Principles of Investing

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By A. W. Moser

The value of a security at any given time may be regarded as a combination of such variable factors as credit standing of the debtor, the conditions of the financial market, the rate of interest carried by the security, maturity date and the price to be paid on redemption. Among these variables the credit standing of the debtor occupies a peculiar position, inasmuch as it does not itself enter into the calculations as a numerical quantity, but merely expresses the expectation that the borrower will be willing and able to repay the values borrowed when due, and the degree of that expectation is mostly a matter of estimate only. While this implies that there is no absolute certainty of repayment and hence that the purchaser of a security or grantor of credit exposes himself to a certain pecuniary risk, however small, it will also be found that the other valuation elements are usually so fixed as to bear a definite relation to the credit risk. It is particularly the rate of interest to be earned on an investment that indicates diverse investing conditions. In other words, the credit element with regard to a given security being to a large extent determined by the guaranties, physical and moral, behind an issue, the interest rate an investor considers as an equitable return on his capital will normally vary, all other things being equal, with the value of those guaranties, or with the degree of safety of the principal and interest thereon. Whether or not a security carries definite redemption provisions does not alter that fundamental principle; there can only be differences of degree.

It follows, therefore, that the rate of interest practically includes a certain weighting to cover the so-called credit risk, which in its broader meaning is the possibility of both principal and interest, or any part thereof, becoming impaired at one time or another. That weighting, consequently, has the characteristics of an insurance premium and ought to be so fixed, in case that it were possible mathematically to regulate operations, that the value of the weightings would become in the long run equal to the losses that occurred, as the premiums charged by an insurance company make up for its losses. Such an exact procedure can not, of course, take place in the investment and credit-granting field, because, if for no other reason, the risk involved is relatively much smaller, generally speaking, although this does not mean that it is always negligible. The extent of the increase of the interest rate on account of the credit risk, however, is probably in most instances the outgrowth of mere estimates, based on personal experience, and may prove more or less sufficient according to the ability of an investor correctly to gauge the soundness of a security or of the credit to be granted.

In cases of actual impairment, it would be a problem of simple statistics for an investor to determine the relative number of his losses, their range, the proportion of their total to his total investments and the time of their occurrence. Barring all minor losses occasioned by market fluctuations, etc., it is no doubt safe to state that individual losses are relatively infrequent, on the one hand, and heavy, on the other hand. This is due to the fact that a man with funds for investment at his disposal, instead of buying a few hundred dollars' worth of securities of each of a correspondingly large number of different debtors will be more likely to invest larger sums in the securities of relatively few concerns, i. e., the number of different issues in a portfolio is usually small compared with the total amount. To carry the idea a little further by assuming that the weightings referred to are correctly fixed in relation to the risk they are intended to cover, one will at once find it apparent that unless a given portfolio be large and extremely diversified it would be fallacy to expect a balancing of surplus interest received and losses within short intervals, such as a year, for instance. Hence relatively few but heavy losses.

If it be conceded that the returns in the form of interest on an investment contain a weighting specifically included, although not segregated and usually not conceived as such, in order to cover an existing risk, it may justly be argued that this surplus interest should be husbanded with the purpose of compensating for losses when they occur.

A similar situation exists, of course, in most transactions involving the extension of credit, not only when buying securities. There is a difference in form to be noted as far as purely commercial credits are concerned, inasmuch as the rate of interest in the latter case is regulated more by customs of trade than by degree of safety, the latter factor, however, being taken care of by other means, such as terms, for instance.

In view of the circumstance, as follows from the foregoing remarks, that to investing and to credit transactions in general attaches a certain possibility of loss, it may truly be said that they contain an element of hazard. The question may therefore be asked whether it would be useful to apply to the subject certain laws of the theory of probability. If by doing so some additional knowledge could be gained with respect to the importance of certain principles often put forth, or a basis established for comparisons, the procedure would have proved its usefulness. Thus if there is value in a well known rule that diversification in investments is advantageous to safety, the same rule would gain in value if some measure of its influence could be obtained.

In order that the subject may be treated as a problem of probability it will be necessary, in the first place, to assume that the possible ways of occurrence of the losses be independent of each other. This means that the impairment of one security should in no way affect the degree of safety of other securities contained in a portfolio, and also that there be no systematic influences at work. These requirements will rarely be completely fulfilled in practice, as bad business for one company may easily affect in an adverse way the profits of other companies represented in a portfolio and thereby the value of their securities; and the consequences of systematic influences, such as those arising from a widespread, general depression, can only with difficulty be avoided. The possibility of such contingencies, however, shall for the purpose of the present investigations be disregarded as of negligible importance.

Accordingly, let the weightings in the form of interest be so regulated in extent that they will ultimately cover the losses normally to be expected in a given portfolio within a sufficiently long interval.

It is desirable, at this point, to insert definitions of a few terms used in the theory of probability.

"Mathematical expectation" is defined as the product of an expected gain in actual value and the mathematical probability of obtaining such a gain. The danger of loss may in this case be regarded as a negative gain. Thus if a person may expect a gain A from an event F whose probability of happening is p, then

$$E = pA$$

will be his mathematical expectation. For $p = 1/5$, $A = 100$, for instance, which means that the event will occur once on an average in each 5 trials, E would result as 20. E may also be considered as the price or equitable premium that a person (player) would have to pay to another person (contractor) who offers the prize A on the result of the event F, in order to be afforded the privilege to take the chance. In the above case, the player will have paid 100 units in 5 trials, thus making up for the prize he could expect in the same number of trials. So the player may be said to face two possibilities when taking the chance: he can either make the net gain $A-E$ with the probability

$$p(A-E)=p(1-p)A=pqA=qE=R,$$
$$\text{where } q=1-p,$$

or lose the premium paid with the probability q,

$$qE=R'.$$

R and R' are equal as to amount, in the example given $=4/5\times20=16$. In a similar position he finds himself the contractor, R' being the expression for his gain and R the expression for his loss.

The quantity R has variously been termed "average risk" or "mathematical risk."

The equation $\frac{R}{E}=\frac{R'}{E}$ indicates the "relative risk," or the intensity of the expectation of gain and of the expectation of loss, respectively, without regard to the importance of the sums at stake. Since $\frac{R'}{E}=q$, the relative expectation of loss, or the danger of the game, increases in the same ratio as the probability of loss.

The practical significance of R and R' is as follows: R' is the equitable premium which the player would have to pay to the contractor referred to, or to another contractor, for the purpose that the latter compensate him for the losses he actually suffers (by losing the premium paid E), while R represents the premium by means of which the contractor on his part could insure himself against loss (when having to pay the prize A).

However, such a contract would not cover the player against every possible risk. As a consequence, merely his expectation of loss as well as, in equal degree, his expectation of net gain will change, i. e., diminish, since his payment is now $E+R'$, his possible net gain

$$R_1=p(A-E-R')=q^2E$$

and his expectation of loss equally $R'_1=q\ qE=q^2E$, because in the event of loss he will only get back his original payment E, but not the additional premium $R'=qE$.

To cover himself against this loss, the player would have to pay the new premium R'_1, whereupon his expectation of net gain would amount to

$$R_2 = p(A - E - R' - R'_1) = q^3E$$

in the face of an equally large expectation of loss

$$R'_2 = q^3E.$$

Continuing so to insure himself, the player could reduce his risk as far as he wished until nearing the meaningless limit where he pays a total amount A and certainly recovers an equally large sum, since his payments at the limit are expressed by

$$E + R' + R'_1 + R'_2 + \ldots\ldots = E(1 + q + q^2 + \ldots\ldots) = \frac{E}{1-q} = \frac{E}{p} = A.$$

If the contractor extended his activities to a number of mutually exclusive events $F_1, F_2, \ldots\ldots F_n$, whose respective probabilities of occurrence conform to the condition

$$p_1 + p_2 + \ldots\ldots p_n = 1,$$

by promising prizes $A_1, A_2, \ldots\ldots A_n$ on the occurrence of the corresponding events the total premium would be

$$E = p_1A_1 + p_2A_2 + \ldots\ldots p_nA_n,$$

whereupon the player's expectation of net gain, or his average risk, would amount to

$$R = \Sigma p_g(A_g - E)$$

if A_g designates all those prizes which are larger than E.

Conversely, his expectation of loss would be

$$R' = \Sigma p_s(E - A_s)$$

if A_s denotes all prizes smaller than E.

It further follows

$$R - R' = \sum_1^n p_iA_i - E\sum_1^n p_i = E - E = 0.$$

Hence $R = R' = \frac{1}{2}\sum_1^n p_i \left| A_i - E \right|$.

In an analogous manner the average risk of the second order could be derived:

$$R_1 = R'_1 = \frac{1}{2}\sum_1^n p_i \left| A_i - E - R' \right|, \text{ etc.}$$

It is important to note that with one and the same premium E the risk R may be quite different, according to the relative amounts of the individual prizes and the probability of obtaining them.

For example: Let a prize be offered of as many dollars as each face of a die bears points. The equitable premium to be exacted from the player amounts to

$$E=\tfrac{1}{6}(1+2+3+4+5+6)=\$3\tfrac{1}{2},$$

the risk of the game is

$$R=\tfrac{1}{6}[(4-3\tfrac{1}{2})+(5-3\tfrac{1}{2})+(6-3\tfrac{1}{2})]=\tfrac{1}{6}(\tfrac{1}{2}+1\tfrac{1}{2}+2\tfrac{1}{2})=\$\tfrac{3}{4},$$

and the relative risk $\frac{3}{4}:3\frac{1}{2}=\frac{3}{14}$, so that the player's premium will amount to $\frac{3}{14}E$, or $21-{}^{3}/_{7}\%$ of $E=\$\frac{3}{4}$, to protect himself against loss of his original payment E.

His total outlay will then be $\$3\frac{1}{2}+\frac{3}{4}=\$4\frac{1}{4}$, which is only exceeded by the prizes of \$5 and \$6. Hence the risk is reduced to

$R_1=\frac{1}{6}(\frac{3}{4}\times 1\frac{3}{4})=\$^{5}/_{12}$ and the relative risk to

${}^{5}/_{12}:4\frac{1}{4}={}^{5}/_{51}$. The insurance to cover the possible loss of the premium $R=\$\frac{3}{4}$ will then be ${}^{5}/_{51}$ or $9^{41}/_{51}\%$ of $\$4\frac{1}{4}=\$^{5}/_{12}$, etc.

Let now a general problem be formulated as follows: Establish a measure for the risk created by the possibility that a given capital A outstanding at the time t may not be restored unimpaired at the date of redemption n years hence.

It is obvious from the outset that any part of the principal A may become irretrievable by the end of the n years' period, so that part must be considered a loss. Thus a loss may constitute, when expressed in per cent. of A, rounded up or down to the nearest whole percentage, 0%, 1%, 2%, ... or 100% of A, each one of these possible events being associated with a certain probability p_i. Since one of the events must necessarily take place, it follows that

$$p_0+p_1+p_2+\ \ldots\ldots\ p_{100}=1$$

and $$E=(p_1+0.01+p_2 0.02+\ \ldots\ldots\ p_{100}\times 1)\,A=pA.$$

The probabilities p_1, p_2, ... are unknown quantities; nor do they need to be known, since the expression in parenthesis may be replaced by an average probability p, as indicated, this average value being either known from experience or determinable by statistical methods. It designates the most probable loss to be expected in the course of a given period. If the principal A represents a single debt and if p be understood as the most probable *annual* average loss, then the total loss in n years will be npA. This may occur at one time, perhaps in the course of the n^{th} year, while the preceding $n-1$ years would not have brought any loss. If on the other hand the principal is made up of many smaller sums, the total loss npA may also be conceived as the result of a number of part losses having taken place at different dates, each part loss either representing a total loss of a single one of the smaller sums or being composed of a number of part losses of such shares. In the second case there is a chance for the losses to be more or less evenly distributed over the given term.

The probability p may be assumed as standing in inverse proportion to the ability of an investor safely to manage his investments; hence if this ability be taken as constant, p can also be regarded as constant for any year within a specified period. A measure of the risk to be considered is then the average risk

$$R=\tfrac{1}{2}\sum_{1}^{n} p_r \left| A_r - E_r \right|$$

or, when only either positive or negative value combinations have to be considered

$$R=\sum_{1}^{\nu} p_r\,(A_r - E_r),$$

in which expression A_r and E_r indicate the outlays and receipts, respectively, which have taken place during the term under consideration, both discounted to the moment t. While the principal A is entitled to a fair interest, the corresponding credit may be regarded as balanced by the dividends due on the security.

To illustrate the problem two examples are presented.

Let g designate the weighting supposedly included in the annual interest and let g be assumed to be payable at the beginning of each year, while a loss would be charged off as of the end of the year. *No sinking fund is provided for.* Recalling that $E=pA$, $R=p(A-E)=qE$ and $p+q=1$, then, if the term is one year,

$$R_{(A)}=p(Av-g),$$

and if the term is $\nu=n$ years,

$$R=_{(A)}\sum_{1}^{n} p_r(Av^r-gv^{r-1}).$$

For $A=1000$, $n=10$ years, $p=\text{constant}=0.01$ and $i=5\%$, then $E=g=10$ and

$$R_{(A)} = \frac{1}{100}\left[1000\ a_{\overline{10}|} - 10(a_{\overline{9}|}+1)\right]$$

where $a_{\overline{n}|} = \frac{1-v^n}{1-v}$ = present value of an immediate annuity of one per annum payable at the end of each year;

$$R_{(A)} = \frac{7721.73-81.08}{100} = 76.4065, \text{ or } A\times 0.0764,$$

$$\frac{R_{(A)}}{E} = \frac{76.40}{10} = 7.64,$$

i. e., the risk for the whole 10 years' period amounts to 7.64 times the annual weighting g.

For the second example, the same data as above are assumed, except that a sinking fund is formed, to which no further risk is attached by annual contributions of $\frac{A}{s_{\overline{n}|}}$, the principal remaining intact throughout the term. The amount at stake is therefore gradually reduced, being at any time equal to the difference between the sum outstanding and the sinking fund, or

$$R_{(A\text{-}S.A.)} = \sum_{1}^{10} 0.01\left[\left(1-\frac{s_{\overline{r}|}}{s_{\overline{n}|}}\right)1000\ v^r - 10\ v^{r-1}\right]$$

$$= \sum_{1}^{10} 0.01\left(1000\ v^r \frac{a_{\overline{n}|-r}}{a_{\overline{n}|}} - 10\ v^{r-1}\right)$$

r	$\frac{a_{\overline{n}\|-r}}{a_{\overline{n}\|}}$	$1000\ v^r \frac{a_{\overline{n}\|-r}}{a_{\overline{n}\|}}$
1	0.9205	876.70
2	0.8370	759.20
3	0.7493	647.20
4	0.6573	540.80
5	0.5607	439.30
6	0.4592	342.60
7	0.3527	250.60
8	0.2408	163.00
9	0.1233	79.50
10	......	......
		4098.90

Hence $R_{(A.\text{-}S.F.)} = \frac{4098.90-81.08}{100} = 40.17$, or $A\times 0.04017$

$$\frac{R_{(A.\text{-}S.F.)}}{E} = \frac{40.17}{10} = 4.017,$$

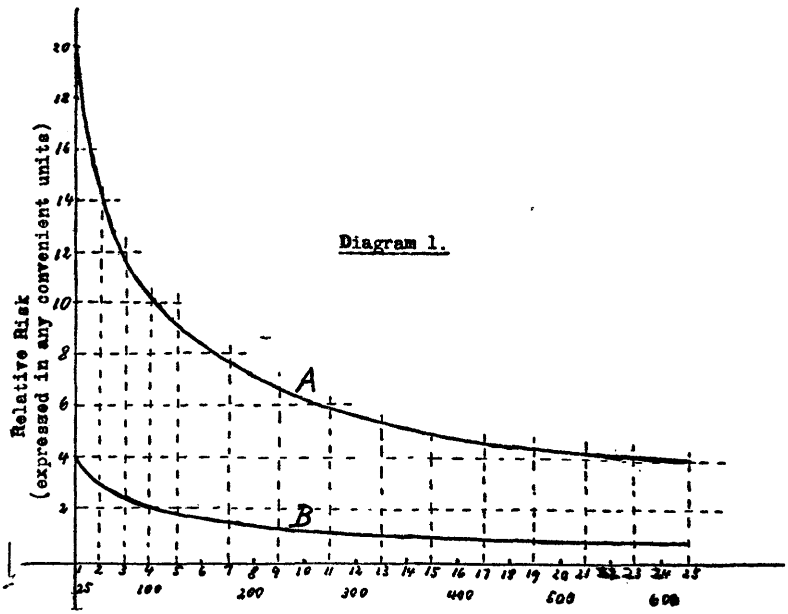
which means that the whole 10 years' risk would be covered by a single premium of 4.017 times the annual weighting g.

The two results reveal that in the case of a sinking fund the risk has been reduced 47.5%, or nearly 50%, as one would also expect on the ground of a general reasoning.

If there be a number of securities or outstanding credits, all identical as to class, term and amount, and consequently with the same average risk per unit of capital, then it may be shown that the total average risk is given by the equation

$$R=\sqrt{s\times A\overline{R}_0^{2}}=AR_0\sqrt{s}$$

and the relative risk $\dfrac{AR_0\sqrt{s}}{sgA}=\dfrac{R_0}{g\sqrt{s}}$ (see diagram 1).



Outstanding units of principal or accounts (all alike)

Curve A corresponds to numbers 1, 2, 3, ...
" C " " " 25, 100, 200, ...

The pace of the risk in diminishing is considerably slowed down with increasing number s, the relative risk nearing zero for limit $s=\infty$.

Suppose the s securities or credits equal in all respects except the amounts A_1, A_2, ... A_s, then

$$R=R_0\sqrt{A^2_1+A^2_2+\ldots\ldots A^2_s}$$

With regard to these formulæ some important deductions may be drawn:

(a) The total average risk of a group of equal securities or credits is obtained by multiplying the average risk of a single one with the square root of their number. The relative risk, which is derived by dividing the former with the total amount of premiums, is inversely proportional to the square root of the number s, as this number appears as factor in the denominator, and may consequently be reduced to any desired limit by increasing s.

It is this sentence that contains the mathematical basis for the correctness of the rule of diversifying investments, often recommended as favorable to safety, whereby the diversification should be of such a nature as to leave the individual contracts independent of each other.

These considerations also bring out the fundamental strength of the so-called investment trusts. In fact, to apply efficiently the principle just referred to, the investment trust is fundamentally in a far better position than the average individual investor, due to its greater material resources, representing a combination of those of many investors, and due also to the circumstance that with this advantage is likely to be coupled the one of the management more expert in the particular field, in general, than the average investor. An ordinary investor, in order to fare as well, should have something equivalent to those qualities inherent in investment trusts, such as specialized knowledge in a certain field, ability and opportunity to participate actively in the management of an enterprise, etc. On the other hand, of course, investment trusts also could not escape, should they fail to recognize it, the operation of that mathematical law as far as their purpose as a purely investing business is concerned.

(b) The sum of the squares of the expression

$$R_0\sqrt{A^2{}_1+A^2{}_2+\ .\ .\ .\ .\ .\ A^2{}_s},$$

if the total of $A_1+A_2+\ .\ .\ .\ .\ .\ .\ A_s$ be constant, reaches a minimum when $A_1=A_2=\ .\ .\ .\ .\ .\ A_s$, i. e., when the individual investments or credits are of equal amount. Inequalities in this respect increase the risk the more sharply single sums deviate from the average.

In a previous example the average risk for a given security was found to be $A \times 0.0764$. Suppose now that there are 100 such securities or credits, each of \$1000. Then

$$R = 1000 \times 0.0764\sqrt{100} = 764.$$

In another instance with the same total of \$100,000, but composed of

1	at	\$20,000
2	"	10,000
5	"	5,000
20	"	1,000
2	"	500
70	"	200
100		

the value of R will result as

$$R = 0.0764\sqrt{748,300,000} = 2090,$$

and if it were all one sum,

$$R = 7640,$$

i. e., the risk has grown to $\sqrt{s}$ times the value that resulted in the case of uniform diversification.

The premiums included in interest rates under the previous assumptions will, if properly husbanded, provide the investor with the means of absorbing the losses that may occur according to the most probable hypothesis. Any other amount of loss, greater or smaller, is to be expected with less probability. In so far as the actual losses do exceed the most probable amount, they must be covered by other means than the weightings g, or a corresponding deficiency of regular income will result. On the other hand, the average investor or money lender is naturally interested in maintaining his annual income as constantly as possible with regard to the units of capital employed. This in fact constitutes, if not an obligation, at least the tendency of any financial organism, and may assume in certain cases the features of a real necessity, thus acting as inducement to adoption of operating policies that will best assure the regularity of the returns.

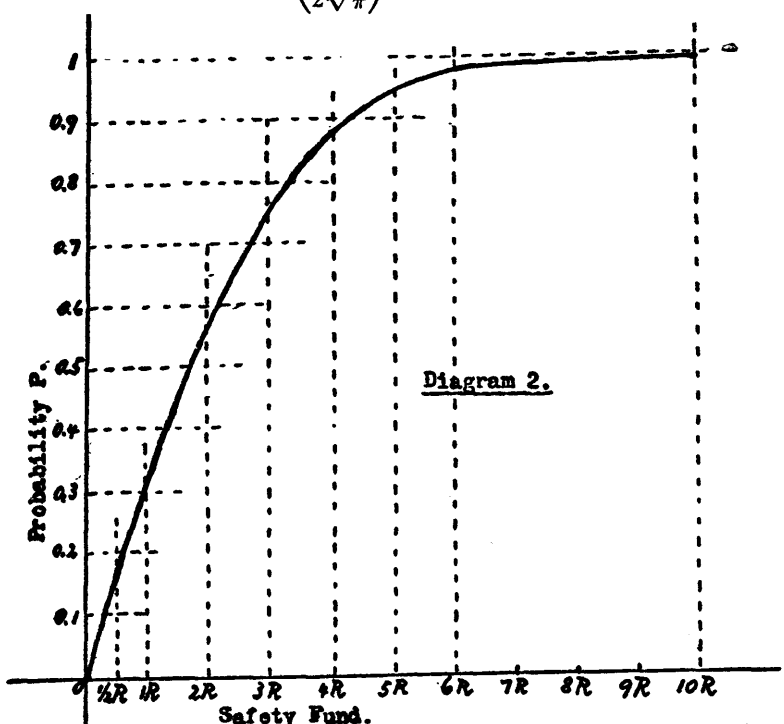
It may, therefore, prove of interest to examine the probability P that the total deviation of the actual losses from their most probable value, namely spA, will not exceed a given limit, say k times the average risk R. The theory of probability teaches that a connection exists between the values of k and P such that

$$P=\frac{2}{\sqrt{\pi}}\int_0^{\frac{k}{2\sqrt{\pi}}} e^{-t^2}dt=\phi\left(\frac{k}{2\sqrt{\pi}}\right),$$

where $t=\dfrac{l}{\sqrt{2\pi\, spq}}$ and l = a limit of the deviations, the symbols s, p and q corresponding to previously established definitions.

In the light of this relation the significance of R is that the loss (in excess of spA), or the total excess deviation, within a given period, if such a loss does occur, will not exceed the amount of kR with the probability

$$R_P=\phi\left(\frac{k}{2\sqrt{\pi}}\right) \text{ (see diagram 2)}$$



As the curve ascends rapidly for values of the abscissa up to $4R$ or $5R$ and then flattens out, it follows that any part of an extra fund in excess of $4R$ to $5R$ is fast losing in importance as a precautionary measure against losses. Hence, unless created for other purposes, reserves exceeding a certain determinable limit become rapidly far less commendable than one remaining within that limit.

Otherwise stated, a fund of kR units will protect an investor with even that probability against the risk of casual deviation of his actual losses from their most probable course.

To constitute such a safety fund, as it might be termed, the interest rates may be thought of as containing besides the premiums g an additional weighting g', which will be reserved for taking care of excess losses. There arises then for the investor the expectation of a gain even in case an unfavorable deviation from the most probable course should take place, as long as this does not exhaust the corresponding contributions $s \times g' \times A$. His gain from this source would even be greater, of course, in the event of the deviation resulting favorably to his interests.

The value of P for a few k's is given in the list below:*

k	Probability P
1	0.31006
2	0.57498
3	0.76863
4	0.88945
5	0.95392
.	. . .
10	0.99993

Creating for the diversified portfolio totaling $100,000, heretofore mentioned, for which an average risk of 2090 was obtained, a safety fund of 5 times this amount, i. e., of $10,450, gives 0.95392 as the probability that the total deviation, if one does occur during the specified term of 10 years, will remain within the limits of the constituted fund or, which is the same thing, that an eventual loss (in excess of spA) will not exceed $5R$. Similarly to cover the uniformly diversified portfolio of $100,000 with a total average risk of 764, a fund of only $3,820 would be required.

Instead of making use of the function $\phi\left(\frac{k}{2\sqrt{\pi}}\right)$ one may obtain approximate results by means of a theorem established by Tchebycheff, which may be expressed as follows:

"The probability that the absolute value of the total deviation will not exceed 0.399α times the total risk

$$R = \sqrt{R^2_1 + R^2_2 + R^2_3 + \ldots\ldots}$$

is $> 1 - \frac{1}{0.16\alpha^2}$,

where α is a number merely subject to the condition that $0.16\alpha^2 > 1$."

*Derived by means of table I in *Wahrscheinlichkeitsrechnung* of Emanuel Czuber.

The results thus derived are approximate in so far as they do not represent the narrowest limits corresponding to a given probability. For $a=5$, for instance, the theorem would only make known that the probability P of the total deviation not to exceed $5R$ is greater than ¾, while this probability actually amounts to $\frac{95392}{100,000}$.

The larger the fund kR, the more secure, naturally, is the position of an investor with regard to casual unfavorable influences. It should be noted, too, that for a given k the fund will result differently according to whether it is intended to cover the risk in question for only one year, for instance, or for a more extended period. The fund will obviously be smaller in the former instance than in the latter case. Their ratio, however, is quite diverse from that of the time intervals on account of the fact that a larger interval offers a greater possibility for a balancing of losses and gains.

From the preceding theoretical considerations the following conclusions of practical consequence may be deduced:

(1) Losses will always occur in investment and credit transactions in general because of the credit factor, this being merely the expression of a moral expectation, thus introducing the element of incertitude. Hence, in order best to assure regularity of returns, reserves should be established at a rate which experience indicates as desirable.

(2) The risk of loss is considerably reduced for a given debt if a sinking fund is provided.

(3) In the case of a number of units of principal the relative risk will be smaller, and consequently the relative premium or reserve required to cover it will be smaller,

(a) the larger the number of units of principal (see diagram 1, page 30)

(b) the less single sums deviate from the average.

These points particularly apply also to the accounts receivable of any commercial enterprise.

(4) The percentage of losses of a business firm supposedly maintaining the same credit policies should gradually, although slowly, diminish with increasing number of accounts, due to the general relation $R=AR_0\sqrt{s}$ (exact equality if all accounts were of the same kind and subject to the same risk).

(5) Reserves built up on the basis of a ratio derived from a sufficiently long experience may be taken to cover, in the future, as the most probable case, the same proportion of losses in relation to the units of capital employed. To offset any excess loss would require, of course, additional reserves (safety fund), which would cover the risk within the limits of a like amount with probabilities as indicated in diagram 2 (page 33). Such reserves, as will also be noticed, are relatively most useful in so far as they are kept within certain, rather definite limits. Reserves to this extent are consequently the most commendable.