

# Digital Phase Estimation Method based on Karhunen-Loève series expansion for Coherent Phase Diversity Detection

Josep M. Fabrega, *member OSA*, Josep Prat, *member OSA*  
 Universitat Politècnica de Catalunya (UPC), Dept. Signal Theory and Communications,  
 Jordi Girona 1, D4-D5, 08034 Barcelona (Spain).  
 E-mail: jmfabrega@tsc.upc.edu

**Abstract:** A novel phase estimation method for coherent systems is presented and demonstrated. It outperforms 5.5 times the phase noise and spectral width tolerance respect to the other methods.

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## 1. Introduction

Homodyne coherent optical reception is considered the ideal method to detect ultra dense Wavelength Division Multiplexing (udWDM) optical signals, because of its excellent wavelength selectivity, and receiver sensitivity. Since they offer OSNR benefit and compensation of signal impairments, they are also considered as a key contender for future 100 GbE transmission. However, its implementation has not been commercially deployed in part because of its stringent requirements in terms of laser spectral linewidth [1].

Along a different line, fast digital signal processing (DSP) has become available in the past years and now can be successfully applied to optical transmission systems. In particular, coherent detection can benefit from these new technologies. Precisely, carrier synchronization can be performed by digital phase estimation techniques when using a phase diversity detection scheme. They allow a free running local oscillator, which hasn't to be phase-locked by any optical phase-locked loop. Some reception approaches combining both, phase diversity architecture and digital signal processing, have been proposed in the last years obtaining similar results [2-4]. Up to now, a popular technique is the phase estimation based on Wiener filter [2], which can achieve good results with a simple digital filter. Another option, more complex, is to use digital regenerative frequency dividers [3]. A third option would be to use a Viterbi&Viterbi algorithm [4], on which the complexity is even higher and the results are also similar to the Wiener filter.

In this paper, for the first time, a phase estimation method based on Karhunen-Loeve series expansion is presented. It can be implemented using standard DSP devices, since its complexity is not very high. Also, it clearly outperforms the Wiener filter reference algorithm .

## 2. Receiver Scheme

A scheme of the receiver to be used is a typical intradyne architecture [2], shown in Fig. 1. There, the optical input signal is interfered with a free-running optical local laser in a  $2 \times 4$   $90^\circ$  hybrid. The output signals of the hybrid are then detected by two balanced detectors, and the I and Q outputs are digitized by an Analog-to-Digital Converter (ADC). After, a Digital Signal Processing (DSP) module performs the phase estimation and data detection.

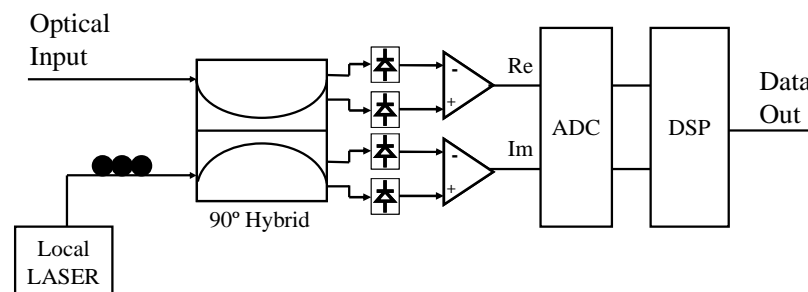


Fig. 1 Scheme for a standard intradyne receiver.

### 3. Phase estimation algorithm

Laser phase noise characterized by a certain spectral width ( $\Delta\nu$ ) can be modeled as a Wiener process ( $w(t)$ ), and it can be expanded into a Karhunen-Loève (KL) series form:

$$w(t) = \sum_{n=1}^{\infty} c_n \varphi_n(t) \quad (1)$$

where  $\varphi_n(t)$  is a set of orthonormal functions (eigenfunctions) in the interval  $(0, T)$  and  $c_n$  are the series coefficients, being random variables. For the Wiener process case, it is shown that [5]:

$$\varphi_n(t) = \sqrt{\frac{2}{T}} \sin(\omega_n t) \quad (2)$$

$$c_n = \sqrt{\frac{2}{T}} \int_0^T w(t) \sin(\omega_n t) \quad (3)$$

where  $\omega_n = \sqrt{\frac{2\pi\Delta\nu}{\lambda_n}} = \frac{(2n+1)\pi}{2T}$ , being the eigenvalues:

$$\lambda_n = E\{c_n^2\} = \frac{8T^2\Delta\nu}{(2n+1)^2\pi} \quad (4)$$

Since larger eigenvalues are those of lower  $n$ , the series can be truncated at a relatively short number of terms,  $M$ . Precisely,  $\lambda_1$  is more than 10 times  $\lambda_5$ , then truncating at  $M = 5$  should be enough. Please note that for  $M = 5$  and  $M = 15$  the output phase error deviation of the estimator is almost the same, independently of the time interval squared per spectral width product. This is shown in Fig. 3 (left), for the proposed performances evaluation.

In order to estimate and cancel the laser phase noise, what we propose is to process block by block the received phase. Thus, a priori, the observable interval length  $T$  is known, and the eigenfunctions can be easily calculated. Consequently, we can estimate the phase noise as:

$$\hat{w}(t) = \underline{c}^T \underline{\varphi} = [c_1 \ c_2 \ c_3 \ \dots \ c_M] \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \\ \vdots \\ \varphi_M(t) \end{bmatrix} \quad (5)$$

From (4), we can see that the lower the interval squared per spectral width product is, the lower the eigenvalues are. Also, inside a block, the phase noise will have a limited variance of only  $2\pi\Delta\nu T$  [5]. So, for small blocks and same spectral width, the estimator will work better. That is the reason why we propose to work with blocks of only 1 symbol.

From one block to another, phase noise is expected to be highly correlated. Thus, the series coefficients also present small changes. So, coefficients are going to be calculated by using an adaptive algorithm. In our case we will use the Least Mean Square (LMS) method.

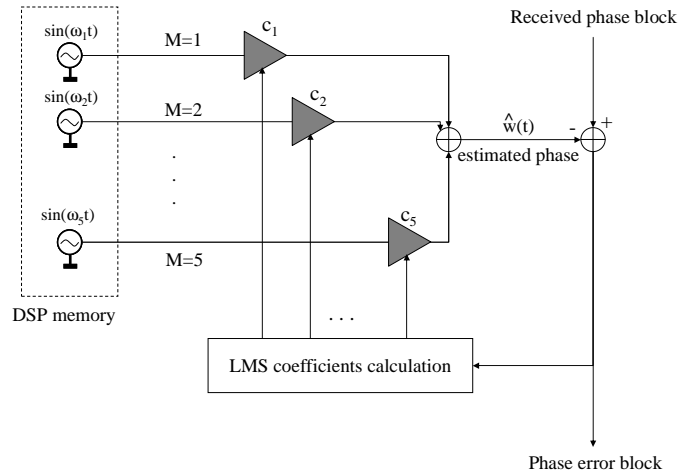


Fig. 2 Block diagram of the phase estimation algorithm.

The estimation algorithm diagram used in the following section is the one depicted in Fig. 2. There, the  $M$  sinusoidal waveforms needed for phase estimation are stored in the DSP memory. Next, a coefficient (coming from the LMS estimation block) is applied to each of the waveforms. The sum of all the waveforms multiplied by the coefficients results to be the estimated phase, and the phase error is used as the LMS input.

#### 4. System performances

In order to evaluate the performances of the proposed receiver architecture, firstly we simulated a BPSK data stream running at 10 Gbps with variable phase noise, from 100 kHz to 10 GHz. It was used to compare the proposed algorithm performances respect to a Wiener filter with a lag of 10 symbols, as described in [2]. In both cases we simulated a data stream of  $2^{20} = 1048576$  symbols, and the received signal was resampled at 16 samples per symbol. Then, the digitized I and Q signals were used to reconstruct the received optical signal and its phase was packet into blocks of 16 samples (one symbol). We used the phase error deviation after phase estimation as a measure for the quality of the algorithm. Results are shown in Fig. 3.

Regarding the term where we truncate the KL expansion, in Fig. 2 we can see that there is almost no difference on the estimation for  $M = 5$  and  $M = 15$ ; so we decided to keep it at 5.

From Fig. 3, it is shown that when using the KL series estimation for a  $13^\circ$  maximum phase error deviation (limiting to a BER of  $10^{-10}$  [6]) we are limited working at a maximum spectral width of 5 % of the bit rate. Also, if Forward Error Correction (FEC) codes are used and a BER of  $10^{-3}$  is operable with only 7 % overhead [7], and a maximum phase error deviation of about  $28^\circ$  is allowed [6], leading to a maximum spectral width of 25 % of the bit rate.

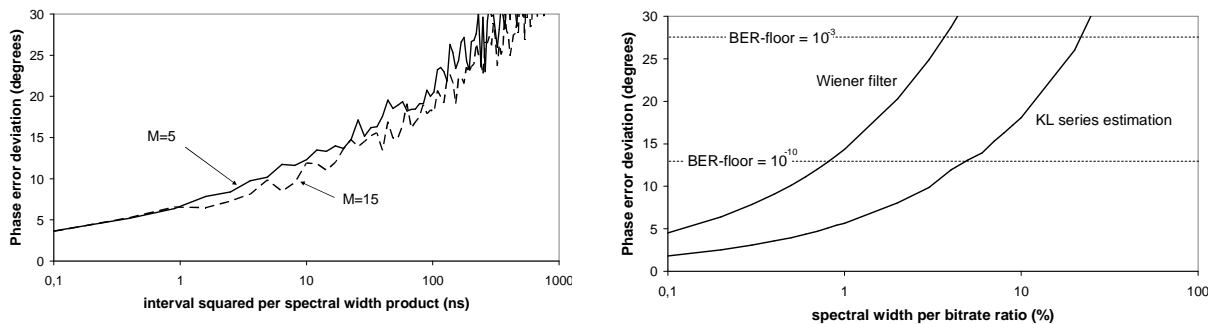


Fig. 3. Left: Phase error deviation as a function of time interval squared per spectral width product. Right: Phase error deviation as a function of the spectral linewidth per bit rate ratio.

#### Conclusions

We have proposed and demonstrated a novel phase estimation algorithm. Due to its spectral properties, it remarkably increases the phase noise tolerance of conventional coherent homodyne receivers, up to a linewidth of 5 % bit rate for a BER of  $10^{-10}$ , and it also avoids the use of oPLL. Precisely, it outperforms by 5.5 times the phase noise tolerance of the Wiener filter estimate.

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