Low-frequency variations in the wake of a circular cylinder at $Re = 3900$

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Abstract. Flow around cylindrical structures is of relevance for many practical applications. Knowledge of flow-related unsteady loading of such structures is crucial for hydro - and aerodynamic control and design. In order to obtain a deeper knowledge of this kind of flow, a DNS have been performed at $Re_D = 3900$ ($Re_D = U_{ref}D/\nu$). The instantaneous velocity signals of probes located in the separated shear-layer and in the vortex formation region exhibit the presence of low-frequency variations. The statistical analysis of these signals suggest that low-frequency variations in the vortex formation length, suction base pressure and intermittencies in the shear layer are closely related. It is shown that these variations are the responsible of the large scattering of data obtained in different experimental and numerical results, as well as the U-shape and V-shape stream-wise velocity profiles observed in the very near wake of the cylinder.

1. Introduction

The flow over a circular cylinder has been subject of several numerical and experimental studies. From a theoretical point of view, this is a canonical case to perform studies of the turbulence behavior of these regions and to learn how they interact between them. On the other hand, from a practical point of view, this case is also of interest since flow around cylindrical structures is of relevance for many practical applications i.e. heat exchangers, bridge piers, chimneys, towers, antennae, wires... Knowledge about flow-related unsteady loading of such structures is crucial for hydro and aerodynamic control and design.

Although the flow past a circular cylinder at $Re_D = 3900$ have been extensively investigated (see for instance Ma *et al.* (2000); Parnaudeau *et al.* (2008) and the citations therein), there is a large scattering in the mean flow solutions in the near wake, with mean flow configurations of the stream-wise velocity varying from a U-shape to a V-shape profile. According to Ma *et al.* (2000), both states reflect the dynamics of the flow in the very near wake which is very sensitivity to disturbances and they concluded that U-shape profile emerges if the background fluctuations are relatively low or the span-wise extend the domain is small ($L_z/D = \pi$). However, the DNS of Tremblay, Manhart & Friedrich (2004) seems to contradict that hypothesis. More recently, Parnaudeau *et al.* (2008) performed experimental and numerical studies and found that about 1200 time-units (250 shedding cycles) were required for obtaining a converged value but in this case they obtained a

U-shaped stream-wise velocity profile. On the other hand, low-frequency variations in the wake of bluff bodies have been observed by several investigations and J.J.Miau *et al.* (1999) have found that such variations in the wake of a trapezoidal cylinder and a circular cylinder at Reynolds numbers above 10^4 are associated with the unsteady variations of the vortex formation length.

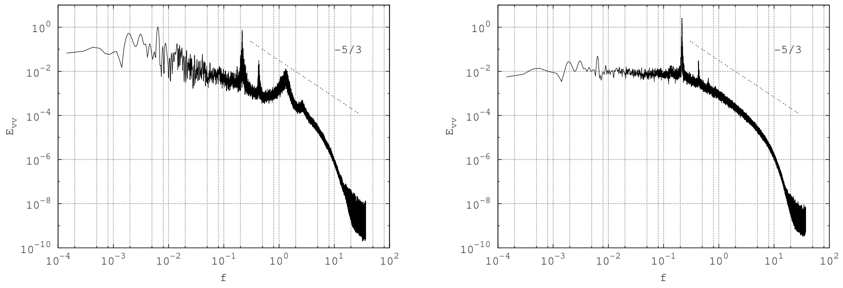
In this paper we report on detecting the low-frequency unsteadiness of the recirculation bubble past a circular cylinder at $Re_D = UD/\nu = 3900$ (based on the free-stream velocity and the cylinder diameter). To do this, we have performed several DNS for different blockage and aspect ratios. The co-existence of two different wake configurations is discussed in detail by means of the analysis of the power spectra of several probes at different locations and averaged statistics.

2. Problem definition and computational domain

The methodology used for solving the flow over bluff bodies with massive separation is described in Lehmkuhl *et al.* (2009) and Rodríguez *et al.* (2011). We consider here the DNS of the flow past a circular cylinder at $Re_D = 3900$. Although several computations have been carried out considering different domain sizes and computational grids, for the sake of brevity, the results presented in this paper have been obtained using a computational domain of dimensions [-8D,16D];[-10D,10D];[0,πD] in the stream-, cross- and span-wise directions respectively, with a circular cylinder of diameter D at (0,0,0). As for the span-wise size of the domain, in a previous study Lehmkuhl *et al.* (2009) it was shown than doubling the domain in the span-wise direction was not influence the statistical data. As the main concern of the present work is about the low-frequency variations in the near wake and on the time-dependence of turbulent statistics, it is required a large integration time. Hence, the computational effort has been focused on the long-term average statistics instead of a larger span-wise domain. The boundary conditions at the inflow consist of a uniform velocity (u,v,w)=(1,0,0), slip conditions in the top and bottom boundaries of the domain, while at the outlet a pressure-based condition is used. At the cylinder surface, no-slip contiditons are prescribed. As for the span-wise direction, periodic boundary conditions are imposed. As mentioned before, the governing equations are discretised on an unstructured mesh generated by the constant-step extrusion of a two-dimensional unstructured grid. The use of an unstructured grid for the plane has allowed to cluster more control volumes around the cylinder surface and in the near wake. As it has been commented, several grids have been considered, but for brevity the results presented here have been computed with a grid of about 9.3 M CVs (72700×128). With this ratio between grid-size and Kolmogorov scale, the resulting grid density obtained should be fine enough for solving the smallest flow scales in the near wake.

3. Results

The simulations have been started from homogeneous flow and initially some random perturbations have been introduced. In order to ensure temporal converged statistically steady state, the flow field has been advanced in time for an initial duration of about 100 tU/D. Once the initial transient has been washed out, statistics have been collected and averaged over approximately 3900 tU/D, which is about 836 shedding cycles. This time integration results in a long simulation time and is by far the largest numerical experiment carried out since now for this flow, but it ensure not only converged statistics bu talso a large time span to analyse low-frequency variations in the wake.



(a) (b)

Figure 1. Energy spectra of the cross-stream velocity at different zones(a) PA and (b) PB

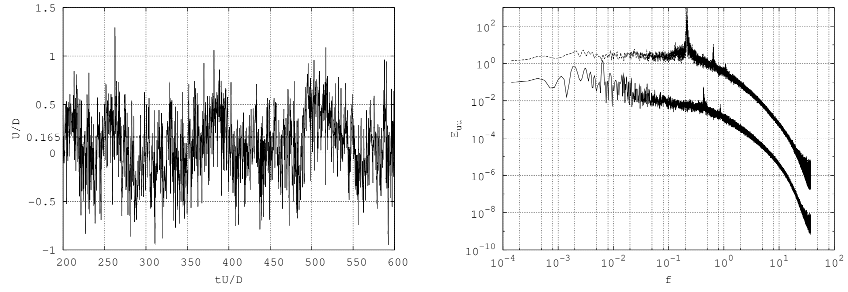
3.1. Energy spectrum

Single-points measurements have been carried out by positioning probes at different locations. Measurement at those ports were taken over the whole simulation time. For the sake of brevity we are presenting here results for probes located at $PA \equiv [x/D = 0.71, y/D = 0.66]$, in one of the shear layers, at $PB \equiv [x/D = 2.0, y/D = 0.6]$ in the wake, $PC \equiv [x/D = 2, y/D = 0.0]$, on the centreline of the wake near the stream-wise position for the recirculation bubble closure.The main frequencies of the fluctuations of the cross-stream velocity component at PA and PB stations have been obtained from their power spectra computed by using the Lomb periodogram technique. The resulting spectra have also been averaged in the azimuthal direction. At PA, the spectrum exhibits a dominant peak at the vortex shedding frequency $f_{vs} = 0.2145$ and its second harmonic. In addition, a broadband peak centered at $f_{KH} = 1.34$ which corresponds with the Kelvin-Helmholtz instabilities of the separating shear-layer is also observed. This value is in fair agreement with Prasad & Williamson (1997*a*) predictions for this Reynolds number ($f_{KH} = 0.0235 Re^{0.67} f_{vs} = 1.29$). This secondary peak desappears as we move downstream and the wake becomes turbulent. However, on the top of these frequencies, there is also a peak at a much lower frequency than of the vortex shedding at $f_m = 0.0064$. This peak is also observed at other stations located in the vortex formation zone.

A piece-wise of the time series for the stream-wise velocity component at PC and its time-averaged value is shown in figure 2(a). At this location can be clearly seen how there is a quasi-periodic wave in the fluctuations of the variable with a frequency which is quite lower than of the vortex shedding. This large-scale quasi-periodic motion registered seems to point out the existence of a modulation of the recirculation bubble which causes its shrinking and enlargement over the time. The spectra of both velocity fluctuations are plotted in figure 2(b). At this location, the energy spectrum of cross-stream velocity fluctuations exhibits a clear peak at the vortex-shedding frequency $f_{vs} = 0.214$ and its 3rd harmonic. On the contrary, the energy spectrum of the stream-wise fluctuations shows a peak at $f_m = 0.0064$ and a weak one at the second harmonic of f_{vs}. While the harmonic peak is characteristic of the vortex-shedding measured at the wake centreline, the dominant one at f_m is the footprint of the quasi-periodic motion observed in the stream-wise velocity.

3.2. Averaged statistics in the wake

The low-frequency behaviour of the velocity fluctuations within the recirculation zone can be physically interpreted as the wake variation between to different modes: i) a high-energy mode dominated by strong fluctuations in the shear-layer, and in general, large-amplitude fluctuations in the vortex formation zone and, ii) a low-energy mode with weaker fluctuations in the shear layer. The large-amplitude fluctuation in the shear layers



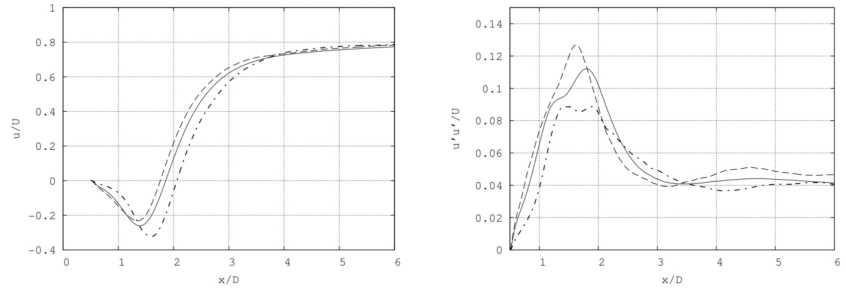
(a) (b)

Figure 2. (a) Time-series for the stream-wise velocity u and its time-averaged value $\overline{u}$ at PC (b) (from bottom to top) Power spectrum of the stream-wise and cross-stream velocity components at PA (for clearness the latter is shifted)

is accompanied with a shrinkage of the recirculation region, while when weaker fluctuations are observed there is also an enlargement of the recirculation region behind the cylinder. As expected, there is also a linkage with the suction base pressure, thus there is an increase in the suction base pressure as the recirculation region becomes larger. Hereafter, we will refer to these modes as Mode S (short recirculation zone) and Mode L (large recirculation zone). The existence of different states in the near wake zone behind a normal flat plate has been studied by Najjar & Balachandar (1998). Similar fluctuations in the recirculation zone behind bluff-bodies has been also observed experimentally and numerically (see Berger *et al.*, 1990; J.J.Miau *et al.*, 1999; Rodríguez *et al.*, 2011).

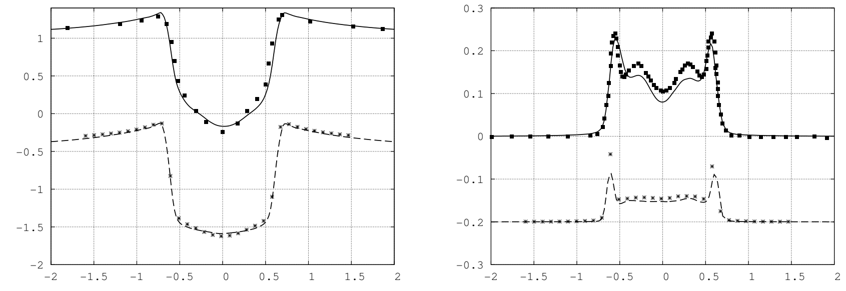
In order to analyse the wake configuration we have computed the time-average statistics during both modes. In figure 3 the stream-wise velocity profile and its fluctuation along the wake centreline is depicted. In the figure are plotted the averaged values for both modes together with the long-term average solution. As can be observed, the profiles of stream-wise velocity and its fluctuations are quite different. As expected, recirculation zone goes from $L_r/D = 1.26$ during Mode S to a larger value of $L_r/D = 1.55$ in Mode L. The long-term average solution (with 3900 tU/D) is within both modes yielding a length of the recirculation region of $L_r/D = 1.36$. In Mode S the velocity deficit also is less than in Mode L. However, it is remarkable the fact that this behaviour is just restricted to the vortex formation zone, as the wake recovers after $x/D > 4$. These results support the hypothesis of the quasi-periodic modulation of the recirculation bubble and further demonstrates that there is an shrinking and enlargement of the recirculation zone which produce different configurations in the wake. Another striking fact is the profile of the stream-wise velocity fluctuations u_{rms}. In Mode L, it exhibits a two-lobed peak with maximums at $x/D = 1.43$ and $x/D = 1.9$. The second one occurs just upstream the location of the recirculation closure ($x/D = 1.95$), pointing the length of the vortex formation zone. This profile is similar to that observed by Norberg (1998) for $Re_D = 3000$. On the contrary, u_{rms} along the wake centreline in Mode S is quite different with only one peak at $x/D = 1.62$. Furthermore, Mode S presents a higher level of fluctuation, suggesting that it is more energetic and turbulent than Mode L. On the top of that, u_{rms} profile for the long-term average solution is similar to that described by Norberg (1998) for $Re_D > 8000$, presenting a peak at $x/D = 1.8$ and (what Norberg called) an inflection point upstreams this peak.

In figure 4, the stream-wise velocity profile and its fluctuation at $x/D = 1.06$ are plotted. For comparison, the experimental results from Parnaudeau *et al.* (2008) and the numerical results of Case I from Ma *et al.* (2000) (finer grid and larger span-wise domain) are also included. These results have been selected as both of them point out contradictory wake



(a) (b)

Figure 3. Effects of the recirculation in the wake configuration. (a) Averaged streamwise velocity and (b) its averaged fluctuation along the wake centreline. (solid line) long-term averaged solution; (dashed line) Mode S; (dash dotted line) Mode L.



(a) (b)

Figure 4. Effects of the recirculation in the wake configuration and comparison with literature results. (a) Averaged streamwise velocity profile and (b) its averaged fluctuation at x/D=1.06.(solid line) Mode S; (dashed line) Mode L; (■) Case I from Ma *et al.* (2000); (*) Experimental results from Parnaudeau *et al.* (2008)

statistics due to their differences in the recirculation zone ($L_r/D = 1.51$ and $L_r/D = 1.12$, respectively). As can be observed, there is a good agreement for first- and second-order statistics with both data for each mode.

Norberg (1987), in his work, suggested that at $Re_D = 5000$ overcomes a change in the wake configuration. This transition was later confirmed by Prasad & Williamson (1997*b*), while Norberg (1998) attributed this change to a transition between a high- and low-quality vortex shedding mode. The results here presented suggest that at $Re_D = 3900$ both modes co-exist and the wake is oscillating between them at a very low-frequency, which causes the large scattering in the experimental and numerical results observed since now. What still remains unclear is which mechanism triggers such oscillation.

4. Conclusions

A study of the wake configuration of the flow past a circular cylinder has been carried out by means of the DNS at $Re_D = 3900$. The analysis of the spectra of the velocity at different locations in the vortex formation zone, suggests that together with the vortex-shedding frequency and the small-scale Kelvin-Helmholtz instabilities frequency, there is also a low-frequency which can be attributed to the shrinkage and enlargement of the recirculation region. This modulation of the near wake can be observed as two different modes in the wake configuration. A high-energy mode, with larger fluctuations in the shear-layers and in the near wake which shortens the recirculation bubble and a low-energy mode with a larger recirculation zone. The average profiles of the wake at the different modes confirm the existence of both modes. Considering that previous experimental results point out to a change in the wake configuration at $Re_D = 5000$, it seems probably that at this Reynolds numbers the wake is fluctuating between both modes. The mechanism of such behaviour remains still unclear.

Acknowledgments

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