Combined and I Indices Based on Principal Component Analysis for Damage Detection and Localization

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ABSTRACT

In this paper, two indices (*combined* or *phi index* and *I* index), different to the presented in [1][2][3] are used to detect damages; these indices are calculated from the information obtained from the projection of the experiments into the PCA models (baseline). They give us a measurement about the difference between the tested and the healthy structure. The experiments are taken from an active piezoelectric system which is excited with lamb waves in different phases and the fact that any defect in the structure changes its vibrational response is exploited.

For localization, five different methods of contribution analysis are used (complete decomposition contribution, partial decomposition contribution, angle based contribution, reconstruction based contribution and diagonal contribution). With these methods, the contribution of each sensor to each index is analyzed, in this way, sensor with largest contribution suggests the path where the damage could be located (from the actuator to this sensor). The combination of all indices and all contributions (a total of 2 x 5) are analyzed and compared. To validate the approaches, they are applied to an aircraft turbine blade instrumented with seven PZT's. Different damages are simulated.

INTRODUCTION

The problem of monitoring defects in structures can be tackled from different points of view, generally they are focused on analysis of physical/mathematical models or analysis of a set of data (experimental or by simulation). Systems which do not use physical models the problem can approach as pattern recognition, some features from the dynamical response of the healthy structure are used as pattern. The fact that any defect in the structure changes its vibrational response is exploited. To perform global detection, classification and localization of damages, techniques such as Principal Component Analysis (PCA) are useful because by means of multivariable analysis, it

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provides arguments for discerning which dynamic is the most important in the system. The use of PCA as pattern recognition tool using time based signals have shown good results for structural damage detection, localization and classification [1][2][3].

These approaches combine the use of an active piezoelectric system, which contain piezoelectric transducers (PZT's) that can be used as actuators or sensors to produce and collect lamb waves in structures in different phases. In each phase of the diagnosis procedure, one PZT is used as actuator (a known electrical signal is applied) and the others are used as sensors (collecting the wave propagated through the structure at different points). The number of phases is related to the number of PZT's. In each phase, an initial baseline model for undamaged structure is built applying PCA to the data collected in several experiments. Current structure (damaged or not) is subjected to the same excitation, and the collected data are projected into the PCA models (one by each phase). This paper present a damage detection and localization methodology based on principal component analysis and two damage indices. The methodology includes the use of a piezoelectric active system which allow to excite and collect the signal from the PZTs attached on the surface of an aircraft turbine blade in different phases. Five methods for damage localization are performed and its results are presented.

PRINCIPAL COMPONENT ANALYSIS

Introduction

Principal Component Analysis (PCA) is a technique of multivariable and megavariate analysis [4] which may provide arguments for how to reduce a complex data set to a lower dimension and reveal some hidden and simplified structure/patterns that often underlie it. The goal of Principal Component Analysis is to obtain the most important information from the data [5]. In order to develop a PCA model it is necessary to arrange the collected data in a matrix **X.** This $m \times n$ matrix contains information from n sensors and m experimental trials [6]. Since physical variables and sensors have different magnitudes and scales, each data-point is scaled using the mean of all measurements of the sensor at the same time and the standard deviation of all measurements of the sensor. Once the variables are normalized the covariance matrix $\boldsymbol{C_x}$ is calculated. It is a square symmetric $m \times m$ matrix that measures the degree of linear relationship within the data set between all possible pairs of variables (sensors). The subspaces in PCA are defined by the eigenvectors and eigenvalues of the covariance matrix as follow:

$$C_x P = P\Lambda \qquad (1)$$

Where the eigenvectors of $\mathbf{C_x}$ are the columns of P, and the eigenvalues are the diagonal terms of Λ (the off-diagonal terms are zero). Columns of matrix P are sorted according to the eigenvalues by descending order and they are called the Principal Components of the data set. The eigenvectors with the highest eigenvalue represents the most important pattern in the data with the largest quantity of information. Geometrically, the transformed data matrix **T** (score matrix) represents the projection of the original data over the direction of the principal components **P**.

$$\mathbf{T = XP} \qquad (2)$$

In the full dimension case, this projection is invertible (since $\mathbf{PP^T=I}$) and the original data can be recovered as $\mathbf{X=TP^T}$. Now, with the given **T**, it is not possible to fully recover **X**, but **T** can be projected back onto the original *m*-dimensional space and obtain another data matrix as follow:

$$\hat{X} = TP^T = X(PP^T) \tag{3}$$

Considering $\hat{X}$ as the projection of the data matrix **X** onto the selected *r* principal components and $\overline{X}$ as the projection onto the residual left components, the following decomposition can be performed:

$$X = \hat{X} + \overline{X} \tag{4}$$

$$\hat{X} = X(PP^T) \tag{5}$$

$$\overline{X} = X\left(I - PP^T\right) \tag{6}$$

Damage Detection Indices

There are several definitions of fault detection indices [8]. Two well-known indices are commonly used to this aim: the Q-index (or SPE-index), the Hotelling's T^2-statistic (D-statistic). There exist another type of indices reported in the literature as combined index [10] and I index [11]. The first one is a combination of the *Q-index* and *T^2-index* and is used to monitor the behavior of a process, the second one is used in meta-analysis and can be interpreted as a percentage of heterogeneity.

Q-statistic, T2-statistic, Combined Index (φ or phi), I index of the i-th sample (or experiment) are defined as follows:

$$Q = (\mathbf{I} - \mathbf{PP^T}) \tag{7}$$

$$T = \mathbf{P}\Lambda^{-1}\mathbf{P^T} \tag{8}$$

$$\varphi_i = Q + T = (\mathbf{I} - \mathbf{PP^T}) + \mathbf{P}\Lambda^{-1}\mathbf{P^T} \tag{9}$$

$$I = \begin{cases} 0 & for \quad Q \le (k-1) \\ \dfrac{Q-(k-1)}{Q} * 100\% & for \quad Q > (k-1) \end{cases} \tag{10}$$

where, k is the number of experiments.

CONTRIBUTION METHODS FOR LOCALIZATION

According to [8] five methods can be used for fault detection in process monitoring. Authors of this work adapted these methods for use in damage detection and localization in structures. These methodologies are used to calculate the contribution of each sensor to each index in each experiment trial. In this way, the damage will be located between actuator and sensor with largest contribution. All the indices can determine if there are damages and distinguish between them, however they do not provide reasons for it. The main idea is to determine which variable or variables are responsible. The variables with the largest contribution are considered major contributors to the damage.

Complete Decomposition Contribution (CDC)

Complete decomposition Contributions also called contribution plots are well known diagnostic tools for fault identification [7]. In each index is indicated the significance of the effect of each variable on the index. The contribution of the variable (or sensor) j to the index is defined as in the equation 11.

$$CDC_j^{Index} = x^T M^{\frac{1}{2}} \xi_j \xi_j^T M^{\frac{1}{2}} x \tag{11}$$

where ξ_j is the j^{th} column of the identity matrix, and represents the direction of $\mathbf{x_i}$.

Partial Decomposition Contributions (PDC)

This method decomposes a damage detection index as the summation of variable contributions.

$$PDC_j^{Index} = x^T M^{\frac{1}{2}} \xi_j \xi_j^T x \tag{12}$$

Diagonal Contributions (DC)

The diagonal contributions remove the cross-talk among variables. The DC is defined as:

$$DC_j^{Index} = x^T \xi_j \xi_j^T M \xi_j \xi_j^T x \tag{13}$$

Reconstruction Based Contributions (RBC)

The Reconstruction Based Contribution [9] is an approach that uses the amount of reconstruction of a damage detection index along a variable direction as the contribution of that variable to the index. The RBC is defined as:

$$RBC_j^{Index} = \frac{(\xi_j^T M x)^2}{(\xi_j^T M \xi_j)} \tag{14}$$

Angle-Based Contributions (ABC)

The ABC measures the cosine between a measure and a variable direction.

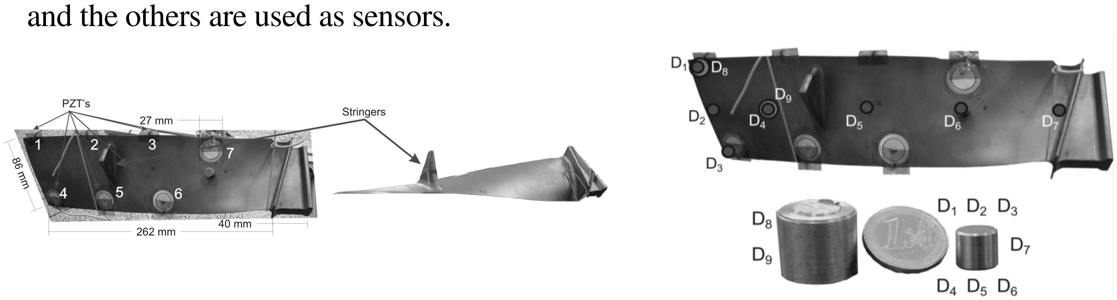
The ABC of variable j is the squared cosine of the angle between $\overline{x}$ and $\overline{\xi_j}$

$$ABC_j^{Index} = \left(\frac{\xi_j^T \overline{x}}{\left\|\overline{\xi_j}\right\| \left\|\overline{x}\right\|}\right)^2 = \frac{(\xi_j^T M x)^2}{\xi_j^T M \xi_j x^T M x} = \frac{RBC_j^{Index}}{Index(x)} \tag{15}$$

where $\overline{\xi_j} = M^{\frac{1}{2}} \xi_j$ and $\overline{x} = M^{\frac{1}{2}} x$

EXPERIMENTAL MOCKUP

This work involves experiments with an aircraft turbine blade. This blade was instrumented with seven piezoelectric transducer discs (PZT's) attached on the surface: three of them were distributed in one face and the others on the other face (see Figure 1). The experiment to assess the structure is performed in several phases [4]. In every phase, just one PZT is used as actuator (an electrical excitation signal is applied) and the others are used as sensors.



a. **b.**

Figure 1 Aircraft turbine blade with the PZT's location and ubication of damages

Adding two masses in different locations, nine damages were simulated. 140 experiments were performed and recorded: 50 with the undamaged structure, and 10 per damage [1]. The PCA model was created using 80% of the whole dataset collected using the undamaged structure. Signals from the other 20% and the whole dataset of the damaged structure were used for testing the approach.

DAMAGE DETECTION

The damage detection methodology includes the use of the *phi* vs *I* plots. Figure 2 shows the plot for the actuator 1. In this figure it is possible to observe that the data from undamaged structure are grouped and separated of the data from the damaged structure, in the same way, the figure shows that there is clustering of results within each damage.

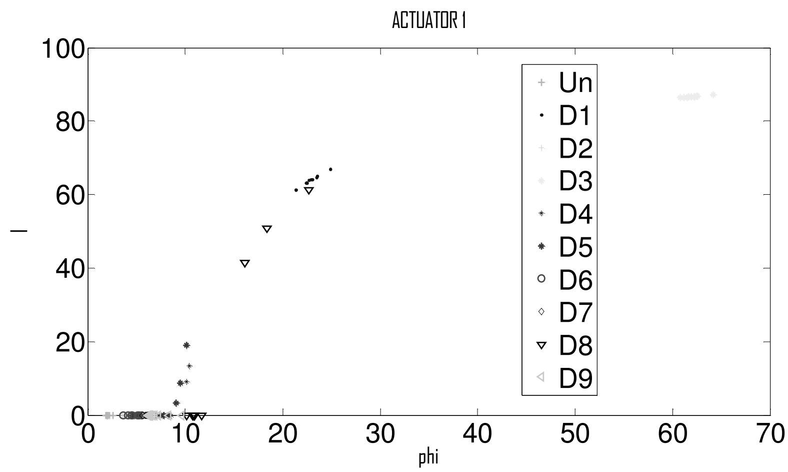


Figure 2 phi vs *I* plot for actuator 1.

Is possible to obtain similar plots with the other actuators, the amplitude and position of the points in the plot depend of the ubication of the actuator in relation with the damage, but in all the plots is possible to detect abnormalities.

DAMAGE LOCALIZATION

The location of damage was made using 5 methods: CDC, PDC, RBC, ABC, DC. As explained in the Contribution Methods for localization section. Figures 3 and 4 show the different contributions in each sensor to *phi* index and *I* index using the CDC method when damage 1 is present. Figure 5 presents the final localization f the damage 1 using I index and CDC. As it is observed in Figure 4, contributions of sensors 3, 6 and 7 are null, this is due to the threshold defined in the index I, which allows to eliminate the less significant contributions. Figures 6 to 9 show the comparison between the five methods (CDC, PDC, RBC, ABC and DC) of contribution of each sensor to each index. Sensors in figure are the PZT that are not working as actuator. In general terms, it shows that the use of all methods allow to locate the damage and, the fact that contributions in most cases have similar results, although it may be noted that in some cases the methods PDC, RBC, ABC & DC provide major contributions compared with the CDC method.

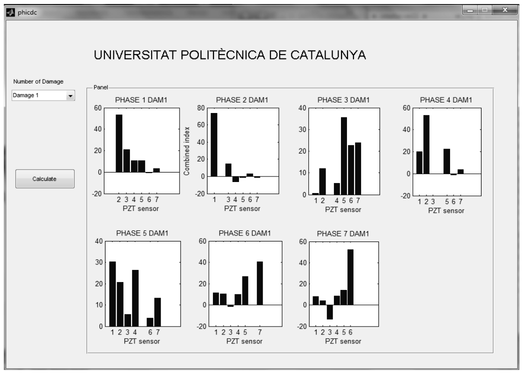


Figure 3 *Combined index* with CDC method

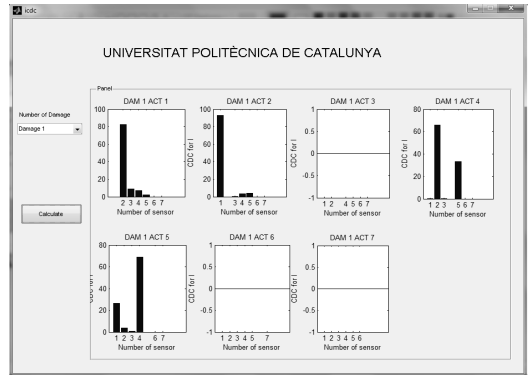


Figure 4 *I index* with CDC method

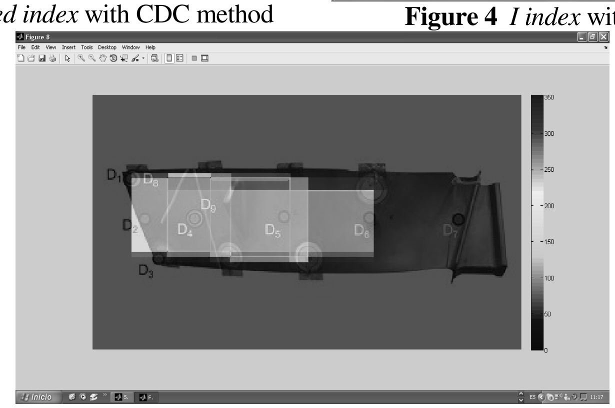


Figure 5 *Damage localization for damage 1using the I-index*

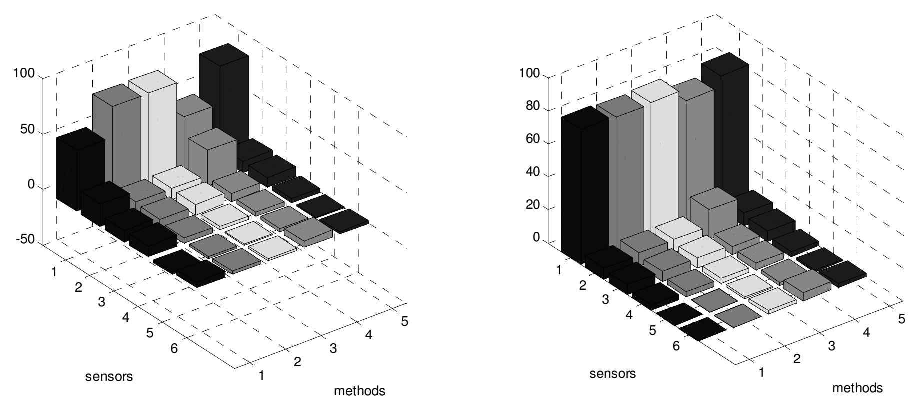


Figure 6 *Comparison between methods of contribution to Phi and I -Index using PZT1 as actuator*

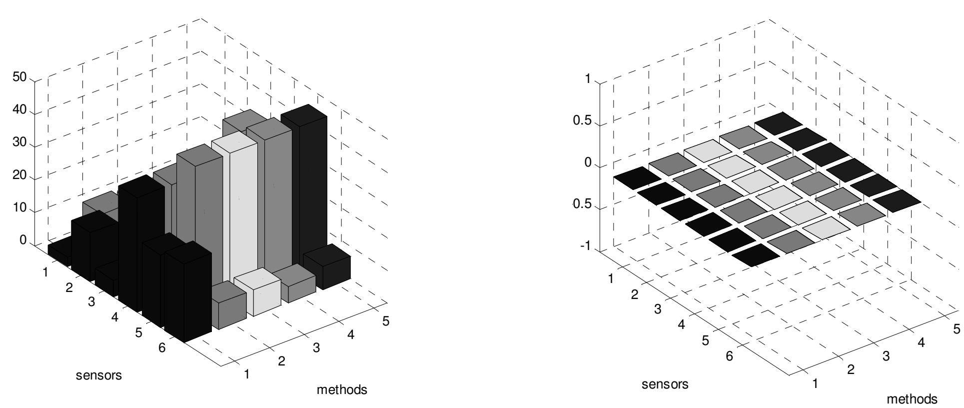


Figure 7 *Comparison between methods of contribution to Phi and I -Index using PZT3 as actuator*

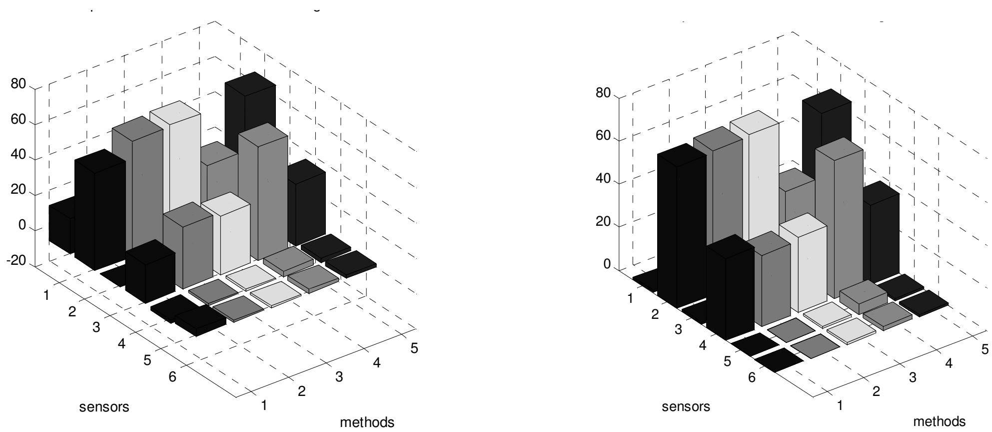


Figure 8 *Comparison between methods of contribution to Phi and I -Index using PZT4 as actuator*

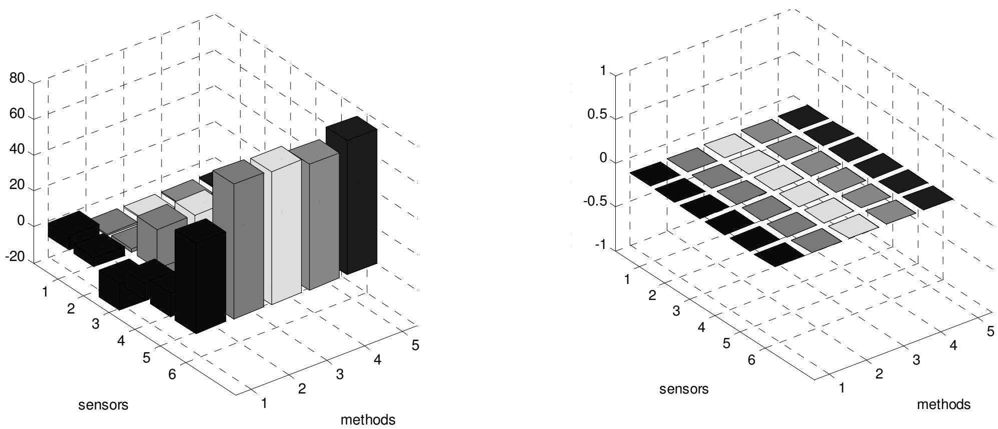


Figure 9 *Comparison between methods of contribution to Phi and I -Index using PZT7 as actuator*

Negative contributions such as those obtained in Figures 6, 8 and 9 for the *phi index* have no physical sense. Also, as is shown for damage detection, also in the damage localization, sensors 3, 6, and 7 have contributions equal to zero since in this case are far from the damage and there is a stringer on the way between the actuator and sensors.

CONCLUSIONS

A novelty methodology in SHM is presented, this include the use of PCA as pattern recognition technique and two indices (*phi* and *I*) for damage detection and 5 methods for damage localization (CDC, PDC, RBC, ABC and DC).

For damage detection, it was showed the utility of *phi vs I* plots, using this kind of plots it is possible to discern the presence of damage in the structure, in general terms, in this figures, the undamaged data and different damages are grouped, it means that similar conditions (damage) share similar space on the figure.

The 5 methods presented allow localizing the damages, in some cases there are less contributions when the CDC method is used.

Results vary for each actuator, obtaining better results in the piezoelectric who are closest to the excitation signal.

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