

# Exponentiated Weibull distribution family under aperture averaging for Gaussian beam waves

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**Abstract:** Nowadays, the search for a distribution capable of modeling the probability density function (PDF) of irradiance data under all conditions of atmospheric turbulence in the presence of aperture averaging still continues. Here, a family of PDFs alternative to the widely accepted Log-Normal and Gamma-Gamma distributions is proposed to model the PDF of the received optical power in free-space optical communications, namely, the Weibull and the exponentiated Weibull (EW) distribution. Particularly, it is shown how the proposed EW distribution offers an excellent fit to simulation and experimental data under all aperture averaging conditions, under weak and moderate turbulence conditions, as well as for point-like apertures. Another very attractive property of these distributions is the simple closed form expression of their respective PDF and cumulative distribution function.

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## 1. Introduction

In recent years increased attention has been given to wireless optical communication systems, commonly known as free-space optics (FSO), as they offer, when compared with radio-frequency (RF) technology, an intrinsic narrower beam; less power, mass and volume requirements, and the advantage of no regulatory policies for using optical frequencies and bandwidth. Nevertheless, the major drawback for deploying wireless links based on FSO technology, where lasers are used as sources, is the perturbation of the optical wave as it propagates through the turbulent atmosphere. At the receiver plane, a random pattern is produced both in time and space. The irradiance fluctuations over the receiver plane resemble the speckle phenomenon observed when a laser beam impinges over a rugged surface, and the *scintillation index* (SI) is the parameter used to quantify such fluctuations.

The performance of a FSO communication system can be greatly reduced by the intensity variations, resulting from atmospheric turbulence. Consequently, deep fading events can occur and eventually complete outages of the signal-carrying laser beam intensity. The reliability of any communication system is mainly characterized by the probability of detection, miss and false alarm; and the probability of fade. All of this criteria demand knowledge of the probability density function (PDF) for the received optical power [1]. Actually, it is rather a difficult task to determine what is the exact PDF that fits the statistics of the optical power received through an atmospheric path.

Historically, many PDF distributions have been proposed to describe the random fading events of the signal-carrying optical beam. Perhaps, the most widely accepted distributions are the Log-Normal (LN) and the Gamma-Gamma (GG) models [2], although, many others have

been subject of study, namely, the lognormally modulated Rician distribution [3], also known as the Beckmann distribution, the lognormally modulated exponential distribution [4], the  $I$ - $K$  distribution introduced by Andrews and Phillips [5] as a generalization of the well-known  $K$  distribution [6].

Experimental studies support the fact that the LN model is valid in weak turbulence regime for a point receiver and works well in all regimes of turbulence for aperture averaged data [7,8]. On the other hand, the GG model is accepted to be valid in all turbulence regimes for a point receiver, nevertheless, this does not hold when aperture averaging takes place [2,7,9].

Nowadays, the search for a distribution capable of model accurately the PDF of irradiance data under all conditions of atmospheric turbulence in the presence of aperture averaging still drives a large amount of theoretical work, as well as efforts in simulation and experimental work.

Here, a family of PDFs alternative to the widely accepted LN and GG distributions is proposed, namely, the Weibull and the exponentiated Weibull distribution. The Weibull distribution have been used recently to propose a double-Weibull process, to describe the PDF of the irradiance fluctuations, in moderate and strong regimes of turbulence [10], although, there was no study in Chatzidiamantis *et al.* [10] on how the Weibull PDF can be used to model the distribution of the received optical power in a FSO communication link. It is noteworthy that the Weibull distribution has been long part of the different fading channel models in RF wireless communications [11,12].

## 2. New PDF family for the received optical power

The Weibull probability density function was introduced as a generalization of the exponential PDF, initially appearing in the field of reliability engineering [13]. This type of PDF rapidly extended to other areas of engineering, specially to model the wind speed distribution [14] and particle size distribution [15]; in radar to model a specific type of clutter [16], and, in wireless communication where some channels are modeled with Weibull fading [11,12]. A generalization of the well-known Weibull distribution, with the addition of an extra parameter, was first proposed by Mudholkar and Srivastava [17] and named exponentiated Weibull (EW) distribution. Here, the Weibull and EW distributions are proposed to model the distribution of the received optical power in free-space optical links.

The typical measure for describing the fluctuations of the received optical power  $I$  in FSO systems is the scintillation index, and it is defined by

$$\sigma_I^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1, \quad (1)$$

where the brackets  $\langle \cdot \rangle$  denote an ensemble average. Therefore, the SI is related to the first and second raw moment of the irradiance, which can be analytically derived from the probability density function governing the random variable  $I$ . Moreover, the scintillation index can be estimated from atmospheric parameters [18].

Next, the Weibull and exponentiated Weibull distribution are presented and their governing parameters are deduced directly from atmospheric parameters.

### 2.1. Weibull PDF

The probability density function (PDF) and cumulative distribution function (CDF) of a random variable  $I$  having the Weibull distribution is defined by

$$f_W(I; \beta, \eta) = \frac{\beta}{\eta} \left( \frac{I}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{I}{\eta} \right)^\beta \right], \quad (2)$$

and

$$F_W(I; \beta, \eta) = 1 - \exp \left[ - \left( \frac{I}{\eta} \right)^\beta \right], \quad (3)$$

respectively; where  $\beta > 0$  is a shape parameter related to the scintillation index of the irradiance fluctuations, and  $\eta > 0$  is a scale parameter, that depends on  $\beta$ , and is related to the mean value of the irradiance. For the special cases of  $\beta = 2$  and  $\beta = 1$ , Eq. (2) reduces to the well-known Rayleigh and negative exponential PDF, respectively.

It is easily proved that the  $n$ -th irradiance moment of the Weibull PDF is given by

$$\langle I^n \rangle = \eta^n \Gamma \left( 1 + \frac{n}{\beta} \right), \quad (4)$$

where the brackets  $\langle \cdot \rangle$  denote expectation, and  $\Gamma(\cdot)$  is the gamma function.

Based on Eq. (1) and Eq. (4), the scintillation index is given by [19]

$$\sigma_I^2 = \frac{\Gamma(1+2/\beta)}{\Gamma(1+1/\beta)^2} - 1 \approx \beta^{-11/6}. \quad (5)$$

For the derivation of the scale parameter  $\eta$ , without loss of generality, it is assumed that  $\langle I \rangle = 1$ , and setting  $n = 1$  in Eq. (4), yields

$$\eta = \frac{1}{\Gamma(1+1/\beta)}. \quad (6)$$

## 2.2. Exponentiated Weibull PDF

The PDF and CDF of a random variable  $I$  described by the exponentiated Weibull (EW) distribution is given by

$$f_{EW}(I; \beta, \eta, \alpha) = \frac{\alpha \beta}{\eta} \left( \frac{I}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{I}{\eta} \right)^\beta \right] \left\{ 1 - \exp \left[ - \left( \frac{I}{\eta} \right)^\beta \right] \right\}^{\alpha-1}, \quad (7)$$

and

$$F_{EW}(I; \beta, \eta, \alpha) = \left\{ 1 - \exp \left[ - \left( \frac{I}{\eta} \right)^\beta \right] \right\}^\alpha, \quad (8)$$

respectively; where  $\beta > 0$  is a shape parameter related to the SI, and  $\eta > 0$  is a scale parameter, that depends on  $\beta$ , and is related to the mean value of the irradiance, as in the precedent case. The additional parameter, compared with Eq. (2),  $\alpha > 0$  is an extra shape parameter that gives more versatility to the EW distribution in the shape of the tails, and it is strongly dependent on the receiver aperture size. It is noteworthy that for  $\alpha = 1$  Eq. (7) reduces to the Weibull distribution, see Eq. (2).

The  $n$ -th irradiance moment of the exponentiated Weibull PDF has recently been derived for any  $\alpha$ , both real and integer, and has the form [20]

$$\langle I^n \rangle = \alpha \eta^n \Gamma \left( 1 + \frac{n}{\beta} \right) \sum_{i=0}^{\infty} \frac{(-1)^i (i+1)^{-(n+\beta)/\beta} \Gamma(\alpha)}{i! \Gamma(\alpha-i)}. \quad (9)$$

As it is readily seen from Eq. (9), the analytical derivation of the EW parameters is rather a complex task. Therefore, a heuristic approach, based on simulation data, was used to obtain a

first approximation to the EW parameters. Thus, the expression for the shape parameter  $\alpha$  was found to follow approximately

$$\alpha \simeq 3.931 \left( \frac{D}{\rho_0} \right)^{-0.519}, \quad (10)$$

where  $D$  is the receiving aperture diameter, and  $\rho_0 = (1.46C_n^2 k^2 L)^{-3/5}$  is the atmospheric coherence radius, being  $k = 2\pi/\lambda$  the wavenumber,  $\lambda$  the optical wavelength, and  $L$  is the distance between the transmitter and receiver planes.

The shape parameter  $\beta$ , as in Eq. (5), was found to be related with the scintillation index as

$$\beta \simeq (\alpha \sigma_I^2)^{-6/11}, \quad (11)$$

and the scale parameter  $\eta$  is given by

$$\eta = \frac{1}{\alpha \Gamma(1 + 1/\beta) g(\alpha, \beta)}, \quad (12)$$

where  $g(\alpha, \beta)$  was introduced to simplify the notation, and is defined by

$$g(\alpha, \beta) = \sum_{i=0}^{\infty} \frac{(-1)^i (i+1)^{-(1+\beta)/\beta} \Gamma(\alpha)}{i! \Gamma(\alpha - i)}. \quad (13)$$

Equation (13) is easily computed numerically as the series converges rapidly, and usually as much as ten terms or less are sufficient for the series to converge.

It is easily verified that for fixed values of the shape parameter  $\beta$  and the scale parameter  $\eta$ , the shape parameter  $\alpha$  controls the lower-tail steepness—when data is visualized in a logarithmic scale. This is an attractive property of the EW distribution as in any communication system, it is precisely the lower-tail of maximum importance because it defines the error-rate and fade probability.

### 3. Simulations

To study the suitability of the Weibull and exponentiated Weibull distributions to model the PDF of the received optical power in free-space optical links, a simulation scenario was set in order to obtain the probability density function for several receiving apertures. The wave optics code used to perform the numerical simulations is based on the fractal method, using the Kolmogorov spectrum of turbulence, where the phase screens are directly reproduced in the spatial domain by first generating an exact low-resolution screen, by means of the covariance method [21], of  $16 \times 16$  points. Next, successive interpolation, using the method by Lane *et al.* [22], were executed to produce the desired grid size  $512 \times 512$ . To minimize the error introduced at each interpolation step, the procedure proposed by Reclons and Dios [23] was followed.

All the simulations for a Gaussian beam were conducted for a link range  $L = 1425$  m using 29 random phase screens, a refractive-index structure constant  $C_n^2 = 2.1 \times 10^{-14} \text{ m}^{-2/3}$  was used, the wavelength and the half-angle beam divergence were set to  $\lambda = 780$  nm and  $\theta = 37$   $\mu$ rad, respectively. The initial beam radius is given by  $W_0 = d/(2\sqrt{2})$ , where  $d = 3.2$  cm is the transmitter aperture. The simulation parameters reproduce conditions of weak to moderate turbulence, with Rytov variances  $\sigma_R^2 = 1.23C_n^2 k^{7/6} L^{11/6}$  from  $2 \times 10^{-3}$  to 1.78.

Receiving apertures of 3, 25, 60 and 80 mm were used to analyze the effects of aperture averaging. Under the simulation conditions used the 3 mm receiving aperture will behave as a point-like receiver, as it is always smaller than the atmospheric coherence radius  $\rho_0$  [24].

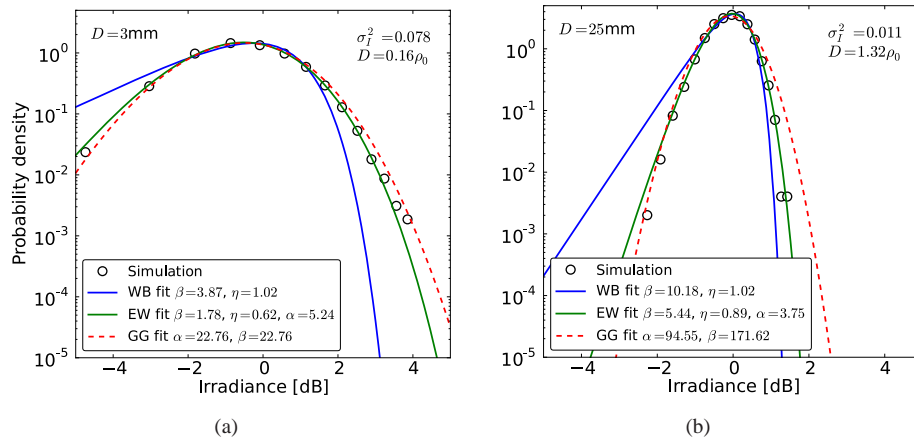


Fig. 1. Weibull (WB) and exponentiated Weibull (EW) models fitted to simulation data for several aperture diameters  $D$ , link distance  $L = 375\text{m}$ , coherence radius  $\rho_0 = 18.89\text{ mm}$  and  $\sigma_R^2 = 0.15$ , under weak turbulence conditions. The Gamma-Gamma (GG) model is shown for comparison purposes.

A total of 30000 realizations were run to reduce the statistical uncertainties in the numerical simulations of the irradiance. The simulated PDF was obtained by sorting the normalized irradiance data into a histogram of 80 bins of equal width, and the count of each bin was associated with the midpoint of its respective bin width. When zero-count bin is found its width is merged with the next bin to the left, thus, unequal bins were used whenever needed. The Weibull and exponentiated Weibull models were fitted to the simulated probability density function, using the Levenberg-Marquardt least-square fitting algorithm [25,26]. The fitted plots presented here are compared to the Gamma-Gamma model in order to show that the EW distribution can reproduce the tail behavior of the GG model, and in some cases it can even outperform the GG model.

To demonstrate the suitability of the Weibull and EW distribution in the weak turbulence regime, the PDF for the two smallest apertures at a distance  $L = 375\text{ m}$  from the receiver is shown in Fig. 1. It is clearly seen that the exponentiated Weibull distribution has the better fit for both receiving apertures, while the GG model is only valid for point-like apertures—i.e. when  $D \ll \rho_0$ . In Fig. 1 the 60 and 80 mm apertures PDF are not shown due to the fact that the histogram is completely concentrated in a bin of 1 dB width or less.

Under moderate turbulence condition, see Fig. 2, the exponentiated Weibull distribution keeps giving the best fit for all the receiving apertures tested. It is evident how the GG model and the EW distribution have a fit very close to the simulation PDF in Fig. 2(a) and Fig. 2(b), although, the EW distribution have a better fit for the 60 and 80 mm receiving apertures. Here, the Weibull distribution exhibits a close fit to the data in Fig. 2(c), and a perfect fit, as well as the EW distribution, in Fig. 2(d).

The simulation data suggest that the Weibull distribution gives a good fit to the probability density function when the receiver aperture is much greater than the atmospheric coherence radius, i.e.  $D \gg \rho_0$ . In this situation, both the Weibull and the exponentiated Weibull distributions give a close fit to the simulated PDF, hence, either can be used.

Figure 3 shows the evolution of the exponentiated Weibull parameters obtained from the fitting algorithm (solid lines), and the estimated parameters following the heuristic formulas presented in Sec. 2.2 (dashed lines), for several receiving apertures and Rytov variances. The

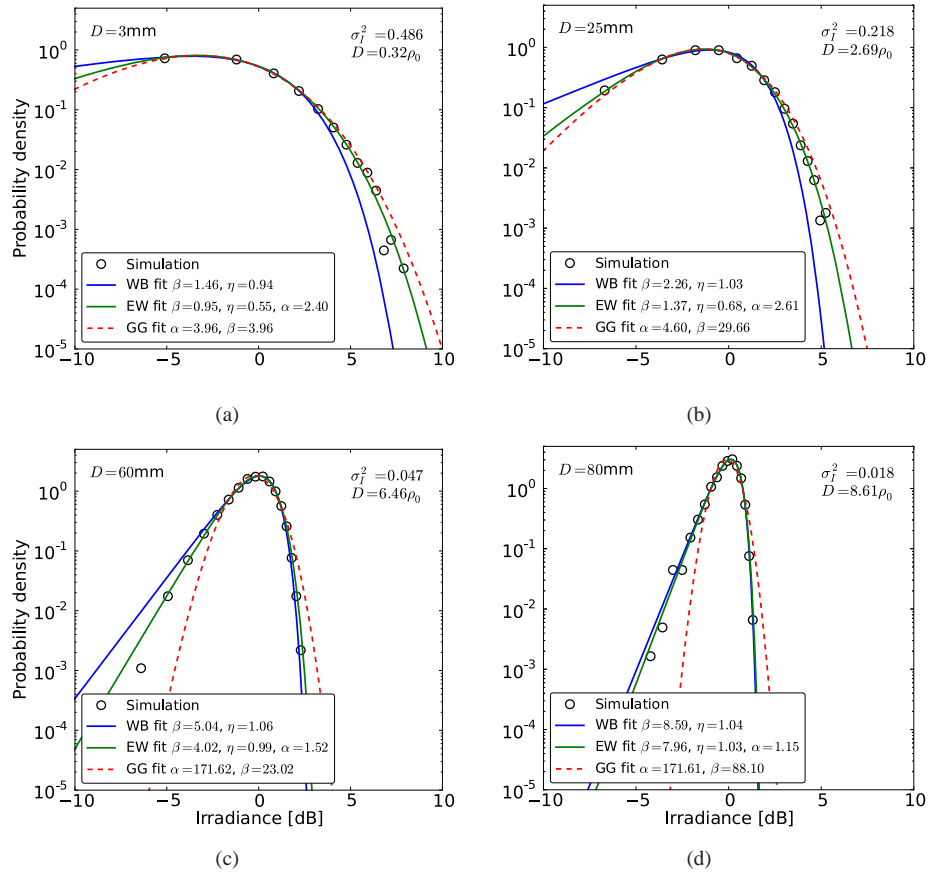


Fig. 2. Weibull (WB) and exponentiated Weibull (EW) models fitted to simulation data for several aperture diameters  $D$ , link distance  $L = 1225$  m, coherence radius  $\rho_0 = 9.27$  mm and  $\sigma_R^2 = 1.35$ , under moderate turbulence conditions. The Gamma-Gamma (GG) model is shown for comparison purposes.

plots are truncated, and points where the simulated PDF was found to be concentrated in a bin of 1 dB width or less are not shown. It is immediately seen that the parameters are strongly affected by the receiving aperture size. This behavior suggests that the EW distribution is a good candidate to model the probability density function of aperture averaged data. Additionally, it can be seen how the heuristic formulas found—see Eq. (10), Eq. (11) and Eq. (12)—to deduce the exponentiated Weibull parameters closely follow the parameter values estimated from the fitting algorithm. Nevertheless, for the smallest aperture this expressions start to deviate from the expected values. This could be produced by the large dynamic range of the received optical power for such a small aperture.

#### 4. Experiments

The experiments were conducted at Barcelona, Spain, between the rooftops of two buildings along a medium density residential terrain. A 780 nm continuous-wave diode laser at 15 mW (12 dBm) from LISA Laser (HL25/MIII), with built-in collimator, was used. A beam expander of diameter 32 mm was mounted with the laser to produce a beam divergence of 37  $\mu$ rad.

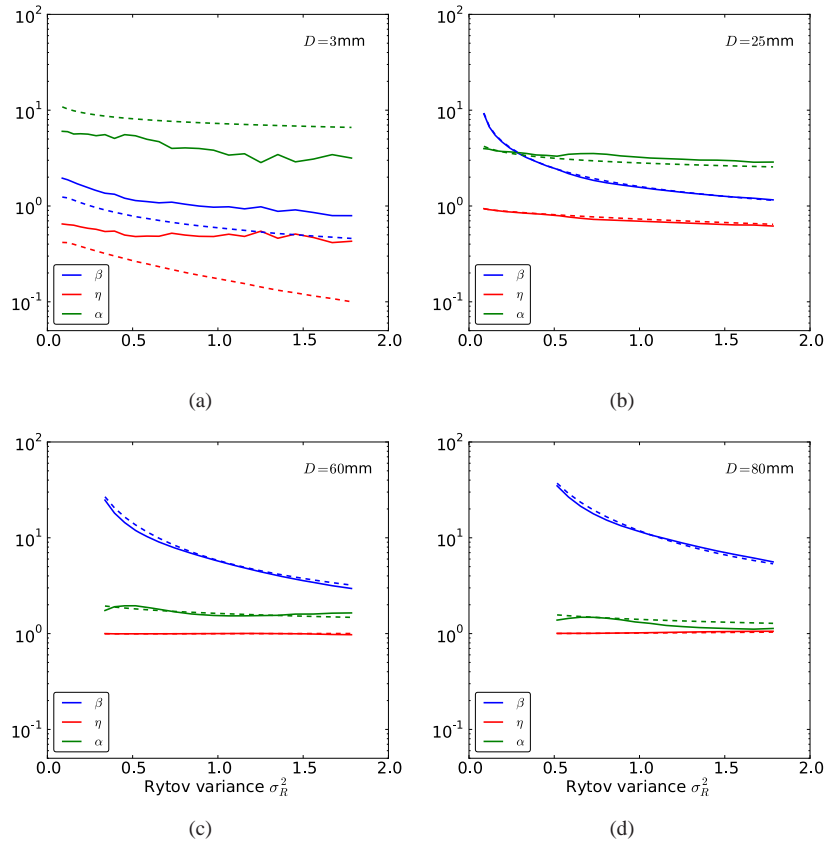


Fig. 3. Exponentiated Weibull fitted parameters (solid line) and estimated parameters (dashed line) for several aperture diameters  $D$ .

The testbed selected for the experiments consisted in a nearly horizontal 1.2 km optical path with the transmitter and receiver on either side of the optical path. On the receiver side the light was detected using a 15 cm focal length Fresnel lens, along with bandpass interference filter with a 3 dB bandwidth of 10 nm to remove the out-of-band background radiation. A set of diaphragms was used so measurements for different aperture diameters were possible. A complete description of the experimental setup can be found in Barrios *et al.* [27].

The irradiance data were collected at the receiver side with a PIN photodetector, and the detected signal was captured at 10 kHz of sampling rate. Data were taken in individual runs for the receiving apertures of 5 min each, hence,  $3 \times 10^6$  samples were available to calculate the experimental probability density function. The diaphragms used for the experiments had aperture diameters of 3, 25, 60 and 80 mm. The estimated value for the refractive-index structure constant for the experiments was found to be  $C_n^2 = 2.1 \times 10^{-14} \text{ m}^{-2/3}$  [27].

Figure 4 shows the probability density function obtained from the experimental data. The same procedure presented in Sec. 3 to estimate the PDF was applied to the experimental data set. Here, it can be seen how the exponentiated Weibull distribution offers an excellent fit to experimental data under all of the aperture averaging conditions tested. In Fig. 4(a) the EW distribution has the ability to reproduce the shape of the Gamma-Gamma model, and it gives a better fit for all other cases.



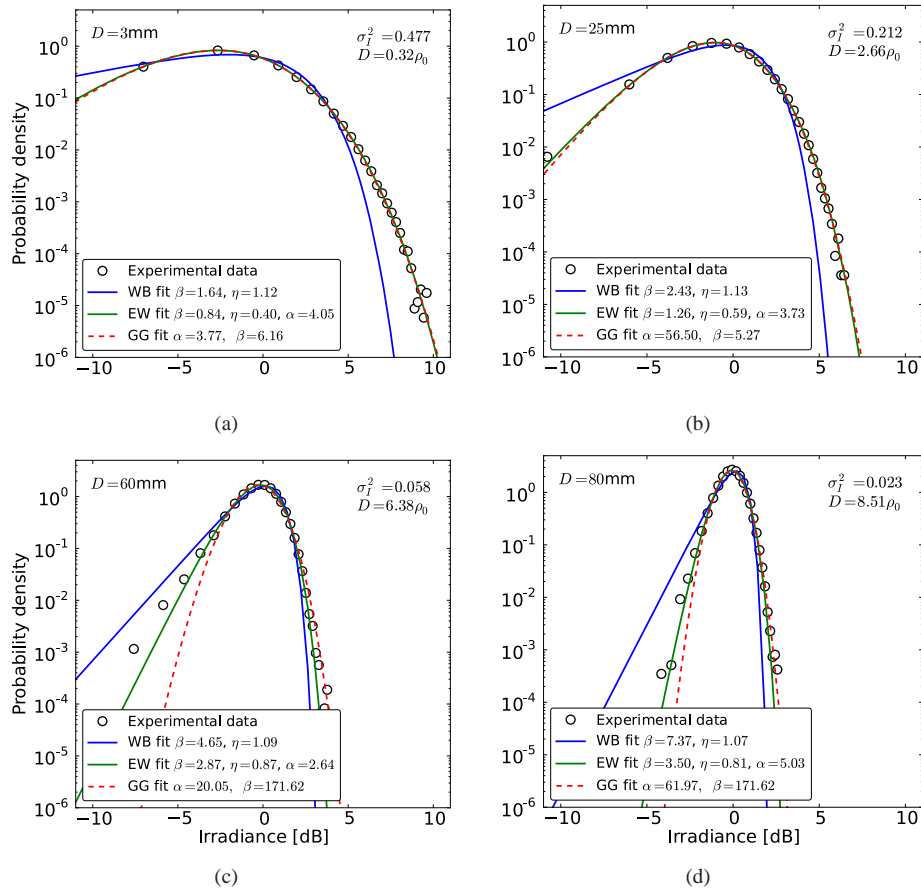


Fig. 4. Weibull (WB) and exponentiated Weibull (EW) models fitted to experimental data for several aperture diameters  $D$ , link distance  $L = 1200\text{m}$ , coherence radius  $\rho_0 = 9.40\text{ mm}$  and  $\sigma_R^2 = 1.30$ , under moderate turbulence conditions. The Gamma-Gamma (GG) model is shown for comparison purposes.

Albeit the simulation data, presented in Sec. 3, suggest that the Weibull distribution can be used under moderate turbulence, when  $D \gg \rho_0$ , in the experimental data set there were problems to reproduce the lower-tail behavior, associated with deep fading events. There is an overestimation of the experimental data lower-tail when fitting to the Weibull distribution.

## 5. Discussion

A new family of probability density functions, known as the exponentiated Weibull distribution, has been presented to model the probability density function under aperture averaging for Gaussian beam waves. A very attractive property of the EW distribution is the simple closed form expression of its PDF and CDF. Moreover, its suitability has been studied in comparison with the widely used Gamma-Gamma model, introduced by Al-Habash *et al.* [2], which has been proven to give excellent fit for data under all turbulence conditions for a point aperture [1, 2, 24, 28]; however it has unpredictable results for aperture averaged data.

In Sec. 3 and Sec. 4, it was shown how the proposed EW distribution offers an excellent fit to simulation and experimental data under all aperture averaging conditions, under weak

and moderate turbulence conditions, as well as for point-like apertures. Currently work is in progress to extend the study of the EW distribution to the strong turbulence regime.

On the other hand, the GG model presents an unpredictable performance when  $D \geq \rho_0$ . Specially it fails to reproduce the lower-tail shape when aperture averaging takes place. On the contrary, the exponentiated Weibull model gives a perfect fit in both tails of the probability density function of the irradiance data.

The exponentiated Weibull model reduces to the classic Weibull distribution when the extra shape parameter  $\alpha$  equals unity, in which case, the distribution parameters  $\beta$  and  $\eta$  can be directly related with atmospheric data, as demonstrated in Sec. 2.1. On the other hand, for the EW model the analytical derivation of the distribution parameters is rather a complex task, hence, a heuristic approach was used to obtain a first approximation to the EW parameters from atmospheric data.

Finally, it is well known that the PDF of irradiance becomes the negative exponential distribution, i.e.  $f(I) \sim \exp(-I)$ , in the limit of saturated scintillation [4, 29]. In this regime the Rytov variance  $\sigma_R^2 \rightarrow \infty$ , the scintillation index  $\sigma_I^2 \rightarrow 1$  and the atmospheric coherence radius  $\rho_0 \rightarrow 0$ . Thus, for a Weibull distribution the shape parameter  $\beta \rightarrow 1$  and the scale parameter  $\eta \rightarrow 1$ , and Eq. (2) reduces the negative exponential distribution. In contrast, in the case of the exponentiated Weibull model, and the heuristic expressions found here, Eq. (7) does not reduce to the negative exponential distribution, as the shape parameter  $\alpha \rightarrow 0$ . Nevertheless, if an appropriate general expression to deduce the parameters of the EW distribution is found, the exponentiated Weibull model becomes an excellent candidate to model the PDF of irradiance data under all condition of atmospheric turbulence in the presence of aperture averaging. Additional work is currently being conducted with the aim of obtaining a more general solution for the EW parameters.

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