Peripheral twists for torus topologies with arbitrary aspect ratio

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Abstract—**A torus is a common topology used in supercomputer networks. Asymmetric Tori suffer from resource usage imbalance, which translates to reduced performance. Twisted Tori employ a twist in the peripheral links of one or more dimensions to improve the topological parameters and overall performance of asymmetric networks. 2D and 3D twisted tori with aspect ratios 2:1 and 2:1:1 have been studied in detail.**

However, commercial machines do not necessarily employ those aspects ratios. In this work we present an early study of the effect of peripheral link twisting in multidimensional twisted tori with arbitrary aspect ratios. We observe that, in the general case, it is impossible to find a specific twist that minimizes all the interesting topological parameters of the network. We also introduce a requirement for the use of several twists in multidimensional torus with adaptive routing.

Keywords—**Twisted torus, network topology**

I. INTRODUCTION

An N-dimensional torus is the Cartesian product of N rings. The torus topology has been widely employed for the interconnection of large-scale supercomputers, since it provides competitive topological properties, it fits naturally to the task mapping of many supercomputing problems and is simple to understand from the programmer's view. Symmetric tori are built from rings of the same length, what under uniform traffic leads to a balanced use of the network resources. A restriction of symmetric tori is that the number of nodes must be a certain power, D^N, where D is the number of nodes in the ring.

Asymmetric (or mixed-radix) tori are those generated from the product of N rings with different lengths $D_1, D_2, \ldots, D_N$. This builds a torus topology with variable number of network nodes, $D_1 \times D_2 \times \ldots \times D_N$, and thus has been commonly used in commercial machines such as the IBM BlueGene [1], the Cray XK6 [5] or the Tofu interconnect in the K computer [2], currently the Top1. There are several reasons to use asymmetric torus, ranging from desiring a given number of network nodes, increasing the size of an existing machine or even mechanical limitations such as in the Cray T3D network [6]. However, asymmetric tori suffer from congestion in the longest dimension, what can cause performance bottlenecks in the network. Under random uniform traffic, the average number of hops traversed on each dimension of the torus is proportional to its length, but the number of links per dimension on each router is constant. Therefore, the links on the longest dimension are the first ones to reach saturation, limiting the performance of the overall network. Other types of traffic are also limited by the difference between the different lengths.

The use of a twist in the peripheral links of one of the dimensions was first proposed in [7] as a mechanism to improve the topological properties and the performance of the network. Subsequent work [3], [4] has formally characterized such topology, called Twisted Torus (TT), in the specific cases of $2{:}1$ (2D) or $2{:}1{:}1$ (3D) aspect ratios. Specifically, $2{:}1$ Rectangular Tori (RT) have twice as many nodes in the long dimension than in the short one; therefore, under uniform traffic, the links in the long dimension are saturated, but the utilization of the links in the shortest dimension is limited to 50% [4]. In this case, the *optimal twist* is the length of the short dimension: Adding a twist of a columns to the peripheral vertical links of a $2a \times a$ RT regains the symmetry of the X and Y dimensions, and allows for full link utilization. The resulting layout of these topologies is presented in Figure 1 for a 8×4 torus. The twisted vertical links in Figure 2 modify the node distance distribution and resulting link utilization, leading to throughput increases of 50% under uniform traffic. Other traffic patterns are also improved with different factors [4].

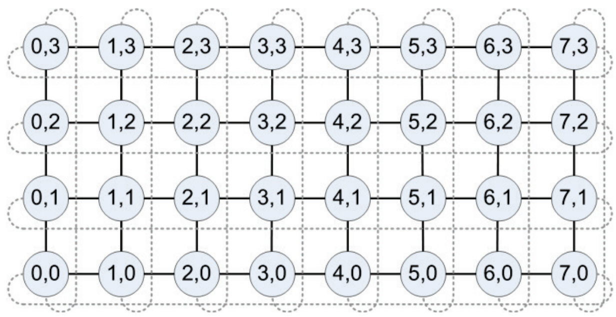

Fig. 1. 8×4 Rectangular Torus.

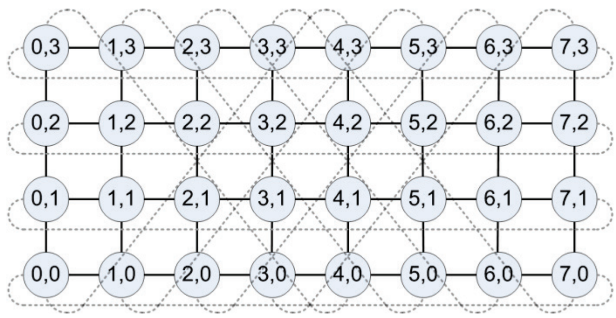

Fig. 2. 8×4 Rectangular Twisted Torus, with a twist of 4 columns.

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Recent technological improvements have largely increased the router pin bandwidth, allowing the construction of high-degree routers [5]. In the case of using torus topologies, this leads to the construction of multi-dimensional torus. Nowadays, machines using 5D or 6D torus are already being deployed, such as the newest BlueGene/Q [9] or the Tofu 6D torus [2]. Any asymmetry in the dimensions of these topologies is even more important, since under uniform traffic it is the single longest dimension which limits performance.

The optimal application of twists to the peripheral links of asymmetric torus of arbitrary aspect ratio has not been studied yet. In this paper we present an early study of the topological properties of asymmetric twisted torus based on exhaustive search of the optimal twist values. Specifically, this paper has two main contributions:

- We perform an exhaustive search of the optimal twist values and observe that there is no single twist that optimizes all the relevant topological parameters of asymmetric tori with arbitrary aspect ratio, such as diameter, average distance or link imbalance.
- We show how certain combinations of twists in a multidimensional twisted tori lead to topologies which are not node-symmetric, and thus do not allow for adaptive routing.

The rest of the paper is organized as follows. In Section II we introduce the parameters and notation that will be used in the rest of the paper. Section III studies the twist values that optimize different network parameters of 2D TT, showing that for arbitrary aspect ratios there is no optimal twist for all parameters. Section IV deals with multiple twists in multidimensional TT, showing that not any combination of twists leads to node-symmetric networks. Finally, Section V concludes the paper and presents the ongoing work.

II. NOTATION AND NETWORK PARAMETERS

In this paper, we consider *N*-dimensional torus. Typical values for N are $N = 2$ for the Rectangular Torus (RT) or $N = 3$ for the Prismatic Torus (PT), following the terminology introduced in [4]. Higher values of N lead to Hypertorus (HT), or, in general, torus with N dimensions.

The different dimensions are typically labeled using the letters X, Y and Z. The number of nodes on each dimension will be denoted as d_X, d_Y, d_Z. With hypertorus the dimensions are typically labeled $D_1, D_2 \ldots, D_N$ and the size of each dimension $d_1, d_2, \ldots, d_N$. The aspect ratio represents the relation between the number of nodes on the different dimensions.

Node labeling — Each node in the network will be labeled with a tuple (x, y, z) or $(x_1, x_2, \ldots, x_N)$, with each element indicating the coordinate in the corresponding dimension in the range $[0, d_i)$. An example of this notation in a 2D torus is shown in Figures 1 and 2, with $d_X = 8$ and $d_Y = 4$.

Peripheral links — In a traditional torus, a link in dimension J joins nodes with coordinates $(x_1, \ldots, x_j, \ldots, x_N)$ and $(x_1, \ldots, (x_j \pm 1)\ mod\ d_J, \ldots, x_N)$. It can be observed that the modulo operation is only used for peripheral links, and that peripheral links always join nodes from the same row or column.

Peripheral twists — A twisted peripheral link breaks the previous rule. We will use the expression t_{JK} to refer to the twist of dimension J over the dimension K with $J \neq K$. A nonzero value in t_{JK} means that the peripheral link on dimension J also modifies the coordinate in dimension K. The modification is called the value of the twist, or skew. Specifically, considering only the dimensions J and K, the node with coordinates $(d_J - 1, x_K)$ will be connected with $(0, (x_K + t_{JK}) mod K)$ along the peripheral link on dimension J. The twist t_{JK} does not modify other coordinates.

The example in Figure 2 shows a 2D Twisted Torus with $d_X = 8, d_Y = 4, t_{XY} = 0$ (no twist on the horizontal peripheral links) and $t_{YX} = 4$. Observe how the node (0, 3) is connected to (4, 0), while node (0,0) is connected to (4,3). In an N-dimensional torus the number of possible twists is $N(N-1)$, regardless their value. For 2D, these are only t_{XY} and t_{YX}. For 3D, these are $t_{XY}, t_{XZ}, t_{YX}, t_{YZ}, t_{ZX}$ and t_{ZY}. In general, we will only cite those twists that are nonzero.

Considering this notation, the previous work in [4] studied three different twisted topologies:

- Rectangular Twisted Tori (RTT): A 2D twisted torus with $d_X = 2a, d_Y = a$ and $t_{YX} = a$.
- Prismatic Twisted Tori (PTT): A 3D twisted tori with $d_X = 2a, d_Y = d_Z = a$ and $t_{YX} = a$.
- Prismatic Doubly Twisted Tori (PDTT): A 3D twisted tori with $d_X = 2a, d_Y = d_Z = a$, $t_{YX} = a$ and $t_{ZX} = a$.

The interest of the peripheral twists relies on the fact that they allow modifying the topological parameters of the interconnection network, and thus its performance, without altering the internal mesh interconnection pattern. In order to quantify the performance improvement or penalty derived from the introduction of a certain twist, we need to measure its effect.

The key performance indicator of any network is the execution time of a set of parallel applications appropriately tuned. However, such execution time depends on many factors, such as the data partitioning and task mapping mechanisms employed, which should also depend on the specific topology being used. Such study is out of the scope of the current paper. In order to perform an early evaluation, synthetic random uniform traffic is typically used, which reflects on average the topological parameters of the network. The most interesting topological parameters will be:

Diameter — denoted k, it is the length of the longest minimum path between any two nodes in the network. The diameter of the network conditions the maximum latency in the network. Then, it can also affect the latency of certain operations, such as collective operations implemented using broadcast trees.

The diameter in a traditional torus without twists is the sum of the diameters of the individual rings, $k = k_X + k_y + k_Z + \cdots$. However, when a twist is applied on any of the dimensions the diameter must be recalculated from the resulting distance distribution.

Average distance — denoted $\bar{k}$, it is the average length of all minimum paths. The average distance is an indicator of the base network latency, this is, the latency without congestion in the network.

The average distance will be an indicator of the maximum throughput in the network. The lower the number of hops a packet has to travel, the higher the number of packets accepted.

In a torus topology there are different classes of links, separated according to their dimension. The average distance $\bar{k}$ can be divided into the individual average distances per dimension, $\bar{k} = \bar{k}_X + \bar{k}_Y + \bar{k}_Z + \cdots$. Each of these individual distances represents the average number of hops that a packet has to traverse along the links in a given dimension. Since there are the same number of links on each dimension, the highest average distance per dimension will indicate the dimension that will first suffer from saturation and will limit performance.

In a perfectly balanced network all the individual per-dimension distances are the same. However, asymmetries in the network dimensions lead to different average distances per dimension. The application of a twist on the peripheral links of a torus modifies the average distances per dimension; the selection of the appropriate twist to minimize these distances is studied in this paper.

Based on the average distances per dimension, we define two additional metrics:

Maximum Average distance per dimension — denoted $\max(\bar{k}_\iota)$. This value is the maximum of the individual distances $\bar{k}_X, \bar{k}_Y, \bar{k}_Z, \ldots$. As argued before, this parameter will determine the saturation limit of the network; therefore, the expected throughput depends on this value as discussed in [4].

Imbalance — The imbalance is defined here as the quotient $I = \frac{N \times \max(\bar{k}_\iota)}{\bar{k}}$. Ideally, $I = 1$ meaning that all links are equally used. A high imbalance value means that there is a significant deviation in the usage of the network dimensions.

The application of a certain twist will modify the distance distribution in the network, and all the previous parameters with it. The next section presents a search for the optimum twists in terms of the different topological parameters introduced in this section.

III. Optimum Twists for 2D Torus with Arbitrary Aspect Ratio

In this section we focus on 2D Twisted Torus (TT) with $d_X \geq d_Y$ (with more nodes on the X dimension than in the Y dimension), and a single twist of dimension Y over dimension X. The work in [4] proves that the optimal twist in terms of the previous parameters for a $2a \times a$ Twisted Torus is $t_{YX} = a$. This twist equals the middle of the longest dimension, connecting the first column with the middle one, thereby reducing the distances in the network. Even more, the twist equals the length of the shorter dimension, what might be beneficial for the resource balancing. This condition only occurs when the aspect ratio is $2:1$.

In this section we study for different aspect ratios how each of these network parameters varies with the selected twist. We will focus on the 2D TT. Our initial tests showed that the results are similar for different network sizes when the aspect ratio is similar. Then, we will fix the number of rows in the topology to a constant reference value (for example $d_Y = 12$ in our experiments), and vary the number of columns d_X from d_Y to $4 \cdot d_Y$, to sweep an aspect ratio ranging from $1:1$ to $4:1$.

For each configuration, we explore the topological parameters of the network with all the possible twists from $t_{YX} = 0$ to $t_{YX} = d_X/2$ (higher values lead to symmetric results). With each twist, we use a breadth-first search algorithm to find the shortest paths between node 0 and any other node in the TT. For each of these shortest paths, we record the number of hops per dimension, and when the same destination can be reached by different routes, we average the result among them all. With all the routes in the network we calculate the diameter, average distance, maximum average distance per dimension and network imbalance for each possible twist t_{YX}. Finally, we calculated the twist that optimized each of the four parameters of interest.

The next plots show the results for each parameter. On each graph we plot the results corresponding to four twisting strategies:

- Not using a twist, $t_{YX} = 0$, labeled *no_tw*.
- The optimum twist for that given parameter, *opt_tw*.
- The twist equals half of the longest dimension, $t_{YX} = d_X/2$, labeled *tw_mid*.
- The twist equals the shortest dimension, $t_{YX} = d_Y$, labeled *tw_rows*.

Figure 3 shows the results for the diameter. The lines for *no_tw* and *opt_tw* are shown in gray. We observe that there is a significant improvement in the diameter when the optimum twist is applied, especially as the aspect ratio grows. It can be also observed that the twist that provides the optimum diameter is the one with $t_{YX} =$

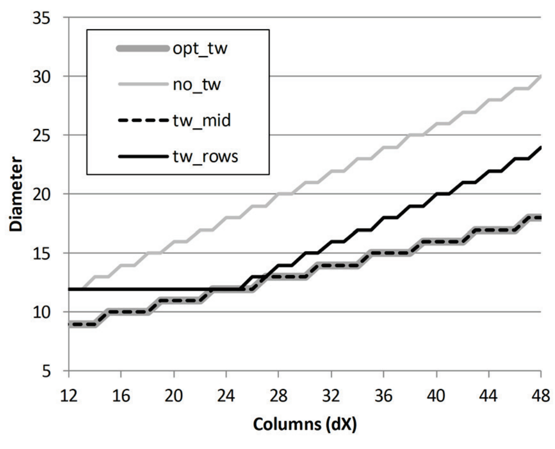

Fig. 3. Diameter of a network with different twisting strategies, as the aspect ratio increases from 1:1 to 4:1. The number of rows is always 12.

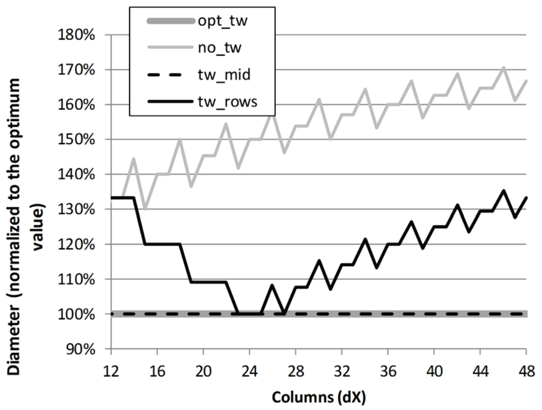

Fig. 4. Diameter of a network with different twisting strategies, as the aspect ratio increases from 1:1 to 4:1, normalized to the optimum value on each case. The number of rows is always 12.

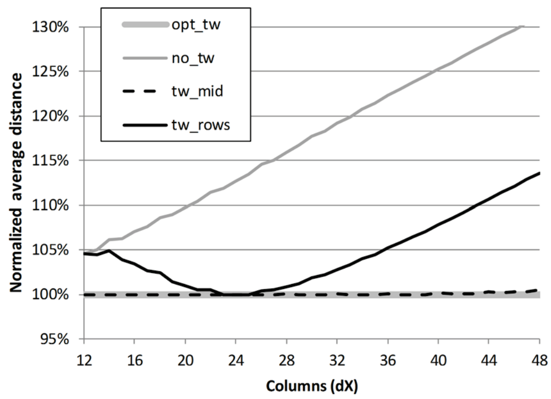

Fig. 5. Average distance with different twisting strategies, as the aspect ratio increases from 1:1 to 4:1, normalized to the optimum value on each case. The number of rows is always 12.

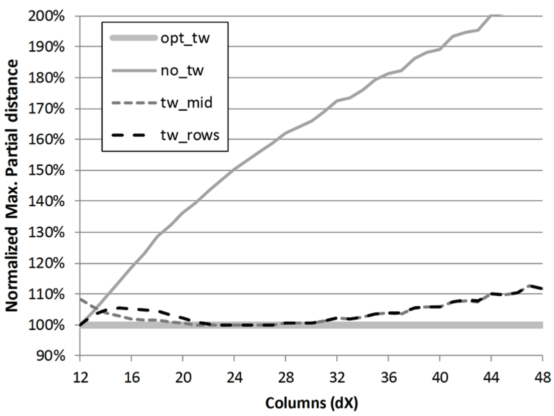

Fig. 6. Maximum partial distance with different twisting strategies, as the aspect ratio increases from 1:1 to 4:1, normalized to the optimum value on each case. The number of rows is always 12

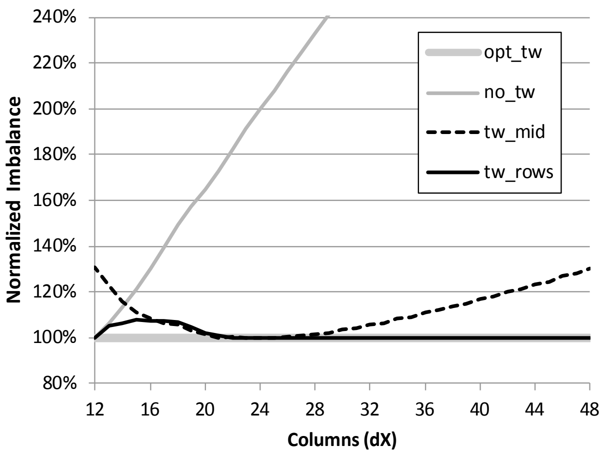

Fig. 7. Imbalance of a network with different twisting strategies, as the aspect ratio increases from 1:1 to 4:1, normalized to the optimum value on each case. The number of rows is always 12.

$d_X/2$, labeled *tw_mid*. Finally, we can observe how with $d_X = 24$ columns, the three values meet: this is precisely the $2{:}\,1$ aspect ratio network studied in [4].

How much diameter reduction can we expect from an optimum twist? Figure 4 shows the previous plots, normalized to the value obtained in each case with the optimum twist. It is clear that a twist equal to half the longest dimension always provides the optimum value in terms of diameter, and that the diameter can be reduced in more than 40%.

Figures 5 to 7 show the results of the other three parameters of interest, in all cases normalized to the optimum value per network size.

Figure 5 shows the results for the average distance. A similar study was already presented in [3]. As with the diameter, the twist to the middle of the longest dimension provides the best performance, except for the case of a very high aspect ratio. The improvement over the RT increases from 5% to 40% as the aspect ratio grows. In the case of the twist *tw_rows*, the performance remains within 5% of the optimal value for aspect ratios lower than $3{:}\,1$, increasing to up to 14% for larger aspect ratios.

The maximum partial distance per dimension is presented in Figure 6. It is very relevant the high difference between the original, untwisted torus, and the best result that can be obtained. Depending on the aspect ratio, the original torus is more than an 80% slower in terms of throughput (which depends on this parameter). A proper twist reduces the maximum distance per dimension and increases throughput. With a twist of *tw_rows* or *tw_mid*, this metric is within 10% of the optimal twist, and within 5% of the optimal twist for aspect ratios lower than $3{:}\,1$.

Finally, Figure 7 shows the imbalance. If no twist is used, the imbalance grows linearly with the aspect ratio as expected. When the proper twist is applied, the usage of the links on each dimension can be almost perfectly balanced. Also, it is interesting to notice that from aspect ratio $2{:}\,1$, the *tw_rows* approach obtains the optimum result.

All the previous figures have presented the results of the different parameters; however, the specific twist that obtains the best result in each case was not presented. Figure 8 shows the twists that get the best results for each of these 4 parameters.

From this figure we can observe that, in general, for a given aspect ratio there is no single twist value that optimizes all the network parameters. For example, diameter and average distance are approximately optimum with *tw_mid.* However, for large aspect ratios the optimum twist to reduce the average distance

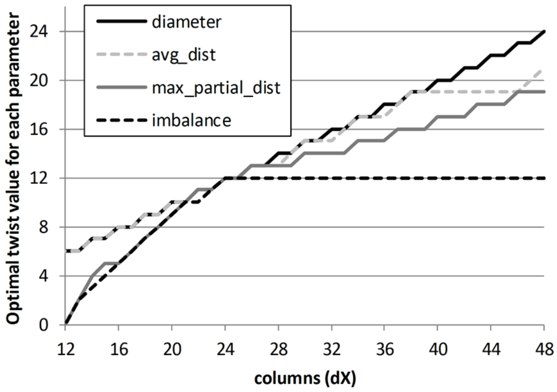

Fig. 8. Optimal twist value for each parameter with different aspect rations. The number of rows is always 12.

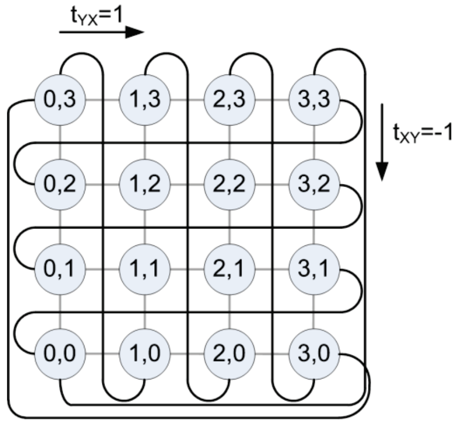

Fig. 9. 4×4 Twisted Torus with $t_{XY} = -1$ and $t_{YX} = 1$.

decreases. Regarding the maximum partial distance per dimension (which conditions the performance under saturation) we observe that the optimum skew is similar to *tw_mid.* up to aspect ratios close to $2{:}1$, but it later decreases. Finally, it is really interesting that the best imbalance in a network with aspect ratio higher than $2{:}1$ is provided by the *tw_rows* approach, regardless the number of columns.

The interesting conclusion to be drawn from this figure is that for arbitrary topologies there is no single twist value that minimizes all the interesting performance related metrics. For example, this implies that, with a large aspect ratio such as 4:1, there is a twist that provides a higher throughput under saturation (maximum partial distance lower) although the traffic is not as balanced as possible (best imbalance); by contrast, the balanced approach makes the same usage of both horizontal and vertical channels, but since the overall average distance in that case is higher, the performance is lower.

IV. USE OF MULTIPLE TWISTS

Section III studies the selection of an optimum twist in a 2D Twisted Torus (TT) depending on different topological parameters of interest. However, the study in that section assumes a 2D TT with $d_X \geq d_Y$ and a single twist t_{YX}. The first assumption can be done without any loss of generality; the selection of the twist (dimension Y over dimension X) is the one that reduces the average distance on the longest dimension X. A similar study using t_{XY} does not provide better results.

In this section we consider the case of applying multiple twists to the same torus network. We will initially study the case in 2D (similarly to [3]), and then consider higher dimensions.

The graph in Figure 9 represents a 4×4 TT with twists $t_{XY} = -1$ and $t_{YX} = 1$. Thus, all the peripheral links have some twist.

Although the topological parameters of this graph could be studied as in the previous section, the problem now is that the graph is not node-symmetric. The node-symmetry property in a graph of this kind implies that the routing between any pair of nodes *source* and *destination* can be computed from the difference of their coordinates, rather than considering the whole topology and the specific identity of the *source* and *destination* nodes. Specifically, without node-symmetry the routing function cannot be performed by means of a routing-record computed at the source node, and adaptive routing is not allowed.

We show this case with an example similar to the one in [3]. Consider the graph in Figure 9 and the routing from node $(0{,}2)$ to $(3{,}0)$. One possibility is to go up $(Y+)$ to the intermediate node $(0{,}3)$ and then left $(X-)$ to the destination node $(3{,}0)$ through the peripheral twisted link. However, if the sequence of jumps is followed in the opposite order, then the first jump goes left $(X-)$ to $(3{,}3)$ using a peripheral link, and then up $(Y+)$ to $(0{,}0)$ using another peripheral link. The final node differs depending on the order of the dimensions followed, because the number of peripheral twisted links varies and each peripheral link modifies the other coordinate. Therefore, routing in the graph of Figure 9 must be either table-based, or use source routing with full knowledge of the network topology, what restricts many of the benefits of the torus topology.

We consider now the case of multiple twists in multidimensional torus. Cámara et al present in [4] two $2a \times a \times a$ 3D twisted torus topologies using one twist (PTT, $t_{YX} = a$) or two twists (PDTT, $t_{YX} = a$ and $t_{ZX} = a$). Both cases are node-symmetric, so they do not suffer from the restrictions considered above for the 2D TT with two twists.

The obvious question now is, which combinations of twists break the node-symmetry of a torus topology? The following result characterizes it.

Theorem — A multidimensional twisted torus is not node-symmetric if there exists a dimension J such that $t_{IJ}, t_{JK} \neq 0$ for some other dimensions I and K.

Proof — The lack of node-symmetry can be proven by finding a source node from which a sequence of jumps leads to different destination nodes depending on the order of the jumps.

Assume the condition is true. For simplicity, we will assume $t_{IJ}, t_{JK} > 0$. First, we consider the case $I \neq K$. Specifically, we will only consider the node labels (i, j, k); the rest of coordinates, if they exist, should remain constant. The source node of the proof will be $(d_I - 1, d_J - t_{IJ} - 1{,}0)$ and the routing record (1,1,0).

If the jump on dimension I is taken first, a twisted peripheral link will be used to reach the intermediate node $(0, d_J - 1{,}0)$. The second jump on the dimension J

uses another twisted peripheral link, leading to the destination node $(0, 0, t_{JK})$.

By contrast, if the jump on dimension J was taken first, then an internal link goes to $(d_I - 1, d_J - t_{IJ}, 0)$, and the second jump on I goes through a twisted peripheral link to $(0,0,0)$.

Finally, the case of $I = K$ also leads to a graph which is not node-symmetric, as presented in the example of Figure 9 with $I = K = X$. As in the previous case, if we consider the source node $(d_I - 1, d_J - t_{IJ} - 1)$ and routing record (1,1), we will reach destination nodes $(t_{IJ}, 0)$ or (0,0) depending on the order of the jumps. ■

We conjecture that the condition in the previous theorem is sufficient and necessary, but do not have a formal proof yet.

With the previous theorem we can consider the number of combinations of twists that maintain the node-symmetry for an *N*-dimensional torus. The number of possible twists is $N(N-1)$ as discussed in Section II. However, with node-symmetry there will be at most $N-1$ concurrent twists; otherwise, some dimension will necessarily *have* a twist in its peripheral links and *receive* the effect of a twist in another dimension, what breaks the node-symmetry according to the theorem. We will consider the lower-grade cases next.

In the 2D case there are two possibilities, t_{XY} or t_{YX}, but not both twists simultaneously. We can assume without loss of generality that the topology is set so that all twists are applied over lower-order dimensions (any other combination will be isomorphic), so the only single possible twist to study will be t_{YX}.

In the 3D torus there are two combinations of two twists: (t_{YX}, t_{ZX}) and (t_{ZX}, t_{ZY}). The first one was applied for torus with $2:1:1$ aspect ratio in [4] to build the PDTT. We do not know of any use of the second combination in previous work. Also, there are three degenerated cases when one of these twists is 0: t_{YX}, t_{ZX} and t_{ZY}. Without forcing a dimension order in the twists, the number of combinations would be much higher.

For multidimensional torus, the combinations of valid twists grows very quickly. Any study to optimize the parameters of the network using twists should consider all these possibilities with all their possible values. Thus, an empirical study based on a breadth-first search appears very costly as the number of dimensions grow.

V. CONCLUSIONS AND FUTURE WORK

In this paper we have identified four topological parameters of torus topologies that condition different aspects of the network behavior. The use of twists in some of the peripheral links modifies these topological parameters and can improve the performance of the network, but in the general case, it is impossible to optimize all of the parameters at the same time because the required twists differ.

We have also introduced a clear notation for the twists and criteria to consider which combinations of twists build node-symmetric networks from multidimensional torus, which are expected to be more used in the near future.

We have many lines of ongoing work. A formal proof of the conjecture in Section IV is critical to validate the work. Also, the high number of combinations of possible twists (and their specific values) for multidimensional torus makes an empirical study like the one in this paper not feasible. It would be interesting to formalize the twisted torus topology and study its properties using graph theory.

Regarding the impact on performance, we have discussed how the applications should be aware of the underlying topology to optimize data partitioning and task mapping. Performing these tasks for a multidimensional torus with arbitrary twists is not trivial, and currently under study.

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