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Removal of excitations of Bose-Einstein condensates by space- and time-modulated potentials

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We propose that periodically in space- and time-modulated potentials (dynamic lattices) can efficiently remove the excited (the high-energy and large momentum) components of the trapped Bose-Einstein condensates (BECs) and, consequently, can result in efficient cleaning of the BECs. We prove the idea by numerically solving the mean-field models (the Schrödinger equation for noninteracting condensates and the Gross-Pitaevskii equation for interacting condensates of repulsive atoms), and we evaluate parameters and conditions for the efficient removal of excitations.

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I. INTRODUCTION

It is known that *static* in time but periodic in space potentials can substantially modify dispersion characteristics of matter waves in Bose-Einstein condensates (BECs). This alters static and dynamic properties of the wave dynamics of BECs (see, e.g., Refs. [1,2] for theory and numerics, Ref. [3] for experiments, and, e.g., Ref. [4] for a review). In the simplest and most known cases, the static lattice reverses the sign of the curvature for the dispersion curve of the corresponding Bloch mode, which results in the reversion of the sign for the effective mass of the condensate. [The effective mass $m^* = \hbar^2/(\partial^2 E/\partial k^2)$ is related to the local curvature of the dispersion curve $E(k) = \hbar\omega(k)$]. In particular, this allows obtaining bright gap solitons in condensates of repulsive atoms (see Ref. [5] for theory and Ref. [6] for experiments). More recently, the dispersion management was proposed for BECs in *dynamic* lattices, i.e., periodically in space- and time-modulated potentials. The shaken lattices were shown to result in so-called dynamic localization [7] and soliton formation [8]. The blinking lattices also resulted in dynamical localization [9] and in so-called subdiffractive solitons [10]. The dynamic localization as well as the vanishing of diffraction are related to the total or local flattening of the dispersion curves of the Bloch modes of matter waves, which increases the effective mass of the quasiparticles to infinity. The latter effects in BECs are analogs of nondiffractive propagation (self-collimation) of light beams in photonic crystals in optics [11] or of sound beams in sonic crystals in acoustics [12]. The flat segments of the dispersion curve result in the fact that the envelopes of the waves do not change in propagation (in optics or acoustics) or in temporal evolution (in BECs), as the constituting wave components do not dephase during the propagation or temporal evolution.

Contrarily, the strongly *curved* segments of the dispersion curves signify an increase in the dispersion of waves and, therefore, can lead to strong modifications in the field distributions in momentum space. As the flattening occurs on a limited segment of the dispersion curve, the formation of strongly curved segments in neighboring parts of the flat segment is likely, such as happens for the lower branch of the dispersion curve in Fig. 1. The strongly curved segments can result in efficient removal of the corresponding components of the Bloch wave from the condensate of a finite size, as the large curvature corresponds to a small effective mass and equivalently to a large movability of the Bloch wave components. Therefore, it can be expected that the particular Bloch wave components of the condensate rapidly leave the trap and the central momentum components remain unaffected. This situation is illustrated in Fig. 1, which is at the basis of the proposed idea for the removal of the excitations (the cleaning and, perhaps, the cooling) of the condensate.

The solid curves in Fig. 1 illustrate dispersion $\omega(k)$ of the Bloch modes of the noninteracting condensate in the spatiotemporally periodic lattice. The Bloch modes are the coupled states of the wave function of the condensate and of the diffraction components of the condensates on the lattice (see Ref. [10] for details; see also Eqs. (5) and (6) below for details). The proposed idea for the removal of the excitations can also be thought of in terms of a resonant scattering of waves by a dynamical lattice with a certain wave number and frequency, as illustrated by the arrows in Fig. 1. The lattice, in this treatment, perturbs the wave function of the condensate and scatters a part of the wave function into the larger momentum and the higher-frequency states. The scattered waves move with higher group velocities (correspond to larger slopes of the dispersion curve in Fig. 1) and can rapidly leave the condensate. The coupling is efficient between the components of the condensate, which are in mutual resonance (or in phase matching) mediated by the dynamical lattice, as shown by the solid arrows. The nonresonant components are unaffected or are affected weakly, as shown by the dashed arrows. The areas of resonant scattering in Fig. 1 correspond to the segments of the strongly curved dispersion lines of the Bloch modes as follows from the simple geometric considerations.

The above discussion, in terms of strongly curved dispersion curves or in terms of resonant scattering, refers to the idealized case of noninteracting BECs. Below, the idea is substantiated by numerically simulating realistic, i.e., the interacting and trapped, BECs. In this paper, we show and quantitatively describe the cleaning effect in two limits: We start with the simpler case of noninteracting condensates simulated by the Schrödinger equation with trapping and periodic potentials (Sec. III). In the concluding part of this paper (Sec. IV), we show that the phenomenon also persists

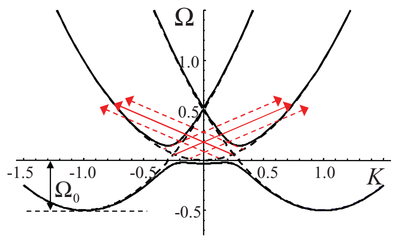

FIG. 1. (Color online) Set of dispersion curves (normalized frequency versus momentum) for the diffraction components of the wave function (dashed lines) and of the Bloch modes (solid lines) for noninteracting BECs in spatiotemporally modulated potentials as obtained from Eq. (6) with $f = 0.1$ and $\Omega_0 = 0.5$. The zero component and two diffraction components of the lowest order are considered. The relative frequency shift Ω_0 and the momentum shift $K = \pm 1$ of parabolas correspond to the modulation frequency and wave number of the periodic potential. The solid arrows illustrate efficient resonant scattering of particular momenta components from the zero-diffraction component of the condensate (central dashed parabola). The nonresonant components remain unaffected or weakly affected by scattering (shown by the dashed arrows). The relatively large detuning from the resonance at $\Omega_0 = 1$ is used in order to increase the values of the resonant momenta ($K \approx \pm 0.35$) for illustrative purposes.

for the interacting (repulsive) condensates by simulating the Gross-Pitaevskii (GP) equation.

II. MODEL

A one-dimensional (1D) BEC, with N particles, subjected to potential $V(x,t)$ in the longitudinal direction x is considered by solving the effective 1D GP equation (see, e.g., Ref. [13] for this 1D case),

$$i\hbar\partial_t\psi = \left[-\hbar^2/(2m)\partial_x^2 + V(x,t) + g|\psi|^2\right]\psi, \qquad (1)$$

The potential consists of a stationary trap and a harmonically periodic (in space and time) modulation,

$$V(x,t) = V_{\text{trap}}(x) + 2V_0\cos(k_0x)\cos(\omega_0t) \qquad (2)$$

The normalizations in Eqs. (1) and (2) are adapted from Ref. [13]; here, $g = \hbar\omega_\perp k_0 a$ is the effective two-body interaction coefficient, $\omega_\perp$ is the angular frequency of the strongly confining radial trap, and a is the interatomic s-wave scattering length ($a > 0$ for a repulsive BEC). Equation (1) is complemented with the normalization condition $k_0\int dx|\psi(x,t)|^2 = N$, which renders ψ adimensional. The paper is facilitated by adopting the following dimensionless quantities:

$$X = k_0x, \quad T = \varpi t, \quad f = \frac{V_0}{2\hbar\varpi}, \quad \Omega_0 = \frac{\omega_0}{\varpi}, \quad G = \frac{g}{\hbar\varpi} \qquad (3)$$

where $\varpi = \hbar k_0^2/(2m)$, in terms of which, Eq. (1) becomes

$$i\partial_T\psi = \left[-\partial_X^2 + V_{\text{trap}}(X) + 4f\cos(X)\cos(\Omega_0T) + G|\psi|^2\right]\psi, \qquad (4)$$

with normalization $\int dX|\psi(X,T)|^2 = N$. In physical units, $p = \hbar k = \hbar k_0 K$ and $E = \hbar\omega = \hbar\varpi\Omega$ are the quasimomentum and the quasienergy of the BEC, respectively. A parabolic trap is considered $V_{\text{trap}}(X) = \Omega_{\text{trap}}^2 X^2/4$ defining the axial trap frequency Ω_{trap}. Absorptive boundary conditions for the matter wave function are imposed at $|X| = X_0$ in order to remove (to recycle) the scattered and rapidly moving components of the field. It is important, for the proposed technique, that the scattered and rapidly moving condensate components leave the trap or are absorbed due to some mechanism. In the corresponding experiment, the recycling can be realized by the traps of a finite depth (with the upper cutoff), such as in those used for evaporative cooling, which justify the absorptive boundary conditions adapted in numerics.

III. NONINTERACTING BECs

First, a condensate of noninteracting atoms $G = 0$ is considered, and the GP degenerates into a Schrödinger equation. The initial conditions are set by either considering a narrow Gaussian distribution of the wave function, which is called deterministic excitation or by a random ensemble of narrow Gaussian excitations, which is called random excitation. In the absence of the periodic lattice, the condensate evolves periodically in the trap with the Ω_{trap} frequency, which corresponds to the trap dipole oscillation frequency. However, symmetric initial distributions lead to the oscillations with the $2\Omega_{\text{trap}}$ frequency, which is the breathing frequency of the trap. In the presence of the dynamic lattice, the periodic evolution is broken. The typical evolution of the wave function of the condensate is shown in Fig. 2 in the coordinate and in the momentum domain.

In temporal evolution, the resonant components of the condensate are scattered efficiently at $K \to K \pm 1$ and eventually are absorbed by the absorbing boundaries at $|X| > X_0$ (or, in the corresponding experiment, the scattered waves leave the trap if the cutoff of the trap is fixed at $X = \pm X_0$). Care needs to be taken that the depth of the trap $V_{\text{depth}} = \Omega_{\text{trap}}^2 X_0^2/4$ is less than the recoil energy of the lattice, which, in normalized variables, is unity. [The recoil energy in the variables of Eq. (1) reads $E = \hbar^2k_0^2/(2m)$]. The wave number of the efficiently scattered mode K_0 can be evaluated from the resonance condition $|K_0| = |(\Omega_0 - 1)/2|$. In particular, the central components $K_0 = 0$ are efficiently removed for $\Omega_0 = 1$, when the central and two first-order components are in resonance at $K_0 = 0$, or speaking in geometric terms, when the three dashed parabolas of Fig. 1 cross at $K_0 = 0$. The removal of the excitations from the BECs is obtained in Fig. 2 for $\Omega_0 < 1$ when the band of the excitation modes, centered at $K < 0$ ($K > 0$), are diffracted to the right (left) directions. We note that the cleaning is also possible for $\Omega_0 > 1$ (not presented here numerically) when the excitation modes with $K < 0$ ($K > 0$) are scattered to the left (right), respectively.

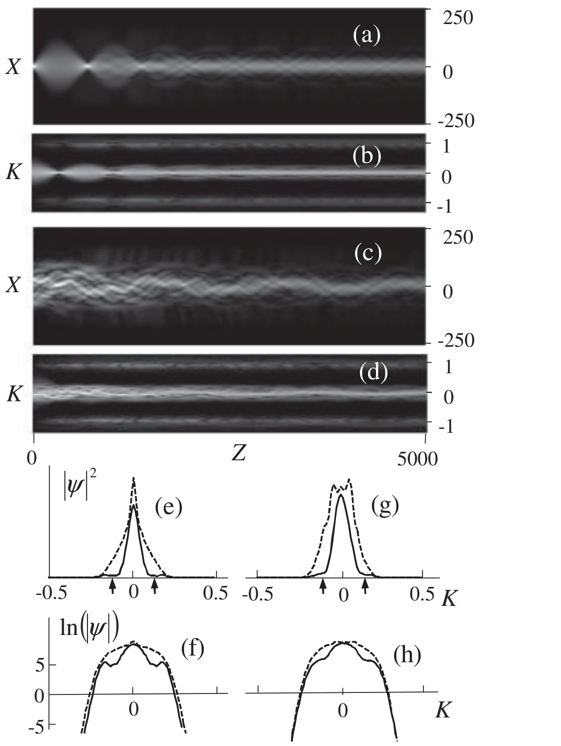

FIG. 2. Evolution of the wave-function density of the trapped condensate in (a) and (c) the space and (b) and (d) the momentum domain in the presence of a dynamic optical lattice as obtained by the numerical integration of Eq. (4). The amplitude of modulation f is linearly ramped until $f_0 = 0.01$ from $T = 0$ to $T = 10^3$ and, then, is kept constant during the rest of calculation. The other parameters are as follows: $\Omega_0 = 0.8$, $\Omega_{\text{trap}} = 5 \times 10^{-3}$, $X_0 = 150$. The initial excitation of the condensate is (a) and (b) deterministic (a narrow Gaussian spot) or (c) and (d) random (random ensemble of Gaussian spots) of the half-width $\Delta X = 5$. The initial (dashed lines) and the final (solid lines) distributions in momentum space are given for (e) and (f) deterministically and (g) and (h) randomly exited condensates in linear and logarithmic scales. The arrows in (e) and (g) indicate the position of the cross points of the dispersion curves of the harmonic components.

In order to understand and to estimate the process of the BEC cleaning, the wave function of the condensate $\psi(X,T)$ is expanded into the central and diffracted harmonic components. The expansion is legitimate when the diffraction components are separated in the momentum domain, i.e., when $\Delta K \ll 1$ where ΔK is the width of the condensate in momentum space,

$$\psi(X,T) = \sum_m \sum_n \psi_{m,n}(X,T) e^{i(mX - n\Omega_0 T)}. \qquad (5)$$

Note, that the expansion (5) considers the spatial and the temporal harmonics. Substitution of Eq. (5) into a linear version of Eq. (4), $(G = 0)$, leads to the following system of equations [14]:

$$[i\partial_T + n\Omega_0 + (\partial_X + im)^2 - V_{\text{trap}}(X)]\psi_{m,n} = f \sum_{p=m\pm 1} \sum_{q=n\pm 1} \psi_{p,q}. \qquad (6)$$

In the absence of the trap $V_{\text{trap}}(X) = 0$, the solution of Eq. (6) results in a set of dispersion curves for the Bloch modes of the condensate $\Omega(K)$ (which is obtained by substitution of $\partial/\partial T \to -i\Omega$ and $\partial/\partial X \to iK$ and by diagonalization of the corresponding set of algebraic equations). In a limit of vanishing coupling, $f \to 0$, the system (6) describes the dispersion of diffraction components (dashed parabolas in Fig. 1). The lattice of nonzero amplitude $f \neq 0$ results in coupled modes (Bloch modes) with the dispersion in Fig. 1 shown by solid lines. The dispersion curves in Fig. 1 are obtained by truncating Eq. (6) to the three lowest modes at or close to mutual resonance: zero mode $\psi_{0,0} \equiv \psi_0$ and two first-order diffracted components $\psi_{1,-1} \equiv \psi_1$ and $\psi_{-1,-1} \equiv \psi_{-1}$, as the excitations of these three resonant modes interact most efficiently and directly participate in the process of the cleaning of the BEC.

IV. RESONANT SCATTERING

Next, I concentrate on the process of scattering the wave components in the vicinity of the cross point between two dispersion curves, e.g., of zero component $\psi_{0,0} \equiv \psi_0$ and of one of the diffracted components $\psi_{-1,-1} \equiv \psi_{-1}$. This alters static and dynamic properties of the wave dynamics In this way, the formation of one sink in the momentum domain is considered, assuming that both sinks are separated in k space. A phenomenologic approach is used in order to simplify Eq. (6): The presence of the trapping potential is neglected, however, it is taken into account that the diffracted components move away and disappear from the trap, i.e., are recycled. As the group velocity of the diffracted components is $v \approx \pm 2$ and as the half-width of the trap is X_0, then, the phenomenologically introduced decay rate is $\alpha = 2/X_0$, and the coupled equation system simplifies to

$$i\partial_T \psi_0(K) = f\psi_1(K) \qquad (7a)$$

$$(i\partial_T - \Delta\Omega_0 + i\alpha)\psi_1(K) = f\psi_0(K). \qquad (7b)$$

Here, $\Delta\Omega(K) = 2K + \Omega_0 - 1$ depends on the distance from the resonance point [from the center of the emerging sink $K_0 = (\Omega_0 - 1)/2$] in the momentum domain.

Equation (7) possesses an analytical, however, cumbersome solution, therefore, the typical results are illustrated and are summarized in Fig. 3. The character of formation of the sink in the density of field component $|\psi_0(K)|$ depends on the ratio between coupling f and decay α (efficiency of recycling). For efficient recycling $\alpha \geqslant f$, the formation of the sink is smooth,

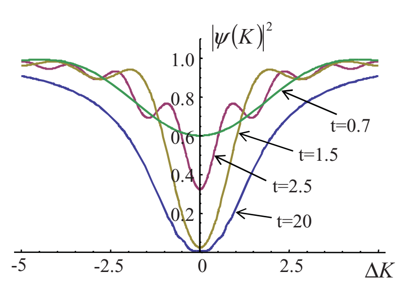

FIG. 3. (Color online) Evolution of the sink of the wave-function density in the momentum domain as obtained from Eq. (7) for $f = 1$ and $\alpha = 0.2$. The profiles at different times $t = 0.7, 1.5, 2.5, 20$ are presented. The resonance is centered on K_0, i.e., $\Delta K = K - K_0$. Note that the width of the sink scales with the parameter f.

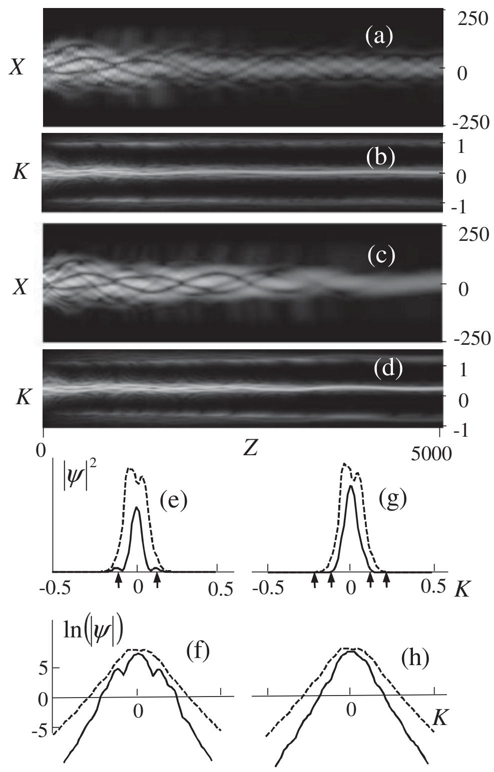

FIG. 4. Evolution of the wave-function density of the trapped condensate in (a) and (c) the space and (b) and (d) the momentum domain in the presence of a dynamic optical lattice with (a) and (b) constant frequency and with (c) and (d) a linear sweep of frequency as obtained by numerical integration of Eq. (4). The parameters in (a) and (b) are the same as in Fig. 2; the frequency in (c) and (d) is linearly increased from $\Omega_1 = 0.6$ at time $T_1 = 0$ to $\Omega_2 = 0.8$ at $T_2 = 5 \times 10$. The initial excitation of the condensate is random. The interacting condensate with $GN = 1.5$ is considered. The initial and final distributions in the momentum space are given for (e) and (f) constant frequency and for (g) and (h) the linear frequency sweep in linear and logarithmic scales. The arrows in (e) indicate the position of the cross points of the dispersion curves (the resonance), and in (g), they indicate the initial and final positions of the cross point.

however, for $\alpha \leqslant f$, the revivals and the fringed structure of the sink develop at the intermediate stage of the evolution $f^{-1} \leqslant T \leqslant \alpha^{-1}$ (Fig. 3). Generally, the width of the sink scales as $\Delta K \approx f$, and the characteristic time of the formation of the sink is $T_0 \approx f^{-1}$. For long times $T \geqslant \alpha^{-1}$, the radiated field components are already well recycled, and again, the sink is smooth.

V. INTERACTING BECS

The interacting condensates (the ensembles of repulsive atoms $G > 0$) can also be efficiently cleaned as Fig. 4 evidences. In fact, the interaction does not substantially alter the process of cleaning in all calculated cases. In the calculations shown in Fig. 4, the strongly interacting limit is considered as the healing length [approximately the half-width of the dark solitons visible in Figs. 4(a) and 4(c)] is significantly smaller than the size of the condensate itself. Concretely, the nonlinearity parameter was $GN = 1.5$, whereas, the strongly interacting regime starts from $GN \geqslant \Omega_{\rm trap}^{1/2}$.

Finally, an improvement in the performance of the above-described cleaning process is also possible. In the best case, the purely periodic lattices remove a finite band from the energy and momentum spectra of the condensate as the resonances in the momentum space are of finite width. In order to remove a broad- or even a semi-infinite band of excitations (to obtain the low-pass cleaning), the parameters (essentially the frequency) of the periodic potential can be varied slowly in time. The cross point between the two dispersion curves of the harmonic components (equivalently, the position of the curved segment on the dispersion curve of a Bloch mode) then moves over a finite distance in momentum (and energy) space, respectively. This allows: (i) preventing the revivals and the fringing of the sink as the resonance condition is continuously violated for the diffracted wave components if the sweep occurs with a particular velocity. In the latter case, the backscattering for the central wave components is suppressed; (ii) to mechanically broaden the sink, as the resonance point moves through a particular range in momentum space. The minimum velocity, roughly speaking, is such that, during the formation of the sink (characteristic time of formation is $T_0 \approx f^{-1}$), the sink moves over its own half-width in momentum space $\Delta K \approx f$, which results in the estimation $dK/dT \approx \Delta K/T_0 \approx f^2$.

In numerical integrations, the frequency of the lattice was linearly increased (and decreased in calculations not shown here) in order to move the resonance point over a substantial region of the momentum space. The results in Fig. 4 compare the cleaning in periodically time-modulated lattices [Figs. 4(a) and 4(b)] and the lattices with a sweep of temporal frequency [Figs. 4(c) and 4(d)] and evidence a more efficient cleaning in the presence of sweep.

VI. CONCLUSIONS

To conclude, a different method has been proposed and numerically demonstrated for the removal of the momentum and energy components from the trapped BECs by dynamic (periodic in space and time) lattices. The sweep of the frequency of the lattice enhances the cleaning effect and can result in a complete low-pass cleaning.

The method of cleaning is demonstrated for the 1D condensates, however, in the case of the noninteracting condensates, can be straightforwardly generalized to the two-dimensional (2D) and three-dimensional cases by simple factorization of the wave function [15]. This generalization, however, is not straightforward for the interacting condensates and for the geometries of the lattices, which are different from the square ones (e.g., hexagonal and of n-fold symmetry in the 2D case).

The enhancement of range and efficiency of cleaning by using a linear sweep of the frequency has also been demonstrated. The process of cleaning could be optimized by choosing a particular shape of the frequency sweep: In particular, as numerical simulations show (not presented here), the double sweep (the increase and then a subsequent decrease in the frequency of the lattice) can lead to a more efficient cleaning. Also, the repetitive cleaning process is possible.

It could be noted that the signatures of the analogous effect of spatial cleaning were recently demonstrated experimentally in optics [16], giving additional weight to the present paper. The optical case corresponds to the noninteracting and nontrapped condensate, as described by the linear Schrödinger equation.

The question, however, remains open whether the proposed process only removes the coherent (Bogoliubov–de Gennes) excitations of the wave function or is also applicable for the cooling of condensed atomic ensembles, i.e., for removing the thermal part of the condensed atomic ensemble of nonzero temperature or even for condensing the atomic ensembles. The answer is, however, beyond the scope of the GP model.

ACKNOWLEDGMENTS

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[14] A nonlinear analog of Eq. (6) is also possible for the interacting condensate, which contains additional terms of self- and cross-nonlinear interactions.

[15] If the 2D profile of the dynamic potential can be expressed as $V(x,y,t) = V_x(x,t) + V_y(y,t)$ (e.g., square lattices), then the solutions of the 2D GP equation [analog of Eq. (1)] can be searched in the factorized form $\psi(x,y,t) = \psi_x(x,t)\psi_y(y,t)$, which, in the linear (noninteracting condensate), allows for completely separating the evolution for $\psi_x(x,t)$ and $\psi_y(y,t)$.

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