

# A tool for the design of public transportation services

Esteve Codina, Ángel Marín and Lúdia Montero

**Abstract**—In this paper a model is described in order to determine the number of lines of a public transportation service, the layout of their lines amongst a set of candidates, their service capacity, and the resulting assignment of passengers to these facilities so as to minimize the total costs of the system. The model takes into account the delays for passengers that queue at the stations reflecting congestion effects of the transport service system and also, the abandonment of waiting queues at stations by passengers. Passengers are assumed to choose the lines they ride on by selecting the most convenient service line following a user equilibrium formulation.

**Index Terms**—Public transportation services, network design.

## I. INTRODUCTION

SETTING properly the required services to attend transportation demand taking into account available resources of a urban public transportation service is a key aspect in order to keep their good performance as well as to ensure the confidence of users. To this end, the effects of congestion due a high level of demand of the transportation system must be taken into account and evaluated and it is specially in this case when additional services such entertainment, informational and advertisement may help.

Models oriented to management aspects of public transport lines which take into account demand in the design process, have an intrinsic relationship with passengers behavior willing to use these transportation services. The behavior of passengers comprises different aspects, some of them interrelated each other. Thus on the one hand one can find basic aspects related to the trip, such as route choice according to self-perceived travel times and fares. On the other hand, there are aspects related to the use of time of passengers during their trip which are related to added facilities of the transportation service. As regard to these aspects, several empirical studies must be cited which provide a qualitative and quantitative description of these aspects. For the use of time of passengers at bus stops, see the work by Ohmori et al. in [8]. The distribution of the willingness to wait is studied by Hess et al. in [5]. Aspects related to route/line choice for the case of frequency based transportation systems (i.e. regular bus lines

or rapid transit systems) the choice made by the users of transportation lines can be described by assignment models. Within the classical passenger transit assignment models under the concept of strategy, the classical work in [11] must be cited. This initial model is unable to take into account congestion in public transportation systems. It has not been until very recently, that these strategy-based models have been able to reflect how effective frequencies may be altered by congestion ([2], [7]). Frequency setting models have been formulated using transit assignment schemas based on strategies and time table based. Using a strategy based assignment model under a static approach, the works in [9] must be taken into account. In this paper a service setting model for public transportation services is developed which is able to reflect the effects of congestion under a static approach for public transportation lines intended for emergency situations or for supporting special events. The model can be also used to evaluate which of the lines in the system is attractive for the users. It is formulated as a bilevel programming problem and it is solved using the simulated annealing method, showing its performance on a small transportation service operating on four stations under a high level of passengers demand. In this model the underlying passenger assignment schema is a non-strategy based user equilibrium following a congested shortest route choice. Waiting times of passengers at queues is taken into account in the model as well as limitations in the willingness for waiting service.

## II. NOTATION AND NETWORK MODEL

In this section a unified notation is presented for the model presented. The transit network is represented by means of a directed graph  $G = (N, A)$ , where  $N$  is the set of nodes and  $A$  is the set of links. The number of trips from  $p$  to  $q$  will be denoted by  $g_{p,q}$ . By  $C \subset N$  it will be denoted the subset of centroids or trip attraction/generation points. By  $W = \{ (p, q) \in C \times C \mid g_{p,q} > 0 \}$  it is denoted the set of active origin-destination pairs  $\omega = (p, q)$  on the network. The set of destinations in the network for a given origin  $p \in C$  shall be denoted by  $\Lambda(p) = \{ q \in C \mid \exists (p, q) \in W \}$  and the set of origin nodes  $p$  for a fixed destination  $q \in C$  shall be denoted by  $\Pi(q) = \{ i \in C \mid (p, q) \in W \}$ . For a node  $i \in N$ , the set of emerging links will be denoted by  $E(i)$  and the set of incoming links by  $I(i)$ . The representation of transit lines will be in the form of an expanded network, as in [11] (see figure 1 below).

Passenger flows during the specified period of time will be organized in commodities, one per each origin-destination, and

E. Codina and L. Montero are associate professors in the Department of Statistics and Operations Research, at the Universitat Politècnica de Catalunya, 08034 Barcelona, Spain e-mails: esteve.codina@upc.edu and lidia.montero@upc.edu

A. Marín is professor in the Statistics and Applied Mathematics Department at the Universidad Politécnica de Madrid, 28040 Madrid, Spain e-mail: angel.marin@upm.es.

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### III. A SERVICE SETTING MODEL BASED ON USER EQUILIBRIUM OF PASSENGER FLOWS

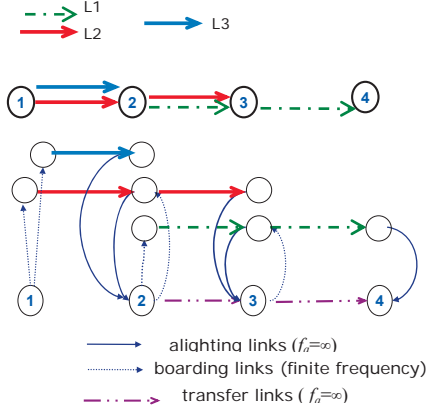


Fig. 1. The transit expanded network assumed in the model.

total flow of passengers on a link  $a \in A$ , during that period of time and for an O-D pair  $\omega \in W$  will be noted by  $v_a^\omega$ . Balance equations for flows per O-D pair  $\omega = (p, q) \in W$  at a node  $i \in N$  will be:

$$\sum_{j \in E(i)} v_{ij}^\omega - \sum_{k \in I(i)} v_{ki}^\omega = \begin{cases} -g_{(p,q)}, & \text{if } i = q \\ g_{(p,q)}, & \text{if } i = p \\ 0, & \text{if } i \neq p \end{cases} \quad (1)$$

By adding non-negativity conditions  $v_a^\omega \geq 0$  for flows on links to previous relationships (1), the following polyhedra are defined:

$$V^\omega = \left\{ v^\omega \in \mathbb{R}_+^{|A|} \mid v^\omega = (\dots, v_{i,j}^\omega, \dots; (i, j) \in A), \right. \\ \left. v_{i,j}^\omega \text{ verify (1)} \right\}, \omega \in W \quad (2)$$

The feasibility set for the congested transit equilibrium problem can be formulated as  $V = \bigotimes_{d \in D} V^\omega$ , being each set  $V^\omega$  defined as in (2) and finally, the polyhedron  $\mathcal{V}$  of total passenger flows on links will be

$$\mathcal{V} = \left\{ v \in \mathbb{R}_+^{|A|} \mid v = \sum_{\omega \in W} v^\omega, v^\omega \in V^\omega \right\} \quad (3)$$

Because of the finite capacity of vehicles, boarding of passengers may not happen at the first arriving vehicle seen by the passenger. Travel times on links are given by functions  $t_a(v)$ ,  $a \in A$  which are finite on  $\mathcal{V}$ . The subset of nodes for which emerging links exist with a finite frequency on  $\mathcal{V}$  will be denoted by  $\hat{N} = \{ i \in N \mid \exists a \in E(i), f_a < +\infty \}$ . Line segments as well as pedestrian, transfer and non transit facilities shall be represented by links  $a \in A$  with either constant or flow dependent travel time functions  $t_a(\cdot)$  and infinite frequencies,  $f_a = +\infty$ . This apply also for links  $a \in I(i)$ ,  $i \in \hat{N}$ , representing alighting at stops.

A first model by Codina and Marín [1], is oriented to set the number of services when passengers have a behavior characterized by two facts: a) no recommendation or regulation is made on the assignment from passengers to lines, b) at each stop passengers choose a transit line accordingly to a route from their origin to their destination that they consider as optimal for them. The choice of a given line at a station does not rely on the concept of strategies, as stated in [11]. Instead, their choice is based on finding the most convenient path for them in order reach their destination, taking into account waiting times at stops and the effect of congestion (a congested version of the path-finding criterion of Dial in [3]). In this paper several modeling components are added to the previous model so that it can be used in order to assess the quality of services for transport in the case of special events.

The design model can be stated as a bilevel programming in which the lower level is an asymmetric traffic assignment problem described briefly in subsection IV. The upper level part of the model minimizes the total costs of the system.

[SUE:]

$$\begin{aligned} \text{Min}_{\nu, z, v} \quad & \sum_{\ell \in L} (\xi^\ell \nu^\ell + \mu^\ell z^\ell) + \chi v^\top F(v, z) \\ \text{s.t.} \quad & v \text{ solves VI in (6)} \\ 1 \quad & \sum_{\ell \in L} \nu^\ell \leq \pi, \nu^\ell \geq 0, \\ 2 \quad & T^\ell \nu^\ell \geq z^\ell C_\ell(v, z), \ell \in L \\ 3 \quad & \lambda^\ell \frac{T}{h_{max}} \leq z^\ell \leq \lambda^\ell \hat{f} T, \ell \in L \\ 4 \quad & \lambda^\ell \in \{0, 1\}, \nu^\ell \in \mathbb{Z}, z^\ell \in \mathbb{Z}, \ell \in L \end{aligned} \quad (4)$$

In the formulation of model [SUE] above, flows  $v$  are assumed to lie in the solution set of an asymmetric traffic model that can be stated as a variational inequality (V.I.), described in section IV. This V.I., which makes up the lower level problem, is parametrized by the number of services  $z^\ell$  assigned at each bus line  $\ell \in L$ . The upper level objective function is composed by two terms. The first one evaluates the operational costs of assigning units to a line plus the operational costs of bus services. The second cost is proportional to the total time spent by all passengers. The coefficient  $\chi$  can be considered as the social cost of time. The model assumes that a fleet of  $\pi$  vehicles is available in order to put into operation a set  $L$  of candidate lines, being the number  $\nu^\ell$  of vehicles assigned to line  $\ell \in L$  with associated cost  $\xi^\ell$ . Vehicles assigned to line  $\ell$  must perform a number of services  $z^\ell$  on a time horizon of length of  $T$  units, with cost per service of  $\mu^\ell$ . The objective function is composed by the investment costs. The cycle length of line  $\ell$ ,  $C_\ell(v, z)$  is assumed to be dependent of the waiting queues of vehicles at stations, which are dependent of the number of passengers served. Cycle length, numbers of vehicles and number of services for each line are linked by relationship 2 in SUE. Relationships 3 and 4 in SUE impose lower and upper values on the number of services. Passenger

flows on lines are assumed to follow a user equilibrium given by V.I. described in section IV.

### A. Computational results

Model [SUE] was solved by means of the simulated annealing algorithm on the transit network at the top of figure 2 and with a passenger's demand given in table I. The set of candidate lines (eleven in total) is depicted at the bottom of figure 2.

	1	2	3	4	Total
1	0	2011	22097	368	24476
2	170	0	3066	230	3466
3	4386	150	0	170	4706
3	2504	150	2438	0	5092
Total	7060	2311	27601	768	<b>37740</b>

TABLE I  
O-D TRIP TABLE FOR A PERIOD OF 3 HOURS.

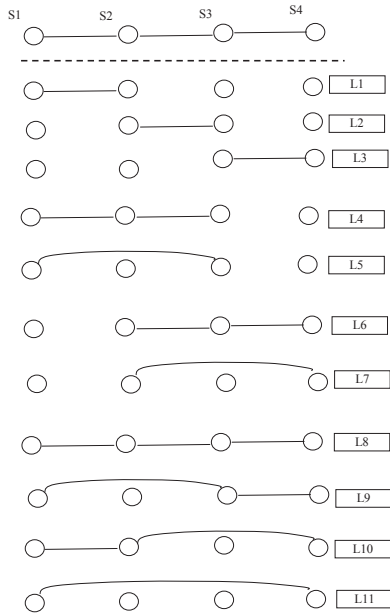


Fig. 2. Set  $L$  of possible transportation lines considered in the computational example. Only lines with the same stopping points in both directions are considered.

Figure 3 below shows the evolution of the objective function for 2000 iterations of the simulated annealing algorithm with low temperature. Execution time on a HP laptop with 2Gb took approximately 1h15min for 2000 iterations. In the computational experiences, the lower level V.I. was solved by means of a diagonalization algorithm using a maximum of 500 iterations. In this case, good objective function values for model SUE above were obtained at a much earlier iteration than the 2000-th one and in practice, a rather acceptable quasi-optimal solution after 15 minutes can be expected. The evolution of the objective function value for model SUE with a high temperature parameter are shown in figure 4. In this case much worse computational results were obtained requiring almost all the 2000 iterations in order to reach very similar objective function values as those of runs with low temperature.

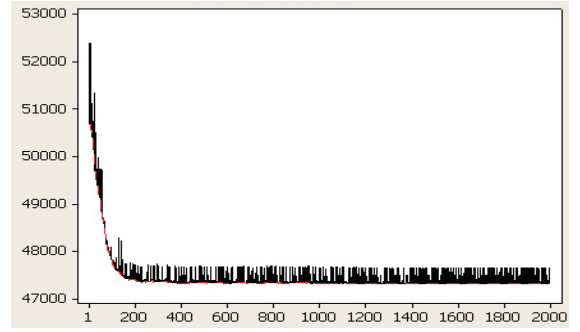


Fig. 3. Evolution of objective function versus iteration number in model SUE using the simulated annealing algorithm for low temperature.

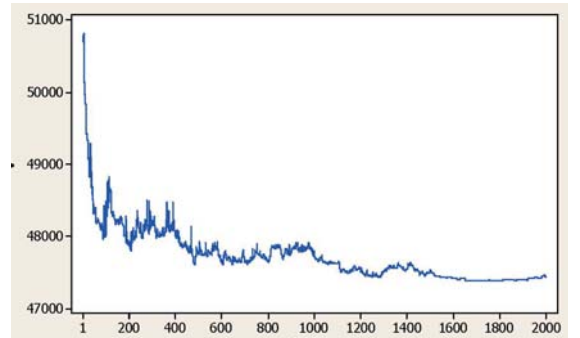


Fig. 4. Evolution of objective function versus iteration number in model SUE using the simulated annealing algorithm for high temperature.

## IV. REPRESENTATION OF THE FLOW OF PASSENGERS ON TRANSPORT LINES IN THE MODEL

Because of the structure of the costs, an asymmetric traffic assignment has been used in order to reproduce the assignment of passengers to possible lines. Asymmetries in costs come from the fact that passenger delays at stations waiting for a bus line to arrive depend not only on passenger's flow arriving at the station to board on that line but also on the unit's occupancy of that line arriving at the station. Because of network structure shown in figure (1), the travel cost functions on boarding links may take into account the passenger capacity  $c$  of transportation units, the coefficient of dispersion  $C_h$  of inter-arrival times of transportation units at stops. Thus, for boarding links  $a$  corresponding to line  $\ell \in L$  the waiting time at a stop will be:

$$s_a(v, z) = \frac{T}{2z^\ell} (1 + C_h^2) \cdot \rho_a \left( \frac{v_a}{cz^\ell - v_{x(a)}} \right), \quad (5)$$

For non-boarding links in the expanded network (i.e. lighting links, transfer links and in-vehicle links) the cost function is an appropriate constant equivalent to the travel time of this operations.

The waiting times for passenger at bus stops takes into account the fact of passenger's renegeing at the queue. To

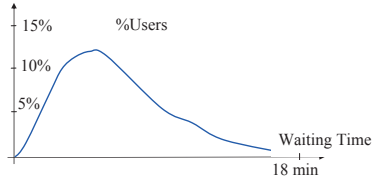


Fig. 5. A typical distribution for the willingness to wait of passengers queuing at a stop.

this end a model of the willingness-to-wait of passenger has been approximated which is described by figure 5. The willingness to wait of passengers at stations can be obtained experimentally in field studies such as [5].

With the previous structure of costs on the expanded network, the assignment of passengers to the transit network can be formulated as the following variational inequality:

$$\text{Find } v^* \in \mathcal{V} \text{ so that: } s(v^*, z)^\top (v - v^*) \geq 0, \quad \forall v \in \mathcal{V} \quad (6)$$

The solution set  $S^*(z)$  of previous problem (6) is considered when solving model SUE by means of the simulated annealing algorithm. A diagonalization algorithm shown below has been implemented in order to solve problem in (6) under several variants. The optimization problem usually in step A of the diagonalization algorithm is solved only approximately with a limited number of iterations. In this kind of problems the level of congestion, which is determined by the demand level, has a direct effect on the number of iterations required in the solution of the optimization problem. Several levels of demand were essayed. For a level of only 10% of the demand shown in table I, just by performing a single iteration of the optimization problem it suffices for the diagonalization algorithm to converge. For 20% or more of the demand specified in table I, ten iterations on the optimization problem were necessary for the diagonalization algorithm to converge. Several types of step lengths were chosen: a)  $\alpha^{(k)} = 1$ , i.e.: pure diagonalization. The solution of the optimization problem was directly the next iterate, b) step lengths based on the method of successive averages (MSA), for instance  $\alpha^{(k)} = 1/(k+1)$  or  $\alpha^{(k)} = 1/(k+1)^{\frac{1}{2}}$  and c) the Harker's acceleration step in [4].

$k$ -th Iteration of the diagonalization algorithm :

$$\begin{aligned} \diamond A - \text{Solve opt. problem } \text{Min}_v \sum_{a \in A} \int_0^{v_a} s_a(v_a; v^{(k)}) dv_a \\ v_a + v_{x(a)} \leq c_a \quad \rightarrow \quad \hat{v}^k \\ v \in \mathcal{V} \\ \diamond B - \text{acceleration step (optional) Determine } \alpha^k \text{ and} \\ v^{(k+1)} = v^{(k)} + \alpha^{(k)}(\hat{v}^k - v^{(k)}) \end{aligned} \quad (7)$$

## V. CONCLUSION AND FUTURE EXTENSIONS

A tool for designing public transportation services has been developed and solved using the simulated annealing method. The computational viability has been shown for small transportation networks. The model presented takes into account the levels of demand and the effects of congestion of the transportation system as well as aspects of users' behavior, such as willingness to wait. Possible extensions include aspects of demand elasticity and waiting models based on trip-oriented information services at stops.

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**Esteve Codina** obtained in 1994 its PhD in Industrial Engineering at Universitat Politècnica de Catalunya (Spain). He is now Associate Professor at the Department of Statistics and Operations Research assigned to the Faculty of Mathematics and Statistics and to the Faculty of Computer Science of this University. His research activity is mainly focused on the application of Operations Research methods and Optimization methods to transportation problems.

**Ángel Marín** obtained in 1982 its PhD in Aeronautical Engineering at Universidad Politécnica de Madrid (Spain). He is now Professor at the Department of Applied Mathematics and Statistics assigned to the Aeronautical Engineering School of this University. His research activity is focused on problems of network design using optimization methods.

**Lidia Montero** obtained in 1993 its PhD in Computer Science at Universitat Politècnica de Catalunya (Spain). She is now Associate Professor at the Department of Statistics and Operations Research assigned to the Faculty of Mathematics and Statistics and to the Faculty of Computer Science of this University.