JOINT TEMPORAL-SPATIAL REFERENCE BEAMFORMING: EIG BEAMFORMING

Miguel Ángel LAGUNAS, Ana I. PEREZ-NEIRA, Miguel Ángel VAZQUEZ

CTTC (Technological Center on Telecommunications of Catalunya)
Avda. Canal Olimpic s/n
09960 Castelldefels (Barcelona)
SPAIN
UPC (Technical University of Catalunya)
c/ Jordi Girona 1-3. Building D5-Campus Nord UPC
08034 Barcelona
SPAIN
m.a.lagunas@cttc.es ana.perez@cttc.es ma.vazquez@cttc.es www.cttc.es

Abstract: Traditional methods for beamforming can be grouped in two families, the time reference beamformer and the spatial reference beamforming. Nevertheless, the increasing demand on receivers for location systems and high bandwidth over frequency selective fading able to properly manage multipath and co-channel interference motivates the need for versatile processing able to cope with both problems. This paper presents a new beamforming procedure, derived from a ML-like framework, which is able to either remove full coherent arrivals, as well as, to enhance them when required yet preserving the receiver architecture and low design complexity. The performance of the beamformer is also tested in the multiuser broadcast scenario.

Key-Words: Array Processing, Eigen-Beamforming, Spatial Reference, Broadcast Multiuser.

1 Introduction

The major beamforming techniques reported in the literature can be encompassed in two families namely Time Reference Beamformer TRB and Spatial Reference Beamformers SRB. Within the TRB framework a reference waveform is framed on the information signal from the desired source, at the receiver the reference is recovered and used for beamforming purposes. On the other hand, SRB beamformers rely in the a priori knowledge of the spatial signature of the desired or Direction Of Arrival DOA in Line Of Sight LOS scenarios [1][2].

Nevertheless, in those communications scenarios with long ISI the ML receiver requires the use of the so-called sequence detectors which entail the joint use of time and spatial reference estimated from data snapshots. Unfortunately, the sequence detectors, inspired on the Viterbi decoder, become quite complex when using spatial diversity at the receiver since they evolve to vector sequence detectors increasing the complexity of the receiver. Furthermore, when a co-channel interference is present it is severely degraded unless the interference is also detected which enlarges considerable the complexity even making it unpractical. The last drawback is that the performance of the sequence detector may degrade the performance due to late arrivals, above the length of the sequence detectors [5]. The elegant solution to these problems is the joint design of a beamformer together with the corresponding Desired Impulse Response DIR of the sequence detector. This solution reduces nicely the complexity of the receiver with minimal impact of the aperture size and with a performance almost equal to the unpractical vector (space-time) ML detector. The beamformer has to remove or properly attenuate both co-channel interferences and late arrivals of the desired, the residual Inter Symbol Interference ISI.

This paper presents a new design of a beamformer able to cope with the mentioned goals. From a ML-like formulation the beamformer objective is to minimize the interference and late arrivals effects at the same time that the Automatic Gain Control AGC at its output acts as a design constrain.

The contributions of this paper is the report of a new beamformer family that, depending on the AGC constrain, solve open problems in GPS/GNSS receivers as well as in those communications links with long ISI in frequency selective channels. This new beamformer family is named as EIG-Beamforming since they are the solution of a generalized eigenvector problem. We claim that the beamforming is new since the design does not coincide with traditional MSE or MV beamforming, just in the special case of rank-one reference covariance and so-called reduced AGC constrain the EIG design reduces to the traditional MSE.

Section II describes the new beamformer in a ML framework and the resulting minimization problem with AGC constrains. Section III shows the versatility of the EIG beamformer to cope the problems associated with multipath both in location applications where it needs to be removed as well as in communication scenarios where energy of early arrivals have to be enhanced to increase the performance of the corresponding sequence detector.

2 The EIG Beamformer

It is clear that the optimum ML receiver to estimate d(n), when its DOA vector are known in a free of interference scenario with white and calibrated noise from the front end is given by (1), where $\underline{X}_n$ is the received snapshot and $\underline{S}_d$ is the DOA vector of the desired.

$$\hat{d}(n)=\min_{c(n)}\left(\underline{X}_n-\underline{S}_d.c(n)\right)^H\left(\underline{X}_n-\underline{S}_d.c(n)\right) \qquad (1)$$

The need for a beamformer appears when interferences show up. Since the optimum receiver for the desired will require detecting both interferences as it is shown in (2a), the optimum receiver is substituted by a suboptimum one formed as (2.b). In this formula, vector $\underline{A}$ plays the role of a forward equalizer (in spatial diversity processing the beamformer) and constant $\underline{A}^H\underline{S}_d$ plays the role of the DIR (Desired Impulse Response, this term was coined several years ago on wired communications)[5][6].

$$[\hat{d}(n),\hat{s}_1(n),..,\hat{s}_{NI}(n)]=$$

$$\min\left(\left|e(n)=\underline{X}_n-\underline{S}_d.d(n)-\sum_{i=1}^{NI}\underline{S}_i.s_i(n)\right|^2\right) \qquad (2.a)$$

$$\hat{d}(n)=\min_{d(n)}\left(\left|e(n)=\underline{A}^H.\left(\underline{X}_n-\underline{S}_d.d(n)\right)\right|^2\right) \qquad (2.b)$$

The design of the beamformer reduces to minimize (3) which is the contribution at the beamformers' output of the contributions of the front-end noise plus interferences.

$$E\left(|e(n)|^2\right)=\underline{A}^H.\left(\underline{\underline{R}}_I+\sigma^2.\underline{\underline{I}}\right)\underline{A} \qquad (3)$$

In order to preclude the trivial solution, a constraint, has to be added to the last equation in order to find a solution for the beamformer. At this point there are several alternatives, being the most trivial the to impose the norm of the DIR equal to one which reduces the design to the traditional MV beamformer. Nevertheless, an engineering justification of this constraint is to control the AGC (Automatic Gain Control) of the receiver, in other words, forcing a giving power level at the beamformer's output. This power level constraint can be set only to the desired contribution (ACG Reduced) or to the contributions of noise and desired (ACG Complete) as it is shown in (4). Note that for implementation issues the complete ACG should be preferred in order to control the dynamic range of signals in the baseband processing board.

$$\begin{array}{ll} AGCr & \underline{A}^H.\underline{\underline{R}}_D.\underline{A}=\phi_0 \\ AGCc & \underline{A}^H.\left(\underline{\underline{R}}_D+\sigma^2.\underline{\underline{I}}\right)\underline{A}=\phi_0 \end{array} \qquad (4)$$

Note that when the covariance matrix of the desired is rank one, the AGCr constraint in solving (3) reduces to the MV or MSE beamformer [1][2].

$$\underline{\underline{R}}_D=P_d.\underline{S}_d.\underline{S}_d^H \qquad (5)$$

Nevertheless both designs, for ACGr and ACGc, produce a beamformer design different from the traditional ones. Only when using ACGr and the desired covariance is rank one, see (6), then the designs coincide.

$$\underline{\underline{R}}_D=P_d.\underline{S}_d.\underline{S}_d^H \qquad (6)$$

In summary the new beamformer, refereed hereafter as EIGc or EIGr (EIG denotes eigen-beamforming), are the solutions of (7a) and (7.b) respectively. Note that the beamformer is basically a generalized eigenvector associated with the maximum eigenvalue[3].

$$\begin{array}{ll} EIG_r & \underline{\underline{R}}_D\underline{A}=\lambda_{\max}\left(\sigma^2\underline{\underline{I}}+\underline{\underline{R}}_I\right)\underline{A} \\ EIG_c & \left(\sigma^2\underline{\underline{I}}+\underline{\underline{R}}_D\right)\underline{A}=\lambda_{\max}\left(\sigma^2\underline{\underline{I}}+\underline{\underline{R}}_I\right)\underline{A} \end{array} \qquad (7)$$

It is evident from (7) that both beamformers do not coincide with MSE or MV, just in case that the covariance of the desired is rank one, i.e. LOS scenario without multipath, the EIG_r beamformer is the same that the MV. Finally, note also that in the literature there is not any similar design. Only in [8] the blind equalization goal leads also to a generalized eigenvector solution, however, only R_D is always accompanied by the R_I in the resulting expressions due to the blind nature of the beamformer.

3. The Role of the Reference on the EIG Design

The information about the reference waveform or location at the receiving site is much richer in the eigen-beamforming than in traditional MSE or MV. In this section, we will explore the possibilities offered by EIG.

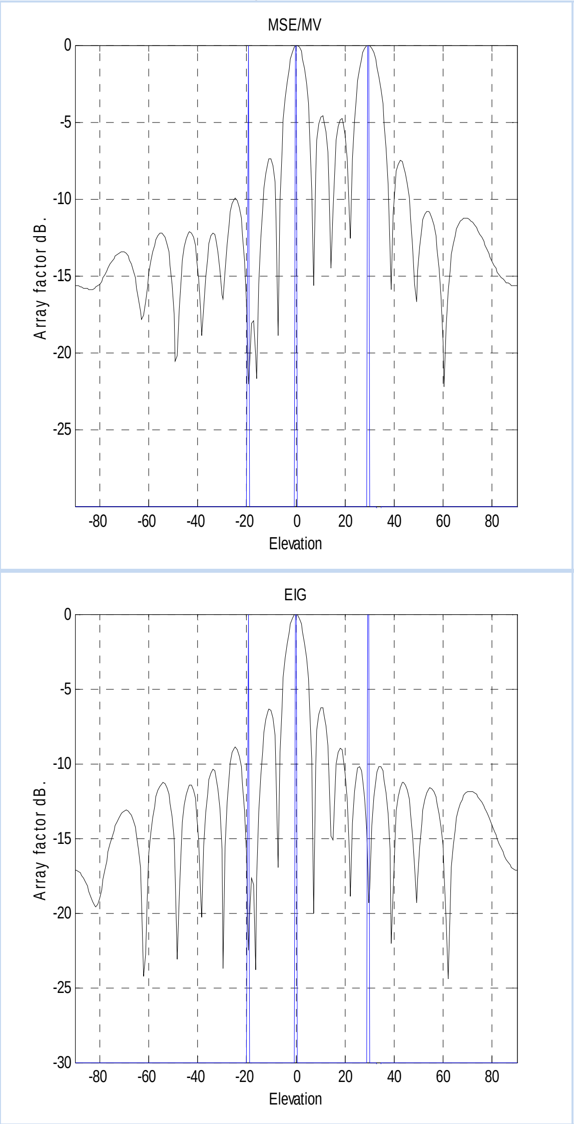

Figure 1. Array Factor Response of traditional MSE/MV (top) and new EIG (bottom).

Considering a point source with reference waveform d(n), usually embedded in the desired source signal for communications applications, and its steering vector $\underline{S}_d$, the major part of beamforming techniques can be grouped either as temporal reference systems TRB where d(n) is known or previously acquired or as spatial reference beamformers SRB where d(n) remains unknown but the steering vector or location of the desired is known. Regardless, TRB is the usual case in communications and SRB is associated with radar/sonar exploration systems, currently both in point to point communications as well as in location receivers (GPS/GNSS) the receiver knows both the steering vector of the desired as well as the reference signal d(n). Under such circumstances the covariance and the interference matrix are available to the beamformer design.

Under such knowledge, the presence of coherent sources, such as early arrival multipath, is not longer a problem for the eigen-beamforming. Just to provide an example, Figure 2.a shows the array factor response corresponding to a traditional MSE or MV beamformer for a scenario where the desired impinges the aperture from the broadside, in addition a full coherent arrival is present at 30º together with an uncorrelated interference at -20º.

The levels of the desired and interference are 10 and 0 dB respectively whereas the coherent arrival is also 10 dB. Vertical lines indicate the location of arrivals and interference. As it can be observed the MV/MSE beamformer captures both the desired and the coherent arrival promoting, due to the time varying character of the multipath severe fading on the array output.

On the other hand, using simultaneously the information desired covariance an interference, the EIGc beamformer response is shown in Figure 2b. Clearly the full coherent source is completely removed proving the robustness of the EIG beamforming to full coherent early arrivals.

Interestingly, removing early arrivals is mandatory for good reception this is not longer the case for late arrivals that contribute to enlarge the energy available for detection. This is the case for the so-called sequence detectors that in a ML basis work on communications scenarios with rich scattering. We will show that the framework behind EIG works perfectly for such receivers.

In a scenario where several replicas, at different and consecutive time samples, arrive to the receiver aperture, the received snapshot is given by (8), where matrix G is the propagation matrix from the desired, vector d_n contains the desired d(n) as well as L replicas until d(n-L+1), vector i_n is the uncorrelated interference and vector w_n the front end noise.

$$\underline{X}_n = \underline{\underline{G}}.\underline{d}_n + \underline{i}_n + \underline{w}_n \qquad (8)$$

The receiver for this case is depicted in Figure 2, where the role of the beamformer is, like in a forward equalizer to remove interference and the DIR which emulates the channel that the reference signal suffers from the transmitter to the beamformers output. When reference is available, it is introduced to the left part and both beamformer and DIR are designed jointly in order to minimize the power of the error signal e(n). It is worthwhile to mention that when L, i.e. the ISI length is unknown, vector do$_n$ may either contain less or more components than vector d$_n$ on the snapshot model. Practical receivers use to underestimate un purpose L mostly due to complexity of the sequence detector. Since this is the case, we will assume that vector do$_n$ contains only L-1 components.

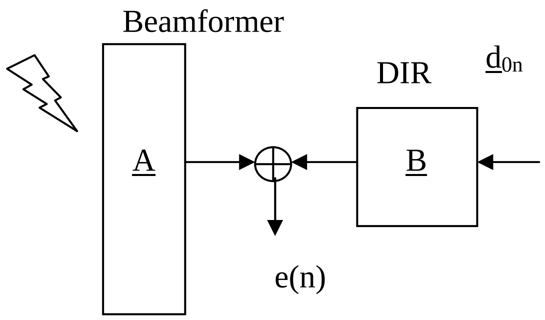

Figure 2. The so-called MDIR receiver showing the beamformer $\underline{A}$, the DIR $\underline{B}$ and the available reference signal vector $\underline{do_n}$.

From the reference signal, the following matrixes are available, where clearly matrix G appears decomposed in two parts G$_0$ that corresponds to the expected multipath and G$_R$ that is the unexpected multipath due to the underestimation of the ISI length L, this second matrix pass to be viewed in the design process as interference:

$$\begin{aligned}
&\underline{\underline{G}}_0 = E\left(\underline{X}_n \underline{d}_n^H\right)\\
&\underline{\underline{R}} = E\left(\underline{X}_n \underline{X}_n^H\right) = \underline{\underline{G}}_0 . \underline{\underline{G}}_0^H + \underline{\underline{R}}_I + \sigma^2 \underline{\underline{I}}\\
&\underline{\underline{R}}_D = \underline{\underline{G}}_0 . \underline{\underline{G}}_0^H \qquad (9)\\
&\underline{\underline{R}}_I = E\left(\underline{i}_n \underline{i}_n^H\right) + \underline{\underline{G}}_R . \underline{\underline{G}}_R^H\\
&\underline{\underline{G}} = \left[\underline{\underline{G}}_0 \underline{\underline{G}}_R\right]
\end{aligned}$$

The design problem can be formulated as (10). Note that the AGC has been set as AGCc defined before.

$$\begin{aligned}
&E\left(\left|\underline{A}^H \underline{X}_n - \underline{B}^H \underline{d}_n\right|^2\right)\Big|_{MIN} \qquad (10)\\
&s.t. \quad \underline{A}^H\left(\underline{\underline{G}}_0 . \underline{\underline{G}}_0^H + \sigma^2 \underline{\underline{I}}\right)\underline{A} = \phi_0
\end{aligned}$$

The solution for the DIR, matched to the channel suffered by the reference (propagation channel G$_0$ plus beamformer) and the beamformer are shown in (11). The snapshots covariance and G$_0$ can be estimated from the reference waveform. Note that this formulation extends largely the direct MDIR receiver [4].

$$\begin{aligned}
&\underline{B} = \underline{\underline{G}}_0^H . \underline{A}\\
&\left(\underline{\underline{G}}_0 \underline{\underline{G}}_0^H + \sigma^2 \underline{\underline{I}}\right)\underline{A} = \lambda_{MAX}\left(\underline{\underline{R}} - \underline{\underline{G}}_0 \underline{\underline{G}}_0^H\right)\underline{A} \qquad (11)
\end{aligned}$$

Note that, with different matrixes, the second equation, i.e. the beamformer design is exactly the previously defined EIGc beamformer. In other words, the new beamformer is the optimum design for such receivers.

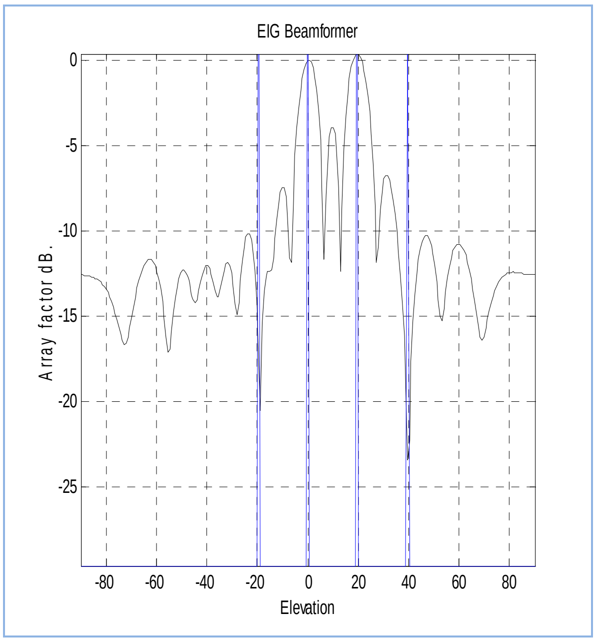

Figure 3. EIG Beamformer response to the scenario formed by the desired (QPSK modulated) d(n) (10dB.,0º), d(n-1) (10dB,20º) and d(n-2) (10dB.,40º). DIR of the receiver set with length equal to 2 taps. Un-coherent interference at -19º and 10 dB.

Figure 3 shows the array factor of the resulting EIG design. The scenario consist on a desired QPSK modulated sampled at the symbol rate composed of 3 paths (L=3), where d(n) , d(n-1) and d(n-2). The DOAs of the arrivals are 0, 20 and 40º respectively. The DIR assumes only two paths, in consequence d(n-2) will be considered late arrival and will cause interference if not properly removed or attenuated by the beamformer. The figure shows clearly that the beamformer performs properly its assigned job. Note that late arrivals use to be unstable, in consequence they do not use to be included in the DIR passing, just in case spatial processing cannot remove them, to the residual ISI term. As an example, GSM receivers set the length of the DIR to a maximum of 5 consecutive arrivals.

4 EIG at the Transmitter: BC Scenario

It is well known that the optimum physical layer processing to achieve capacity in an broadcast scenario, consist in successive pre-cancellation of streams with MSE criteria [11]. Without loss of generality, the case of a transmitter with two antennas broadcasting two streams for the respective users will be used along the rest of this section. Users are considered having single antenna receivers. For this case, it is assumed that matrix H reflects the channel matrix and matrix Q the precoder containing the beamformers for each user as columns. The resulting channel is shown in (12) as matrix R.

$$\underline{\underline{H}}\,\underline{\underline{B}} = \begin{bmatrix} \underline{h}_1^H \\ \underline{h}_2^H \end{bmatrix} \begin{bmatrix} \underline{q}_1 & \underline{q}_2 \end{bmatrix} = \begin{bmatrix} r11 & r12 \\ r21 & r22 \end{bmatrix} \qquad (12)$$

Thus, the signal to noise SNR ratio which every receiver experiences is given in (13) together with the corresponding achievable rate. The SNRs are set in terms of the beamformer and the channel to every user, as well as, the energy the transmitter assigns to every stream.

$$snr1 = \frac{r11}{1+r12} = \frac{\left|\underline{q}_1^H\,\underline{h}_1\right|^2 E_1}{1+\left|\underline{q}_2^H\,\underline{h}_1\right|^2 E_2}$$

$$r1 = \log 2\left(1+snr1\right) \qquad (13)$$

$$snr2 = r22 = \left|\underline{q}_2^H\,\underline{h}_2\right|^2 E_2$$

$$r2 = \log 2\left(1+snr2\right)$$

It is assumed that a TH (Tomlinson_Harashima) precoder pre-cancels the r21 term [7][9].

The goal of the precoder is to maximize the sum-rate of this scenario, in other words, to set the proper selection for the beamformers such that the sum of rate r1 plus rate 2 is maxim for a specific choice of power E_1 and E_2 with global power constraint E_T. In fact, the capacity region for the BC (2x2) channel can be found in reference [10] and reproduced below:

$$C_{BC} = \log 2\left[\left(\frac{E_T}{2}\Delta^{0.5} + \frac{h_1^2+h_2^2}{2\Delta^{0.5}}\right)^2 - \frac{h_{12}^2}{4\Delta}\right]$$

$$where \quad \underline{\underline{H}}\,\underline{\underline{H}}^H = \begin{bmatrix} h_1^2 & h_{12} \\ h_{12}^* & h_2^2 \end{bmatrix} \qquad (14)$$

$$and \quad \Delta = h_1^2 h_2^2 - h_{12}^2$$

Note that the first term, in this case, coincides with the point to point capacity of a MIMO(2,2) channel.

The optimum choice for the beamformers is given in (15) for the case where user 1 experiences better channel than user 2. Note that both cases use the quiescent response for user 1 and differ on the beamformer for user 2. The EIG version used is the so-called AGC complete since for rank one channels the AGC reduced will coincide with an MSE design.

An important remark, concerning the extension to the case of the interference cannel is that the EIG design is autonomous in the sense that it depends only on the energy of his stream. Note that this not longer the case for the MSE.

The following figures show the achievable rates for the optimum design MSE (top) and the EIG design (bottom) for a BC channel with global power constrain of 6 dB.

$$\left|\underline{h}_1\right|^2 \geq \left|\underline{h}_2\right|^2$$

$$EIG\_BC$$

$$\underline{q}_1 = \frac{\underline{h}_1}{\left|\underline{h}_1\right|}$$

$$\left(\underline{\underline{I}} + E_2\,\underline{h}_2\,\underline{h}_2^H\right)\underline{q}_2 = \lambda_{\max}\left(\underline{\underline{I}} + E_2\,\underline{h}_1\,\underline{h}_1^H\right)\underline{q}_2$$

$$\underline{q}_{2EIG} = \frac{\underline{q}_2}{\left|q\right|_2}$$

$$MSE \quad BC$$

$$\alpha = \left(\frac{\Delta E_1 + h_2^2}{1+E_1 h_1^2}\right) \quad \varepsilon^2 = \frac{h_{12}^2}{\left(\Delta E_1 + h_2^2\right)\left(1+E_1 h_1^2\right)}$$

$$\underline{q}_1 = \frac{\underline{h}_1}{\left|\underline{h}_1\right|}\left(1+\varepsilon^2 E_2\right)$$

$$\underline{q}_2 = \left(\underline{\underline{I}} - \left(\frac{E_1}{1+E_1.\left|\underline{h}_1\right|^2}\right)\underline{h}_1\,\underline{h}_1^H\right)\frac{\underline{h}_2}{\alpha} \qquad (15)$$

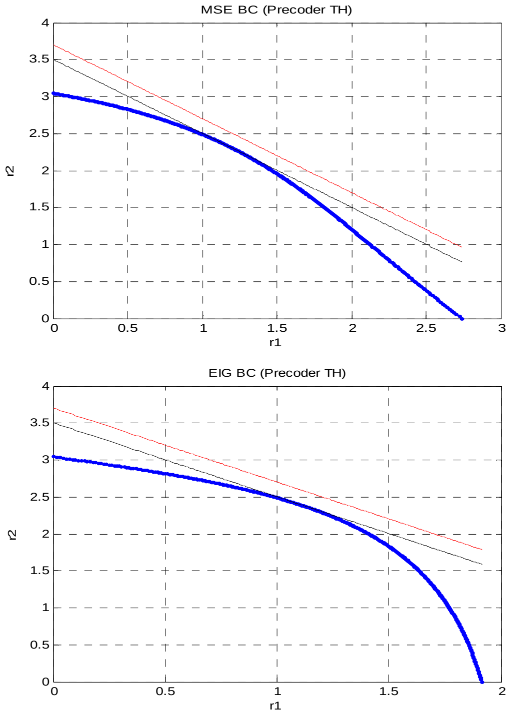

Figure 4. Achievable rates for MSE (Top) and EIG (Bottom). Straight lines represent point to point MIMO(2,2) and BC(2,2) maximum sum-rates.

As it can be seen, EIG also becomes close (tangential) to the maximum sum-rate line for BC as it does the MSE. Nevertheless it should be noted that the region is greater for EIG than the resulting region of achievable rates for MSE. In addition, it is worthwhile to note that the EIG plot stays closer to the maximum rate along more choices of rates for each user. This last feature implies an extra robustness in favor of EIG with respect mismatches on the optimum power allocation.

All the previous comments can be considered advantages of EIG beamforming versus traditional MSE, i.e. autonomous design and power allocation robustness. The only disadvantage has to be found in the fact that the MSE achieves capacity. Nevertheless, the loss that EIG suffers on its maximum rate versus MSE is small enough to justify its use taking in mind the two advantages mentioned. To show up to what degree EIG is close to the optimum performance, Figure 5 shows the difference on the maximum achievable sum rate between MSE and EIG for the scenario mention with 6 dB global power constrain (i.e. total power with respect front end noise power at the receivers.

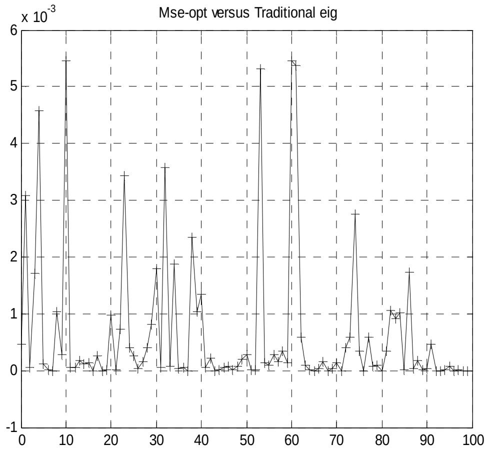

Figure 5. Difference on the maximum achievable sum-rate of MSE versus EIG. Each point of the graphic results from the average performance over 100 Rayleigh channel realizations

Note that the differences stay below 0.006 for sum-rates in the range of 3 to 4.

5 Conclusions

It has been shown that the new beamformer design is able to manage full coherent arrivals to an aperture both in order to remove selective fading at the beamformer output, as well as to properly match the expectation of a sequence detector in terms of accepting early arrivals and properly remove late arrivals which produce residual ISI degrading the performance of the sequence detector. The extension of the versatility and performance of the new beamformer to broadcast scenarios BC as well as to the so-called interference channel IC is already done with promising results that require more formal validation than the mere beamforming factor.

Finally, the performance of the EIG beamforming is shown in a BC multiuser scenario with implementation advantages over optimum MSE-

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