lagrangian point geometrically defined. Another interesting approach could be to insert them into the stable manifold of a libration point orbit.[2] However, due to the high number of possibilities and the number of asteroids considered in the study this has been left for further research. Capturing an asteroid near the Earth would make easier to mine it, as well as to exploit it in terms of studying its behaviour and physical properties.

The problem of interest is to find the fuel optimal low-thrust capture trajectory from the original asteroid orbit to the Sun-Earth L_2 libration point. The departure time has been set between 2456000.5 and 2460000.5 JD and the maximum time of flight allowed is 1800 days.

We have followed a methodology which in this paper is divided in four main sections. The first section of this paper is dedicated to prune the asteroids from the Near Earth Asteroids database and select the smallest NEAs closest to the Earth. The objective is to find a feasible trajectory with a technology already demonstrated, i.e. Variable Specific Impulse Magnetoplasma Rocket (VASIMR)[3] engines. The mass of the asteroids is a parameter which is very difficult to know. However, this parameter is essential for the trajectory computation, so the first section discusses different solutions to estimate this value. Next section is centered to get a rough departure time and time of flight with a global optimization method, propagating with a two-body model forward and backward the trajectory to a mid-point. The third section describes the core of the paper, the trajectory optimization applying the 4th order Runge-Kutta shooting method. It has been noticed that it is very sensible to converge, thus an accurate initial guess for the optimization problem is needed. The last section lists the results of the trajectory optimization for the pruned asteroids of section 2.

2 Asteroids Database Selection

In this research, JPL's Solar System Dynamics Group small-body database (SBDB) has been used to select the asteroids and get the orbital elements and stellar magnitude data for each one. As of 5th of September of 2012, 9049 NEAs (Near Earth Asteroids) have been identified.

An initial pruning of the asteroids has been made with the criteria to select the smallest ones within the Earth's neighborhood[4] , in order to be capable to move them

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with the current technology of a Solar Electric Propulsion (SEP) system. Then, the constraints in the list of the database asteroids are a semi-major axis range between 0.85-1.15 AU and a stellar magnitude bigger than 28. The stellar magnitude represents the brightness of the object, a direct relationship with its size. Applying these constraints, we get a final selection of 40 asteroids with the parameters in Table 1. The asteroid 2004 UH1, has been discarded because it has a very high eccentricity, plus it is a type of asteroid whose orbit intersects with the Earth orbit, but the relative speed is very high. The combination of these facts would need a very big effort to change the orbit.

Name	Class	a [AU]	e	i [deg]	Ω [deg]	ω [deg]	M [deg]	n [deg/day]	H
1991 VG	Apollo	1.03	0.0491	1.45	73.98	24.51	340.17	0.95	28.39
2000 LG6	Aten	0.92	0.1109	2.83	72.55	8.19	185.75	1.12	29.019
2003 SW130	Aten	0.88	0.3043	3.67	176.45	47.80	49.55	1.19	29.117
2003 WT153	Aten	0.89	0.1777	0.37	55.61	148.91	55.61	1.17	28.048
2006 BV39	Apollo	1.15	0.2714	0.74	127.09	74.96	116.33	0.80	28.984
2006 JY26	Apollo	1.01	0.0830	1.44	43.50	273.45	29.55	0.97	28.349
2006 RH120	Apollo	1.03	0.0245	0.60	51.14	10.14	221.25	0.94	29.527
2007 EK	Apollo	1.13	0.2724	1.21	168.58	83.26	181.71	0.82	29.258
2007 UN12	Apollo	1.05	0.0605	0.24	216.11	134.34	238.24	0.91	28.741
2008 CM74	Apollo	1.09	0.1469	0.86	321.58	242.73	339.86	0.87	28.043
2008 GM2	Apollo	1.05	0.1572	4.10	195.11	278.25	121.93	0.91	28.356
2008 HU4	Apollo	1.09	0.0733	1.26	221.34	341.50	327.11	0.86	28.223
2008 JL24	Apollo	1.04	0.1066	0.55	225.82	281.97	124.19	0.93	29.572
2008 KT	Apollo	1.01	0.0848	1.98	240.66	101.86	7.44	0.97	28.215
2008 LD	Aten	0.89	0.1547	6.54	250.90	201.42	202.43	1.17	28.864
2008 UA202	Apollo	1.03	0.0686	0.26	21.06	300.89	330.08	0.94	29.44
2008 UC202	Apollo	1.01	0.0685	7.46	37.43	91.24	230.71	0.97	28.242
2008 WO2	Apollo	1.03	0.1882	2.01	238.15	85.70	331.19	0.95	29.779
2009 BD	Apollo	1.06	0.0516	1.27	253.33	316.73	115.11	0.90	28.236
2009 WQ6	Aten	0.87	0.4087	5.82	55.68	227.27	288.88	1.22	29.186
2009 WW7	Apollo	1.09	0.2618	2.53	57.18	273.71	241.39	0.87	28.894
2009 WR52	Apollo	1.03	0.1551	4.24	61.03	269.88	329.31	0.94	28.32
2009 YR	Aten	0.94	0.1102	0.70	86.95	127.87	257.46	1.08	28.003
2010 JW34	Aten	0.98	0.0548	2.26	49.81	43.61	294.67	1.01	28.148
2010 RF12	Apollo	1.06	0.1882	0.88	163.85	267.56	254.92	0.90	28.369
2010 UY7	Aten	0.90	0.1499	0.46	39.95	210.44	228.37	1.16	28.527
2010 UE51	Apollo	1.06	0.0597	0.62	32.29	47.25	239.36	0.91	28.311
2010 VL65	Apollo	1.07	0.1440	4.40	223.12	253.97	203.68	0.90	28.423
2010 VQ98	Apollo	1.02	0.0271	1.48	46.17	341.60	316.69	0.95	28.2
2011 AM37	Apollo	1.10	0.1473	2.63	291.28	129.20	213.92	0.85	29.69
2011 BQ50	Aten	0.95	0.0982	0.36	281.01	1.27	141.56	1.06	28.341
2011 CA7	Apollo	1.08	0.2888	0.12	311.00	278.61	108.09	0.88	30.319
2011 CH22	Aten	0.88	0.2358	0.13	334.67	27.59	115.28	1.20	28.961
2011 JV10	Apollo	1.14	0.2020	1.40	221.39	297.52	101.68	0.81	29.706
2011 MD	Amor	1.06	0.0371	2.45	271.63	5.84	56.38	0.91	28.073
2011 UD21	Aten	0.98	0.0302	1.06	22.52	208.45	144.91	1.02	28.483
2012 AQ	Apollo	1.07	0.1038	2.86	97.32	316.09	280.99	0.89	30.698
2012 EP10	Apollo	1.05	0.1160	1.03	348.04	105.73	249.95	0.92	29.165
2012 FS35	Apollo	1.10	0.1185	2.34	186.57	42.23	126.35	0.86	30.286

Table 1: Orbital Elements, stellar magnitude and class of the Asteroids selected

The vast majority of the asteroids selected are from the Apollo family, which have a semi-major axis greater than 1 AU and a perihelion distance $q < 1.017AU$. In fact, 38 out of the 39 asteroids selected cross the Earth's orbit and they are rather Apollo or Aten; except one that belongs to the Amor family ($1 < q < 1.3$ AU and $a > 1$ AU). The highest inclination is 7.45 deg from the ecliptic plane and the range of eccentricity among the asteroids selected is between 0.0244 and 0.4086.

2.1 Mass Estimation

The mass of an asteroid is very difficult to obtain as it can only be determined by in-situ measurements or from the observed dynamics (i.e., spacecraft tracking during encounters, natural satellites of asteroids).[5] In this research, two methods to estimate the mass of the selected asteroids have been conducted both assuming an spherical shape of the asteroid and a bulk density[6] of 2.6 g/cm^3.

The mass estimation in the first method has been made from the relationship stated in JPL[7] between the absolute magnitude H and the diameter of the asteroid, which assumes an albedo ranging from 0.25 to 0.05. This method grouped the asteroids based on its absolute magnitude.

H	D [m]	Mass [Kg]
28	$7 - 15$	466945 - 4594579
28.5	$5 - 12$	170169 - 2352424
29	$4 - 9$	87126 - 992429
29.5	$3 - 7$	36756 - 466945
30	$3 - 6$	36756 - 294053

Table 2: Relationship from JPL between H, D and Mass

In practice, as we are studying the asteroids with $H > 28$, there will be 5 groups as it can be seen in Table 2. We have classified the asteroids which have a stellar magnitude between 28-28.3 in the group H=28, the asteroids with an H between 28.3-28.7 in the H=28 group, the asteroids with an H between 28.7-29.2 in the H=29, and so on.

In the second method the diameter has been derived from the Eq. 1, based on the absolute magnitude H, according to Fowler and Chillemi[8] and assuming an albedo range[2] between 0.05 to 0.5.

$$D = \frac{1329}{\sqrt{p_v}} 10^{-0.2H}, \quad (1)$$

where p_v is the geometric albedo and H is the asteroid absolute magnitude from the SBDB database.

In Table 3, we have represented the diameter and the corresponding mass that would have an asteroid with the correspondent stellar magnitude indicated, supposing that it is calculated with the Method 2. This is useful to be able to compare the diameter in both methods. In the upper limit differs up to 37 %, though the lower limit difference is only 6.5 %. If we compare the masses in both methods, we can notice that the upper limit of mass is very different in both methods because in the first method we have considered an albedo equal to 0.25 and in method 2 equal to 0.5.

H	D [m]	Mass [Kg]
28	$4.72 - 14.93$	143249 - 4529934
28.5	$3.75 - 11.86$	71794 - 2270345
29	$2.98 - 9.42$	35982 - 1137868
29.5	$2.37 - 7.48$	18033 - 570284
30	$1.88 - 5.94$	9038 - 285819

Table 3: Method 2 relationship between H, D and Mass considering an albedo range of 0.05 - 0.5

Therefore, two absolute limits for each mass method have been estimated for asteroids, reflecting the uncertainty of the mass determination and being able to give a range of the asteroid mass that the mission could find.

In the following sections the procedure to be able to capture asteroids is presented. The procedure is not trivial and requires a serial of steps to find an optimal trajectory to capture the asteroids. The objective of these previous steps from the actual refined trajectory optimization is to dispose of suitable good initial guessed parameters such as the departure time, the time of flight and the history of control of the trajectory to be able to converge to the optimal solution.

3 Transfer opportunity search

The goal of the initial orbit search step is to find a suitable departure time and time of flight. In this research, we concluded that it is very important to have a good first estimation of them because they are very sensible in the trajectory optimization. A low-thrust trajectory modeled as a series of impulses connected by conic arcs proposed by Sims and Flanagan[9] has been applied.

The model consists on a low-thrust trajectory divided into two segments and modeled as a N series of impulsive maneuvers connected by conic Lambert arcs, separated in an equal time step nodes. The trajectory is propagated (two-body regime) forward in time from the asteroid to the connection point and backward from the Sun-Earth Lagrangian Point L_2 to the connection point. The Lagrangian Point has been geometrically defined using JPL ephemerides DE405. The trajectory is shown in Figure 1. The propagation between impulses is according to a sequential Kepler model, avoiding in this case the numerical integration, which is the most time consuming part of the trajectory generation.

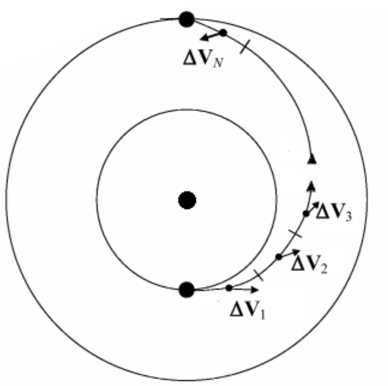

Fig. 1: Trajectory model[10]

The impulsive maneuver ΔV on each node is defined by three parameters α, β and k, which represent the magnitude and the direction of the thrust.

$$\Delta V = \Delta V_{\max} k [\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta]^T$$

where $\alpha \in [0, 2\pi)$, $\beta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $k \in [0, 1]$.

The ΔV at each node should not exceed maximum magnitude $\Delta V_{\max} = a t_{step}$, where a is the acceleration offered by the low-thrust engine when operated at full thrust and t_{step} is the time span between the two nodes. In our computations, the maximum magnitud of the acceleration has been set to $10^{-4} m/s^2$.

The global search of the trajectory[10] ,[11] is based on Differential Evolution algorithm. The version of Differential Evolution algorithm implemented is from Storn and Price[12] , with a weighting factor $F = 0.8$ and a crossover constant $CR = 0.8$. For each run, the number of maximum iterations has been set to 20000 and the population size to 50. It is worth to mention that the sampling has been made uniformly distributed over the surface of the sphere.

The number of optimization parameters is the sum of the Time of Flight (TOF), Departure Time (DT) and N*3, the number of nodes of the ballistic segment multiplied by the magnitude plus the angles of thrust. Upper and lower bounds are set for the variables between 0 and 1, except for the departure time and time of flight. The departure time in our problem has been set between $2456000.5 - 2460000.5$ JD and the time of flight between $200 - 1800$ days.

The matching conditions of the global optimization consists in the position and velocity of the encounter point. The positions should be concurrent and the velocities mismatch should be less than a tolerance error.

The performance index of the global optimization problem includes the constraints in the objective function. Thus, the objective function to be minimized is the sum of the impulses in each node and the state vector mismatches:

$$J = \sum_{i=1}^{n} k_i + \lambda_r ||\mathbf{R}_{conn}||_2 + \lambda_v ||\mathbf{V}_{conn}||_2$$

where λ_r and λ_v are the weighting factors, $\mathbf{R}_{conn}$ is the position error at the connection point, and $\mathbf{V}_{conn}$ is the velocity error at the connection point. The variables are expressed in canonical units, where the distance unit for the position mismatches is in AU. The weighting factors affect the feasibility and optimality of the trajectory found. The higher the weighting parameters are, the optimizer focuses more on the constraints. In our program, they have been set to 10^5.

By using the Differential Evolution algorithm, a solution can be found very fast. However, the position and the velocity mismatches at the connection point are rather big. In fact, the velocity error is bigger than the capability of low-thrust engine. Therefore, the trajectory obtained is not a feasible solution and only the rough departure time and time of flight have been used to find the optimal trajectory.

The option to improve the results using a local optimizer has also been studied. The time history of control gets smoother using this method, but the improvement in terms of matching is insignificant. Therefore, we have decided to skip this step and go directly to the next one because the outputs from the global optimization method are already acceptable initial seeds for the trajectory optimization.

4 Trajectory Generation

By using the method introduced in section 3, the rough transfer trajectory, which is approximated by a series of Lambert arcs, is obtained. However, this transfer trajectory is inaccurate and its optimality also cannot be guaranteed (The general performance index contains the penalty function. The weight of the penalty function changes the characteristics of the optimization problem). In this section, the accurate trajectory optimization technique will be introduced. The sequential keplerian approximation is replaced by the integration of the ordinary differential equations of motion, which includes a more accurate low-thrust orbital dynamic model. A series of piecewise continuous control histories is generated by using the conjugated equations in calculus of variation. Then, these control histories are optimized by Runge-Kutta 4th order shooting method, and a series of local optimal solutions are obtained (The Runge-Kutta 4th order shooting method is a kind of gradient based optimization method. Therefore, like all the rest of gradient based algorithms, it can only obtain the local optimal solution). The solution which has the best performance index will be chosen, and considered as the global optimal solution.

4.1 Initial guess generation

The typical dynamical equations, expressed in cartesian coordinates, the conjugated equations and the optimal control equations to obtain the initial seed of the optimization problem are presented below. In this case the mass is propagated along the trajectory, while in the global optimization it was assumed to keep it constant throughout the trajectory.

$$\begin{aligned}
\dot{\mathbf{r}} &= \mathbf{v} \\
\dot{\mathbf{v}} &= -\frac{\mu}{r^3}\mathbf{r} + \frac{T}{m}\boldsymbol{\gamma} \\
\dot{m} &= -\frac{T}{g_0 I_{sp}} \\
\dot{\boldsymbol{\lambda}}_r &= \boldsymbol{\lambda}_v \frac{\mu}{r^3} - \frac{3\boldsymbol{\lambda}_v^T \mathbf{r}}{r^5}\mathbf{r} \\
\dot{\boldsymbol{\lambda}}_v &= -\boldsymbol{\lambda} \\
\dot{\lambda}_m &= -||\boldsymbol{\lambda}_v|| \frac{T}{m^2}
\end{aligned} \tag{2}$$

where, T and $\boldsymbol{\gamma}$ are the control variables. The performance index is defined as

$$J = \frac{m_f}{m_0} \longrightarrow \max \tag{3}$$

where, m_f and m_0 are the final mass and the initial mass of the spacecraft-asteroid assembly, respectively. By using the Pontryagin's maximum principle, the optimal control can be obtained.

$$\boldsymbol{\gamma} = -\frac{\boldsymbol{\lambda}_v}{||\boldsymbol{\lambda}_v||}$$

T is determined by the switching function:

$$H_T = -\frac{||\boldsymbol{\lambda}_v||}{m} - \frac{\lambda_m}{g I_{sp}} \tag{4}$$

$$\begin{array}{lll}
T = 0 & \text{if} & H_T > 0 \\
T = T_{\max} & \text{if} & H_T < 0 \\
0 < T < T_{\max} & \text{if} & H_T = 0
\end{array}$$

where $g = 9.8m/s^2$ and the engine parameters are $I_{sp} = 3000s$ and T_{max} is set to when the engines operate at full thrust, which will depend on the case studied.

The direct method does not need the conjugated equations in Eq. 2, but in this research the conjugated equations have been used to generate the initial guesses of the piecewise continuous time history of control. The procedure is listed as follows:

1. We generate the initial values of the conjugated states $[\boldsymbol{\lambda}_r^T, \boldsymbol{\lambda}_v^T, \lambda_m]$ randomly, setting the limits of the values between -100 to 100 to have the same order of magnitude.
2. Then, we use these initial values, the departure time and the time of flight, which are obtained from the global search step, to propagate the set of Eq. 2 ;
3. If the error of the final states is smaller than a given tolerance (position error less than 0.5 AU and velocity less than 0.2 VU), the set of initial conjugated states $[\boldsymbol{\lambda}_r^T, \boldsymbol{\lambda}_v^T, \lambda_m]$ is accepted, otherwise it is rejected and step 1-3 are repeated, until a set of acceptable conjugated states is obtained;
4. Generate the discrete time history of control by using the control equation and switching function of Eq. (4) (In all computations the number of nodes that has been used is 81).

Using this procedure, a series of trajectories whose final states are not very far from the target state can be obtained, and meanwhile the time histories of control are also obtained. In this research, 50 sets of time history of control are obtained in this step.

4.2 Local optimization

The next step is to refine the control using the Runge-Kutta 4th order shooting method.[13] The basic idea of the Runge-Kutta 4th order shooting method is to convert optimal control problem to constrained parameter optimization problem. The optimization parameters of this problem can be denoted as $\mathbf{z} = [\mathbf{u}_0, \mathbf{u}_1, \cdots, \mathbf{u}_N]^T$, where $\mathbf{u}_i = [\alpha_i, \beta_i, T_i]^T$, whose initial guesses are already obtained by using random generation method. The lower bound and upper bound of $[\alpha_i, \beta_i, T_i]^T$ are $[0, -\frac{\pi}{2}, 0]^T$

and $[2\pi, \frac{\pi}{2}, T_{\max}]^T$, respectively. The constraints can be written as

$$\Delta = [\mathbf{r}(t_f), \mathbf{v}(t_f)]^T - [\mathbf{r}_f, \mathbf{v}_f]^T \quad (5)$$

where, $[\mathbf{r}(t_f), \mathbf{v}(t_f)]^T$ is the final state obtained by numerical integration, and $[\mathbf{r}_f, \mathbf{v}_f]^T$ is the target state. The performance index is the same as Eq. 3.

Using the fixed step-size 4th order Runge-Kutta integral formula, the state equations can be propagated. Meanwhile, the derivatives of the constraints and performance index with respect to optimization parameters, which are denoted as $\frac{\partial \Delta}{\partial \mathbf{z}}$ and $\frac{\partial J}{\partial \mathbf{z}}$, can be obtained. All derivatives are expressed anallytically. In our research, the local optimal solution of this parameters optimization problem can be found using sequential quadratic programming (SQP) algorithm.[14]

5 Results

In the previous sections a methodology to capture asteroids has been described. Starting from the estimation of the asteroids mass, finding a rough time of flight and departure time. Then, setting the procedure to generate a suitable seed for the calculation of the optimal trajectory to capture an asteroid. This section is devoted to present the method of the computations itself and discuss the results found.

The transfer opportunity search has been done one time for each asteroid, needing about 4 minutes per computation. In section 4, a total of 156 trajectories has been computed, which is the combination of the 39 selected asteroids among the two methods and the two boundaries of each one. The computational time of the trajectory generation strongly depends on the time of flight of each computation case. For instance, 2010 UY7 Method 2 upper limit of mass estimation, 12:41.15 min time is needed when executed in dual core Intel Xeon 3.2 GHz CPU. However, the case of 2012 FS35 Method 2 lower limit only needed 1:34.02 min of computational time.

The outputs from global optimization (GO) (listed in Table 4) problem are rough estimates for two reasons. On the one hand, in some cases we did not get any converged result using directly the outputs from GO. Therefore, we decided to change the time of flight between 6-33 Time Units (348.79 - 1918.37 days) with a time step of 3, which is approximate one orbit, and keep the converged result with the lowest time of flight. On the other hand, the parameters also change because the transfer of the asteroid is considered from the first time that the engine opens until it stops for the last time. Therefore, as the control profile we have obtained is different for each method, the time of flight and departure time vary. Although, the objective function in this research is the maximization of the final mass.

In the trajectory optimization problem, the thrust of the engine is one input parameter. The criteria that we have followed is to start with the lowest one (5 N) and search if for different time of flight it converged. Otherwise, we increased the thrust.

As we can see in Table 5 and 6, the thrust needed in the lowest mass limit is in the range between $5 - 25$ N. However, in the upper limit mass cases the thrust needed is much bigger, in the range between 15 - 200 N. This one order of magnitude difference is obviously due to the mass increment of the mass needed for the transfer.

Therefore, if the asteroids would have the lower limit of the mass estimated in Method 2, only using one VASMIR engine would be enough to capture them. Otherwise, more than one engine would be required to move them.

The time when the thrust is open is a very interesting parameter because it gives us an idea of the real cost to do the transfer.

Global Optimization					
Name	DT_{gl} [JD]	TOF_{gl} [d]	Name	DT_{gl} [JD]	TOF_{gl} [d]
1991 VG	2457589.62	976.84	2009 WW7	2457444.51	1192.99
2000 LG6	2458636.74	1498.08	2009 WR52	2456024.82	1154.47
2003 SW130	2456128.74	912.35	2009 YR	2458046.12	1220.78
2003 WT153	2456220.47	1569.58	2010 JW34	2456006.76	929.11
2006 BV39	2457303.08	1192.22	2010 RF12	2459962.67	1499.15
2006 JY26	2456976.17	1500.00	2010 UY7	2456970.35	1278.91
2006 RH120	2460000.50	1618.54	2010 UE51	2459989.98	574.26
2007 EK	2458014.42	1220.78	2010 VL65	2458982.79	1245.14
2007 UN12	2458639.10	883.32	2010 VQ98	2456000.50	1046.38
2008 CM74	2457264.83	1256.63	2011 AM37	2457412.01	1307.15
2008 GM2	2458974.37	1453.31	2011 BQ50	2459410.93	1084.44
2008 HU4	2457224.88	1278.91	2011 CA7	2458886.04	1064.23
2008 JL24	2459947.31	1628.75	2011 CH22	2456895.09	1119.30
2008 KT	2456408.20	1046.38	2011 JV10	2458967.43	1351.09
2008 LD	2459989.80	985.79	2011 MD	2460000.50	1005.69
2008 UA202	2459999.64	1443.84	2011 UD21	2456000.50	1226.28
2008 UC20	2460000.50	1395.18	2012 AQ	2458872.57	1337.28
2008 WO2	2456000.50	1799.58	2012 EP10	2456000.75	1453.31
2009 BD	2459425.83	1046.38	2012 FS35	2458396.83	1213.16
2009 WQ6	2457438.15	1569.58			

Table 4: List of global optimization section results (departure time (DT_{gl}) and time of flight (TOF_{gl})) of the 39 pruned asteroids ($H > 28$ and 0.85 <semi-major axis< 1.15).

Table 5: Outputs of the Method 1 in trajectory optimization section of the 39 pruned asteroids ($H > 28$ and 0.85 <semi-major axis< 1.15). A column of the asteroid stellar magnitude is written, specifying in parenthesis in which group of Method 1 mass estimation belongs. A is in the group of $H = 28$, B in the group of $H = 28.5$, C in the group of $H = 29$, D in the group of $H = 29.5$ and E in the group of $H = 30$. Next to that, the following 5 columns belong to the minimum limit range of Method 1 and the last 5 columns to the maximum limit range. These 5 columns for each limit are the Thrust (T), acceleration (a), departure time (DT), time of flight (TOF) and time when the thrust is on (t_{ON}).

Table 6: List of results from Method 2 trajectory optimization in upper and lower limit of mass estimation, from the 39 pruned asteroids ($H > 28$ and 0.85 <semi-major axis< 1.15). The outputs specified in the table are the thrust (T), the acceleration (a), the mass, the departure time, the time of flight and the time when the thrust is on (t_{ON}).

	Method 1:	Lower Limit Results					Upper Limit Results				
Name	H	T [N]	a $[m/s^2]$	DT [JD]	TOF [d]	t_{ON} [d]	T [N]	a $[m/s^2]$	DT [JD]	TOF [d]	t_{ON} [d]
1991 VG	28.39 (B)	15	8.81E-005	2457591.64	771.70	126.99	50	2.13E-005	2457590.97	849.85	595.87
2000 LG6	29.02 (C)	15	1.72E-004	2458638.80	1363.25	209.73	50	5.04E-005	2458638.18	1311.47	627.83
2003 SW130	29.12 (C)	15	1.72E-004	2456128.90	884.98	392.31	100	1.01E-004	2456129.19	845.83	662.71
2003 WT153	28.05 (A)	25	5.35E-005	2456220.47	1491.10	674.92	150	3.26E-005	2456220.56	517.96	423.79
2006 BV39	28.98 (C)	15	1.72E-004	2457304.51	858.40	274.21	50	5.04E-005	2457303.08	1353.32	1074.29
2006 JY26	28.35 (B)	15	8.81E-005	2456980.30	1260.00	285.00	50	2.13E-005	2456976.17	1500.00	1215.00
2006 RH120	29.53 (D)	5	1.36E-004	2460000.50	1602.35	129.48	15	3.21E-005	2460000.50	1602.35	420.82
2007 EK	29.26 (C)	15	1.72E-004	2458016.94	854.55	305.20	25	2.52E-005	2458014.42	1918.37	1841.64
2007 UN12	28.74 (C)	15	1.72E-004	2458639.71	786.15	97.16	25	2.52E-005	2458639.10	883.32	582.99
2008 CM74	28.04 (A)	15	3.21E-005	2457266.12	1181.23	1130.97	100	2.18E-005	2457266.48	1822.45	1285.31
2008 GM2	28.36 (B)	15	8.81E-005	2458977.62	1249.85	479.59	100	4.25E-005	2458975.77	1082.90	1024.68
2008 HU4	28.22 (A)	15	3.21E-005	2457224.88	1266.12	767.35	100	2.18E-005	2457224.88	1569.58	1255.66
2008 JL24	29.57 (D)	5	1.36E-004	2459955.43	1091.26	114.01	15	3.21E-005	2459949.55	1465.88	586.35
2008 KT	28.22 (A)	15	3.21E-005	2456408.92	1004.53	805.72	100	2.18E-005	2456408.51	1782.00	1170.00
2008 LD	28.86 (C)	15	1.72E-004	2459992.17	739.34	285.88	50	5.04E-005	2459989.80	1569.58	1334.14
2008 UA202	29.44 (D)	5	1.36E-004	2460002.13	1169.51	115.51	15	3.21E-005	2459999.89	1357.21	765.24
2008 UC20	28.24 (A)	25	5.35E-005	2460000.50	1395.18	1088.24	150	3.26E-005	2460000.50	655.73	418.55
2008 WO2	29.78 (E)	5	1.36E-004	2456005.45	1511.64	287.93	15	5.10E-005	2456003.20	1365.53	784.79
2009 BD	28.24 (A)	15	3.21E-005	2459427.81	931.28	481.34	100	2.18E-005	2459426.96	1188.24	822.63
2009 WQ6	29.19 (C)	15	1.72E-004	2457438.15	755.72	619.69	150	1.51E-004	2457438.15	802.23	671.43
2009 WW7	28.89 (C)	15	1.72E-004	2457446.36	1025.97	357.90	50	5.04E-005	2457445.47	1339.37	1102.19
2009 WR52	28.32 (B)	15	8.81E-005	2456026.61	1050.57	565.69	100	4.25E-005	2456026.37	1413.07	1127.45
2009 YR	28.00 (A)	15	3.21E-005	2458047.17	1147.53	793.51	125	2.72E-005	2458046.12	1220.78	1062.08
2010 JW34	28.15 (A)	15	3.21E-005	2456006.76	929.11	724.71	100	2.18E-005	2456006.76	1381.23	948.72
2010 RF12	28.37 (B)	15	8.81E-005	2459967.57	989.44	359.80	100	4.25E-005	2459962.67	1615.42	726.94
2010 UY7	28.53 (B)	15	8.81E-005	2456971.89	1176.60	345.31	100	4.25E-005	2456970.80	2042.30	793.50
2010 UE51	28.31 (B)	15	8.81E-005	2459989.98	333.07	126.34	50	2.13E-005	2459989.98	871.99	828.39
2010 VL65	28.42 (B)	15	8.81E-005	2458983.86	1058.37	510.51	100	4.25E-005	2458983.43	1207.79	996.11
2010 VQ98	28.20 (A)	15	3.21E-005	2456000.68	1035.92	523.19	100	2.18E-005	2456001.02	1469.87	674.94
2011 AM37	29.69 (D)	5	1.36E-004	2457413.81	928.08	274.50	15	3.21E-005	2457412.01	1307.15	1176.43
2011 BQ50	28.34 (B)	15	8.81E-005	2459411.30	1062.75	260.26	100	4.25E-005	2459411.49	1051.90	531.37
2011 CA7	30.32 (E)	5	1.36E-004	2458887.32	936.52	372.48	15	5.10E-005	2458886.31	1412.62	1020.22
2011 CH22	28.96 (C)	15	1.72E-004	2456898.36	917.83	257.44	100	1.01E-004	2456895.24	863.27	531.91
2011 JV10	29.71 (D)	5	1.36E-004	2458967.43	1067.36	283.73	15	5.10E-005	2458967.43	1351.09	756.61
2011 MD	28.07 (A)	15	3.21E-005	2460000.85	985.58	683.87	100	2.18E-005	2460000.74	1381.23	1018.48
2011 UD21	28.48 (B)	15	8.81E-005	2456003.88	993.29	171.68	50	2.13E-005	2456000.50	1214.02	649.93
2012 AQ	30.70 (E)	5	1.36E-004	2458874.87	922.72	240.71	15	5.10E-005	2458873.49	1283.79	575.03
2012 EP10	29.17 (C)	15	1.72E-004	2456005.25	1191.72	130.80	25	2.52E-005	2456000.75	1569.58	1130.09
2012 FS35	30.29 (E)	5	1.36E-004	2458399.55	788.56	254.76	15	5.10E-005	2458396.83	1152.51	655.11

Method 2:	Lower Limit Results						Upper Limit Results					
Name	T [N]	a $[m/s^2]$	Mass [kg]	DT	TOF [d]	t_{ON} [d]	T [N]	a $[m/s^2]$	Mass [kg]	DT	TOF [d]	t_{ON} [d]
1991 VG	5	5.98E-005	83578	2457591.64	771.70	234.44	50	1.89E-005	2642968	2457589.62	928.00	722.86
2000 LG6	5	1.43E-004	35050	2458636.74	685.96	370.42	50	4.51E-005	1108388	2458638.03	1423.17	764.02
2003 SW130	5	1.63E-004	30612	2456130.63	766.37	419.68	125	1.29E-004	968037	2456130.72	920.82	481.34
2003 WT153	10	7.46E-005	134058	2456221.22	776.07	523.19	200	4.72E-005	4239278	2456221.22	1515.84	866.20
2006 BV39	5	1.36E-004	36787	2457304.31	882.24	369.59	50	4.30E-005	1163300	2457303.08	1395.18	1311.47
2006 JY26	5	5.65E-005	88449	2456979.79	1290.00	420.00	50	1.79E-005	2796996	2456976.17	1691.65	1639.33
2006 RH120	5	2.88E-004	17374	2460000.50	515.93	175.41	50	9.10E-005	549404	2460000.78	1602.35	161.85
2007 EK	5	1.98E-004	25194	2458015.02	662.71	279.04	50	6.28E-005	796695	2458014.42	1046.38	983.60
2007 UN12	5	9.72E-005	51463	2458639.71	794.98	159.00	50	3.07E-005	1627391	2458639.10	874.48	441.66
2008 CM74	5	3.70E-005	134987	2457266.56	1156.10	892.21	100	2.34E-005	4268663	2457264.83	1256.63	879.64
2008 GM2	5	5.71E-005	87598	2458974.37	871.99	767.35	100	3.61E-005	2770077	2458975.81	1311.47	1171.95
2008 HU4	5	4.75E-005	105267	2457225.03	854.55	497.03	100	3.00E-005	3328832	2457224.88	1035.92	753.40
2008 JL24	5	3.06E-004	16326	2459950.31	523.19	90.69	25	4.84E-005	516288	2459949.27	1433.30	325.75
2008 KT	5	4.70E-005	106437	2456408.82	1710.00	486.00	100	2.97E-005	3365828	2456408.51	1782.00	630.00
2008 LD	5	1.15E-004	43420	2459989.97	975.93	453.46	125	9.10E-005	1373068	2459989.80	1241.71	613.88
2008 UA202	5	2.55E-004	19593	2459999.64	592.95	188.35	50	8.07E-005	619572	2460002.62	1169.51	216.58
2008 UC20	5	4.88E-005	102540	2460000.50	1569.58	1224.27	150	4.63E-005	3242589	2460000.50	1743.97	1255.66
2008 WO2	5	4.08E-004	12266	2456000.50	455.76	176.72	25	6.45E-005	387875	2456003.60	1601.62	593.86
2009 BD	5	4.84E-005	103393	2459429.43	1031.55	313.38	50	1.53E-005	3269580	2459426.64	1506.79	1145.79
2009 WQ6	5	1.80E-004	27829	2457438.15	784.79	540.63	150	1.70E-004	880019	2457438.15	793.51	566.79
2009 WW7	5	1.20E-004	41657	2457446.15	1014.04	477.19	100	7.59E-005	1317322	2457444.81	1726.53	1430.06
2009 WR52	5	5.43E-005	92064	2456026.01	1085.21	969.76	100	3.43E-005	2911333	2456026.02	1674.21	1360.30
2009 YR	5	3.50E-005	142657	2458046.12	1162.65	732.47	100	2.22E-005	4511198	2458046.12	1495.43	807.53
2010 JW34	5	4.28E-005	116759	2456006.76	1014.99	429.02	100	2.71E-005	3692257	2456006.76	929.11	631.80
2010 RF12	5	5.81E-005	86038	2459962.67	952.21	742.93	100	3.68E-005	2720770	2459962.67	1569.58	1083.01
2010 UY7	5	7.23E-005	69166	2456971.82	1086.50	427.27	100	4.57E-005	2187217	2456970.61	1485.00	1110.00
2010 UE51	5	5.36E-005	93216	2459989.98	459.41	246.93	100	3.39E-005	2947759	2459989.98	574.26	511.09
2010 VL65	5	6.26E-005	79853	2458982.79	1245.14	722.18	100	3.96E-005	2525177	2458982.79	1395.18	1269.61
2010 VQ98	5	4.60E-005	108666	2456001.55	1123.12	317.40	100	2.91E-005	3436307	2456001.27	1454.87	419.96
2011 AM37	5	3.60E-004	13870	2457413.09	627.83	111.61	25	5.70E-005	438624	2457412.01	1267.93	575.15
2011 BQ50	5	5.59E-005	89432	2459411.49	1051.90	368.71	100	3.54E-005	2828081	2459411.12	1073.59	694.04
2011 CA7	5	8.60E-004	5817	2458886.04	232.53	223.23	25	1.36E-004	183947	2458886.58	947.17	393.77
2011 CH22	5	1.32E-004	37974	2456897.04	749.91	374.95	125	1.04E-004	1200859	2456895.09	1220.78	964.42
2011 JV10	5	3.69E-004	13567	2458967.43	610.39	140.39	25	5.83E-005	429034	2458969.75	1202.47	702.57
2011 MD	5	3.86E-005	129507	2460001.19	965.46	543.07	100	2.44E-005	4095358	2460000.74	1381.23	878.96
2011 UD21	5	6.80E-005	73501	2456003.88	993.29	220.73	50	2.15E-005	2324299	2456000.50	1214.02	637.67
2012 AQ	0.1	2.90E-005	3446	2458872.57	523.19	292.99	50	4.59E-004	108966	2458872.57	348.79	136.03
2012 EP10	5	1.75E-004	28648	2456000.75	697.59	174.40	25	2.76E-005	905924	2456000.75	1495.44	1181.40
2012 FS35	5	8.21E-004	6088	2458396.83	232.53	137.19	25	1.30E-004	192527	2458399.34	824.95	254.76

6 Conclusions

In this research, a procedure to capture an asteroid in the L_2 libration point has been described. It has been divided in the transfer opportunity search and the trajectory generation sections, obtaining a low-thrust trajectory for each case with an on-off profile control. A list of the asteroids capture opportunities has been obtained indicating the results of the most important parameters such as the acceleration, the time of flight and the duration time of the thrust open. A converged solution has been found for all asteroids with all mass estimation combinations.

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References

[1] H. Baoyin, Y. Chen and J. Li Capturing Near Earth Objects. *Research in Astronomy and Astrophysics, 2011.*

[2] J.P. Sanchez., D. Garcia Yarnoz and C.R. McInnes Near-Earth Asteroid Resource Accessibility and Future Capture Mission Opportunities. *Global Space Exploration Conference*, 2012.

[3] B.W. Longmier, J.P. Squire, L.D. Cassady, M.G. Ballenger, M.D. Carter, C. Olsen, A. V. Ilin, T. W. Glover, G. E. McCaskill, F. R. Chang Díaz, E.A. Bering and J. Del Valle. VASIMR VX-200 Performance Measurements and Helicon Throttle Tables Using Argon and Krypton. *32nd International Electric Propulsion Conference*, 2011.

[4] J.P. Sanchez. and C.R. McInnes On the Ballistic Capture of Asteroids for Resource Utilisation. *International Astronautical Congress*, 2011.

[5] J.L. Hilton. Asteroid Masses and Densities. *U.S. Naval Observatory*.

[6] Chesley, S. R., Chodas, P. W., Milani, A., Valsecchi, G. B. and Yeomans, D. K. Quantifying the Risk Posed by Potential Earth Impacts. *Icarus, Vol. 159, 2002, pp.423-432.*, 2002.

[7] Jet Propulsion Laboratory. Near Earth Observation Program http://neo.jpl.nasa.gov/glossary/h.html, [Accessed on 5th of September 2012].

[8] Fowler J.W. and Chillemi J.R. The IRAS Minor Planet Survey. *17,43, 1992.*

[9] J.A. Sims and S.N. Flanagan. Preliminary Design of Low-Thrust Interplanetary Missions. *AAS/AIAA Astrodynamics Specialist Conference*, 1999.

[10] C. H. Yam, D. Izzo and F. Biscani Global Optimization of Low-Thrust Trajectories via Impulsive Delta-V Transcription. *ESA - Advanced Concepts Team*, 2009.

[11] C. H. Yam, D. Izzo and F. Biscani Towards a High Fidelity Direct Transcription Method for Optimisation of Low-Thrust Trajectories. *ESA - Advanced Concepts Team*, 2010.

[12] K. Price and R. Storne Differential Evolution (DE)for Continuous Function Optimization. http://www1.icsi.berkeley.edu/ storn/code.html, [Accessed on 5th of September 2012].

[13] Y. Gao Advances in low-thrust trajectory optimization and flight mechanics. *PhD Thesis*, 2003.

[14] Philip E. GILL. Users Guide for SNOPT Version 7: Software for Large-Scale Nonlinear Programming. *Department of Mathematics. University of California, San Diego, La Jolla, CA 92093-0112, USA.*