Sound transmission through double walls: statistical and deterministic models

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Summary
Double walls are increasingly used in construction. Due to this, there is interest in reliable models of their sound insulation for the frequency range required in regulations (50 - 5000 Hz). These models can be either statistical or deterministic. In this work, the finite layer method (FLM) is presented as a numerical technique for solving the problem in a deterministic way. It is used for discretising the Helmholtz equation in the cavity and combines a finite element method (FEM) discretisation in the direction perpendicular to the wall with trigonometric functions in the two in-plane directions. The FLM exploits the simple geometry of the double wall and accounts for all its boundary and interface conditions with a reasonable computational cost. The statistical energy analysis (SEA) is a more suitable framework of analysis for vibroacoustic problems in large domains such as buildings. However, the best SEA approach for modelling double walls is not clear in the literature. The cavity is considered as a subsystem or treated as a connecting device between the two leaves depending on the author. The finite layer method is used to evaluate the performance of these two approaches, concluding that both considerations have to be taken into account together to reproduce the real behaviour. Finally, the FLM is used to define a combined deterministic-energy based approach to deal with this kind of problems.

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1. Introduction

Double walls are structural elements consisting of two leaves with an air cavity (which may be totally or partially filled with absorbing material) between them (see Figure 1). They cheaply provide load-bearing configurations with good acoustic properties and a minimal mass. The increasing use of these elements leads to the interest in reliable models of their sound insulation. These models should cover a wide frequency range (50 to 5000 Hz) in order to evaluate the outputs defined in regulations [1, 2].

Models of the sound transmission through double walls couple the structural vibration of the leaves with the sound propagation through the cavity. To do so, there are different approaches: on the one hand, deterministic models can be used. They consist on solving the structural dynamics equation for the leaves and the Helmholtz equation for the cavity. These equations are expressed in the frequency domain and can be solved either numerically [3, 4] or analytically with the help of assumptions and simplifications [5, 6]. On the other hand, energy-based formulations such as Statistical Energy Analysis (SEA) [7] can be used.

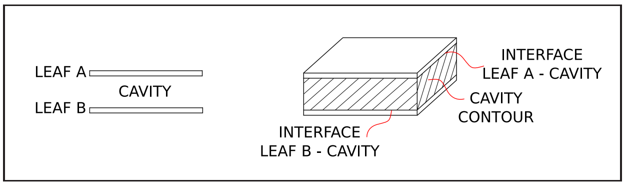

Figure 1. Sketches of the double wall and its parts.

Deterministic computations at the higher frequencies required by regulations have a large computational cost when dealing with large domains as those used in building design. SEA seems to be the best alternative for this kind of problems. However, some parameters required by this technique, such as the coupling loss factor, are not straightforward to obtain for certain configurations. Either experiments or simulations have to be performed for fitting their values.

In this work, the finite layer method (FLM) is presented as a good technique for discretising the pressure field when modelling double walls deterministically. However, for studying the sound transmission in a building, the best choice is to use SEA. The FLM is used to provide the SEA coupling loss factors associated to the double wall. The goal is to solve large

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vibroacoustic problems with SEA, using the data obtained from small vibroacoustic problems solved deterministically.

An outline of the paper follows. The bases of the finite layer method are explained in Section 2 and particularised for modelling the sound reduction index of double walls. In Section 3, different approaches to the sound transmission through double walls with statistical energy analysis are shown: on the one hand existing analytical expressions of the coupling loss factors involved and on the other hand the technique for estimating these parameters numerically. Section 4 shows some validation examples for the different models of the sound transmission through double walls presented in this work. The conclusions of Section 5 close the paper.

2. The finite layer method

2.1. Method

The FLM is a discretisation technique that has been used for modelling layered problems such as the vibration of thick plates [8] or certain groundwater flow problems [9]. In this work it is used for discretising the pressure field in the cavity of the double wall. This technique combines a FEM-like discretisation in the direction perpendicular to the wall with trigonometric functions in the two in-plane directions. It leads to less computational cost than the FEM but is still detailed enough to enforce the interface conditions between fluid and structure. Thus, it is specially suitable for computing the noise transmission through layered configurations of finite dimensions.

The pressure field is modelled with the Helmholtz equation

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0, \tag{1}$$

where k is the wave number and $p(\mathbf{x})$ is the pressure field, which is interpolated by means of layer functions. These can be understood as standard FEM interpolation functions [10] in the z direction $N_j(z)$, multiplied by appropriate interpolation functions $\Phi_s(x, y)$ in the xy plane (see Figure 2)

$$p(\mathbf{x}) = \sum_{s=1}^{n_{xy}} \sum_{j=1}^{n_z} p_{js}\, N_j(z)\, \Phi_s(x, y)\,. \tag{2}$$

In equation (2), n_z is the number of nodes in the z direction as shown in Figure 2, n_{xy} is the number of interpolation functions considered in the xy plane and p_{js} is the pressure phasor value at node j for the interpolation function $\Phi_s(x, y)$. In this case, $\Phi_s(x, y)$ is chosen such as to fulfill the reflecting condition

$$\nabla p \cdot \mathbf{n} = 0 \tag{3}$$

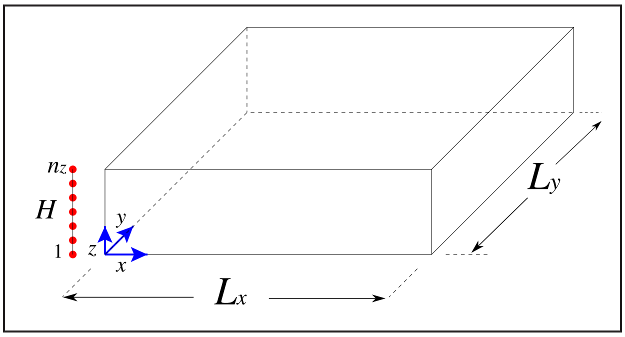

Figure 2. Sketch and notation used in the finite layer method.

at the cavity contour:

$$\Phi_s(x, y) = \cos\left(\frac{s_x \pi x}{L_x}\right) \cos\left(\frac{s_y \pi y}{L_y}\right) \qquad s_x, s_y = 0, 1, 2, \dots \tag{4}$$

2.2. Modelling the sound transmission through double walls with FLM

The leaves of the double wall are considered to be simply supported. They are modelled with the thin plate equation expressed in the frequency domain

$$D\nabla^4 u(x, y) - \omega^2 \rho_s u(x, y) = q(x, y), \tag{5}$$

where $D = Eh^3/12\,(1-\nu^2)$ is the bending stiffness of the leaf (with h, E and ν the thickness, Young's modulus and Poisson's ratio of the leaf respectively), ρ_s its mass per unit surface, $\omega = 2\pi f$ (with f the frequency of vibration) and $u(x, y)$ the displacement of the leaf.

The displacement field is expressed in terms of the eigenfunctions ϕ_r of a simply supported plate as

$$u(x, y) = \sum_{r=1}^{n_{\text{modes}}} a_r\, \phi_r(x, y) \tag{6}$$

where n_{modes} is the number of modal functions used in the interpolation, a_r is the phasor modal contribution of mode ϕ_r and

$$\phi_r = \sin\left(\frac{r_x \pi x}{L_x}\right) \sin\left(\frac{r_y \pi y}{L_y}\right), \qquad r_x, r_y = 1, 2, \dots \tag{7}$$

The discretised vibration and pressure fields are replaced in the weak form of equations (5) for the leaves and (1) for the cavity, respectively. They are coupled imposing weakly the force equilibrium and the continuity of normal velocity at the cavity-leaf interfaces.

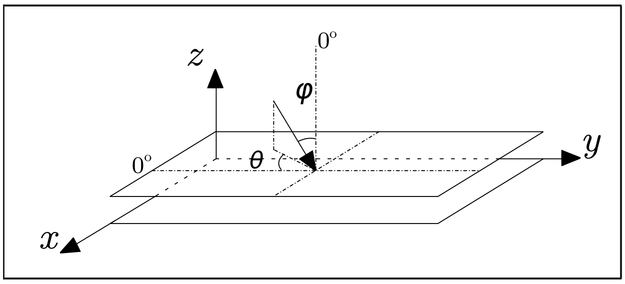

Figure 3. Angles defining the incident pressure wave.

2.3. The sound reduction index

The sound transmission through the double wall is measured in this work with the sound reduction index R between two rooms. This value is computed in terms of the incident and radiated powers, $\Pi_{\rm in}$ and $\Pi_{\rm rad}$, of the structure.

The computation of this value requires the excitation to be a pressure wave impinging on one of the leaves, modelled as

$$p(\mathbf{x}) = p_0 \mathrm{e}^{-\mathrm{i}(k_x x + k_y y + k_z z)} \tag{8}$$

where $k_x = k \sin\varphi \cos\theta$, $k_y = k \sin\varphi \sin\theta$ and $k_z = k \cos\varphi$.

This wave may have several orientations, defined by angles θ and φ as shown in Figure 3. Four different values of θ, equispaced between $\theta = 0$ and $\theta = 45^o$ due to the symmetry of the problem, are considered. If the leaf was rectangular instead of square, this limit would be 90^o. Also ten different values of φ have been considered, equispaced between $\varphi = 0$ and $\varphi_{\rm lim} = 90^o$.

The final value of the sound reduction index is computed as

$$R = 10 \log_{10} \left(\frac{1}{\tau_d} \right) \tag{9}$$

where

$$\tau_d = \frac{\int_0^{45^o} \int_0^{90^o} \tau \cos\theta \sin\theta \cos\varphi \sin\varphi \,\mathrm{d}\varphi \,\mathrm{d}\theta}{\int_0^{45^o} \int_0^{90^o} \cos\theta \sin\theta \cos\varphi \sin\varphi \,\mathrm{d}\varphi \,\mathrm{d}\theta} \tag{10}$$

and

$$\tau(\theta, \varphi) = \frac{\Pi_{\rm rad}(\theta, \varphi)}{\Pi_{\rm in}(\theta, \varphi)}. \tag{11}$$

In equation (11), $\Pi_{\rm rad}(\theta, \varphi)$ is obtained from the vibration field with the technique described in [11] and

$$\Pi_{\rm in}(\theta, \varphi) = \frac{\left\langle P^2_{\rm RMS} \right\rangle L_x L_y \cos\varphi}{\rho_{\rm air} c}, \tag{12}$$

where $\left\langle P^2_{\rm RMS} \right\rangle$ is the mean square pressure exciting the leaf and $\rho_{\rm air}$ and c are the density and the speed of sound in the air respectively.

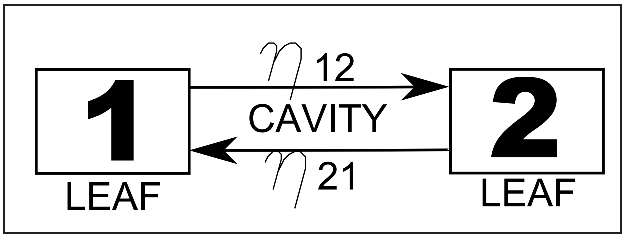

Figure 4. Sketch of a SEA model where the cavity is considered as a connecting device.

3. Statistical energy analysis for double walls

3.1. Existing approaches

Different SEA references [12, 13] do not coincide in the optimal way to deal with double walls. Neither the coupling loss factor expressions nor the identification of subsystems is clear. The two leaves should be considered as separated subsystems but there is not a unified criterion about the treatment recommended for the cavity. Here, two of the suggested techniques for this kind of problems are shown.

The first one consists on considering the air cavity as a connection between the leaves (see Figure 4), in particular as a spring with stiffness $K_{\rm air} = \rho_{\rm air} c^2 / H$ where H is the thickness of the cavity. The coupling loss factor η_{ij} between leaf i and leaf j is obtained with the electrical circuit analogy used by Hopkins [12]

$$\eta_{ij} = \frac{Re\{Y_j\}}{m_i |Y_i + Y_j + Y_c|^2}, \tag{13}$$

where

$$Y_i = \frac{1}{8\sqrt{D_i \rho_{si}}} \tag{14}$$

and m_i are the point mobility and the mass of leaf i respectively and

$$Y_c = \frac{\mathrm{i}\omega}{K} \tag{15}$$

is the mobility of the spring.

The other option is to consider the cavity as an SEA subsystem itself [13] (see Figure 5), and obtain its own modal density

$$n_i = \frac{4\pi f^2 V_{\rm cav}}{c^3} + \frac{2\pi f S_{\rm cav}}{4c^2} + \frac{L_{\rm cav}}{8c} \tag{16}$$

and internal loss factor

$$\eta_{ii} = \frac{c \alpha S_{\rm cav}}{8\pi f V_{\rm cav}}, \tag{17}$$

where $V_{\rm cav}$ is the cavity volume, $S_{\rm cav}$ is the surface of the cavity boundary, $L_{\rm cav}$ is the sum of the length of

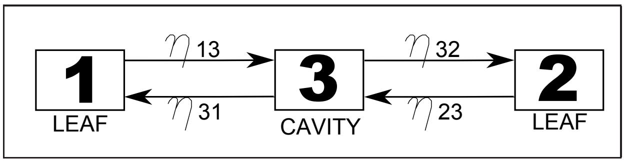

Figure 5. Sketch of a SEA model where the cavity is considered as a subsystem.

all the cavity edges and α is the absorbing factor at the cavity boundary.

Then, the coupling loss factors between leaves and cavity are obtained

$$\eta_{ij} = \frac{\rho_{\mathrm{air}} c \sigma f_c}{4\pi f^2 \rho_s}, \tag{18}$$

where f_c is the coincidence frequency between the leaf and the air and σ is the radiation efficiency of the leaf, computed with the expressions defined in [14].

3.2. Proposed technique

The technique proposed in this work consists on estimating the coupling loss factors of the energetic analysis from the numerical solution of deterministic problems. In particular, for a double wall, the cavity is considered as a connection between the two leaves and the coupling loss factor between them is estimated from the numerical simulation of the deterministic problem, computed with the FLM as described in Section 2.2.

The coupling loss factor (CLF) estimation requires the computation of the averaged energy of each leaf. Once the displacement field $u(x, y)$ in a leaf is known, the velocity of the leaf is obtained as $v(x, y) = \mathrm{i}\,\omega\, u(x, y)$, where $\mathrm{i} = \sqrt{-1}$. Then, the averaged energy of the leaf is computed as

$$E = m \left\langle v_{RMS}^2 \right\rangle \tag{19}$$

where m is the mass of the leaf and $\left\langle v_{RMS}^2 \right\rangle$ is the spatial mean square value of the velocity in that leaf.

The calculation of the coupling loss factor between the two leaves is based on the SEA formulation for the 2-subsystem case with only the first subsystem excited

$$\left\{ \begin{array}{c} \Pi_1^{\mathrm{in}}/\omega \\ 0 \end{array} \right\} = \left[\begin{array}{cc} \eta_{11} + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_{21} + \eta_{22} \end{array} \right] \left\{ \begin{array}{c} E_1 \\ E_2 \end{array} \right\} \tag{20}$$

and the consistency relationship

$$\eta_{ij} n_i = \eta_{ji} n_j, \tag{21}$$

where E_i is the energy of subsystem i and Π_1^{in} is the external power applied to leaf 1.

Once the numerical simulation is performed and the energies of the leaves are known, the coupling loss factor can be isolated from the second equation of the SEA system as

$$\eta_{12} = \frac{\eta_{22} E_2}{E_1 - \frac{n_1}{n_2} E_2}. \tag{22}$$

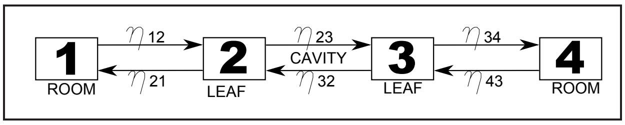

Figure 6. SEA sketch for the room-double wall-room system.

In this work, the sound reduction between two rooms separated by a double wall is computed. To do so, not only the coupling loss factor between the two leaves forming the double wall must be computed, but also the coupling loss factor between each leaf of the wall and its adjacent room (see Figure 6). This factor is obtained from the SEA system consisting of two subsystems: the room and its adjacent leaf. The problem with an excitation in the leaf is solved numerically and the coupling loss factor is obtained from the energies computed for the leaf and the room.

These estimated factors are applied later in the SEA analysis of the whole room-double wall-room system and the sound reduction index is obtained from the resulting energies of the rooms. The other required parameters such as the internal loss factors are obtained with the analytical expressions available in the literature [13].

The main idea of the proposed technique is to solve every small deterministic problem once and to use the estimated factors to model large vibroacoustic problems based on repetitions of the same elements, as happens with buildings. The small deterministic problems usually consist only of two of the susbystems forming the global system (i.e. room and leaf). Therefore, they can be approached deterministically with a reasonable computational cost.

4. Examples and comparisons

4.1. Comparison of analytical CLF expressions with the numerical estimations

The SEA approaches described in Section 3.1 are checked here. The coupling loss factor between the two leaves of a double wall provided by the analytical expressions is compared with the CLF computed from numerical simulations in Figure 7. Also an alternative technique resulting from combining the two analytical expressions is included in the comparison.

The basic properties of the leaves of the double wall used for the comparison are summarised in Table I.

For comparing the approach that considers the cavity as a third subsystem, an equivalent coupling loss

Table I. The assumed properties for a GN plasterboard leaf, used for all analyses.

Variable	Symbol	Value
Leaf length in x	L_x	2.4 m
Leaf length in y	L_y	2.4 m
Thickness	h	0.013 m
Young's modulus	E	2.5×10^9 N m^{-2}
Density	ρ	692.3 kg m^{-3}
Poisson's ratio	ν	0.3
Loss factor	η	3%

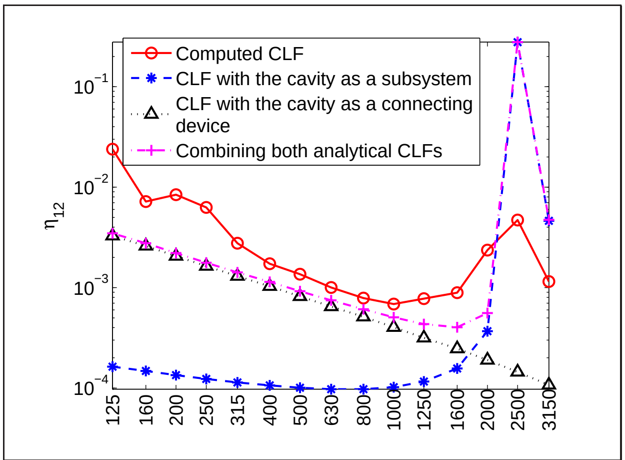

Figure 7. Comparison of the η_{12} estimations and analytical expressions for the cavity in double walls.

factor between the leaves is obtained. Considering the cavity as subsystem 3, then

$$\eta_{12}^{\mathrm{equi}} = \frac{\eta_{32}\eta_{13}}{\eta_{33} + \eta_{31} + \eta_{32}}. \tag{23}$$

Leaving the low-frequency discrepancies aside, the estimated CLF law shows two main features: on the one hand, the importance of the equivalent stiffness of the air, specially at mid frequencies; on the other hand, the coincidence phenomenon that takes place at 2500 Hz. This phenomenon is only considered by SEA when the cavity is treated as a subsystem. In fact, SEA overestimates the transmission at the coincidence frequency. Figure 7 shows that the two analytical expressions for computing the CLF miss some physical information if used separately. The fourth curve shows a more complex SEA model, which considers the cavity both as a connecting device and as a subsystem (see Figure 8). The SEA system with three subsystems is solved, including altogether the coupling loss factor between the two leaves defined in equation (13) and the coupling loss factors between leaves and cavity described in equation (18). Another option would be to choose one behaviour or the other depending on the frequency but the appropriateness of considering both behaviours together

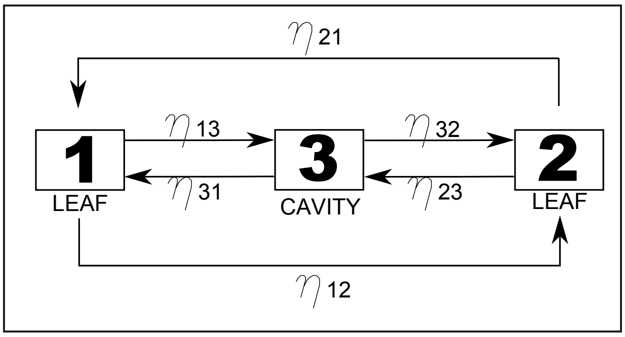

Figure 8. SEA sketch for the combination of the two techniques.

Table II. Properties of the double glazing.

Variable	Symbol	Value
Leaf length in x	L_x	1.2 m
Leaf length in y	L_y	1.2 m
Thickness	h	0.004 m
Young's modulus	E_{leaf}	7.2×10^{10} N m^{-2}
Density	ρ_{leaf}	2500 kg m^{-3}
Poisson's ratio	ν	0.22
Loss factor	η	4%

along the whole frequency range becomes evident in Figure 7.

4.2. Comparison of the combination of SEA and estimated CLFs with experimental values

The technique described in Section 3.2 for computing the sound reduction index of a double wall with SEA is tested by comparing it with available experimental data. In [15], Tadeu *et al.* show the sound reduction index measured in the lab for a double glazing. In Table II the properties of the glass leaves are shown. The cavity between them is 0.012 m thick.

The experimental results in [15] are depicted averaged in 1/10 octave bands. For the comparison, their sound energies have been averaged in order to provide the sound reduction index law in one-third octave bands

$$\langle R \rangle = 10 \log_{10} \left[\frac{1}{n} \sum_{i=1}^{n} 10^{0.1 R_i} \right]. \tag{24}$$

To obtain the curve corresponding to the analysis with SEA, the CLF between the two leaves of the double wall has been computed with the technique described in Section 3.2. The small numerical problem has been solved with the finite layer method as described in Section 2.2. Moreover, the CLF between each room and its adjacent leaf has also been computed numerically solving a small problem based on

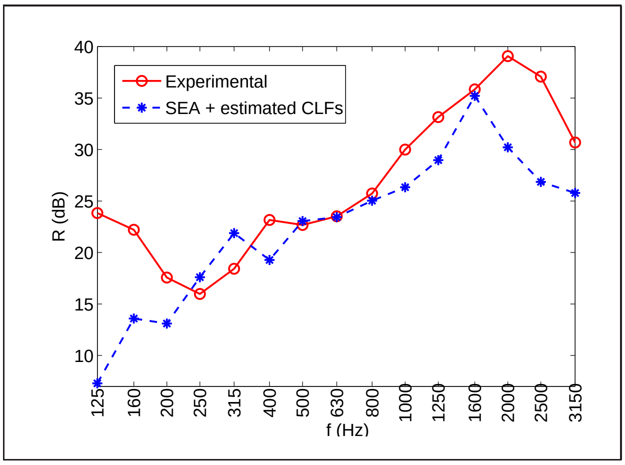

Figure 9. Comparison of the sound reduction index computed numerically and experimental measurements.

the sound propagation between one leaf and a room in contact with it.

Despite the discrepancies at low frequencies, where the SEA hypotheses are not fulfilled, the experiment is well reproduced. The room-double wall-room system is modelled in an uncoupled way, which allows reaching the whole frequency range with an affordable computational cost.

The main conclusion is that, once the coupling loss factors are computed, large vibroacoustic systems can be solved with few degrees of freedom. The system of two rooms separated by a double wall has four subsystems (the two rooms and the two leaves of the wall) and, therefore, 4 degrees of freedom if approached with SEA. However, the deterministic problem has more than 100 000 degrees of freedom at high frequencies if approached numerically. Besides, the estimation of coupling loss factors from numerical results is performed without any additional physical simplification than those included in the deterministic model. No assumptions on the dominant transmission path are done and therefore the estimated coupling loss factor accounts for all the transmission phenomena involved in the vibroacoustic problem.

5. Conclusions

- The finite layer method is a reliable technique to discretise the pressure field when modelling the sound transmission through double walls. It reduces the computational cost of the finite element method but is complete enough to respect the interface conditions between layers.
- Results show the main deficiencies of the standard techniques used in SEA for estimating the coupling loss factor associated to double walls. A numerical estimation of the coupling loss factor in these situations allows to take into account all the transmission phenomena involved in the problem. It also allows to detect that considering the two analytical expressions for the CLF together provides a good model of the real behaviour.
- The combination of numerical and statistical methods is useful to solve realistic vibroacoustic problems. It allows reaching the whole frequency range required by regulations with a reasonable computational cost for large domains based on the repetitions of smaller elements.

Acknowledgement

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