When Gossip Meets Consensus: Convergence in Correlated Random WSNs*

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ABSTRACT

We study the convergence of consensus algorithms in wireless sensor networks with random topologies where the instantaneous links exist with a given probability and are allowed to be spatially correlated. Aiming at minimizing the convergence time of the algorithm, we adopt an optimization criterion based on the spectral radius of a positive semidefinite matrix for which we derive closed-form expressions. The general formulations allow the computation of the optimum parameters ensuring almost sure convergence to a consensus under these topology conditions. We show that the expressions derived for consensus algorithms can be particularized for gossip algorithms, particularly for the broadcast gossip algorithm and for the pair-wise gossip algorithm.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Distributed Systems—*Distributed Applications*; C.3 [**Special-Purpose and Applications-Based Systems**]: Signal Processing Systems

General Terms

Algorithms, Theory, Performance.

1. INTRODUCTION

The study of probabilistic convergence of the consensus algorithm is usually addressed considering statistically independent links which exist with a given probability. Substantial results have been reported in literature, either considering undirected instantaneous topologies, i.e. bidirectional communication links [1–3], or considering directed instantaneous topologies, i.e. directional communication links [4–7]. The communication links among the nodes of a network may be however spatially correlated not only due to an intrinsic correlation of the channels, but also to the communication protocol itself. An example is the random gossip algorithm [8], where at each iteration a node wakes up randomly and either establishes a bidirectional communication link with one node randomly chosen as in pair-wise gossip of geographic gossip, or establishes directional links will all the nodes within its connectivity radius as in broadcast gossip. The difference between gossip and standard consensus is that in the former, only one node is transmitting at each iteration, whereas in the latter, several nodes are broadcasting their state values at the same time. Gossip algorithms can be therefore seen as asynchronous versions of the consensus algorithm with spatially correlated links.

General contributions on consensus in correlated random networks include [9], which considers the MSE convergence rate as an optimization criterion in undirected topologies. For directed topologies, [10] derives closed-form expressions for the asymptotic convergence factor assuming fixed out-degree networks. Furthermore, [11] studies the asymptotic convergence rate of randomized protocols and [12] derives a sufficient condition for almost sure (a.s.) convergence of the broadcast gossip algorithm as a particular case of the consensus algorithm in random directed topologies.

This paper presents results on convergence of consensus algorithms in wireless sensor networks (WSNs) with random topologies and links exhibiting spatial correlation, where the more general model in [13] is considered. A.s. convergence to a consensus under these connectivity conditions is established by the spectral radius of a positive semidefinite matrix for which we derive closed-form expressions for constant link weights. We consider the minimization of this spectral radius as the optimization criterion to reduce the convergence time of the algorithm and show that the closed-form expressions derived can be particularized to obtain the results for random gossip algorithms.

Graph Theory Concepts: The information flow among the nodes of the network is described by a graph $\mathcal{G}(k)=\{\mathcal{V},\mathcal{E}(k)\}$ where $\mathcal{V}$ is the constant set of vertices (nodes) and $\mathcal{E}(k)$ is the set of edges (links) e_{ij} at time k $\forall i,j\in\{1,\cdots,N\}$ where the information flows from j to i [14]. e_{ij} belongs to $\mathcal{E}(k)$ with probability $0\leq p_{ij}\leq 1$, and we assume that $p_{ij}=p_{ji}$ and $p_{ii}=0$ $\forall i$. The set of neighbors of node i at time k is $\mathcal{N}_i(k)=\{j\in\mathcal{V}: e_{ij}\in\mathcal{E}(k)\}$. Let $\mathbf{P}\in\mathbb{R}^{N\times N}$ denote the connection probability matrix with entries $\mathbf{P}_{ij}=p_{ij}$.

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The instantaneous adjacency matrix $\mathbf{A}(k) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}(k)$ is random with entries: $[\mathbf{A}(k)]_{ij} = 1$ with probability p_{ij}, $[\mathbf{A}(k)]_{ij} = 0$ with probability $1-p_{ij}$, and symmetric mean $\bar{\mathbf{A}} = \mathbf{P}$. The degree matrix $\mathbf{D}(k) \in \mathbb{R}^{N\times N}$ is diagonal with entries $[\mathbf{D}(k)]_{ii} = \sum_{j=1}^{N} [\mathbf{A}(k)]_{ij}$ and $\mathbf{L}(k) = \mathbf{D}(k) - \mathbf{A}(k)$ denotes the instantaneous Laplacian. By construction, $\mathbf{L}(k)$ is random with smallest eigenvalue in magnitude $\lambda_N(\mathbf{L}(k)) = 0$ with associated left eigenvector $\mathbf{1} \in \mathbb{R}^{N\times 1}$, an all-ones vector of length N. If $\mathcal{G}(k)$ is connected, $\lambda_N(\mathbf{L}(k))$ has algebraic multiplicity one. The graph is assumed connected in expectation with associated Laplacian $\bar{\mathbf{L}} = \bar{\mathbf{D}} - \mathbf{P}$. $\bar{\mathbf{L}}$ is therefore symmetric by construction and irreducible.

Spatially Correlated Links: The entries of $\mathbf{A}(k)$ are assumed independent over time but correlated in space, and this information is arranged in the matrix $\mathbf{C} \in \mathbb{R}^{N^2\times N^2}$ with $\{st\}^{th}$ entry

$$\mathbf{C}_{st} = \begin{cases} 0, & s=t \\ \mathbb{E}[a_{ij}a_{qr}] - p_{ij}p_{qr}, & s\neq t \end{cases} \quad \text{with} \begin{cases} s = i+(j-1)N \\ t = q+(r-1)N \end{cases} \tag{1}$$

for nodes i, j, q, r, where $a_{ij} = [\mathbf{A}(k)]_{ij}$, and the time indexing is omitted since it does not affect the computations.

2. CONSENSUS IN RANDOM NETWORKS

Consider a WSN composed of N nodes indexed with $i \in \{1, \cdots, N\}$ and let $\mathbf{x}(k) \in \mathbb{R}^{N\times 1}$ denote the vector of all states at time k, initialized at time $k=0$ with the values of the measurements. The evolution of $\mathbf{x}(k)$ can be written in matrix form as follows

$$\mathbf{x}(k+1) = \mathbf{W}(k)\mathbf{x}(k), \quad \forall k > 0 \tag{2}$$

where $\mathbf{W}(k) \in \mathbb{R}^{N\times N}$ is the weight matrix at time k. We consider constant link weights ϵ such that

$$\mathbf{W}(k) = \mathbf{I} - \epsilon\mathbf{L}(k), \quad \forall k \geq 0, \tag{3}$$

where $\mathbf{L}(k)$ is the random Laplacian. Due to the random nature of $\mathbf{L}(k)$, the matrices $\{\mathbf{W}(k), \forall k \geq 0\}$ are *i.i.d.* and have by construction at least one eigenvalue equal to 1 with associated right eigenvector $\mathbf{1}$. For directed topologies $\mathbf{W}(k)$ satisfies: *i)* $\mathbf{W}(k)\mathbf{1} = \mathbf{1}$, *ii.a)* $\mathbf{1}^T\mathbf{W}(k) \neq \mathbf{1}^T$, $\forall k$ and *iii)* $\bar{\mathbf{W}}\mathbf{1} = \mathbf{1}$, $\mathbf{1}^T\bar{\mathbf{W}} = \mathbf{1}^T$, whereas for undirected topologies $\mathbf{W}(k)$ satisfies: *i)*, *ii.b)* $\mathbf{1}^T\mathbf{W}(k) = \mathbf{1}^T$ $\forall k$ and *iii)*. A right eigenvector $\mathbf{1}$ in *i)* implies that after reaching a consensus the network will remain in consensus, and a left eigenvector $\mathbf{1}$ in *ii.b)* implies that the average of the state vector is preserved from iteration to iteration.

A Sufficient Condition for a.s. Convergence

Consider the distance to the instantaneous average, i.e. the vector of deviations at time k given by

$$\mathbf{d}(k) = (\mathbf{I}-\mathbf{J})\mathbf{x}(k) \tag{4}$$

where $\mathbf{I}$ denotes the $N\times N$ identity matrix and $\mathbf{J}_N = \mathbf{1}\mathbf{1}^T/N$ is a normalized all-ones matrix. $\mathbf{d}(k)$ specifies how far the nodes are from a consensus at time k. Using property *i)* of $\mathbf{W}(k)$ we have

$$\mathbf{d}(k+1) = (\mathbf{I}-\mathbf{J})\mathbf{W}(k)\mathbf{d}(k) \tag{5}$$

Remark that $\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2\right] = \mathbb{E}\left[\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2 | \mathbf{d}(k-1)\right]\right]$, and the expected norm of $\mathbf{d}(k+1)$ given $\mathbf{d}(k)$ is

$$\mathbb{E}\left[\|\mathbf{d}(k+1)\|_2^2 | \mathbf{d}(k)\right] = \mathbf{d}(k)^T\mathcal{W}\mathbf{d}(k) \leq \lambda_1(\mathcal{W})\|\mathbf{d}(k)\|_2^2$$

where we have used the fact that for any unitary vector $\mathbf{u}$ yields $\mathbf{u}^T\mathbf{X}\mathbf{u} \leq \lambda_1(\mathbf{X})\mathbf{u}^T\mathbf{u}$ and

$$\mathcal{W} = \mathbb{E}\left[\mathbf{W}(k)^T(\mathbf{I}-\mathbf{J})\mathbf{W}(k)\right] \tag{6}$$

since $(\mathbf{I}-\mathbf{J})$ is symmetric and idempotent. Repeatedly conditioning and replacing iteratively for $\mathbf{d}(k)$ we obtain

$$\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2\right] \leq \lambda_1^k(\mathcal{W})\|\mathbf{d}(0)\|_2^2. \tag{7}$$

Clearly, the right-hand side of the inequality above converges if

$$\lambda_1(\mathcal{W}) < 1. \tag{8}$$

If the condition in (8) is satisfied, convergence of the upper bound in (7) is ensured, and a smaller $\lambda_1(\mathcal{W})$ will result in a faster convergence. The minimization of $\lambda_1(\mathcal{W})$ is a convex optimization problem and its optimum value satisfies (8) [16], ensuring almost sure convergence of the consensus algorithm in (5). Therefore, the minimization of $\lambda_1(\mathcal{W})$ is the criterion chosen to reduce the convergence to a consensus.

The next step consists in deriving closed-form expressions for the matrix $\mathcal{W}$ assuming constant link weight matrices of the form in (3) satisfying *i)* and *iii)* with temporally independent but spatially correlated entries, and searching for the value of ϵ that minimizes $\lambda_1(\mathcal{W})$. Replacing (3) in (6) yields

$$\mathcal{W} = \mathbb{E}\left[\mathbf{L}(k)^T(\mathbf{I}-\mathbf{J})\mathbf{L}(k)\right]\epsilon^2 - 2\bar{\mathbf{L}}\epsilon + \mathbf{I} - \mathbf{J}. \tag{9}$$

Considering instantaneous directed communication links, the matrix $\mathcal{W}$ in (9) has a closed-form expression given by [16]

$$\mathcal{W} = \left(\bar{\mathbf{L}}^2 + \frac{2(N-1)}{N}(\bar{\mathbf{L}} - \tilde{\mathbf{L}}) + \mathbf{R}\right)\epsilon^2 - 2\bar{\mathbf{L}}\epsilon + \mathbf{I} - \mathbf{J} \tag{10}$$

where

$$\tilde{\mathbf{L}} = \tilde{\mathbf{D}} - \mathbf{P} \odot \mathbf{P} \tag{11}$$

$\odot$ denotes Schur product, $\tilde{\mathbf{D}}$ is diagonal with entries

$$\tilde{\mathbf{D}}_{ii} = [(\mathbf{P}\odot\mathbf{P})\mathbf{1}]_i$$

and $\mathbf{R}$ is a symmetric $N\times N$ matrix built with the covariance terms in $\mathbf{C}$ as follows

$$\begin{aligned} \mathbf{R}_{mm} &= \mathbf{g}_m^T\mathbf{C}\,\mathbf{g}_m + \tfrac{1}{N}(\mathbf{g}_m^T - \mathbf{q}_m^T)\mathbf{C}\,(\mathbf{q}_m - \mathbf{g}_m) \\ \mathbf{R}_{mn} &= \textstyle\sum_i \mathbf{e}_{in}^T\mathbf{C}\,\mathbf{e}_{im} - \mathbf{g}_n^T\mathbf{C}\,\mathbf{e}_{nm} - \mathbf{g}_m^T\mathbf{C}\,\mathbf{e}_{mn} \\ &\quad + \tfrac{1}{N}(\mathbf{g}_n^T - \mathbf{q}_n^T)\mathbf{C}\,(\mathbf{q}_m - \mathbf{g}_m) \end{aligned} \tag{12}$$

with

$$\mathbf{g}_m = \mathbf{1}\otimes\mathbf{e}_m, \quad \mathbf{q}_m = \mathbf{e}_m\otimes\mathbf{1}, \quad \mathbf{e}_{mn} = \mathbf{e}_m\otimes\mathbf{e}_n, \tag{13}$$

$\mathbf{e}_i$ is the i^{th} column of $\mathbf{I}$ and $\otimes$ denotes Kronecker product. Considering instead instantaneous undirected communication links, the matrix $\mathcal{W}$ in (9) has a closed-form expression given by [16]

$$\mathcal{W} = \left(\bar{\mathbf{L}}^2 + 2(\bar{\mathbf{L}} - \tilde{\mathbf{L}}) + \mathbf{R}'\right)\epsilon^2 - 2\bar{\mathbf{L}}\epsilon + \mathbf{I} - \mathbf{J} \tag{14}$$

where $\tilde{\mathbf{L}}$ is as defined in (11) and

$$\begin{aligned} \mathbf{R}'_{mm} &= \mathbf{g}_m^T\mathbf{C}\,\mathbf{g}_m \\ \mathbf{R}'_{mn} &= \textstyle\sum_i \mathbf{e}_{in}^T\mathbf{C}\,\mathbf{e}_{im} - \mathbf{g}_n^T\mathbf{C}\,\mathbf{e}_{nm} - \mathbf{g}_m^T\mathbf{C}\,\mathbf{e}_{mn}, \end{aligned} \tag{15}$$

with $\mathbf{g}_m$ and $\mathbf{e}_{mn}$ defined in (13). We particularize now the closed-form expressions in (10) and (14) for the broadcast gossip algorithm and for the pair-wise gossip algorithm respectively in the following sections.

The Broadcast Gossip Algorithm

We particularize the closed-form expression in (10) for the broadcast gossip algorithm, a particular case of consensus in directed topologies with correlated links. In the implementation of this algorithm, at each iteration a single node is randomly chosen with probability $p = 1/N$ to transmit its state to the nodes within its connectivity range. The rest remain silent forcing the following correlation among links

$$\mathbf{C}_{\substack{ij\\qr}} = \begin{cases} p(1-p) & \text{if } i \neq q \text{ and } \{j = r\} \in \{\mathcal{N}_i, \mathcal{N}_q\} \\ -p^2 & \text{if } j \neq r, \text{ and } j \in \mathcal{N}_i \text{ and } r \in \mathcal{N}_q \\ 0 & j = r \text{ and } i = q \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where for simplicity $\mathbf{C}_{\substack{ij\\qr}}$ denotes the entry $\mathbf{C}_{st}$ with s and t as defined in (1). For notation convenience we let $\mathbf{L}$ denote the Laplacian matrix of the fixed graph, and $\bar{\mathbf{L}} = p\mathbf{L}$. A closed-form expression for $\mathbf{R}$ in (12) can be found after a series of computations using the values given in (16) [16], obtaining

$$\mathbf{R} = 2\tilde{\mathbf{L}} - \bar{\mathbf{L}}^2 + \frac{1}{N}\left(2(\bar{\mathbf{L}} - \tilde{\mathbf{L}}) - p\mathbf{L}^2\right)$$

where $\tilde{\mathbf{L}}$ is as defined in (11). Replacing $\mathbf{R}$ in (10) we obtain

$$\mathcal{W}_{BG} = \left(2\bar{\mathbf{L}} - \bar{\mathbf{L}}^2\right)\epsilon^2 - 2\bar{\mathbf{L}}\epsilon + \mathbf{I} - \mathbf{J} \tag{17}$$

which coincides with the expression for $\mathcal{W}$ in [12, Lemma 2] after replacing for $\epsilon = 1-\gamma$, where γ is denoted the mixing parameter. Summing up, the expressions for the matrix $\mathcal{W}$ given in [12] can be obtained using the general expression in (10), showing that the broadcast gossip algorithm can be treated as a particular case of the consensus algorithm with spatially correlated random links. Furthermore, since all the matrices in (17) are diagonalized by the same set of unitary eigenvectors, we find that the optimum ϵ is the value minimizing the following function

$$f_{N-1}(\epsilon) = \left(2\lambda_{N-1}(\bar{\mathbf{L}}) - \lambda_{N-1}(\bar{\mathbf{L}})^2\right)\epsilon^2 - 2\lambda_{N-1}(\bar{\mathbf{L}})\epsilon + 1 \tag{18}$$

where $\lambda_{N-1}(\bar{\mathbf{L}}) = p\lambda_{N-1}(\mathbf{L})$ is the second smallest eigenvalue of the average Laplacian matrix and where we have replaced $\lambda_{N-1}(\mathbf{I} - \mathbf{J}) = 1$. A graphical representation is depicted in Fig. 1. The optimum ϵ is therefore given by

$$\epsilon^* = \frac{1}{2 - \lambda_{N-1}(\bar{\mathbf{L}})}.$$

Then, replacing for ϵ, $\bar{\mathbf{L}}$, and p we obtain the optimum mixing parameter γ^* derived in [12, Corollary 1].

The Pair-wise Gossip Algorithm

In this section we particularize the closed-form expression in (14) for the pair-wise gossip algorithm, a particular case of consensus in undirected topologies with correlated links. In this algorithm, at each iteration a single node i wakes up randomly with probability $p = \frac{1}{N}$ and establishes a bidirectional communication link with another node chosen at random within its connectivity range with probability $1/\mathcal{N}_i$. For symmetry, $p_{ij} = p_{ji}$ and equal to $p(\frac{1}{\mathcal{N}_i} + \frac{1}{\mathcal{N}_j})$. The rest remain silent forcing the following correlation among links

$$\mathbf{C}_{\substack{ij\\qr}} = \begin{cases} p\left(\frac{1}{\mathcal{N}_i} + \frac{1}{\mathcal{N}_j}\right) - p^2\left(\frac{1}{\mathcal{N}_i} + \frac{1}{\mathcal{N}_j}\right)^2 & \text{if } j = q, i = r, j \in \mathcal{N}_i \\ -p^2\left(\frac{1}{\mathcal{N}_i} + \frac{1}{\mathcal{N}_j}\right)\left(\frac{1}{\mathcal{N}_q} + \frac{1}{\mathcal{N}_r}\right) & \text{if } j \in \mathcal{N}_i \text{ and } r \in \mathcal{N}_q \\ 0 & j = r \text{ and } i = q \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

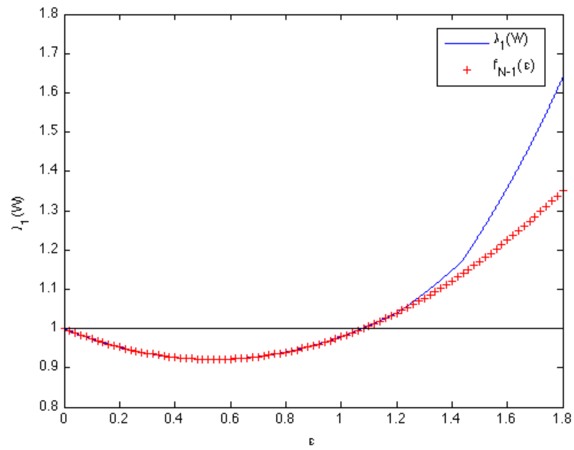

Figure 1: $\lambda_1(\mathcal{W})$ as a function of ϵ along with the curve for $f_{N-1}(\epsilon)$ for the broadcast gossip algorithm in a network with $N = 10$.

with zero elements on the main diagonal. Using the values given in (19), the matrix of correlation terms defined in (15) is given by

$$\mathbf{R}' = 2\tilde{\mathbf{L}} - \bar{\mathbf{L}}^2$$

where $\bar{\mathbf{L}} = \bar{\mathbf{D}} - \bar{\mathbf{A}}$ with $\bar{\mathbf{A}}_{mn} = p(\frac{1}{\mathcal{N}_m} + \frac{1}{\mathcal{N}_n})$, and $\tilde{\mathbf{L}} = \tilde{\mathbf{D}} - \tilde{\mathbf{A}}$ with $\tilde{\mathbf{A}}_{mn} = p^2(\frac{1}{\mathcal{N}_m} + \frac{1}{\mathcal{N}_n})^2$. Replacing for $\mathbf{R}'$ in (14) we obtain

$$\mathcal{W}_{PG} = 2\bar{\mathbf{L}}\epsilon^2 - 2\bar{\mathbf{L}}\epsilon + \mathbf{I} - \mathbf{J}.$$

Analogously to the previous case, the largest eigenvalue of the matrix $\mathcal{W}_{PG}$ has a closed-form expression, given in this case by

$$\lambda_1(\mathcal{W}_{PG}) = 2\lambda_{N-1}(\bar{\mathbf{L}})\epsilon^2 - 2\lambda_{N-1}(\bar{\mathbf{L}})\epsilon + 1$$

and the value minimizing the function above is $\epsilon^* = 1/2$, which is the optimum mixing parameter for pair-wise gossip algorithms. In the following section, the analytical results are supported with the simulations of a general case, namely a random geometric network network with instantaneous directed links existing with different probabilities of connection and assuming different correlation among pairs of links.

3. SIMULATION RESULTS

We simulate a random geometric network composed of $N = 20$ nodes randomly deployed in a unit square and with fixed position, where two nodes are connected only if the euclidean distance between them is smaller than 0.37. The entries of $\mathbf{x}(0)$ are modeled as Gaussian random variables (r.v.'s) with mean $x_m = 20$ and variance $\sigma_0^2 = 5$ and the links are generated as correlated Bernoulli r.v.'s with different probabilities chosen uniformly between $[0, 1]$. For the spatial correlation we consider the autoregressive model in [18, Sec. 2.2-2.4] with $p = \max\{\mathbf{P}\}$ and $\theta = 0.3$. A total of 10.000 independent realizations were run to compute the expected norm of the error vector $\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2\right]$, where $\mathbf{P}$ was kept fixed while a new non-symmetric $\mathbf{L}(k)$ was generated at each iteration. Fig. 2 shows the empirical $\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2\right]$ in log-linear scale as a function of k for three different values of ϵ: 1) $\epsilon = 1/(N-1) = 0.0526$ (dashed-dotted line); 2) $\epsilon_{bound} = 0.1850$, i.e., the value minimizing the MSE upper bound defined in [15] (dashed line); 3) $\epsilon^* = 0.3367$ minimizing ${}_1(\mathcal{W})$ (dotted-line). Fig. 3 shows the empirical MSE w.r.t the statistical mean of the initial values in log-linear scale averaged over all nodes for the three cases, along with the benchmark value σ_0^2/N (solid line). The results depicted in Fig. 2 verify that the choice of the optimum ϵ^* reduces the convergence time of the algorithm, whereas the results in Fig. 3 are more useful to evaluate the deviation of the state w.r.t. the statistical mean of the initial measurements caused by asymmetric links.

4. CONCLUDING REMARKS

The convergence of the consensus algorithm in random WSNs with spatially correlated links has been studied, where a useful criterion for the reducing of the convergence time has been adopted. This criterion states a sufficient condition for almost sure convergence to a consensus and is based on the spectral radius of a positive semidefinite matrix for which we derive closed-form expressions for constant link weight matrices, assuming both directed and undirected instantaneous links. The general expressions provided subsume existing protocols found in literature and greatly simplify the derivation of the optimum link weights. The closed-form expressions have been particularized for the broadcast gossip algorithm and for the pair-wise gossip algorithm, showing that they can be seen as particular cases of the consensus algorithm with spatially correlated links. The analytical results are further validated with computer simulations of a general case with different probabilities of connection for the links and different correlations among pairs of links.

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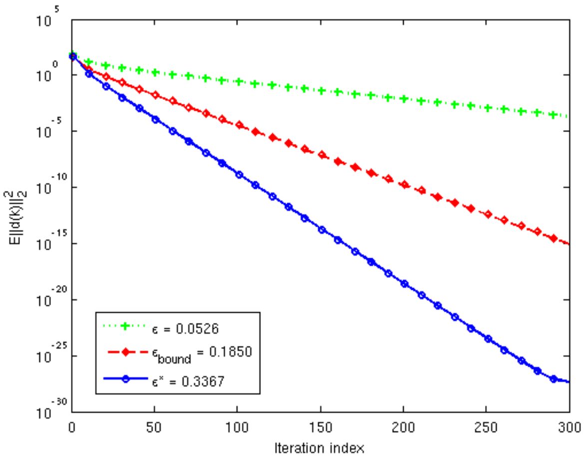

Figure 2: $\mathbb{E}\left[\|\mathbf{d}(k)\|_2^2\right]$ as a function of the iteration index for different link probabilities and different link weights in a random geometric graph.

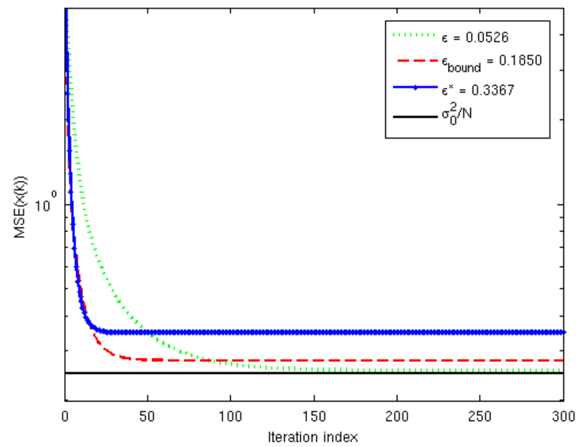

Figure 3: Empirical $\mathsf{MSE}(x(k))$ as a function of the iteration index for the same deployment with different link probabilities and different link weights.

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