SIMULATION OF DAMAGE ON LAMINATES

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Abstract. *In this paper, we present a recently developed progressive damage model for composites (CPDM) that allows the switching between different damage modes during the numerical process to mimic real physical behaviour. The modelled damage modes are ranging from matrix cracking to fibre breakage or delamination. Moreover, the CPDM is flexible in the sense that provides the basic structure to include variations on the evolution of damage modes whenever appropriate by modifying basic second-order tensors. Ongoing work for potential linkage to the extended finite element method (XFEM) will be also shown.*

1 INTRODUCTION

The modelling of failure of high performance materials in general –and composites in particular– is still a recurrent problem in many aspects. Amongst others:

- The existing techniques or methods often diverge before the actual failure of the material.

- There are different concepts about how to model failure. This could be based on failure criteria. In general, these criteria are nonlinear functionals of the stress components, see [1] for an overview of these. They have been extensively used in the past decades. Their main shortcomings range from abortion of the numerical process to poor replication of switching –if any at all– between distinct damage modes in a time–marching numerical method. There is the most recent option of using progressive damage models (PDM) that overcome some of the problems of failure criteria but PDM cannot replicate discontinuities on its own.

- Different strategies for modelling the discontinuities exerted by the cracks. Nowadays, a number of computational methods have proved their reliability in many areas of simulation. In particular, linear stress analysis is a successful story. However, the debate about numerical strategies for modelling discontinuities such as a cracks remains open.

In this paper, we present a recently developed PDM for composites (CPDM) that allows the switching between different damage modes during the numerical process, ranging from matrix cracking to fibre breakage or delamination. Moreover, the proposed PDM is flexible in the sense that provides the basic structure to include variations of damage mode evolutions if appropriate through modification of basic second-order tensors as described below. To avoid the nonhomogeneous stress field –measured at Gauss points– in the computation of damage, a mapping to the strain space of the damage surfaces that conform the undamaged domain is performed to allow a smoother numerical process, eventually, diminishing the risk of ill-convergence in some grade.

Our main concern at this stage of the research is the link with a convenient numerical method for replication of discontinuities. There are already interesting numerical methodologies to perform this step. Thus, the partition of unity finite element methods (PUFEM) [2] such as the extended finite element method (XFEM) [3] or the phantom node method (PNM) [4] offer potential infrastructure for this potential linkage. We have used XFEM for modelling mode I delamination with orthotropic enrichment functions and the results are excellent, see [5]. Some of these results will be also shown in the minisymposyum even if they are not using the proposed PDM at this moment but to highlight the advantages of XFEM. A potential way to explore is indeed the link PDM–XFEM or PDM–PNM. However, some barriers, such as the flexibility of the codes to allow a PDM based on continuum damage mechanics to be linked to a technique based on discontinuities, must be clarified.

2 DAMAGE TENSOR AND EFFECTIVE STRESS

The internal damage state is characterised by a set of damage internal variables contained within the vector $\boldsymbol{\eta}_n = [\eta_{11}, \eta_{22}, \eta_{33}, \eta_{12}, \eta_{23}, \eta_{31}]_n^T$. Each damage variable is responsible for the deterioration of stiffness. Each component of the stiffness tensor can be reduced by one of the internal variables. Note that a damage internal variable should be computed for different damage modes depending of the loading state and type of failure. This is modelled in CPDM to

acknowledge this fact as described below. The damage tensor containing the damage internal variables is defined as,

$$D_{ij}(\boldsymbol{\eta}_n) = \delta_{ij}/(1 - \eta_{ij}|_n) \tag{1}$$

Then, the relationship between effective stress and nominal stress is,

$$\hat{\boldsymbol{\sigma}}_n = \mathbf{D}(\boldsymbol{\eta}_n) \cdot \boldsymbol{\sigma}_n \tag{2}$$

Material coordinates at ply level are defined according to:

1: longitudinal to fibres.

2: in-plane perpendicular to fibres.

3: out-of-plane perpendicular to fibres.

The stress is introduced as a contracted vector, $\boldsymbol{\sigma}_n = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]_n^T$, and subscript n denotes the time step t_n.

The nominal stress-strain relationship may be written through a constitutive matrix that integrates the internal damage state, as follows,

$$\boldsymbol{\sigma}_n = \mathbf{D}^{-1}(\boldsymbol{\eta}_n) \cdot \mathbf{C_0} \cdot \boldsymbol{\varepsilon}_n = \mathbf{C}_n \cdot \boldsymbol{\varepsilon}_n \tag{3}$$

with C given in material local coordinates,

$$\mathbf{C}(\boldsymbol{\eta}) = \begin{bmatrix} \frac{(1-\eta_{11})(1-\nu_{23}\nu_{32})}{E_{22}E_{33}\gamma} & \frac{(1-\eta_{11})(\nu_{12}+\nu_{32}\nu_{13})}{E_{11}E_{33}\gamma} & \frac{(1-\eta_{11})(\nu_{13}+\nu_{12}\nu_{23})}{E_{11}E_{22}\gamma} & 0 & 0 & 0 \\ \frac{(1-\eta_{22})(\nu_{12}+\nu_{32}\nu_{13})}{E_{11}E_{33}\gamma} & \frac{(1-\eta_{22})(1-\nu_{13}\nu_{31})}{E_{11}E_{33}\gamma} & \frac{(1-\eta_{22})(\nu_{23}+\nu_{21}\nu_{13})}{E_{11}E_{22}\gamma} & 0 & 0 & 0 \\ \frac{(1-\eta_{33})(\nu_{13}+\nu_{12}\nu_{23})}{E_{11}E_{22}\gamma} & \frac{(1-\eta_{33})(\nu_{23}+\nu_{21}\nu_{13})}{E_{11}E_{22}\gamma} & \frac{(1-\eta_{33})(1-\nu_{12}\nu_{21})}{E_{11}E_{22}\gamma} & 0 & 0 & 0 \\ 0 & 0 & 0 & (1-\eta_{12})S_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\eta_{23})S_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\eta_{31})S_{31} \end{bmatrix}$$

$$\gamma = \frac{(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13})}{E_{11}E_{22}E_{33}}$$

3 UNDAMAGED DOMAIN AS A FUNCTIONAL OF STRAIN

Each damage mode ξ is represented by a damage surface on the strain space. These surfaces are mapped from stress functionals, Equation (4), which are linear combination of normalised energy release rates (ERRN), Equation (5). The intersection of the damage surfaces on the strain space creates the undamaged domain which shrinks with the progression of damage due to the ERRN dependence upon damage.

$$\bar{Y}_{ij}(\boldsymbol{\sigma}_n, \boldsymbol{\eta}_n) := \begin{cases} \left.\frac{\sigma_{ii}^2}{2\,E_i(1-\eta_{ii}^2)X_{ii}^{(t)}}\right|_n & \text{if } i = j \text{ and } \sigma_{ii} \geq 0, \\[2ex] \left.\frac{\sigma_{ii}^2}{2\,E_i(1-\eta_{ii}^2)X_{ii}^{(c)}}\right|_n & \text{if } i = j \text{ and } \sigma_{ii} < 0, \\[2ex] \left.\frac{\sigma_{ij}^2}{2\,S_{ij}(1-\eta_{ij}^2)X_{ij}}\right|_n & \text{if } i \neq j \end{cases} \tag{4}$$

where E_i is the Young's modulus in direction i, S_{ij} is the elastic shear modulus associated to directions i and j, $X_{ii}^{(t)}$ represents the direct tensile strength of the laminate in direction i and, X_{ij} denotes the shear strength.

$$f^\xi(\boldsymbol{\sigma}_n, \boldsymbol{\eta}_n) := f^\xi(\bar{Y}_{ij}(\boldsymbol{\sigma}_n, \boldsymbol{\eta}_n)) \qquad \xi = 1, 2, \ldots, m \tag{5}$$

Equation(5) is rearranged as,

$$f^\xi(\boldsymbol{\sigma}_n, \boldsymbol{\eta}_n) := \boldsymbol{\sigma}_n^T \cdot \mathbf{F}^\xi(\boldsymbol{\eta}_n) \cdot \boldsymbol{\sigma}_n - 1 \qquad \xi = 1, 2, \ldots, m \tag{6}$$

where $\mathbf{F}^\xi$ denotes the tensor associated to the damage mode ξ and m is the total number of damage modes modelled. The mapping in Equation(7) is obtained by previous computation of these damage surfaces on the stress space. Thus, these stress damage surfaces are built as function of the so-called ERRN in Equation(4) due to the propagation of damage variables. The stress damage surfaces f^ξ shrink with the progression and development of any damage mode affecting them. Once f^ξ are calculated the corresponding g^ξ damage surface on the strain space is computed as follows,

$$\mathbf{G}^\xi(\boldsymbol{\eta}_n) = \boldsymbol{C}_n^T \cdot \mathbf{F}^\xi(\boldsymbol{\eta_n}) \cdot \boldsymbol{C}_n \qquad \xi = 1, 2, \ldots, m \tag{7}$$

$$g^\xi(\boldsymbol{\varepsilon}_n, \boldsymbol{\eta}_n) := \boldsymbol{\varepsilon}_n^T \cdot \mathbf{G}^\xi(\boldsymbol{\eta}_n) \cdot \boldsymbol{\varepsilon}_n - 1 \qquad \xi = 1, 2, \ldots, m \tag{8}$$

For instance for inter-ply damage, mode I ($\xi = 5$),

$$\mathbf{F_n^5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/(2E_3(1-\eta_{33}^2)X_{33}^{(t)}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(2E_{23}(1-\eta_{23}^2)X_{23}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(2E_{31}(1-\eta_{31}^2)X_{31}) \end{bmatrix}_n \tag{9}$$

or for matrix crushing in 3-direction through thickness, i.e. out-of-plane perpendicular to fibres, ($\xi = 6$),

$$\mathbf{F_n^6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/(2E_3(1-\eta_{33}^2)X_{33}^{(c)}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(2E_{23}(1-\eta_{23}^2)X_{23}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(2E_{31}(1-\eta_{31}^2)X_{31}) \end{bmatrix}_n \tag{10}$$

The interested reader is referred to [6] for more details on these tensors associated to distinct damage modes.

4 DAMAGE CRITERIA

Following Curiel Sosa et al. [6] the criteria used are,

$$\nabla_\varepsilon g_n^\xi \cdot \dot{\boldsymbol{\varepsilon}}_n > 0 \qquad g^\xi(\boldsymbol{\varepsilon}_n, \boldsymbol{\eta}_n) \geq 0 \qquad \xi = 1, 2, \ldots, m \tag{11}$$

5   DAMAGE PATHS

The characterisation of directional vectors $\mathbf{d}^{\xi}$ is performed according to [6] as follows,

$$\mathbf{d}_n^{\xi} := \boldsymbol{\varepsilon}_n^T \cdot (\mathbf{G}_n^{\xi T} + \mathbf{G}_n^{\xi})/\|\nabla_{\varepsilon}\, g_n^{\xi}\| \qquad \xi = 1, 2, \ldots, m \tag{12}$$

6   COMPUTATION OF DAMAGE RATE

Once the criterion in Equation(11) is satisfied the progression of the damage mode ξ concerned is performed following [6],

$$\psi_n^{\xi} := (\nabla_{\varepsilon} g_n^{\xi} \cdot \dot{\boldsymbol{\varepsilon}}_n)^{1/p} \qquad \xi = 1, 2, \ldots, m \tag{13}$$

where ψ^{ξ} denotes the corresponding increment. p is a parameter depending upon the composite material and the geometry that must be adjusted for mesh independency. Finally, the damage rate vector is computed according to [7],

$$\dot{\boldsymbol{\eta}}_n = \sum_{\xi=1}^{m} \psi_n^{\xi}\, \mathbf{d}_n^{\xi} \tag{14}$$

7   CPDM EMBEDDED INTO EXPLICIT–FEM

The algorithm of the explicit time-stepping scheme deployed is provided below in some detail.

  i Initialisation:

    – $n = 0$, $t_0 = 0$; $\dot{\mathbf{u}}_0 = 0$; $\boldsymbol{\eta}_0 = 0$, $\boldsymbol{\sigma}_0 = 0$; $\mathbf{X}_0$

  ii Compute lumped mass matrix $\mathbf{M}$

  iii Loop over time steps:

    iii.1

$$\mathbf{X}_n = \mathbf{X}_{n-1} + \mathbf{u}_n \tag{15}$$

    iii.2 Loop over elements e:

      iii.2.1 Compute strain:

$$\boldsymbol{\varepsilon}_n = \mathbf{B} \cdot \mathbf{u}_n \tag{16}$$

      iii.2.2 Call to subroutine:

$$\boldsymbol{\varepsilon}_n,\ \boldsymbol{\eta}_{n-1} \longrightarrow \mathbf{C}_n,\ \boldsymbol{\sigma}_n,\ \dot{\boldsymbol{\eta}}_n$$

      iii.2.3 Update increment of damage:

$$\Delta\boldsymbol{\eta}_n = \Delta t_n\, \dot{\boldsymbol{\eta}}_n \tag{17}$$

      iii.2.4 Damage internal variables:

$$\boldsymbol{\eta}_{n+1} = \boldsymbol{\eta}_n + \Delta\boldsymbol{\eta}_n \tag{18}$$

      iii.2.5 Element stiffness matrix:

$$\mathbf{k}_n^{(e)} = \int_{\Omega_e} \mathbf{B}^T \cdot \mathbf{C}_n \cdot \mathbf{B}\ \mathrm{d}\Omega \tag{19}$$

    iii.2.6 End loop over elements.

  iii.3 Assembly for all elements:

$$\mathbf{K}_n^{(int)} = \bigwedge_{e=1}^{numel} \mathbf{k}_n^{(e)} \tag{20}$$

  iii.4 Update nodal external force vector:

$$\mathbf{f}_n^{(ext)} = \bigwedge_{e=1}^{numel} \left\{ \int_{\Omega^e} \mathbf{N}^T \boldsymbol{b}_n \mathrm{d}\Omega + \int_{\Gamma^e} \mathbf{N}^T \boldsymbol{q}_n \, \mathrm{d}\Gamma \right\} \tag{21}$$

  iii.5 Compute acceleration :

$$\ddot{\mathbf{u}}_n = \mathbf{M}^{-1} \cdot [\mathbf{f}_n^{(ext)} - \mathbf{f}^{(int)}(\boldsymbol{\eta}_n)] \tag{22}$$

  iii.6 Compute the nodal mid-step velocities $\dot{\mathbf{u}}_{n+1/2}$ as,

$$\dot{\mathbf{u}}_{n+1/2} = \dot{\mathbf{u}}_{n-1/2} + \Delta t_n \, \ddot{\mathbf{u}}_n \tag{23}$$

  iii.7 Nodal displacement vector:

$$\mathbf{u}_{n+1} = \mathbf{u}_n + \Delta t_{n+1/2} \, \dot{\mathbf{u}}_{n+1/2} \tag{24}$$

  iv New critical time step $\Delta t_n^{(crit)}$, update time step $\Delta t_n = \alpha \Delta t_n^{(crit)}$, $\alpha \subseteq]0, 1]$, and check convergence (relative norm of the residual).

  v End loop over time steps.

Above, $\mathbf{N}$ is the shape functions tensor, $\mathbf{b}_n$ are body forces, $\mathbf{q}_n$ the traction forces applied over the boundary of the body, and $\mathbf{B}$ is the strains operator.

8 NUMERICAL RESULTS

Single element tests were used for validation [1]. CPDM was able to detect the right modes of damage in tension (matrix cracking and fibre breakage) and compression tests (matrix crushing and fibre kinking) performed over a single element. The proposed technique was used to simulate maps of damage in a cross-ply laminate when impacted by a projectile at low velocity. The results of delamination and matrix crushing from Curiel Sosa et al. [6] will be shown in the conference presentation in some detail and compared with experiments by [8].

9 CONCLUSION

The formulation of a technique for composite laminate damage simulation was presented in this note. The main characteristics of CPDM are enumerated as follows:

1. Calculation of damage directions.

2. Growth functions of ERRN mapped onto strain space for greater stability.

3. Criteria of damage based on shrinkage of strain damage surfaces and positive accumulation of damage.

CPDM is able to address initiation of distinct damage modes and damage evolution afterwards. A further development is intended in relation to use PUFEM in conjunction with CPDM. At the moment, that PUFEM-CPDM connection is ongoing. Complete results or in-development work will be presented at the time of the conference.

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