Experimental demonstration of bistable phase locking in a photorefractive oscillator

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We report experimental evidence of bistable phase locking in nonlinear optics, in particular, in a photorefractive oscillator emitting in few transverse modes. Bistable phase locking is a recently proposed method for converting a laserlike system, which is phase invariant, into a phase-bistable one by injecting a suitable spatially modulated monochromatic beam, resonant with the laser emission, into the optical cavity. We experimentally demonstrate that the emission on the fundamental TEM_{00} mode becomes phase bistable by injection of a beam with the shape of the TEM_{10} mode with appropriate frequency, in accordance with recent theoretical predictions [K. Staliunas *et al.*, Phys. Rev. A **80**, 025801 (2009)]. The experimental observations are supported by an analytical study of a few-transverse-mode photorefractive oscillator model.

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Free-running lasers and photorefractive oscillators (PROs), which, in many aspects, are analogous to free-running class-A lasers [1,2], are phase-invariant systems, like any other self-oscillatory system undergoing a Hopf bifurcation. It is well known that a monochromatic signal externally injected into a laser cavity can lock the frequency and the phase of the latter to that of the injection. The laser-field phase then locks to a single value resulting in a *monostable* phase locking [3]. For some purposes, however, it is important that the laser field locks not to a single value but to two different values resulting in *bistable* phase locking. Phase bistability is different from conventional optical bistability of passive nonlinear interferometers where the laser intensity (not only the phase) can take two different values for fixed parameters.

There are mechanisms for converting a phase-invariant nonlinear system into a phase bistable one. Parametric driving [4, 5], which consists of a periodic forcing of the self-oscillatory system at a frequency around two times its natural oscillation frequency, can lead to phase bistability. Another mechanism, termed rocking, has been proposed [6], which consists of a nearly resonant forcing of the system (at a frequency close to that of the free-running oscillations) with a signal whose amplitude changes periodically (e.g., sinusoidally) or even randomly [7] *in time*. This *temporal* rocking has been proven experimentally to induce phase bistable locking in laserlike systems [8] and in other self-oscillatory systems, such as electronic circuits [9]. In this technique, the variational potential of the system (in a mechanical analog) is tilted periodically in phase space, hence, the term rocking [6]. Roughly speaking, when the injection follows with alternating opposite values of phases, say 0 and π, the phase of the slave laser does not know which phase to choose, therefore, it chooses any one of the two intermediate phases, which are $\pm\pi/2$.

More recently, *bistable phase locking* was proposed [10] where the injection amplitude is not time but space modulated on a relatively small spatial scale. Bistable phase locking has been demonstrated theoretically for spatially extended systems described by a complex Ginzburg-Landau equation, which is a reasonable model for class-A lasers or PROs with infinite extension in the transverse space (i.e., with an infinite number or continuum of transverse modes). An experimental implementation of bistable phase locking for a system with an infinite number of transverse modes is problematic, but it turns out, however, that bistable phase locking also can be realized in systems with a small number of spatial degrees of freedom. In fact, just two spatial modes suffice. In Ref. [11], some of us theoretically considered a laser cavity with spherical mirrors tuned to (and oscillating at) its TEM_{00} mode, subjected to the injection of a TEM_{10} mode (which displays two opposite phase values in adjacent spatial domains) with a frequency nearly resonant with that of the TEM_{00} mode. It was predicted that, in this situation, the phase of the lasing TEM_{00} mode can lock to any of two opposite values.

Here, we report an experimental demonstration of bistable phase locking in a few-transverse-mode nonlinear optical system and present an analytical justification of the results. We use not a laser but a laserlike system—the PRO. Despite their different microscopic descriptions, both systems share the same order parameter equations [2]. This means that the spatiotemporal effects in such PROs, including multi-transverse-mode dynamics, are analogous or very similar to those occurring in lasers. In particular, the modal equations for PROs are analogous to those for lasers, except for some additional nonlinear phase-shift effects for the drift-type PROs [12,13] (not for diffusion-type PROs, such as the one considered in this Brief Report), which just modify the coefficients in the modal equations without altering the main character of the solutions.

The PRO cavity is formed by two spherical mirrors [labeled M and PZM in Fig. 1] with radii of 0.8 m separated by 1.2 m. Mirror PZM is mounted on a piezoelectric crystal that permits the tuning of the cavity length on a submicrometer scale. The cavity free-spectral range is $\nu_{FSR} = 125$ MHz, and the frequency difference between the adjacent TEM_{00} and the TEM_{01} modes is $\Delta\nu = \nu_{FSR}/3$ for the considered configuration. The TEM_{00} mode, see Fig. 1(a), has a beam-waist diameter of 0.242 mm. The $BaTiO_3$ photorefractive crystal is placed at the beam waist and is pumped by a monochromatic Gaussian beam from a single-mode 532-nm ion-argon laser. With this cavity configuration, the PRO supports high-order

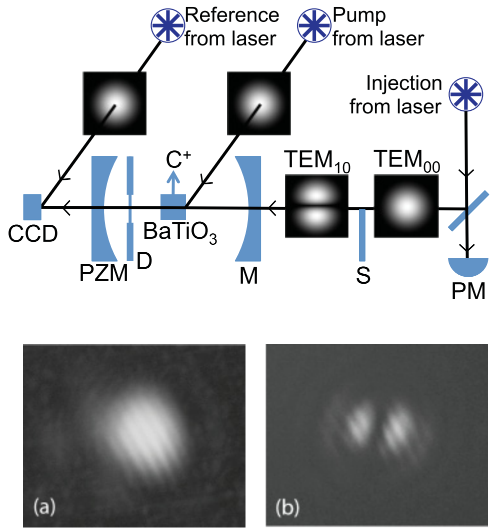

FIG. 1. (Color online) Scheme of the experiment to observe bistable phase locking (see explanations in the main text). The insets show (a) the emission of photorefractive oscillator without external injection (the TEM_{00} mode) and (b) the field prepared for injection (approximately the TEM_{10} mode). M and PZM are mirrors, D is a diaphragm, S is a $\lambda//2$ slab, and PM is a power meter. The CCD camera records the output, and the c^+ axis of the nonlinear crystal is marked.

transverse modes, therefore, a diaphragm D is used to restrict the oscillation to only the TEM_{00} and TEM_{10} modes (obviously the TEM_{01} mode is allowed as well, but it is not excited in the experiment). A part of the pumping laser beam is split and is used for injection. The injection is prepared by diffraction on the edge of a flat $\lambda/2$ glass slab (S in Fig. 1), thus, forming two lobes with opposite phases in the far field. After spatial filtering, the injection beam gets a spatial distribution similar to that of a TEM_{10} mode, see Fig. 1(b), and then is injected into the cavity through mirror M. The cavity itself is tuned so that its TEM_{00} mode is resonant with the pump (which does not resonate inside the cavity, see Fig. 1(a), but creates the oscillating field). A power meter (PM in Fig. 1) measures the intensity of the injected field. Finally, a camera records the interference between the output field, through the mirror PZM, and the reference beam from the injection in order to measure the phase and intensity of the emitted beam.

The experimental results are summarized in Fig. 2 where we plot the boundary of the region where bistable phase locking is observed by using the injection power and the cavity detuning as control parameters. Inside the closed balloon (dots come from the experiment, and the continuous line is a fit to the theory developed below), the PRO emission is cw (monochromatic) with a frequency equal to that of the injected beam and consists of the superposition of the (generated) TEM_{00} mode and the injected TEM_{10}-shaped beam. Inside the balloon, the phase of the TEM_{00} mode can take any of two opposite values (separated by π) as explained below. Outside the balloon locking disappears, but the scenario is different, depending on the injection power as we explain below. The dots delimit the locking region as experimentally obtained. They were obtained by scanning the cavity detuning (through the voltage applied to the piezomirror) for fixed injection power. These points are scattered, especially for weak

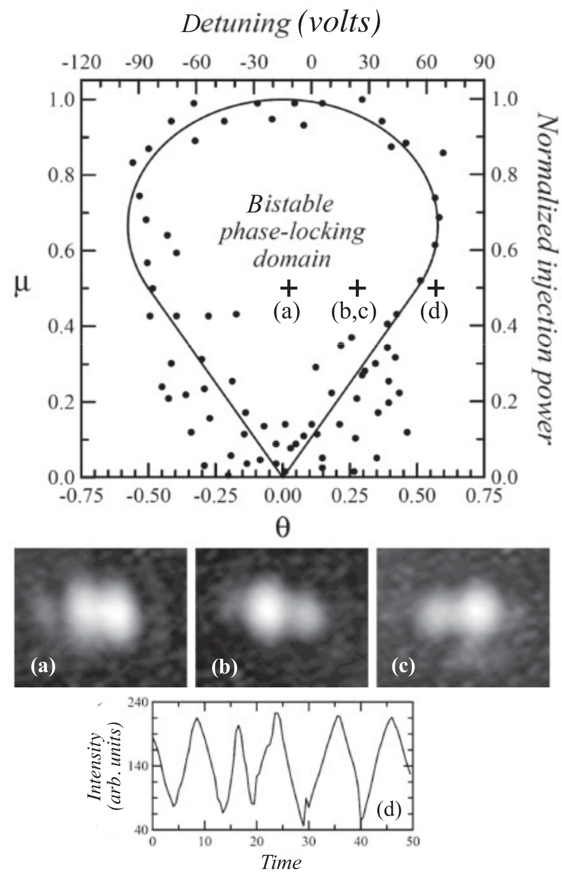

FIG. 2. Rocking area in the plane defined by cavity detuning and injection pump as predicted theoretically (full line) and obtained experimentally (dots). The experimental values of μ are obtained by normalization (unity is the highest value); the experimental detuning values are the voltages applied to the piezomirror. Subfigures (a)–(d) display experimental results corresponding to the points marked with crosses in the main figure. (a)–(c) are snapshots of the output pattern for different detunings, corresponding to the middle of the (a) rocking area and close to the (b) and (c) right boundary. In (d), intensity pulsations obtained outside the locking region are shown. The experimental detuning values are -10 V for (a), 40 V for (b) and (c), and 60 V for (d).

injections, as we are showing the results of different scans. We attribute this scattering to the fact that, in the experiment, the cavity length was not actively stabilized [14], which could have led to slight variations in the cavity resonance from run to run, especially when the lapse between runs was longer than a few minutes. In any case, for a given run, bistable phase locking was observed between a left point and a right point, which are those marked in the figure. As for the representation, the injection power has been normalized to its maximum value, and the scale in the detuning axis has been chosen in order to obtain the best fit with the theoretical prediction (see below).

We prove that the emission is phase bistable inside the balloon by extracting the phase information from the recorded interferograms (see Ref. [15] for a detailed explanation of the method). However, one can detect this phase bistability simply by inspecting the intensity pattern of the emitted radiation: As inside the balloon the field is a superposition of a TEM_{00} mode and a TEM_{10} mode, its transverse distribution can be expressed as $E(x,y) = e^{-r^2}(\sqrt{I_0}e^{i\phi} + \sqrt{I_1}x)$, where $r = \sqrt{x^2+y^2}$ is the (dimensionless) radial coordinate, x is the Cartesian coordinate along which the TEM_{10} mode is oriented, I_0 and I_1 are normalized intensities of modes TEM_{00} and TEM_{10}, respectively, and ϕ is the relative phase between them. Hence, the total intensity $I(x,y) = e^{-2r^2}(I_0 + I_1 + 2\sqrt{I_0 I_1}x \cos\phi)$ is, in general, asymmetric with respect to the inversion $x \to -x$ (but for $\varphi = \pm\pi/2$), and changing $\varphi \to \varphi + \pi$ produces a mirror-symmetric pattern. At the middle of the locking area [inset (a) in Fig. 2], both bistable patterns nearly have the same spatial intensity distribution because the generated TEM_{00} mode is in phase quadrature ($\pm\pi/2$) with the injected TEM_{10} mode as checked by interference techniques. Then, the intensity pattern is mirror symmetric (this experimental result shows that the setup cavity+injection has good symmetry). As we move from the central region toward the locking boundaries, the two phase-bistable intensity patterns gradually become distinguishable because the relative phase ϕ varies monotonically by varying detuning and takes the values $\pi/4$ or $3\pi/4$ at the lateral boundaries of the balloon (discussed below). Hence, close to the boundaries of the locking region, the intensity distributions of both bistable states maximally are asymmetric [insets (b) and (c) in Fig. 2] being mirror images one of another in accordance with Ref. [11].

Passing the lateral boundaries (say, for normalized injections from 0.0 to ≈0.7, see Fig. 2), periodic oscillations in the PRO output appear, which are the result of a beat between the now unlocked TEM_{00} and TEM_{10} modes [see inset (d) in Fig. 2]. Finally, above the locking region, the TEM_{00} switches off, and the PRO output reduces to the injected TEM_{10}-shaped beam. In this way, above the rocking area, a monostable phase locking is obtained.

Next, we pass to a theoretical interpretation of the experimental results based on the models considered in Refs. [1,2,12,13]. Adopting the notation of Ref. [13], the dimensionless model reads

$$\sigma^{-1}\partial_t F = -(1 + i\Delta - ia\hat{L})F + N + E_{\rm in}, \quad (1a)$$

$$\partial_t N = -N + g\frac{F}{1+|F|^2}, \quad (1b)$$

where $\hat{L} = \nabla^2/4 - r^2 + 1$ is a linear operator governing diffraction in a curved mirror cavity. In Eqs. (1), F and N are proportional to the field and photorefractive grating complex amplitudes, respectively, a is the diffraction coefficient (equal to the frequency separation between modes belonging to consecutive transverse-mode families, normalized to the cavity linewidth), $E_{\rm in}$ is proportional to the injected field amplitude, g is the gain parameter ($g > 1$ for a PRO driven above its oscillation threshold), σ is the ratio of the grating to the field decay rates (a very large number), and time t is normalized to the decay time of the photorefractive grating. Equations (1) are written in the frequency frame of the pump, hence, the actual electric field of the light emitted by the PRO is proportional to $F\exp(-i\omega_p t)$, ω_p being the pump frequency. Accordingly, Δ is the difference between the frequency of the cavity fundamental mode and ω_p, normalized to the cavity linewidth. In our case, as the injected field and the pump field come from the same laser, both have the same frequency ω_p.

Let $\psi_{lm}(\mathbf{r})$ represent the Hermite-Gauss modes of the linear cavity, which verify $\hat{L}\psi_{lm}(\mathbf{r}) = -(l+m)\psi_{lm}(\mathbf{r})$. Following the experimental scheme, we consider that ω_p is tuned close to a TEM_{00} cavity resonance and that the injected field has the shape of the cavity TEM_{10} mode, i.e., $E_{\rm in} = f_{\rm in}\psi_{10}(\mathbf{r})$, where $f_{\rm in}$ is taken to be real without loss of generality (it sets the phase reference). If one assumes that any cavity mode of higher order than the TEM_{00} and TEM_{10} is suppressed due to large cavity losses, as in the experimental setup, then, the truncated expansion,

$$F(\mathbf{r},t) = f_0(t)\psi_{00}(\mathbf{r}) + f_1(t)\psi_{10}(\mathbf{r}) \quad (2)$$

is a good enough description of the intracavity field. We further assume that: (i) g is close above unity (the PRO is close above its oscillation threshold) resulting in $|F|^2 \ll 1$, which allows for approximating the saturating nonlinearity in the equation for N by a cubic nonlinearity; (ii) ω_p is sufficiently close to the TEM_{00} mode frequency (i.e., Δ is of order unity at most), and (iii) the frequency separation between the actual TEM_{10} and the TEM_{00} cavity modes (equal to parameter a in our model) is large (much larger than unity).

Our goal is, starting from model (1), to derive a simple evolution equation for the TEM_{00} mode amplitude $f_0(t)$, see Eq. (2). We give the main steps but skip the details. First, the field envelope F is eliminated adiabatically from Eq. (1a) as $N = (1 + i\Delta - ia\hat{L})F - E_{\rm in}$, and expression (2) is substituted therein. Using Eq. (1b), two equations, one for $f_0(t)$ and one for $f_1(t)$, are obtained after projection onto $\psi_{00}(\mathbf{r})$ and $\psi_{10}(\mathbf{r})$. Finally, the largeness of a allows the adiabatic elimination of $f_1(t)$ as $f_1 = -if_{\rm in}/a$. The final result is a single equation for amplitude $f_0(t)$, which has a simpler looking form if we define a new complex amplitude A through

$$f_0(t) = -i\sqrt{(g-1)/\pi g}\,A(t).$$

We have included the factor $-i$ in order that $\arg A = \arg(f_0/f_1)$, which we denote by ϕ, measures the relative phase between modes and, thus, governs the intensity pattern of the emitted field as discussed above. The equation for A is a Stuart-Landau equation,

$$\dot{A} = \frac{g-1}{1+i(g-1)\theta}[(1-2\mu-i\theta)A - |A|^2A - \mu A^*], \quad (3)$$

with broken phase symmetry because of the term proportional to A^* and with rescaled injection and detuning parameters given by

$$\mu = \frac{3}{4\pi}\frac{g}{g-1}\frac{f_{\rm in}^2}{a^2}, \qquad \theta = \frac{\Delta}{g-1}. \quad (4)$$

Equation (3) is similar to the one derived in Ref. [11] for spatially rocked class-A lasers. The only difference with the one in Ref. [11] is the complex coefficient on the right-hand side of Eq. (3), which actually does not alter the character of the solutions.

Equation (3) has the following stable nontrivial steady states:

$$|A|^2 = 1 - 2\mu + \sqrt{\mu^2-\theta^2}, \qquad \mu\exp 2i\phi = -\sqrt{\mu^2-\theta^2} + i\theta, \quad (5)$$

with $A = |A|\exp(i\phi)$. The second equation in Eq. (5) is solved by two values of ϕ (differing by π) and, thus, proves the phase bistability. The domain of existence of the above phase-locked solutions is independent of g because Eq. (5) does not depend on g [although both μ and θ depend on g through Eqs. (4)] and is represented in Fig. 2 as a continuous line. The lateral boundary is given by $|\theta| = \mu$, whereas, the top boundary, where $|A| = 0$, is given by $\mu = (2 + \sqrt{1 - 3\theta^2})/3$ as follows from Eq. (5). A linear stability analysis of the phase-locked solutions shows that they are stable in all their existence ranges, which is the rocking balloon.

Concerning the phase behavior at exact injection resonance ($\theta = 0$), $\phi = \pi/2$ or $3\pi/2$, i.e., the TEM_{00} mode amplitude is in phase quadrature with respect to the intracavity TEM_{10} mode. Away from the resonance, phase ϕ changes and deviates maximally from its resonant values just at the lateral boundaries of the rocking balloon, becoming $\phi = \pi/2 \pm \pi/4$ or $3\pi/2 \pm \pi/4$ (the $\pm$ signs correspond to the left and right lateral boundaries, respectively). These different phase relations imply different transverse distributions of the total laser intensity $|F(\mathbf{r},t)|^2$: At resonance, symmetric solutions (with respect to the y axis) are found, whereas, those observed at the lateral boundary are maximally asymmetric, in good agreement with the experimental results (see the insets in Fig. 2). Finally, we note that the reduced model (3) correctly describes (not shown here) the dynamical regimes observed outside the locking region, similar to Ref. [11].

Concluding, we have demonstrated experimentally and justified theoretically through a relatively simple model that the proper injection of a TEM_{10} signal beam into a PRO tuned to its TEM_{00} mode can lock the phase of the latter to one of two equivalent opposite values in this way experimentally certifying the idea of bistable phase locking in general [10] and in systems with few transverse modes in particular [11].

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