

NONLINEAR MODELLING OF REINFORCED CONCRETE STRUCTURES

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1 INTRODUCTION

Non-linear modelling of structures is a crucial tool in order to understand complex structural behaviour, improve new design and to face the problem of assuring specific performance levels under complex loading.

In this paper three main research lines are described which focus on different aspects of non-linear modelling of reinforced concrete structures and to improve knowledge of the material behaviour.

The first topic deals with the simulation of coupled nonlinear response of cross-sections of arbitrary geometry to fully 3D loading. Since frame element analysis is still the most recurrent simulation technique in civil engineering, it is of interest to overcome some of the intrinsic limitation of traditional fibre elements: tangential forces and other 3D effects. A sectional modelling capable of handling this problem is presented and its application to several situations is demonstrated.

On the other hand, it is well known that when load discontinuity exists or the structural geometry cannot be assimilated to a set of bars beam theories are not applicable and the structure, or part of it, needs to be considered as a D-Region (*disturbed region*). Modern design of these regions is based on the Strut-And-Tie method currently included in most concrete codes. However, in general it doesn't exist a unique Strut-And-Tie scheme for a given structure and loading and it may be difficult to find a plausible scheme for new cases. Moreover, the methodology lacks of an explicit approach for damage control under different load levels. This has motivated the second research line to be presented which deals with a methodology for automatic generation of Strut-And-Tie models, with the possibility of considering constructability requirements, and the corresponding assessment of the nonlinear response of such structures.

Finally, a brief description of some experimental work currently carried out with the goal of improving knowledge nonlinear concrete mechanics are given.

2 BEAM-COLUMN SECTIONAL MODEL FOR 3D LOADING

2.1 *Motivation*

Structural modelling by means of beam-column elements is the more extended analysis method used today even though computational cost of full solid models tends to be more accessible. The main reasons for this are that model construction is easier and quicker and also is the interpretation of results which can be directly used for ULS design. Moreover,

accuracy is very good, in linear and non-linear stages, for *B-regions* (*beam-regions*) when they are governed by normal stresses. Current state-of-the-art includes fibre sectional analysis; which enables versatility in geometry definition, stage construction, etc.; and force-based elements which assure virtually exact solutions independently of the mesh density. Some special applications of frame element modelling in nonlinear regime may be found in Mari and Bairan (2008).

Fibre cross-section models have, however, some drawbacks derived from their starting hypotheses. These may be summarised as follows:

- Uniaxial constitutive equations
- Limited confinement modelling
- Limited modelling of tangential forces (shear and torsion) under general loading and cross-section shapes

Correct handling of tangential forces is of particular interest for current state-of-knowledge since it reduces the possibilities of frame-element modelling when these type of forces are important. It should be emphasised that in most failures of modern designed structures under late earthquakes are related to some type of shear loading producing an unexpected structural behaviour.

Most cross-section models are based on a kinematical hypothesis which is considered valid for all load stages; for instance, plane-section hypothesis in Navier-Bernoulli elements, or Timoshenko constant shear hypothesis, also parabolic shear strain distribution as has been used. One important problematic of reinforced concrete when comes to shear modelling is the big differences in the material response under different load levels. Moreover, its cracked biaxial response is known to be anisotropic, hence ideas that are extrapolated from linear isotropic elasticity are likely to fail in some load stage. This is the case of using a fixed shear strain profiles which typically do not reproduce correct shear stress distributions and failure modes. As a result, although maximum load is sometimes captured in a reasonable manner, steel strains and cracks are likely to be incorrect. These parameters are of interest when comes to evaluation of structural damage or structural performance.

From the abovementioned, it comes that that there is no such thing as a fixed shear strain profile in cracked reinforced concrete since it depends of the material state and section geometry. Instead, correct shear distribution should be deduced from internal equilibrium along fibres, therefore it is state-dependent. This problem can be handled somehow easily handled for symmetric sections under in-plane loading where this equilibrium can be posed in a 1D fashion by discretizing the section in layers. This scheme is followed, for instance, in the well known software RESPONSE-2000 (Bentz, 2000) and in the Dual-Sectional-Analysis Method (Vecchio and Collins, 1986), both developed in the University of Toronto. However, when loading is biaxial, include torsion, or the section geometry is arbitrary equilibrium conditions to be solved among fibres is 3D and hyperstatic and the above scheme cannot be applied. See Bairan and Mari (2007b) for a detailed analysis of the different hypothesis considered for shear and torsion loading.

2.2 Model for Total Interaction Nonlinear Sectional Analysis (TINSA)

2.2.1 Basis

TINSA is a model for the nonlinear analysis of generic cross-sections loaded under six possible degrees of freedom (axial force, biaxial bending, biaxial shear and torsion). The main idea of the model is that starting from the Navier-Bernoulli plane-section hypothesis (PS) the cross-section response can be improved as much necessary in order to satisfy 3D

equilibrium along fibres. This is done by adding a warping-distortion displacement field (\mathbf{u}^w) to the Navier-Bernoulli displacements (\mathbf{u}^{ps}). Afterwards, it comes that displacements, strains and stresses can be decomposed in a plane-section (PS) component and a warping-distortion (W) component as follows:

$$\mathbf{u} = \mathbf{u}^{ps} + \mathbf{u}^w \quad (1)$$

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{ps} + \boldsymbol{\varepsilon}^w \quad (2)$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{ps} + \boldsymbol{\sigma}^w \quad (3)$$

After considering full 3D equilibrium conditions a particular integration can be done along the cross-section domain which drives to the functional shown in Eq. (4) this represents a weak-form of the 3D equilibrium among fibres. A detailed derivation of this process may be found in Bairan and Mari (2006a).

$$R(x) = \iint_A \delta \mathbf{u}^T \mathbf{E}_x^T \boldsymbol{\sigma}' dA - \iint_A \mathbf{L}_{yz} (\delta \mathbf{u})^T \boldsymbol{\sigma} dA = 0 \quad (4)$$

This methodology has been applied to reinforced concrete (Bairan and Mari, 2007a) and other materials including non-isotropic composites laminates (Bairan and Mari, 2006b).

2.3 Implementation

The method was originally implemented by means of a FE formulation in the cross-section domain, therefore combining a 2D FE problem in the cross-section together with a 1D problem along the beam length. In the sectional problem the plain-section strains are the input data (coming from the 1D beam problem) and the internal variables to be solved are the warping and distortion field (\mathbf{u}^w). Here, concrete solid material is represented with planar 2D elements, linear elements are used to simulate transversal reinforcements and point elements represent longitudinal reinforcements, see Fig. 1.

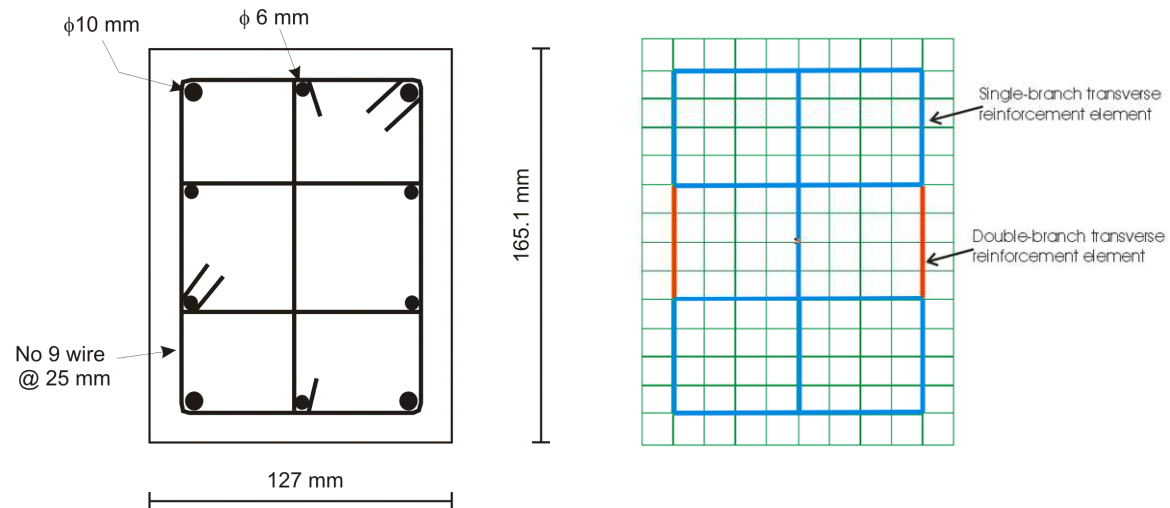


Figure 1 Description of the cross-section domain

Recently, the formulation has also been implemented by means of generalized coordinates method, where the unknown variables are approximated with a summation of predefined shape functions along the complete section as shown in Fig. 2.

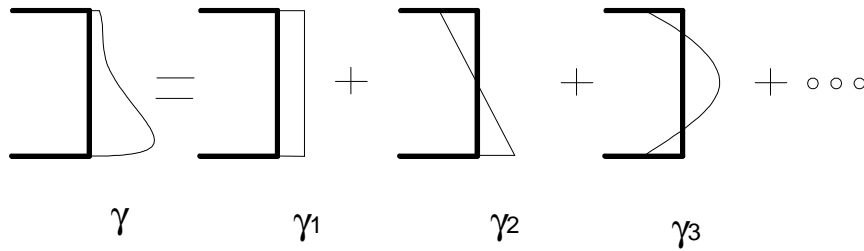


Figure 2 Generalized coordinates applied to cross-section analysis

2.4 Applications

2.4.1 Shear loading

The method has been applied to simulate shear loading until failure of a set of high-strength beams, test reported in Cladera and Mari (2005), with different reinforcement arrangements including beams without stirrups, with stirrups and with longitudinal reinforcement in the web as described in Fig. 3. Shear force-strain response is presented in Fig. 4 for the three referenced cases. In Fig. 5 local strains in longitudinal and transverse reinforcement are compared against those experimentally measured. It is worth noticing that the model agreement is satisfactory both for the overall response and for local measures of strains for different load conditions and failure modes.

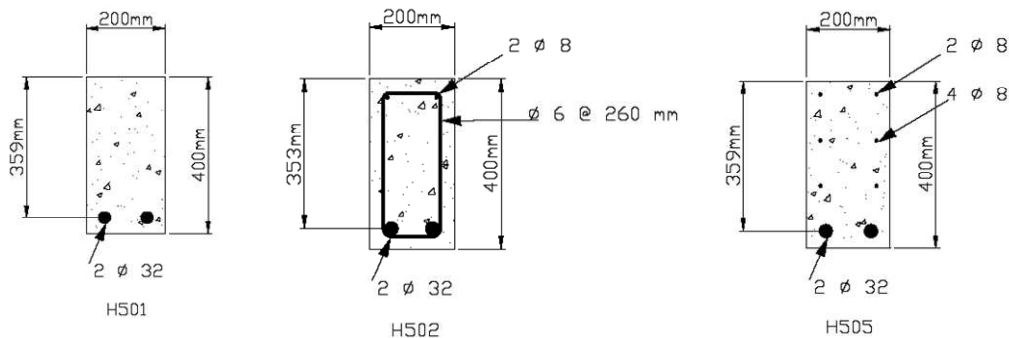


Figure 3 Specimens with different shear reinforcement arrangements

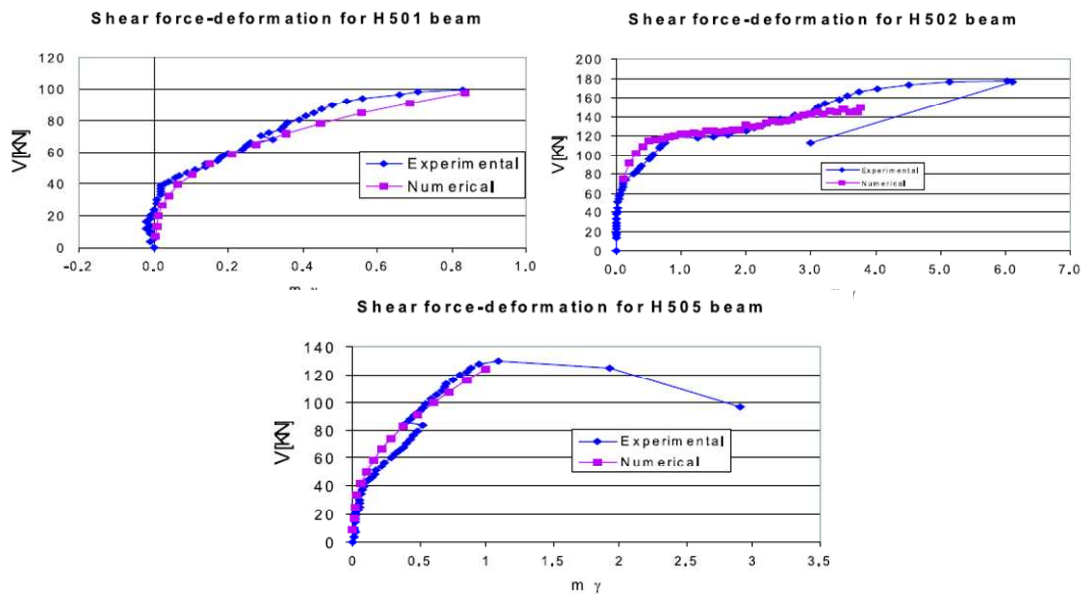


Figure 4 Shear force-strain response for different specimens

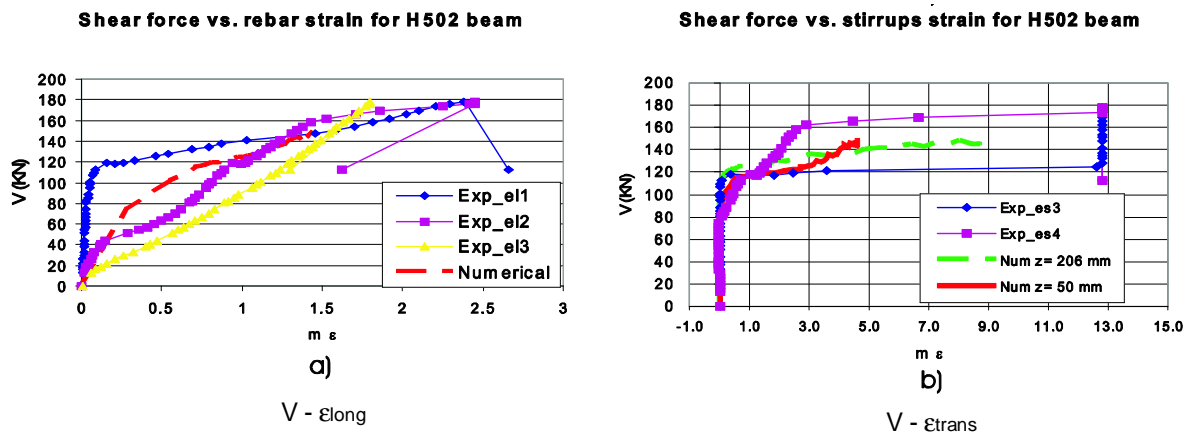


Figure 5 Shear force against strains in longitudinal reinforcement (a) and stirrups (b) for specimen H502

2.4.2 Concomitant moment-curvature and shear force-strain diagram

One interesting application of the proposed model is to obtain moment-curvature diagrams for concomitant tangential forces and vice versa. The former is shown in Fig. 6 for different shear span (M/V) ratios where effects of shear force in the curvature ductility, cracked stiffness, yielding initiation and post-yielding stiffness are noticeable.

Fig. 7 shows moment-longitudinal reinforcement strain and shear force-stirrup strain diagrams for different M/V ratios. It is evident that the presence of high shear forces introduces additional stresses in the longitudinal reinforcement, noticed all over the load history and producing yielding for a lower value of the bending-moment. This effect is known in the structural mechanics as shear-bending interaction. However, usually the effect of bending moment on the shear force is not so well known although it should be expected from reciprocity theorems. This effect is noticed in Fig. 7 (b) where high concomitant bending moments introduce additional stresses in stirrups.

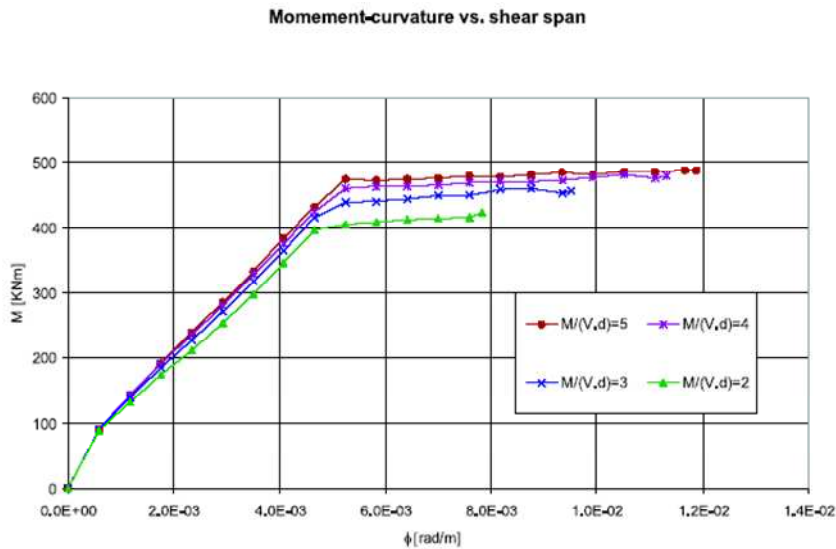


Figure 6 Moment-curvature for different M/V ratios

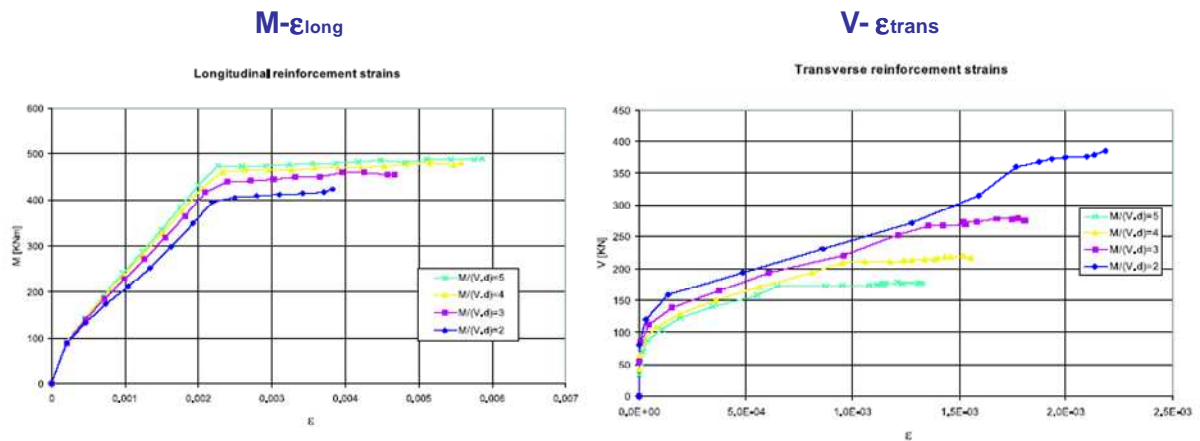


Figure 7 Moment-rebar strain (a) and Shear-stirrup strain (b) for different M/V ratios

2.4.3 Torsion loading

Applicability of the formulation to torsion loading is shown by reproducing the tests of Onsongo (1978) on combined torsion and bending. Specimens of these tests were designed so TBO series was over-reinforced and failure took place as concrete crushing. On the other hand, TBU series was under-reinforced and failure took place as reinforcement yielding. Fig. 8 compares the response of the sectional model against the experimental data for the two series.

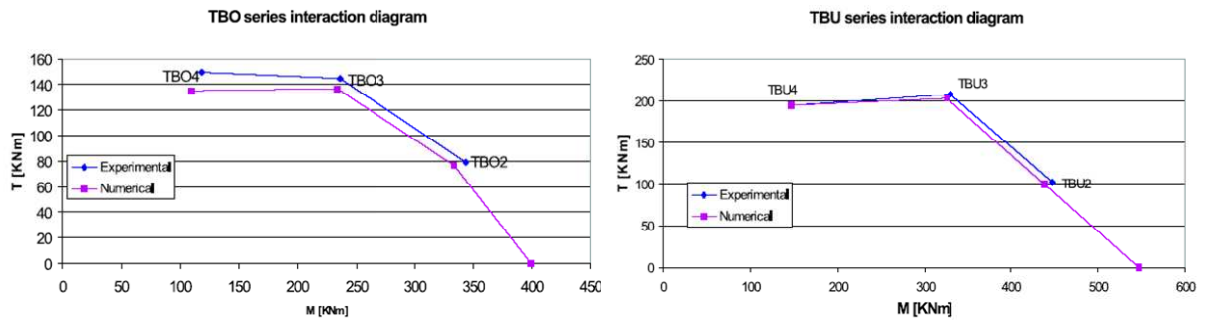


Figure 8 Torsion-bending response for different failure modes

2.4.4 Confinement

Since the sectional model considers shape distortion and 3D constitutive equations in each fibre it is possible to reproduce confinement effects. Fig. 9 show the stress distribution along the cross-section for different load stages of the force-strain curve shown in Fig. 10. At early stages, some confinement already exist in the corners of confined core. At stage IV spalling is starting from the corners and at stage V it has already developed in the whole perimeter, softening is noticed in this point in Fig. 10. Afterwards, load is sustained and a slight hardening is evident in the Fig. 10.

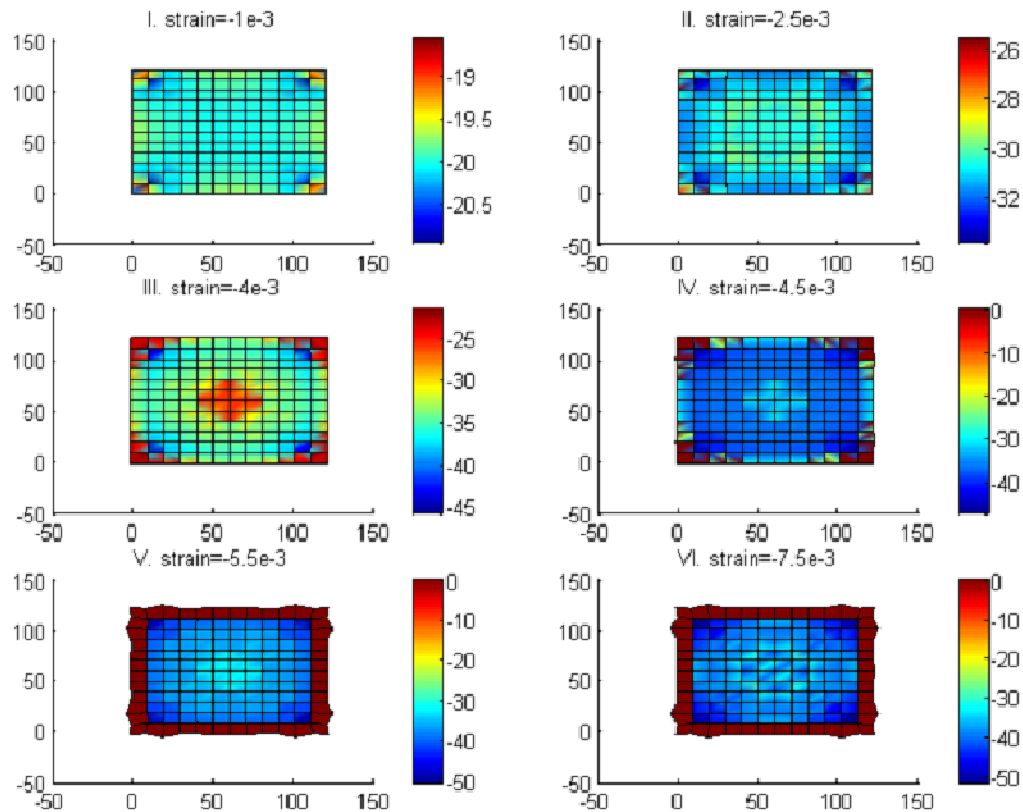


Figure 9 Stress distribution during loading and section distortion of confined section

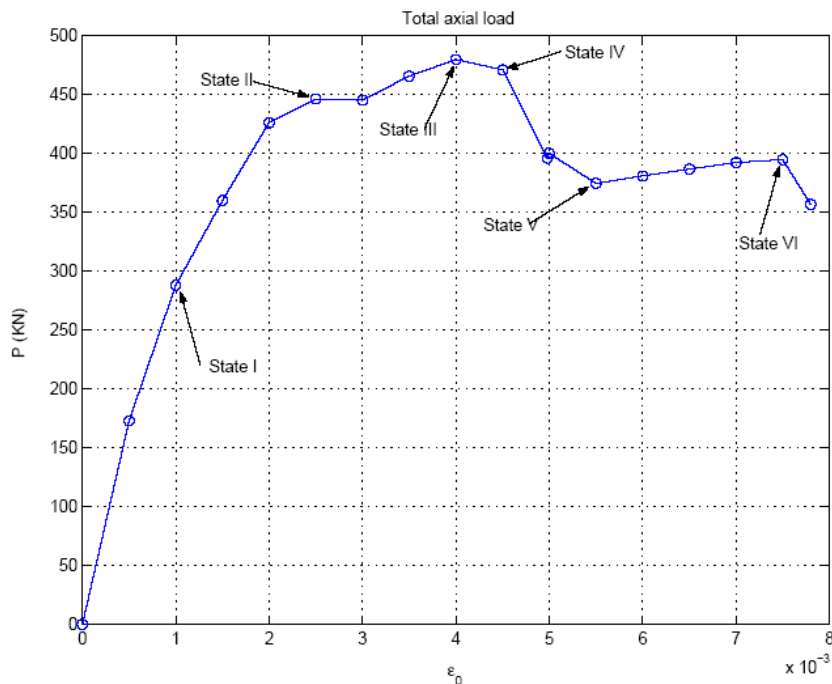


Figure 10 Response of a confined concrete column

2.4.5 Effect of tangential forces in the nonlinear response of hyperstatic structures

The effect of considering tangential forces in the nonlinear response of hyperstatic structures is investigated by analysing the continuous beam of Fig. 11. The structure is analyzed according to three different models; Model 1 corresponds to traditional Navier-Bernoulli formulation neglecting shear effects. Model 2 uses Timoshenko beam elements with linear interpolation and TINSAs cross-section implemented using the generalized coordinates method. That section is transversally reinforced with stirrups of $\varnothing 10$ c/100 mm. On the other hand, Model 3 uses the same numerical formulation but it is reinforced with stirrups of $\varnothing 10$ c/50 mm.

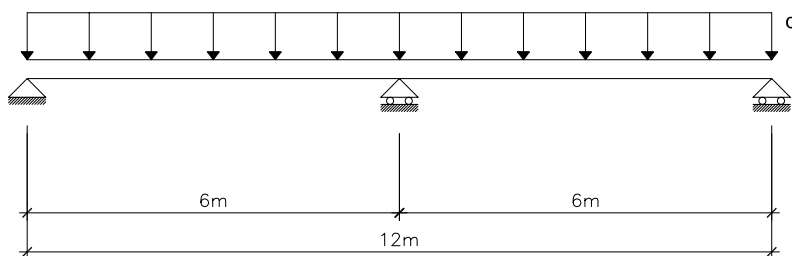


Figure 11 Reference structure

Force displacement responses for the three models are shown in Fig. 12. Effect of shear strains in the total deformation is noticed soon after cracking. On the other hand, first yielding is influenced by the consideration or not of shear forces. Model 2, with less shear reinforcement is the one that yields first and with less load carrying capacity as the stirrups are yielded in the middle support. In model 3 stirrups did not yield, although shear effects are noticed in the non-linear regime. Maximum load is slight lower than for the model that

neglects shear effects but with larger deformations. It should be mentioned that the analysis was conducted under load control and was stopped at 175 kN/m.

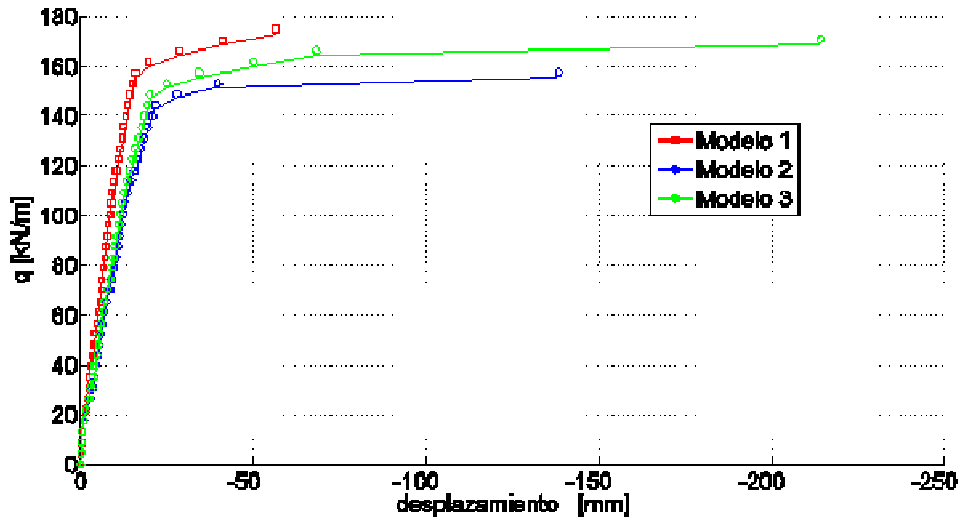


Figure 12 Force-displacement curves for the three models

Shear crack patterns are presented in Fig. 13 where line thickness is proportional to the crack widths. Difference in the crack pattern from considering or not shear forces are evident. More realistic crack patterns are obtained in the models considering shear effects.

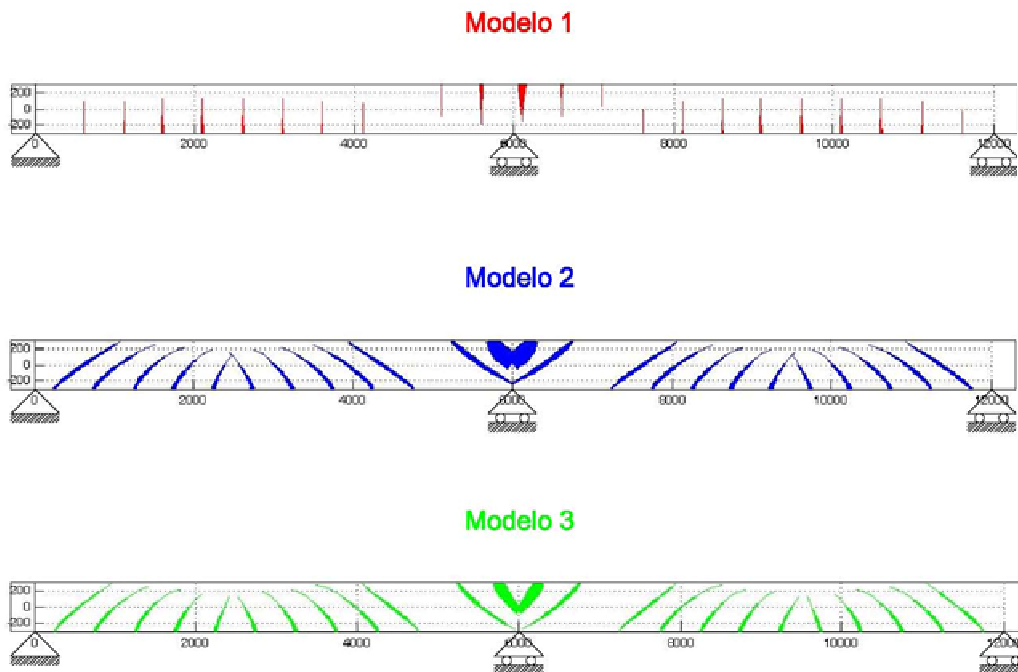


Figure 13 Shear crack patterns for the three models for $q=127$ kN/m

2.5 *Current and future developments*

Current works regarding this research line include:

- Dynamic analysis of structures sensible to shear forces and torsion.
- Inclusion of stage construction process and time effects.
- Simulation of retrofitted and repaired structures.

3 DISTURBED REGIONS

3.1 *Motivation*

Disturbed regions are very frequent in all structures, starting from beam-column joints to coupling beams or deep beams with or without openings. This consideration is also applicable to local zones where loads are introduced, bearing or geometry transition zones.

Current state-of-the-art design of these regions is based on the Strut-and-Tie (SAT) method which is based on the lower-bound theorem from theory of plasticity; hence it is only applicable for ultimate-limit-states (ULS) situations after occurrence of plastic strains. Since reinforced concrete is not a fully plastic material, application of plasticity principles have to be done with care. Therefore, several limitations and practical requirements are usually considered in design codes for the application of SAT method. However, there is no explicit approach in the method to evaluate damage under moderate or high loading and neither under service loads. Moreover, the method requires that an equivalent truss (strut-and-tie scheme) is proposed by the user before applying the design rules. In general, more than one equivalent truss is possible and sometimes it is not direct to obtain a plausible one especially for new structures or elements with complex geometry. Hence, practical applicability of the method is not that extended and it is somehow limited to standard situations.

3.2 *Main idea*

In this research line two main activities are being carried out. Firstly, a numerical methodology, based on structural topology optimization, is being developed in order to automatically derive plausible strut-and-tie schemes. This methodology does not require that the user proposes an initial reinforcement arrangement, but proposes distribution for both struts and ties elements.

The approach starts from the linear elastic solution of the D-region problem derived from a finite element model. Afterwards, some decision criteria based on the energy density distribution and an efficiency factor is assigned to each element of model. The process is repeated, always using linear elastic analysis, until no significant change in the total energy of the structure is found.

This approach is suitable for applying several decision taking criteria in order to produce different equivalent trusses. Particularly, criteria for considering constructability of the final design can be developed.

On the other hand, this research line also deals with the assessment of D-regions through non-linear analysis.

3.3 Generation of Strut-and-Tie models

Fig. 14 shows a typical D-region consisting on a deep-wall with an opening. The figure also shows the elastic response of the region by beams of principal compression and tensile stresses. The formulation is first applied without any constructability criterion. The equivalent truss of Fig. 15 (model 1) results from applying the automatic SAT method.

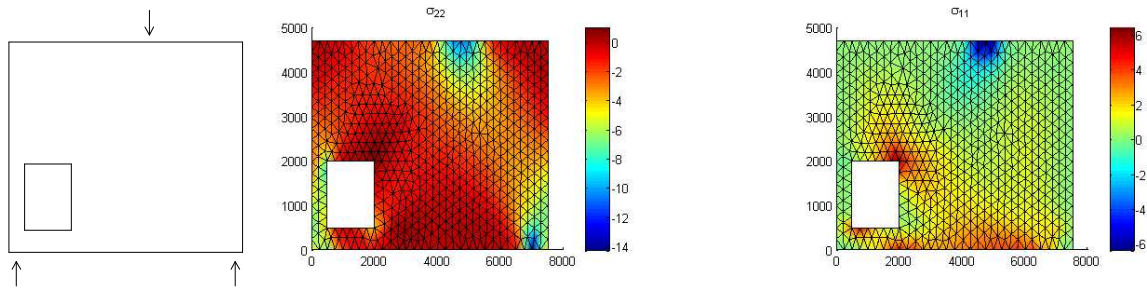


Figure 14 Reference structure and linear elastic response a) loading, b) principal compression, c) principal tension

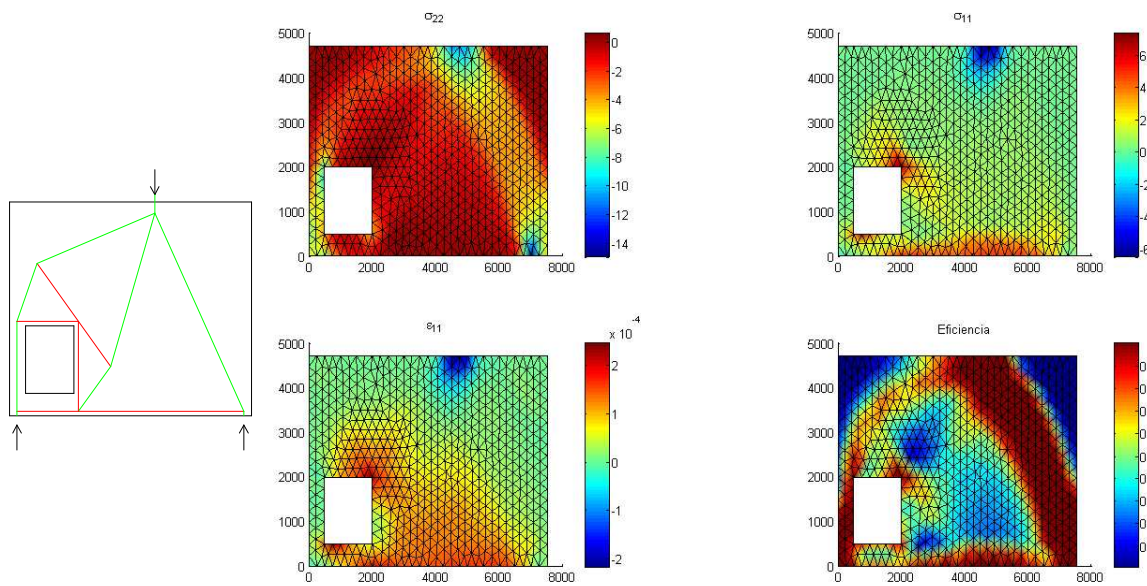


Figure 15 Strut-and-tie model 1 generated without constructability conditions a) Strut and Tie model, b) principal compression stress, c) principal tension stress, d) principal tension strain, e) efficiency parameter

It has to be noticed, however, that design according to model 1 requires the introduction of inclined reinforcements which might be interesting to avoid for constructability reasons. Hence, additional criteria may be given in order to force the use of only horizontal and vertical reinforcements. This is done by assigning orthotropic behaviour in suitable elements. As a result, the strut-and-tie model shown in Fig. 16 (model 2) is obtained.

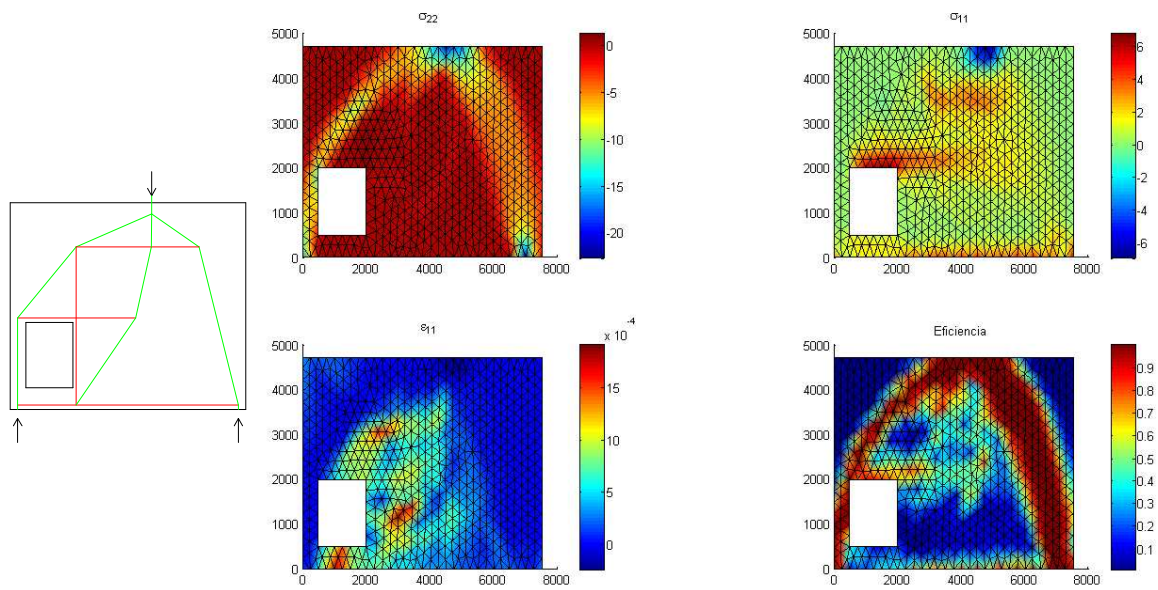


Figure 16 Strut-and-tie model 2 generated with constructability conditions a) Strut and Tie model, b) principal compression stress, c) principal tension stress, d) principal tension strain, e) efficiency parameter

3.4 Assessment of D-Regions

Both strut-and-tie models obtained in the previous section are plausible and may be used for design although differences in their structural behaviour might exist. These can be investigated through non-linear structural analysis of the reinforced concrete region after designing the element. The design was conducted considering a standard design rule of most design codes that limits yielding stress to 400 MPa, in order to indirectly control damage under service loads. However, only principal reinforcement was included, i.e. no minimum steel ratio was considered uniformly distributed in all directions as it is usually required.

Figures 17 and 18 show the cracks distribution for the two designs for 25%, 50%, 75% and 100% of the design load. It can be seen that model 1 is more efficient in controlling crack width for moderate loading than model 2. It is also noticeable that a vertical crack forms in model 1 in the centre span that propagates upwards. This crack is far less significant in model 2 and where it is restricted to the bottom of the wall.

Figure 19 compares the load-displacement curves for the two cases. Both designs resist higher loads than the design load. This is according to the expectations since the strut-and-tie model is based in the lower-bound theorem of the theory of plasticity, hence resistance should be larger than the design load. However, in general model 2 is more flexible and suffers more damage.

It should be mentioned that in this analysis non-linearity of concrete and steel have been considered and also multiaxial behaviour of concrete. However, bond between concrete and steel was considered perfect.

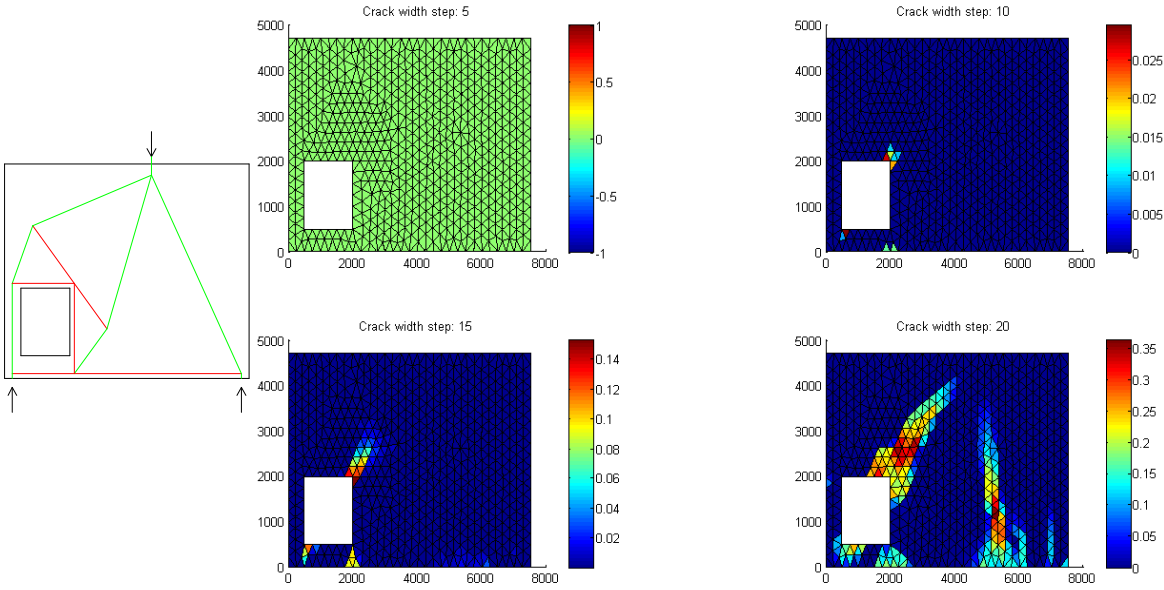


Figure 17 Crack patterns under different load conditions for structure designed according to strut-and-tie model 1 (for 25%, 50%, 75% and 100% of the design load)

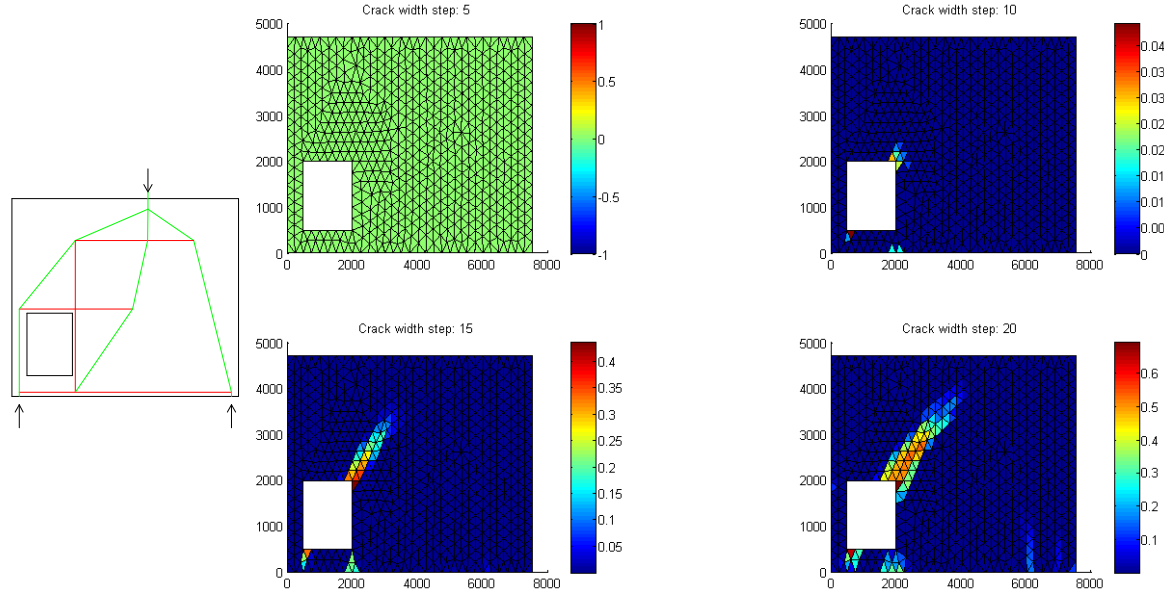


Figure 18 Crack patterns under different load conditions for structure designed according to strut-and-tie model 2 (for 25%, 50%, 75% and 100% of the design load)

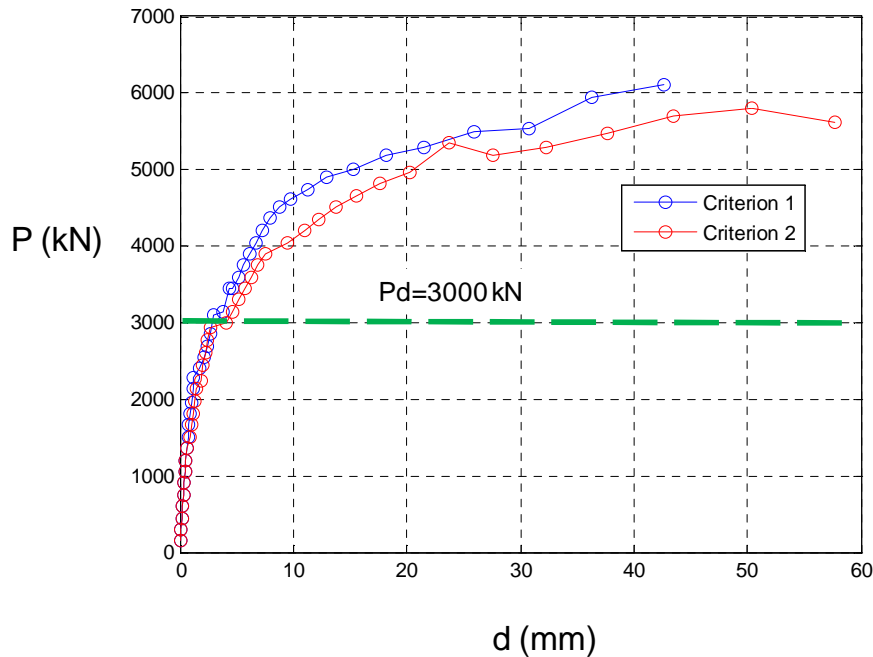


Figure 19 Comparison of the nonlinear response of model 1 and 2

3.5 Current and future developments

This field is being currently under development. Future works to be considered are:

- Optimal design of D-regions
- Performance based design of D-regions assuring damage control
- Consideration of bond-slip
- 3D D-regions

4 EXPERIMENTAL INVESTIGATIONS AND CONSTITUTIVE MODELING

4.1 Motivation

Experimental investigations are carried out in order to improve knowledge of material behaviour, to investigate actual response of complex systems and to validate analytical models. In the following some relevant experimental researches are being described.

4.2 Poisson strains under cyclic nonlinear loading

It is known that after damage, concrete lateral deformation increase a lot more than what predicts Poisson elasticity coefficient. This increment of strains is good for stretching lateral reinforcement and provides confinement stresses. Although some models have been found in the literature to simulate this effect, most of them only describes monotonic loading. A research campaign is being carried out in order to investigate the response of lateral strains of normal and high strength concrete under high cyclic loading.

Fig. 20 shows the laboratory set-up made for this campaign and a typical σ - ϵ_{long} - ϵ_{trans} curve under cyclic loading. Fig. 21 shows measured ϵ_{long} - ϵ_{trans} curves and a comparison with a proposed model.

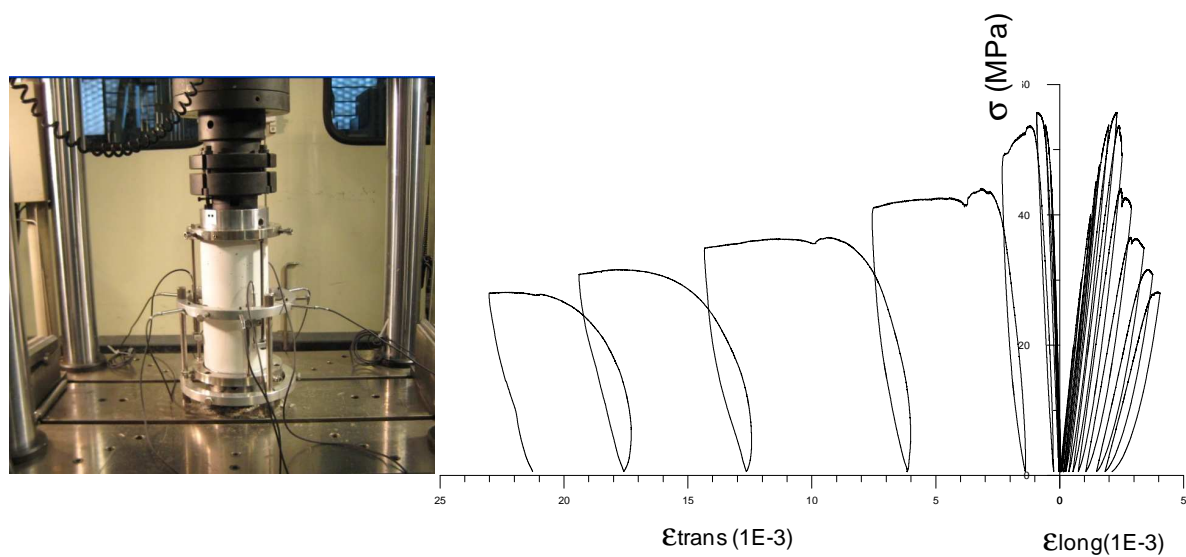


Figure 20 Laboratory set-up for measuring lateral strains(a) and typical σ - ϵ_{long} - ϵ_{trans} curves (b)

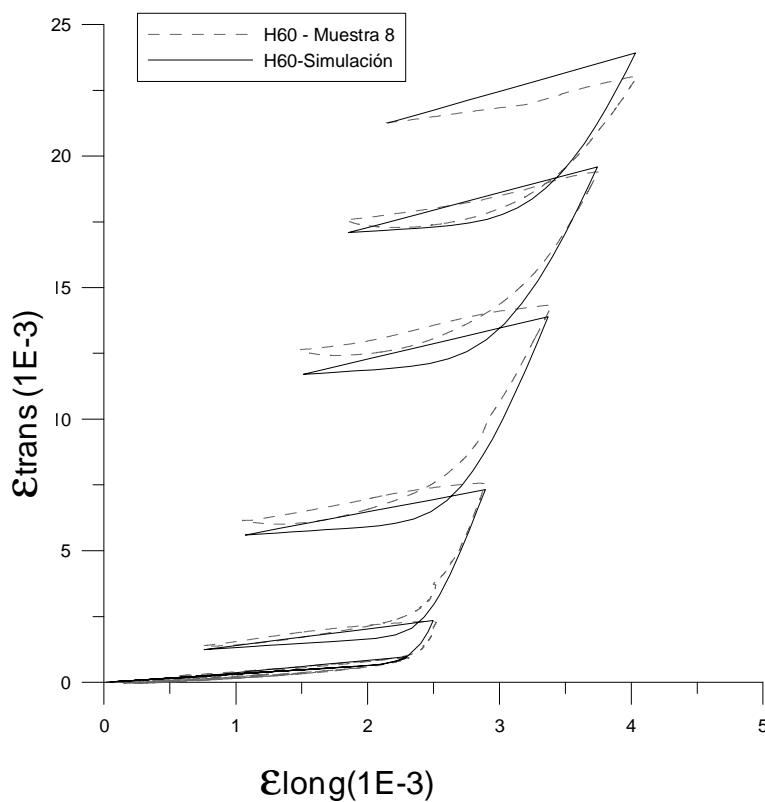


Figure 21 ϵ_{long} - ϵ_{trans} curves. Comparison of experimental and numerical results.

4.3 Cyclic biaxial response of concrete retrofitted piers

An experimental campaign to investigate the behaviour of concrete piers subjected to biaxial bending and shear is being currently prepared. In this campaign efficiency of retrofitting and repairing systems with FRP laminates will also be considered. Parallel nonlinear analyses

with the total interaction sectional model will be carried out with the intention of validating and to provide deeper understanding of the phenomena.

4.4 Effects of straightening of steel reinforcement coils in the mechanical characteristics

Spanish steel industry produces reinforcing bars of normal and very high ductility characteristics, denominated with S and SD grades respectively. In SD steels, a characteristic strain ductility, based on the strain under peak stress, of $\epsilon_{\max,k} > 0.08$ has to be guaranteed ($P(\epsilon_{\max} < \epsilon_{\max,k})=0.05$). Moreover, the new Spanish concrete design code requires this type of reinforcement for high-risk seismic areas.

Currently, the SD steel are also being produced and commercialized as coils, see Fig. 22. With this format, bars have to be straightened and cut in special machines before use. However, they provide many manipulating and storages advantages which are very interesting for constructors; also they are interesting from the optimization point of view since bars can be cut on the required length, avoiding splices and wastes.

An experimental and analytical research was conducted in order to investigate if the fabrication and straightening process produces changes in the mechanical properties, see Bairan et al (2008). Particularly in the ductility characteristics since it conditions the SD denomination. The research also included the development of a particular constitutive model for this steel with mixed-hardening in order to simulate the rolling and straightening processes.

It was noticed that in general ductility is reduced and yielding stress is increased as shown Fig. 23. Curvature ranges that could be applied safely applied to coil in order to keep ductility affection within acceptable limits were identified.



Figure 22 Coil of high-ductility reinforcing bars

Current development in this field include the investigation of the response of this type of steel against fatigue loading.

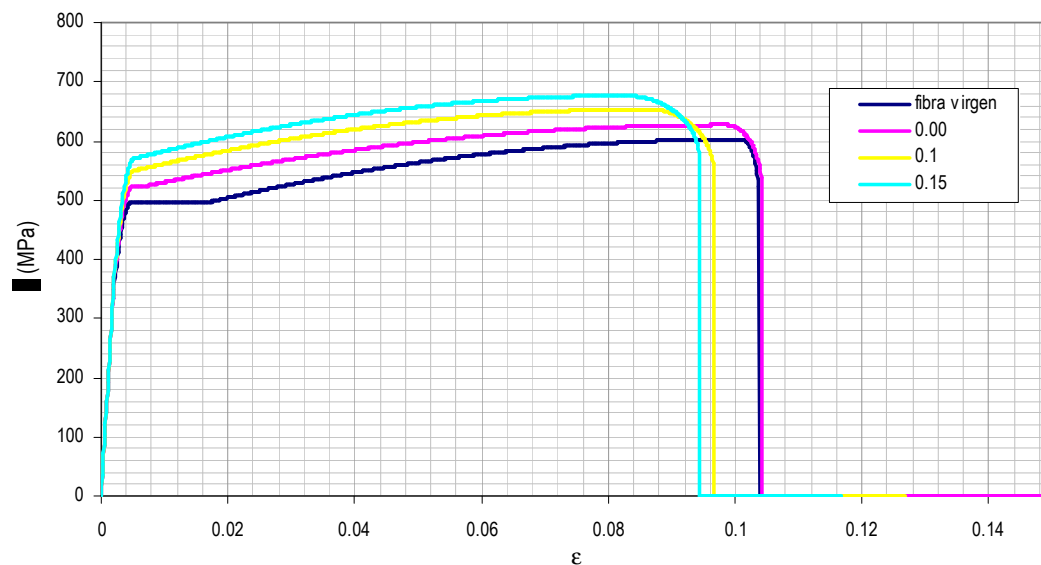


Figure 23 Typical σ - ϵ of straightening bars for different original \varnothing/R ratio (R : radius of the spiral)

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- Institute for the Promotion of Certified Reinforcements (IPAC)
- Spanish Lamination Company (CELSA).

6 REFERENCES

- Bairan, J.M., Mari, A.R. (2006a). “Coupled model for the non-linear analysis of anisotropic sections subjected to general 3D loading. Part 1: Theoretical formulation”. *Computers & Structures*, Vol. 84, 2254-2263, Elsevier. DOI: 10.1016/j.compstruc.2006.08.036.
- Bairan, J.M., Mari, A.R. (2006b). “Coupled model for the non-linear analysis of anisotropic sections subjected to general 3D loading. Part 2: Implementation and validation”. *Computers & Structures*, Vol. 84, 2264-2276, Elsevier. DOI: 10.1016/j.compstruc.2006.08.036.
- Bairan, J.M., Mari, A.R. (2007a). “Multiaxial-coupled analysis of RC cross-sections subjected to combined forces”, *Engineering Structures*, Vol 29, 1722-1738, Elsevier. DOI: 10.1016/j.engstruct.2006.09.007
- Bairan, J.M., Mari, A.R. (2007b). “Shear-Bending-Torsion Interaction in Structural Concrete Members: A Nonlinear Sectional Approach”, *Arch. Comput. Methods Eng.*, 14:249-278, Springer. DOI: 10.1007/s11831-007-9007-5.
- Bairan, J.M., Rosa, J.C., Mari, A.R., Ortega, H. (2008). “Efectos de la fabricación en rollo y desdoblado de armaduras pasivas de acero de alta ductilidad en sus prestaciones mecánicas”, *IVACHE Congress*, Valencia, Spain.

- Bairan, J.M., Mari, A.R., Mohr, S. (2009). "Behavior of reinforced concrete under normal and tangential forces by means of Total Interaction Nonlinear Sectional Analysis (TINSA)", *Informes de la Construcción*, Inst. Eduardo Torroja, CSIC, Madrid, Spain (under revision)
- Bentz, E. (2000). "Sectional analysis of reinforced concrete members", Doctoral Thesis, University of Toronto, Toronto, Canada.
- Cladera, A., Mari, A.R. (2005). "Experimental study on high-strength concrete beams failing in shear", *Engineering Structures*, 10 (27), pp. 1519-1527.
- Mari, A.R., Bairan, J.M. (2008). "Nonlinear analysis of concrete structures under complex loadings", *Handling Exceptions in Structural Engineering*, University of Rome La Sapienza, Rome. DOI: 10.3267/HE2008.
- Vecchio, F.J., Collins, M.P. (1986). "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear", *ACI Struct. Journal*, Vol. 83, 219-231.
- Onsongo, W. M. (1978). "*The diagonal compression field theory for reinforced concrete beams subjected to combined torsion, flexure and axial load*", Doctoral Thesis, University of Toronto, Toronto, pp. 246.