STABILITY OF A PIPELINE HYDRAULIC FLUID WITH ONE END FIXED

B. Mediano-Valiente
Departamento de Matemàtica Aplicada I
Universitat Politècnica de Catalunya
Spain
bego_med@yahoo.es

M. I. García-Planas
Departament de Matemàtica Aplicada I
Universitat Politècnica de Catalunya
Spain
maria.isabel.garcia@upc.edu

Abstract
The dynamics and stability of pipes conveying fluid has been studied thoroughly in the last decades. In this paper we study the stability in the Liapunov sense, of a clamped-pinned pipe conveying fluid at a low speed. After describing the motion of the system by partial differential equations we solve equations using finite element method testing solutions by means of ANSYS, we analyze the characteristic equation and its eigenvalues in order to obtain the stability conditions.

Key words
Stability, eigenvalues, pipe conveying fluid.

1 Introduction

The dynamics and stability of pipes conveying fluid has been studied thoroughly in the last decades see for example [1; 2; 3].

It is well known that the dynamical behaviour of pipes of a finite length depends strongly on the type of boundary. We must distinguish between the type of supports (fixed, one end fixed, etc.) and their location (horizontal, vertical). In this paper we refer to a one end fixed horizontal pipeline, respectively.

The dynamics of the system can be described by a partial differential equation [4; 5]

$$EI\frac{\partial^4 y}{\partial x^4} + (m_f U^2 - T)\frac{\partial^2 y}{\partial x^2} + 2m_f U\frac{\partial^2 y}{\partial x \partial t} + (m_p + m_f)\frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

Considering boundary conditions at ends of a clamped-pinned pipe we find approximate solution using Galerkin's methode obtaining as a result a linear gyroscopic system possessing the properties of linear Hamiltonian systems.

We compute the eigenvalues of this linear Hamiltonian system in order to study the stability. It is known that the stability of a linear Hamiltonian system is not asymptotic, nevertheless the study provides the necessary stability condition for the original non-linear system

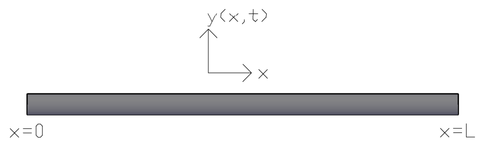

Figure 1. Pipeline

The paper is structured as follows. Section 2 presents a mathematical statement of the problem. Section 3 is devoted to analyze the stability of linear gyroscopic system obtained in section 1. Section 4 presents a simulation of the dynamic system using ANSYS.

2 Preliminaries

The system under consideration is a straight, tight and of finite length pipeline, passing through it a fluid. The following assumptions are taken into account in the analysis of the system:

i) Are ignored the effects of gravity, the coefficient damping material, the shear strain and rotational inertia
ii) The pipeline is considered horizontal
iii) The pipe is inextensible
iv) The lateral movement of $y(x,t)$ is small, and with large length wave compared with the diameter of the pipe, so that theory Euler-Bernoulli is applicable for the description of vibration bending of the pipe.
v) It ignores the velocity distribution in the cross section of pipe.

The equation for a single span prestressed pipeline where the fluid is transported is a function of the distance x and time t and is based on the beam theory:

$$EI\frac{\partial^4 y}{\partial x^4} + m_p\frac{\partial^2 y}{\partial x^2} = f_{int}(x,t) \tag{2}$$

where EI is the bending stiffness of the pipe (Nm^2), m_p is the pipe mass per unit length ($\frac{kg}{m}$) and f_{int} is an inside force acting on the pipe.

The internal fluid flow is approximated as a plug flow, so all points of the fluid have the same velocity U relative to the pipe. This is a reasonable approximation for a turbulent flow profile. Because of that we cant write the inside force as:

$$f_{int} = -m_f\frac{d^2 y}{dt^2}\bigg|_{x=Ut} \tag{3}$$

where m_f is the fluid mass per unit length ($\frac{kg}{m}$) and U is the fluid velocity ($\frac{m}{s}$).

The internal fluid causes an hydrostatic pressure on the pipe wall.

$$T = -A_i P_i \tag{4}$$

where A_i is the internal cross sectional area of the pipe (m^2) and P_i is the hydrostatic pressure inside the pipe (Pa).

Finally if we consider that the total acceleration is equal to the composition of local, carioles and centrifugal acceleration. The resulting equation is (1):

$$\begin{aligned} EI\tfrac{\partial^4 y}{\partial x^4} + (m_f U^2 - T)\tfrac{\partial^2 y}{\partial x^2} + \\ 2m_f U\tfrac{\partial^2 y}{\partial x \partial t} + (m_p + m_f)\tfrac{\partial^2 y}{\partial t^2} = 0 \end{aligned} \tag{5}$$

The boundary conditions at ends of a clamped-pinned pipe are given as:

$$\begin{aligned} &y(0,t) = 0, \qquad y(L,t) = 0 \\ &\frac{\partial^2 y(0,t)}{\partial t} = 0, \quad EI\frac{\partial^2 y(L,t)}{\partial x^2} = -K_{rs}\frac{\partial y(L,t)}{\partial x} \end{aligned} \tag{6}$$

where K_{rs} is the stiffness of the rotational spring at the right end.

Applying Galerkin method and taking $n = 2$ the approximate solution is:

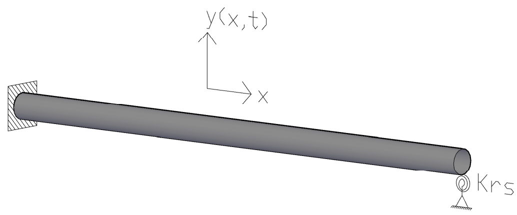

Figure 2. Boundary conditions

$$y(x,t) = q_1(t)\text{sen}\,\frac{\pi}{L}x + q_2(t)\text{sen}\,\frac{2\pi}{L}x$$

Replacing the solution in the equation 5, we get:

$$\begin{aligned} &EIq_1(t)\left(\tfrac{\pi^4}{L^4}\text{sen}\,\tfrac{\pi}{L}x + q_2(t)\tfrac{16\pi^4}{L^4}\text{sen}\,\tfrac{2\pi}{L}x\right) + \\ &(m_f U^2 - T)\left(-q_1(t)\tfrac{\pi^2}{L^2}\text{sen}\,\tfrac{\pi}{L}x - q_2(t)\tfrac{4\pi^2}{L^2}\text{sen}\,\tfrac{2\pi}{L}x\right) + \\ &2m_f U\left(\dot{q}_1(t)\tfrac{\pi}{L}\cos\tfrac{\pi}{L}x + \dot{q}_2(t)\tfrac{2\pi}{L}\cos\tfrac{2\pi}{L}x\right) + \\ &(m_p + m_f)\left(\ddot{q}_1(t)\text{sen}\,\tfrac{\pi}{L}x + \ddot{q}_2(t)\text{sen}\,\tfrac{2\pi}{L}x\right) = 0 \end{aligned} \tag{7}$$

Multiplying by sen $\frac{\pi}{L}\xi$ and sen $\frac{2\pi}{L}\xi$, respectively, we obtain:

$$\begin{aligned} &\tfrac{L}{2}(m_p + m_f)\ddot{q}_1(t) - \tfrac{8}{3}m_f U\dot{q}_2(t) + \\ &\qquad\left(EI\tfrac{\pi^4}{2L^3} - \tfrac{(m_f U^2 - T)\pi^2}{2L}\right)q_1(t) = 0 \\ &\tfrac{L}{2}(m_p + m_f)\ddot{q}_2(t) - \tfrac{8}{3}m_f U\dot{q}_1(t) + \\ &\qquad\left(EI\tfrac{8\pi^4}{L^3} - (m_f U^2 - T)\tfrac{4\pi^2}{L^2}\right)q_2(t) = 0 \end{aligned} \tag{8}$$

The previous equation system can be written as matrix form like:

$$M\ddot{q} + B\dot{q} + Cq = 0$$

Where gyroscopic lineal system is:

$$\ddot{x} + G\dot{x} + Kx = 0$$

with

$$G = M^{-1/2}BM^{-1/2}, \qquad K = M^{-1/2}CM^{-1/2}.$$

Applied to our study is:

$$M^{-1/2} = \frac{1}{\sqrt{\frac{L}{2}(m_p + m_f)}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

so, it remains:

$$G = \frac{16m_f}{L(m_f + m_p)} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$K = \frac{2}{L(m_f + m_p)} \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$$

with:

$$K_1 = EI\frac{\pi^4}{2L^3} - \frac{(m_f U^2 - T)\pi^2}{2L}$$

$$K_2 = EI\frac{8\pi^4}{L^3} - (m_f U^2 - T)\frac{4\pi^2}{L^2}$$

Introducing the vector:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} + Gx/2 \end{pmatrix}$$

and calculating the derivatives of x and y we found $\dot{x} = y - Gx/2$, $\dot{y} = \ddot{x} + G\dot{x}/2$ and considering that $\ddot{x} = -G\dot{x} - Kx$ linearizing the system we get:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -G/2 & I_2 \\ G^2/4 - K & -G/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

The matrix A of this system is Hamiltonian because QA is symmetrical, where Q is antisymmetrical:

$$Q = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

If we consider the following parameters:

$$\Lambda = \frac{EI\pi^4}{L^3}$$
$$\delta = (m_f U^2 - T)\frac{\pi^2}{L}$$
$$\beta = \frac{1}{L(m_f + m_p)} \quad (9)$$

the matrix G and K are written as:

$$G = 16m_f\beta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$K = 2\beta \begin{pmatrix} \frac{1}{2}\Lambda - \frac{1}{2}\delta & 0 \\ 0 & 8\Lambda - \frac{4}{L}\delta \end{pmatrix} \quad (10)$$

Therefore, matrix A is:

$$A = \begin{pmatrix} 0 & a & 1 & 0 \\ -a & 0 & 0 & 1 \\ b & 0 & 0 & a \\ 0 & c & -a & 0 \end{pmatrix} \quad (11)$$

where:

$$a = 8m_f\beta$$
$$b = -64m_f^2\beta^2 - \beta\Lambda + \beta\delta \quad (12)$$
$$c = -64m_f^2\beta^2 - 16\beta\Lambda + \tfrac{8}{L}\beta\delta$$

The characteristic equation of the matrix is:

$$\lambda^4 + (2a^2 - b - c)\lambda^2 + (a^2 + c)(a^2 + b) = 0 \quad (13)$$

that is a biquadratic equation, where we find the eigenvalues of the system:

$$\lambda = \pm\sqrt{\frac{-2a^2 + b + c \pm \sqrt{-8a^2b - 8a^2c - 2bc + b^2 + c^2}}{2}} \quad (14)$$

If we change the previous equation with the parameters, we extract:

$$\lambda^4 + \left(256m_f^2\beta^2 + 17\beta\Lambda - \left(1 + \frac{8}{L}\right)\beta\delta\right)\lambda^2 +$$
$$16\beta^2\Lambda^2 - \left(16 + \frac{8}{L}\right)\beta^2\Lambda\delta + \frac{8}{L}\beta^2\delta^2 = 0 \quad (15)$$

$$\lambda = \pm\sqrt{\frac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}} \quad (16)$$

with

$$\lambda_1 = -256m_f^2\beta^2 - 17\beta\Lambda + \left(1 + \tfrac{8}{L}\right)\beta\delta$$
$$\lambda_2 = 65536m_f^4\beta^2 + 8704m_f^2\beta\Lambda - \left(512 + \tfrac{4096}{L}\right)m_f^2\beta\delta +$$
$$225\Lambda^2 + \left(1 + \tfrac{64}{L^2} - \tfrac{16}{L}\right)\delta^2 + \left(30 - \tfrac{240}{L}\right)\Lambda\delta \quad (17)$$

3 Stability

In this section we study the stability properties of linear dynamic systems representing the pipeline. Also, we will present a detailed explanation of the effect of the stabilization in terms of the bifurcation theory of eigenvalues.

Eigenvalues of the matrix (11) characterize the stability of the Hamiltonian system. The system is stable if the eigenvalues lyes on the imaginary axe.

Taking into account that the values in the system are know only approximately, the matrix A in the system can be considered as a family of matrices depending on parameters a, b, c in a neighborhood of a fixed point p_0, that permit us to study the stability border.

Stability conditions requires that the roots obtained in (16), $\lambda^2 = \dfrac{\lambda_1 \pm \beta\sqrt{\lambda_2}}{2}$ are real and negative. Imposing these conditions we can determine the stability zone in the parameter space.

We observe that the points $p = (a, b, c)$ such that

$$\left.\begin{array}{r} 2a^2 - b - c = 0 \\ (a^2 + c)(a^2 + b) = 0 \end{array}\right\}, \qquad (18)$$

the characteristic polynomial is λ^4,

If $a \neq 0$ the Jordan form of A is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $a = 0$ the Jordan form of A is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

At these points we have singularities of the type 0^4 y 0^3 on the stability boundary.

Also we obtain the eigenvalue 0 at the points such that (a, b, c)

$$\left.\begin{array}{r} (a^2 + c)(a^2 + b) = 0 \\ 2a^2 - b - c \neq 0 \end{array}\right\} \qquad (19)$$

At the points $(a, b, -a^2)$ we have two possibilities depending on b if it is equal or not to $-a^2$

For $b \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3a^2 - b} & 0 \\ 0 & 0 & 0 & -\sqrt{3a^2 - b} \end{pmatrix}$$

For $b = -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3a^2 - b} & 0 \\ 0 & 0 & 0 & -\sqrt{3a^2 - b} \end{pmatrix}$$

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To ensure stability we need the non-zero eigenvalues lye in the imaginary axe we have $3a^2 - b < 0$, so $b = -a^2$ is out of stability space.

At the points $(a, -a^2, c)$ we have two cases depending on c be equal or not to $-a^2$

For $c \neq -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3a^2 - c} & 0 \\ 0 & 0 & 0 & -\sqrt{3a^2 - c} \end{pmatrix}$$

For $c = -a^2$ the Jordan form is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{3a^2 - c} & 0 \\ 0 & 0 & 0 & -\sqrt{3a^2 - c} \end{pmatrix}$$

Analogously, the case $c = a^2$ is out of the stability space

For the case $b \neq -a^2$ and $c \neq -a^2$ we have singularities of the type 0^2 in the boundary of stability.

It remains to study the case that no eigenvalue is zero

The roots of $\mu^2 + (2a^2 - b - c)\mu + (a^2 + c)(a^2 + b) = 0$, are real and negative when

$$\left.\begin{array}{l} 2a^2 - b - c > 0 \\ (a^2 + c)(a^2 + b) > 0 \\ (2a^2 - b - c)^2 \geq 4(a^2 + c)(a^2 + b) \end{array}\right\} \qquad (20)$$

In the case $(2a^2 - b - c)^2 = 4(a^2 + c)(a^2 + b)$ the eigenvalues are $\lambda = \pm i\sqrt{2a^2 - b - c} = \pm i\omega$ double.

It is easy to observe that rank $(A - (\pm i\omega)I) = 3$ so the Jordan form is

$$\begin{pmatrix} i\omega & 1 & 0 & 0 \\ 0 & i\omega & 0 & 0 \\ 0 & 0 & -i\omega & 1 \\ 0 & 0 & 0 & -i\omega \end{pmatrix}$$

At the points (a, b, c) with $2a^2 - b - c > 0$, $(a^2 + c)(a^2 + b) > 0$ and $(2a^2 - b - c)^2 = 4(a^2 + c)(a^2 + b)$ we have singularities of the type $\pm i\omega^2$.

The last case

$$\left.\begin{array}{l} 2a^2 - b - c > 0 \\ (a^2 + c)(a^2 + b) > 0 \\ (2a^2 - b - c)^2 > 4(a^2 + c)(a^2 + b) \end{array}\right\} \qquad (21)$$

determined the stability points (a, b, c) remaining within the area bounded by the above singularities.

Removing the variable change we know that:

$$a = \frac{8m_f}{L(m_f+m_p)}$$

$$b = \frac{-64m_f^2}{L^2(m_f+m_p)^2} - \frac{EI\pi^4}{L^4(m_f+m_p)} + \frac{(m_fU^2+A_iP_i)\pi^2}{L^2(m_f+m_p)}$$

$$c = \frac{-64m_f^2}{L^2(m_f+m_p)^2} - \frac{16EI\pi^4}{L^4(m_f+m_p)} + \frac{8(m_fU^2+A_iP_i)\pi^2}{L^3(m_f+m_p)}$$

Taking as a constant parameters $L = 1000mm$, $I = 2,185 \cdot 10^6$, $A_i = 2500\pi$ due to the geometry of the pipe and $m_f = 2,5\pi \cdot 10^{-6}\frac{Tn}{mm}$ assuming the fluid as water. We also suppose that the study is applied to the inside wall of the pipe so U at these points are zero.

Therefore the values a, b y c are:

$$a = \frac{2\pi \cdot 10^{-8}}{(2,5\pi \cdot 10^{-6}+m_p)}$$

$$b = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6}+m_p)^2} - \frac{2,185 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6}+m_p)} + \frac{2,5 \cdot 10^{-3}P_i\pi^3}{(2,5\pi \cdot 10^{-6}+m_p)}$$

$$c = \frac{-4 \cdot 10^{-16}\pi^2}{(2,5\pi \cdot 10^{-6}+m_p)^2} - \frac{34,96 \cdot 10^{-6}E\pi^4}{(2,5\pi \cdot 10^{-6}+m_p)} + \frac{2 \cdot 10^{-5}P_i\pi^3}{(2,5\pi \cdot 10^{-6}+m_p)}$$

That permit us to obtain the following relation depending only on m_p, E and P_i:

$$\begin{aligned}
&\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi^2 E - 2.52\pi P_i > 0 \\
&15.27752 \cdot 10^{-4}\pi^2 E^2 + P_i^2 - 17.48874 \cdot 10^{-1}\pi E P_i > 0 \\
&\left(\frac{16 \cdot 10^{-13}}{2.5 \cdot 10^{-6}\pi + m_p} + 37.145 \cdot 10^{-3}\pi E - 2.52P_i\right)^2 > \\
&4\pi^2(76.3877 \cdot 10^{-6}\pi^2 E^2 - 87.4437 \cdot 10^{-3}\pi E P_i + \\
&5 \cdot 10^{-2}P_i^2)
\end{aligned} \tag{22}$$

We make this study to show the stability of pipes with different materials assuming in all of them that the fluid transported is water and causes a constant pressure on its walls of 4 bar. The geometrical conditions of the pipe are the inside diameter equal to 50 mm and the thickness of the pipe which is 6 mm. The materials chosen are PVC, Polyethylene and Concrete.

The values of E and m_p of the PVC pipe are:

$$E = 30,581\frac{N}{mm^2}$$

$$m_p = 2,76 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (22) we found that the solution is unstable.

The values of E and m_p of the PE pipe are:

$$E = 9,174\frac{N}{mm^2}$$

$$m_p = 1,91 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (22) we found that the solution is unstable.

The values of E and m_p of the Concrete pipe are:

$$E = 221,203\frac{N}{mm^2}$$

$$m_p = 4,40 \cdot 10^{-6}\frac{Tn}{mm}$$

Applying the inequalities (22) we found that the solution is stable.

We observe that the case of PVC pipe is the furthest away from stability zone.

4 Simulation

To demonstrate the stabilities found in the previous chapter we calculate vibration characteristics of a pipe conveying fluid using a Finite Element package called ANSYS. To determinate the vibration characteristics we used modal analysis, with this analysis you find natural frequencies and mode shapes which are important parameters in the design of a structure for dynamic studies.

In the following pictures it is shown the performance of the first and the second shapes and the natural frequencies of them.

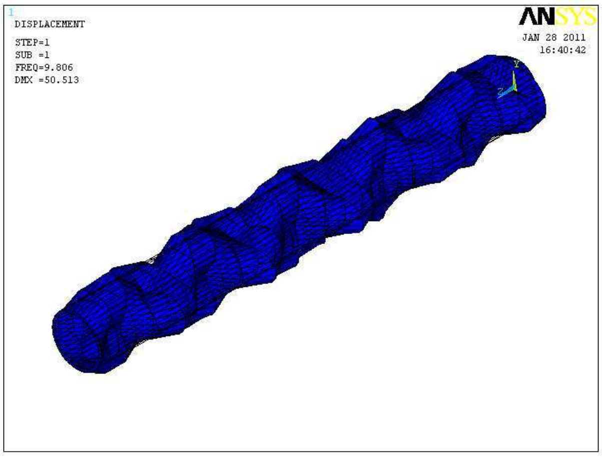

Figure 3. First shape of PVC

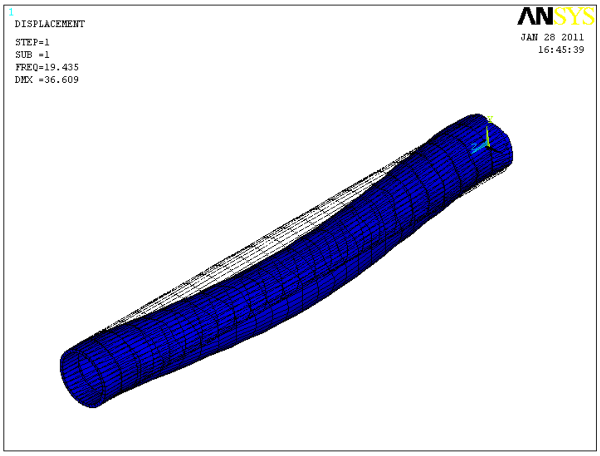

Figure 4. First shape of Polyethylene

As seen in picture 3, 4 and 5 the lowest natural frequency is the concrete pipe (it is not write on the picture because is zero) and the biggest one is the Polyethylene pipe ($19,435$ Hz) but the greater displacement of x axis is the PVC pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete pipe is stable.

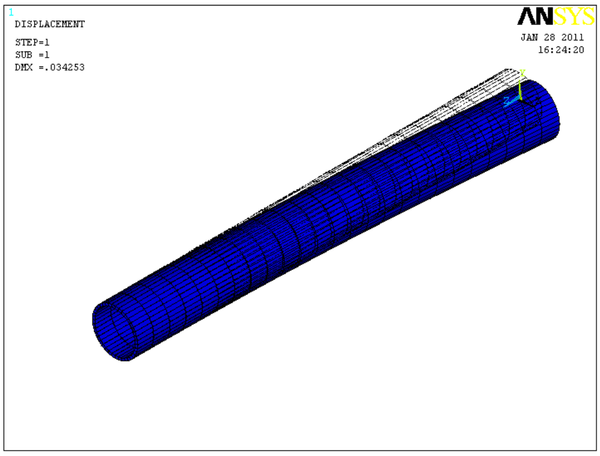

Figure 5. First shape of Concrete

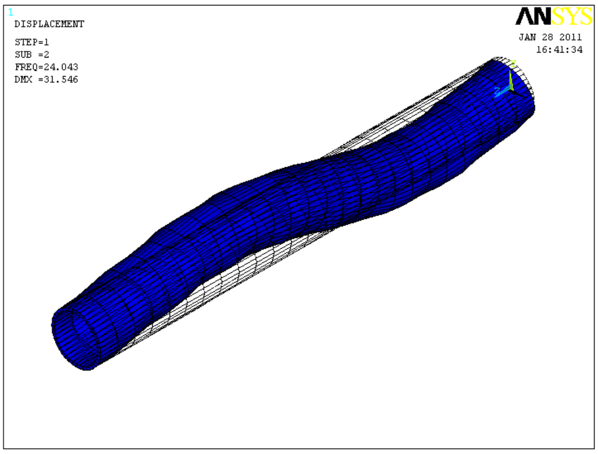

Figure 6. Second shape of PVC

As seen in picture 6, 7 and 8 the lowest natural frequency is the concrete pipe (0,0238 Hz) and the biggest one is the PVC pipe ($24,043$ Hz) but the greater displacement of x axis is the PE pipe. This combination result in instability of Polyethylene and PVC pipe whereas in Concrete pipe is stable.

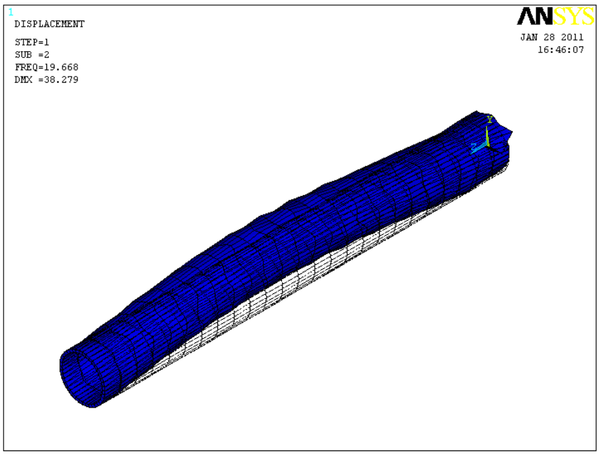

Figure 7. Second shape of Polyethylene

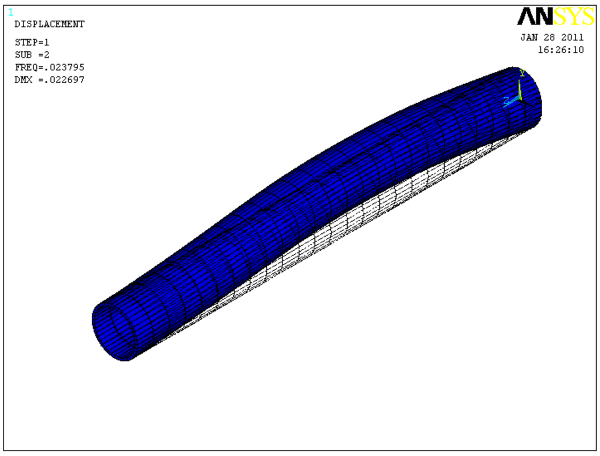

Figure 8. Second shape of Concrete

5 Conclusion

In this paper we have compared the calculations and the simulation of typical materials for a pipe used in public works.

We have shown that the dynamics and stability of pipes conveying fluid not only depends on the boundary conditions but it is also strongly important the material of the pipe and the pressure produced by the fluid.

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