Buck-Boost DC-DC Converter with Fractional Control

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Abstract

This paper deals with the fractional modeling of a DC-DC buck-boost converter, suitable in solar-powered electrical generation systems, and the design of a fractional controller for the aforementioned switching converter. Although the modeling and design of the controller is carried out for this particular DC-DC converter, it can be easily extended to other kind of switching converter. In addition, the comparison between integer-order plant/controller and fractional-order plants/controller is carried out. The article also shows that, under the same design conditions, the fractional-order controller has a better performance and behaviour than the classical integer-order controller in both situations, that is, with integer-order plant and fractional-order plant models.

I.- INTRODUCTION

Photovoltaic (PV) arrays are generally the bulkiest and most expensive parts of solar-powered electrical generation systems. Optimum utilization of available power from these arrays is therefore essential and can considerably reduce the size, weight and cost of such power systems. The controller is usually an essential part of a PV system. It incorporates a DC-DC converter and is used as a controlled energy-transfer-equipment between the main energy source (PV arrays) and an auxiliary energy system based on ultracapacitors. Most converters are based on either the buck converter (step-down), boost convert (step-up) or buck-boost converter setup. This capability of the converter makes it ideal for converting the solar panel maximum power point voltage to the load operating voltage. Problems exist with battery packs including the inability to absorb and discharge large current loads during regenerative braking and boost assist, performance degradation over their life, weight, size and environmental concerns regarding disposal. Ultracapacitors, or electrochemical capacitors (EC), can eliminate these problems.

The performance characteristics of ultracapacitors differ somewhat from those of conventional capacitors. The impedance of any real ultracapacitor can be easily reproduced in any frequency model equation by replacing every *jw* expression with $(j\omega)^{\alpha}$, $0<\alpha<1$, and where $\alpha=1$ represents an ideal capacitor with no frequency dependence [1]. Experimentally, the parameter α is not often smaller than 0.5, the case for a Warburg impedance. A single value of α normally describes an electrochemical system over only a limited frequency range.

This non-ideality is a typical feature of electrochemical charging processes, and may be interpreted as resulting from a distribution in macroscopic path lengths (nonuniform active layer thickness) or a distribution in microscopic charge transfer rates, absorption processes, or surface roughness [3]. For distributed parameter systems, it has been shown that fractional order calculus will play a role in its modeling and analysis. In general, fractional order systems are useful to model various stable physical phenomena (commonly diffusive systems) with anomalous decay, say those that are not of exponential type. It is natural to consider fractional order controls. Clearly, for closed-loop control systems, there are four situations. They are: 1) IO (integer order) plant with IO controller; 2) IO plant with FO (fractional order) controller; 3) FO plant with IO controller and 4) FO plant with FO controller.

In this paper, we focus on the control of a buck-boost converter based on ultracapacitors as an essential element in the optimal use of available energy in the PV arrays. A fractional control approach is motivated by the fractional nature that presents the model of the converter with ultracapacitors as accumulator. Furthermore, FO and IO linear feedback controllers are designed and compared in the control of the FO and IO models that can describe the plant in different frequency ranges.

II.- STATE-SPACE AVERAGING MODEL OF A BUCK-BOOST DC/DC SWITCHING CONVERTER BASED ON ULTRACAPACITORS.

During last years, many scientists and authors have worked in order to obtain different capacitor models that have been reported in previous articles [1]-[4]. In this way, Westerlund and Ekstam in [2] proposed that a better modelling of the impedance of a capacitor *C* could be given by:

$$Z(j\omega)=\frac{1}{(j\omega)^{\alpha}C} \quad ; \quad 0<\alpha<1 \tag{1}$$

Based on this last expression, the current $i(t)$ through the capacitor *C* is a function of the general voltage $v(t)$ across it:

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$$i(t) = CD_t^{\alpha} v(t) \tag{2}$$

It can be noticed that $D_t^{\alpha} v(t)$ is the fractional time derivative of the voltage $v(t)$ across the capacitor. For different capacitors α is not equal to one, but it is close to 0.999. The ultracapacitor can be modelled in intervals in which, at low frequencies, it is similar to a classical capacitor (that is, $\alpha \approx 1$), and at medium frequencies it can be modelled by a diffusion effect and it is better characterized in the Warburg domain $(j\omega)^{1/2}$ than in the classical Laplace domain $(j\omega)$ [3], [4]. At higher frequencies, the resistance as well as the capacitance of a porous electrode decreases, because only part of the active porous layer is accessible at high frequencies. The ultracapacitor may thus be represented by an ideal capacitor [1]. Figure 1 displays the Nyquist diagram for the capacitor models (real and ideal). In these systems, it is natural to consider also fractional order controls. In fact, clearly, for closed-loop control systems, there are four situations; that is: 1) IO (integer order) plant with IO controller; 2) IO plant with FO (fractional order) controller; 3) FO plant with IO controller, and 4) FO plant with FO controller.

In this article, we focus on the control of two DC/DC switching converters based on ultracapacitors as an essential element in the optimal use of available energy in PV arrays. A fractional control approach is motivated by the fractional nature that presents the model of the converter with ultracapacitors as accumulator. Furthermore, in current article, FO and IO linear feedback controllers are designed and compared in the control of the FO and IO models that can describe the plant in different frequency ranges.

II.a.- State-Space Averaging Model of an Ideal Buck-Boost Converter Based On Ultracapacitors.

In order to show the fractional modelling of DC/DC converter, the state-space averaging method to model the buck-boost converter in Figure 2 is considered. The main difference of this converter with the classic DC/DC buck and boost converters is that the output voltage has an opposite sign to the input DC source $E(t)$. The input voltage $E(t)$ is an independent source whose value is defined by the MPPT (maximum power point tracking) of a PV system.

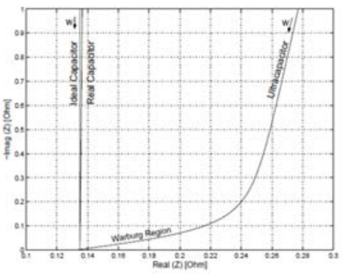

Fig. 1.- Nyquist diagram of a capacitor (real and ideal) and an ultracapacitor.

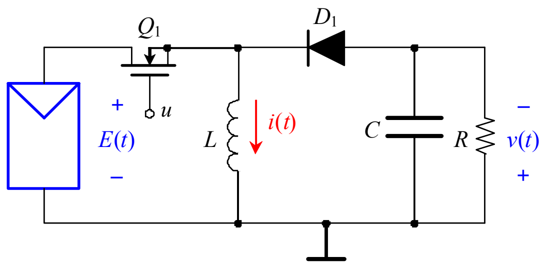

Fig. 2.- DC/DC buck-boost switching converter circuit.

The operation of this system is as follows: On the one hand, when the transistor Q_1 is switched to ON state (conduction state), the diode D_1 is inversely polarized. Thus, considering ideal switches without losses, the circuit topology shown in Figure 3.*a* is obtained. During this period, the inductor current is generated from the source $E(t)$. While the diode remains inversely polarized we say the circuit is operating in the *charging period*. On the other hand, when the transistor Q_1 is switched *OFF*, the diode D_1 is directly polarized generating the circuit topology shown in Figure 3.*b*. This second period is known as the *discharging period* due to the fact that the stored energy in the inductor L is transferred to the system load R [5]. When Kirchoff voltage and current laws are applied to both circuit topologies of figures 3.*a* and 3.*b*, and the obtained models are combined into a single dynamic model, the resulting system of differential equations describing the buck-boost converter is the following:

$$\left.\begin{aligned} LD_t^{1} i(t) &= (1-u)\, v(t) - uE(t) \\ CD_t^{\alpha(\omega)} v(t) &= -\frac{1}{R} v(t) - (1-u)\, i(t) \end{aligned}\right\} \tag{3}$$

where $D_t^{1} i(t)$ is the first time derivative of the current $i(t)$ flowing through the inductor L, and $\alpha(\omega) = \{1, 0.5, 1\}$ describes the electrochemical system over a frequency range ω (low, medium and high frequencies, respectively). Thus, the normalized average model of the ideal buck-boost converter based on ultracapacitors is given by:

$$\left.\begin{aligned} D_\tau^{1} x_1 &= u_x x_2 + (1-u_x) E \\ D_\tau^{\alpha(\omega)} x_2 &= -u_x x_1 - \frac{1}{Q} x_2 \end{aligned}\right\} \tag{4}$$

where the variable x_1 represents the *normalized* inductor current, x_2 is the normalized output voltage, and $u_x = 1-u$ represents the average control variable. Clearly, the underlying transformation is given by:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{L}{C}} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix}, \text{ and } Q = R\sqrt{\frac{C}{L}} \quad ; \quad \tau = \frac{t}{\sqrt{LC}} \tag{5}$$

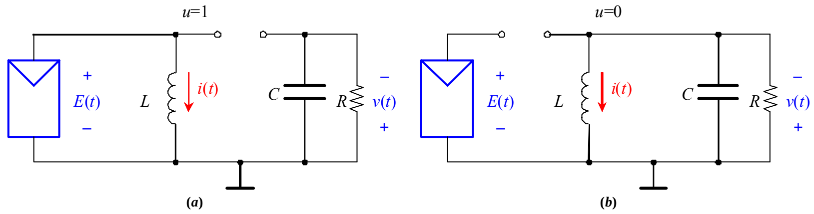

Fig. 3.- States of the DC-DC buck-boost switching converter: (*a*) With u=1: ON state, and (*b*) u=0: OFF state.

II.b.- State Feedback Controller Design for the Buck-Boost Converter Based On Ultracapacitors.

Let us consider the tangent linearization model of the normalized average ideal buck-boost converter system defined by (4) and around the equilibrium point:

$$z_2 = v^* < 0 \;\; ; \;\; z_1 = -\frac{v^*\left(E - v^*\right)}{QE} \;\; ; \;\; u_z = \frac{E}{E - v^*} \tag{6}$$

where v^* is the normalized reference voltage. The linearization of the average model is given by:

$$\begin{aligned} D_\tau^1 e_1 &= \frac{E}{E - v^*} e_2 - \left(E - v^*\right) u_e \\ D_\tau^{\alpha(\omega)} e_2 &= -\frac{E}{E - v^*} e_1 - \frac{1}{Q} e_2 + \frac{v^*\left(E - v^*\right)}{QE} u_e \end{aligned} \tag{7}$$

where:

$$e_1 = x_1 - z_1 \quad ; \quad e_2 = x_2 - z_2 \quad ; \quad u_e = u_x - u_z \tag{8}$$

The objective is to find a stabilizing control law $u_e(t)$ such as:

1. The equilibrium point $e = 0$ of equation (7) is locally and asymptotically stable.
2. The control system must reject constant disturbances, like:

$$\lim_{t\to\infty}\left[v(t) - v^*(t) \right] = 0 \tag{9}$$

3. $0 \le u_x(t) \le 1, \;\; \forall t \ge 0$.
4. The eigenvalues of the average feedback state can be arbitrarily assigned.

II.*b*.1.- IO Controller.

In this case, an average integer and linear state feedback control is found in the form:

$$u_e = -k_1 I_\tau^1 e_1 - k_2 e_1 - k_3 e_2 \tag{10}$$

which drives the average stabilization error state e to zero in an exponentially stable way. Such a controller is designed with the help of the average tangent linearization system and it will use, for the average nonlinear system, the following control input:

$$u_x = u_z - k_1 I_\tau^1 e_1 - k_2 e_1 - k_3 e_2 \tag{11}$$

- **IO Plant (Low and High Frequencies).**

The equivalent closed loop tangent system for α(ω)=1, that is, for low and high frequencies (integer order plant), is given by:

$$\begin{aligned} D_\tau^2 e_1 &= Rk_2 D_\tau^1 e_1 + Rk_1 e_1 + \left[Rk_3 + \frac{E}{R} \right] D_\tau^1 e_2 \\ D_\tau^2 e_2 &= -\left[Pk_3 + \frac{1}{Q} \right] D_\tau^1 e_2 - \left[Pk_2 + \frac{E}{R} \right] D_\tau^1 e_1 - Pk_1 e_1 \end{aligned} \tag{12}$$

where:

$$P = \frac{v^* R}{QE} \tag{13}$$

$$R = E - v^* \tag{14}$$

In matrix form, it is expressed as:

$$D_\tau^1 e_{ioio} = \begin{bmatrix} 0 & 1 & 0 \\ h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \end{bmatrix} e_{ioio} \quad (15)$$

where the different terms of the matrix are:

$$h_1 = Rk_1 \ ; \ h_2 = Rk_2 \ ; \ h_3 = Rk_3 + \frac{E}{R} \ ; \ h_4 = -Pk_1 \ ; \ h_5 = -\left(Pk_2 + \frac{E}{R}\right) \ ; \ h_6 = -\left(Pk_3 + \frac{1}{Q}\right) \quad (16)$$

and

$$e_{ioio} = \left[e_1,\ D_\tau^1 e_1,\ D_\tau^1 e_2\right]^T \quad (17)$$

The characteristic polynomial is given by:

$$p(s) = s^3 + a_{1io}s^2 + a_{2io}s + a_{3io} \quad (18)$$

being:

$$a_{1io} = Pk_3 - Rk_2 + \frac{1}{Q} \ ; \ a_{2io} = Ek_3 + \left(\frac{E}{R}\right)^2 + \frac{2v^* - E}{Q}k_2 - Rk_1 \ ; \ a_{3io} = \frac{2v^* - E}{Q}k_1 \quad (19)$$

- **FO plant (Medium Frequencies).**

The equivalent closed loop tangent system for α(ω)=0.5 is given by:

$$\begin{aligned} D_\tau^2 e_1 &= Rk_2 D_\tau^1 e_1 + Rk_1 e_1 + \left[Rk_3 + \frac{E}{R}\right] D_\tau^1 e_2 \\ D_\tau^{1.5} e_2 &= -\left[Pk_3 + \frac{1}{Q}\right] D_\tau^1 e_2 - \left[Pk_2 + \frac{E}{R}\right] D_\tau^1 e_1 - Pk_1 e_1 \end{aligned} \quad (20)$$

In matrix form it is expressed as:

$$D_\tau^{0.5} e_{iofo} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ h_1 & 0 & h_2 & 0 & h_3 \\ 0 & 0 & 0 & 0 & 1 \\ h_4 & 0 & h_5 & 0 & h_6 \end{bmatrix} e_{iofo} \quad (21)$$

where:

$$e_{iofo} = \left[e_1,\ D_\tau^{0.5} e_1,\ D_\tau^1 e_1,\ D_\tau^{1.5} e_1,\ D_\tau^1 e_2\right]^T \quad (22)$$

and its characteristic polynomial is given by:

$$p(\lambda) = \lambda^5 + a_{1fo}\lambda^4 + a_{2fo}\lambda^3 + a_{3fo}\lambda^2 + a_{4fo}\lambda + a_{5fo} \quad (23)$$

being $\lambda = s^{0.5}$, and:

$$a_{1fo} = Pk_3 + \frac{1}{Q} \ ; \ a_{2fo} = -Rk_2 \ ; \ a_{3fo} = Ek_3 + \left(\frac{E}{R}\right)^2 + \frac{2v^* - E}{Q}k_2 \ ; \ a_{4fo} = -Rk_1 \ ; \ a_{5fo} = \frac{2v^* - E}{Q}k_1 \quad (24)$$

II.*b*.2.- FO Controller.

In this subsection, the degree of freedom yielded by fractional models in state space is used to offer fractional controllers for each plant.

- **IO Plant (Low and High Frequencies).**

In this case, an average non-integer and linear state feedback control is newly found in the form:

$$u_e = -q_1 I_\tau^{0.5} e_1 - q_2 e_1 - q_3 D_\tau^{0.5} e_1 - q_4 e_2 - q_5 D_\tau^{0.5} e_2 \quad (25)$$

which drives the average stabilization error state *e* to zero in a generalized exponentially stable fashion. Such a controller is designed with the help of the average tangent linearization system and it will use, for the average nonlinear system, the following control input:

$$u_x = u_z + u_e \quad (26)$$

The equivalent closed loop tangent system for α(ω)=1 is given by:

$$D_\tau^{1.5}e_1 = Rq_1e_1 + Rq_2D_\tau^{0.5}e_1 + Rq_3D_\tau^{1}e_1 + \left[Rq_4 + \frac{E}{R}\right]D_\tau^{0.5}e_2 + Rq_4D_\tau^{1}e_2$$
$$D_\tau^{1.5}e_2 = -Pq_1e_1 - \left[Pq_2 + \frac{E}{R}\right]D_\tau^{0.5}e_1 - Pq_3D_\tau^{1}e_1 - \left[Pq_4 + \frac{1}{Q}\right]D_\tau^{0.5}e_2 - Pq_5D_\tau^{1}e_2 \quad (27)$$

In matrix form is expressed as:

$$D_\tau^{0.5}e_{foio} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ g_1 & g_2 & g_3 & g_4 & g_5 \\ 0 & 0 & 0 & 0 & 1 \\ g_6 & g_7 & g_8 & g_9 & g_{10} \end{bmatrix} e_{foio} \quad (28)$$

where:

$$g_1 = Rq_1 \ ; \ g_2 = Rq_2 \ ; \ g_3 = Rq_3 \ ; \ g_4 = Rq_4 + \frac{E}{R} \ ; \ g_5 = Rq_5$$
$$g_6 = -Pq_1 \ ; \ g_7 = -\left(Pq_2 + \frac{E}{R}\right) \ ; \ g_8 = -Pq_3 \ ; \ g_9 = -\left(Pq_4 + \frac{1}{Q}\right) \ ; \ g_{10} = Pq_5 \quad (29)$$

$$e_{foio} = \left[e_1,\ D_\tau^{0.5}e_1,\ D_\tau^{1}e_1,\ D_\tau^{0.5}e_2,\ D_\tau^{1}e_2\right]^T \quad (30)$$

and its characteristic polynomial is given by:

$$p(\lambda) = \lambda^5 + b_{1io}\lambda^4 + b_{2io}\lambda^3 + b_{3io}\lambda^2 + b_{4io}\lambda + b_{5io} \quad (31)$$

being:

$$b_{1io} = Pq_5 - Rq_3 \ ; \ b_{2io} = Pq_4 - Rq_2 + \frac{1}{Q} \ ; \ b_{3io} = Eq_5 - Rq_1 + \frac{2v^* - E}{Q}q_3 \ ; \ b_{4io} = Eq_4 + \left(\frac{E}{R}\right)^2 + \frac{2v^* - E}{Q}q_2 \ ; \ b_{5io} = \frac{2v^* - E}{Q}q_1 \quad (32)$$

- **FO plant (Medium Frequencies).**

In this case, an average non-integer and linear state feedback control is found in the form:

$$u_e = -q_1I_\tau^{0.5}e_1 - q_2e_1 - q_3D_\tau^{0.5}e_1 - q_4e_2 \quad (33)$$

which drives the average stabilization error state e to zero in a generalized exponentially stable way. Such a controller is designed with the help of the average tangent linearization system and it will use, for the average nonlinear system, the control input:

$$u_x = u_z + u_e \quad (34)$$

The equivalent closed loop tangent system for α(ω)=0.5 is given by:

$$D_\tau^{1.5}e_1 = Rq_1e_1 + Rq_2D_\tau^{0.5}e_1 + Rq_3D_\tau^{1}e_1 + \left[Rq_4 + \frac{E}{R}\right]D_\tau^{0.5}e_2$$
$$D_\tau^{1}e_2 = -Pq_1e_1 - \left[Pq_2 + \frac{E}{R}\right]D_\tau^{0.5}e_1 - Pq_3D_\tau^{1}e_1 - \left[Pq_4 + \frac{1}{Q}\right]D_\tau^{0.5}e_2 \quad (35)$$

In matrix form is expressed as:

$$D_\tau^{0.5}e_{fofo} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ g_1 & g_2 & g_3 & g_4 \\ g_6 & g_7 & g_8 & g_9 \end{bmatrix} e_{fofo} \quad (36)$$

where:

$$e_{fofo} = \left[e_1,\ D_\tau^{0.5}e_1,\ D_\tau^{1}e_1,\ D_\tau^{0.5}e_2\right]^T \quad (37)$$

and its characteristic polynomial is given by:

$$p(\lambda) = \lambda^4 + b_{1fo}\lambda^3 + b_{2fo}\lambda^2 + b_{3fo}\lambda + b_{4fo} \quad (38)$$

being:

$$b_{1fo} = Pq_4 - Rq_3 + \frac{1}{Q} \ ; \ b_{2fo} = \frac{2v^* - E}{Q}q_3 - Rq_2 \ ; \ b_{3fo} = \frac{2v^* - E}{Q}q_2 - Rq_1 + Eq_4 + \left(\frac{E}{R}\right)^2 \ ; \ b_{4fo} = \frac{2v^* - E}{Q}q_1 \quad (39)$$

Equating these polynomials to a desired closed loop characteristic polynomial, feedback gains for the rational linear controllers can be obtained [6].

III. SIMULATION RESULTS FOR THE BUCK-BOOST CONVERTER BASED ON ULTRACAPACITORS.

Simulations have been carried out in order to assess the effectiveness of the proposed full state feedback controllers. They have been computed on basis of the tangent linearized systems, to accomplish a stabilization around a normalized equilibrium point value for initial conditions set at origin of coordinates. In order to compare the performances of different control laws (IO and FO controllers for IO and FO plants), the same poles placement than in closed loop system is used for determinate the feedback gains. All zeros of the characteristic polynomial are defined by ε. The following practical parameters and design values have been used: Q=0.75, $E(t)$=E=10 V, v^*=–5 V, and ε=0.15.

The average control input initially takes negative values and then a slower response is proposed. This would cause a temporary saturation to zero of the corresponding switched controller. In order to verify that the control system rejects constant disturbances, at t=25 s a step signal is used as disturbance on output. Figure 4 depicts the response of the nonlinear average buck-boost converter circuit based on ultracapacitors for low frequencies using IO and FO control actions by means of state-feedback controllers computed on the basis of the linearized tangent average system complemented with the nominal equilibrium control input.

Similarly, Figure 5 depicts the response of the nonlinear average buck-boost converter circuit for medium frequencies. In both plant models (IO and FO plants), FO controllers show a best behaviour at closed loop system than IO controllers. It can be appreciated that transient responses are smoother and the convergence to the origin is higher.

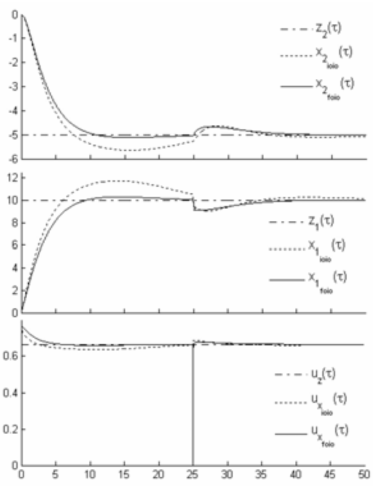

Fig. 4.- Response of average buck-boost converter based on ultracapacitors using IO and FO control actions by means of state-feedback controllers ($\alpha(\omega)$=1).

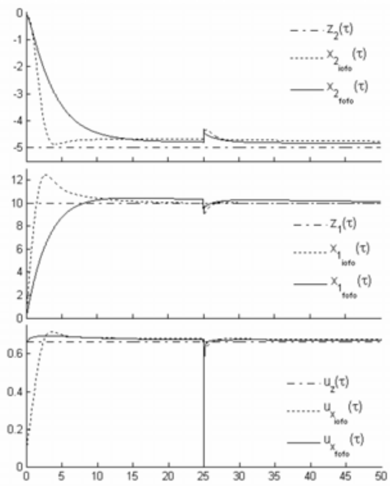

Fig. 5.- Response of average buck-boost converter based on ultracapacitors using IO and FO control actions by means of state-feedback controllers ($\alpha(\omega)$=0.5).

IV. CONCLUSIONS.

In this work, on the one hand, the proposal of fractional modelling for DC-DC converters based on ultracapacitors, suitable for many powered electrical systems, is presented. For the sake of simplicity, a particular example, based on a buck-boost converter, is carried out. Owing to the fact that the fractional order model of the system changes according to the frequencies range, fractional-order and integer-order models are proposed.

In addition, on the other hand, fractional-order and integer-order linear feedback controllers are designed, compared and verified by simulations results in each plant model. Simulation results show that fractional-order controllers are more suitable for both plant models (integer-order and fractional-order plant model) than integer-order controllers. This aspect represents a strong motivation to the modelling and control of powered electrical systems via fractional control techniques.

V. ACKNOWLEDGMENT

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