ELECTRIC SYSTEMS

Escola d'Enginyeria d'Igualada (UPC)

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Edition: September 2010

SISTEMES ELÈCTRICS. SYLLABUS

UNIT 1: Direct Current (DC) Circuits
2 weeks

UNIT 2: Alternating Current (AC) Circuits
2 weeks

UNIT 3: Three-Phase AC Circuits
3 weeks

UNIT 4: Transformers
3 weeks

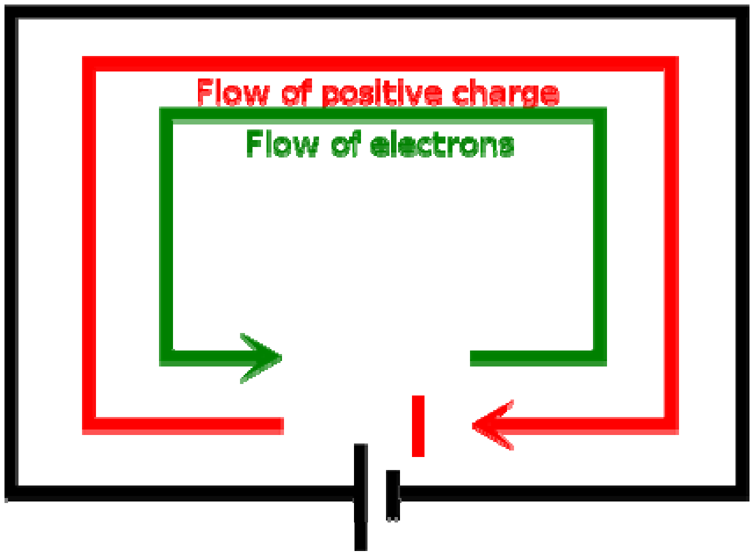
UNIT 5: Asynchronous AC Machines
3 weeks

UNIT 6: Other Electrical Machines
1 week

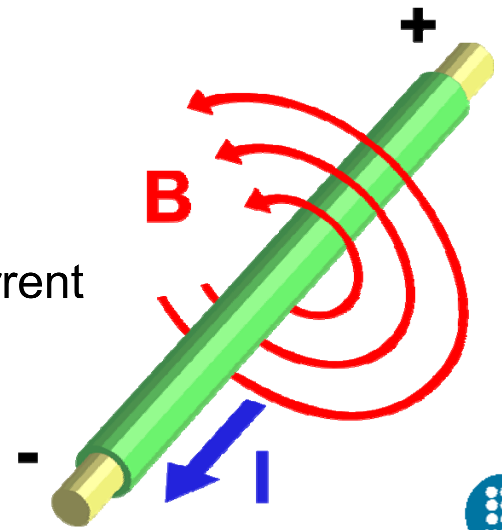
UNIT 7: Electrical Installations and Low Voltage Electrical Protections
1 week

ELECTRICAL CURRENT

When a metal wire is connected across the two terminals of a DC voltage source, the free electrons of the conductor are forced to drift toward the positive terminal. The free electrons are therefore the current carrier in a typical solid conductor.



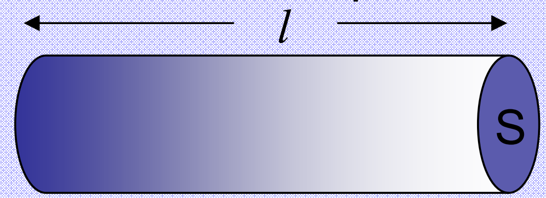
According to Ampère's law, an electric current produces a magnetic field.



Electrical resistance

- The electrical resistance of an object is a measure of its opposition to the passage of a steady electric current.
- It was discovered by George Ohm in 1827
- The SI unit of electrical resistance is the ohm (Ω).
- The resistance R of a conductor of uniform cross section can be computed as:

$$R = \rho \cdot \frac{l}{S}$$



l is the length of the conductor, measured in meters [m]

S is the cross-sectional area of the current flow, measured in square meters [m²]

ρ (Greek: rho) is the electrical resistivity of the material, measured in ohm-metres (Ω m). Resistivity is a measure of the material's ability to oppose electric current.

ELECTRICAL RESISTANCE: temperature dependence

Electrical resistance: temperature dependence

- The electric resistance of a typical metal increases linearly with rising temperature as:

$$R = R_0 \cdot [1 + \alpha \cdot (T - T_0)]$$

Where

- T: metal's temperature
- T_0: reference temperature (usually room temperature)
- R_0: resistance at T_0
- α: percentage change in resistivity per unit temperature. It depends only on the material being considered

Material	Resistivity (Ω·m) at 20 °C	Temperature coefficient K^{-1}
Silver	1.59×10^{-8}	0.0038
Copper	1.68×10^{-8}	0.0039
Gold	2.44×10^{-8}	0.0034
Aluminium	2.82×10^{-8}	0.0039
Tungsten	5.60×10^{-8}	0.0045
Zinc	5.90×10^{-8}	0.0037
Nickel	6.99×10^{-8}	0.006
Iron	1.0×10^{-7}	0.005
Platinum	1.06×10^{-7}	0.00392

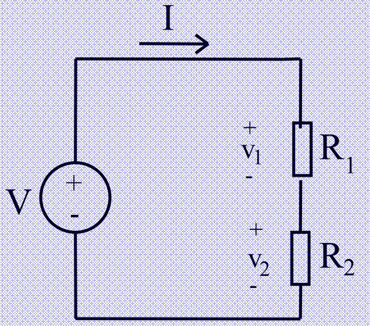
SERIES/PARALLEL CONNECTED LOADS

	SERIES	PARALLEL
Resistors	$R_{equivalent} = \sum_{i=1}^{n} R_i$	$\frac{1}{R_{equivalent}} = \sum_{i=1}^{n} \frac{1}{R_i}$ $R_1//R_2$: $R_{equivalent} = \frac{R_1 . R_2}{R_1 + R_2}$
Inductors	$L_{equivalent} = \sum_{i=1}^{n} L_i$	$\frac{1}{L_{equivalent}} = \sum_{i=1}^{n} \frac{1}{L_i}$
Capacitors	$\frac{1}{C_{equivalent}} = \sum_{i=1}^{n} \frac{1}{C_i}$	$C_{equivalent} = \sum_{i=1}^{n} C_i$

VOLTAGE AND CURRENT DIVIDERS

The voltage divider

- It is a simple linear circuit that produces an output voltage that is a fraction of its input voltage.
- Voltage is partitioned among the components of the divider.

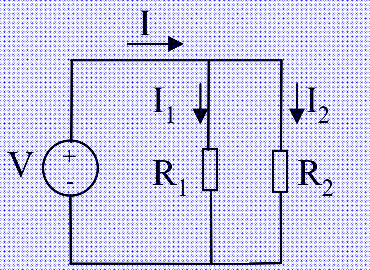


General case: $$V_{Rj} = V \cdot \frac{R_j}{\sum_i R_i}$$

In the example shown: $$V_{R_2} = V \cdot \frac{R_2}{R_1 + R_2}$$

The current divider

- A current divider is a simple linear circuit that produces an output current that is a fraction of its input current (IT).
- Current is split between the branches of the divider.

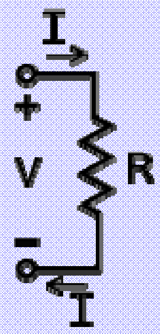


General case: $$I_i = I \cdot \frac{R_1 \cdot R_2 \cdots R_{i-1} \cdot R_{i+1} \cdot R_n}{\sum_i R_i}$$

In the example shown: $$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

Ohm's law

The current through a conductor between two points is directly proportional to the potential difference -or voltage across the two points-, and inversely proportional to the resistance between them.



$$I = V/R \quad \text{or} \quad V = I \cdot R$$

- The electrical resistance of an object is a measure of its opposition to the passage of a steady electric current.
- The electrical resistance was discovered by George Ohm in 1827
- The SI unit of electrical resistance is the ohm (Ω).

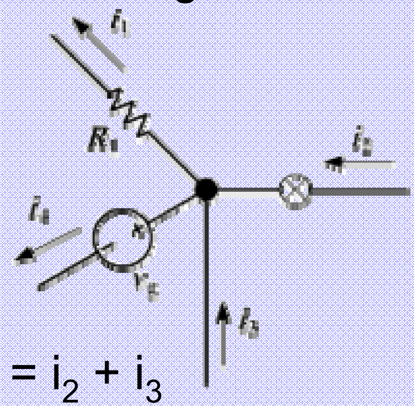
The resistance R of a conductor of uniform cross section can be computed as:

KIRCHHOFF'S CIRCUIT LAWS

Kirchhoff's current law (KCL)

At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

$$\sum_i I_i = 0 \quad \textbf{or} \quad \sum I_{input} = \sum I_{output}$$

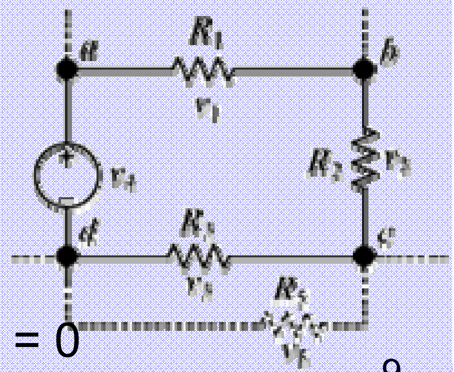


The current entering any junction is equal to the current leaving that junction: $i_1 + i_4 = i_2 + i_3$

Kirchhoff's voltage law (KVL)

The directed sum of the electrical potential differences (voltage) around any closed circuit must be zero.

$$\sum_i V_i = 0$$

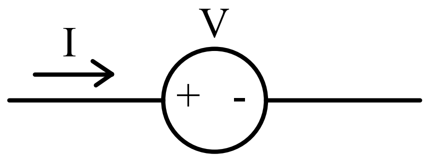


The sum of all the voltages around the loop is equal to zero: $v_1 + v_2 + v_3 + v_4 = 0$

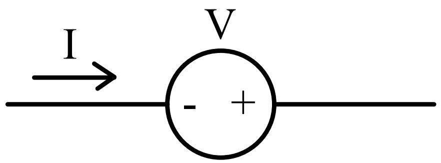
ELECTRIC POWER

Electric power

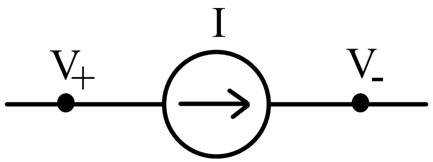
- **Is the rate at which electrical energy is transferred by an electric circuit or consumed by an electric load.**
- The SI unit of power is the **watt**.
- The power consumed by a load is considered as positive
- The power delivered by a source is considered as negative



$P = + V \cdot I$ Absorbed power



$P = - V \cdot I$ Delivered power



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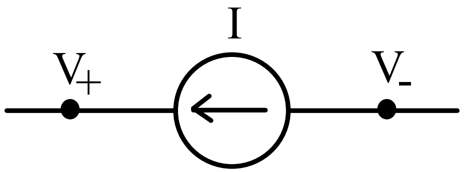


$P = - V \cdot I$ Delivered power

Energy

- Energy results form the integral: $$W = \int_0^t P(t)\cdot dt$$
- If power is constant it results: $$W = P\cdot t = V\cdot I\cdot t$$
- Energy is usually measured in kWh (Kilowatt hour). Energy in watt hours is the multiplication of power in watts and time in hours.
- 1 kWh = 3.6 MJoules
- Electric power meters:

	joule	watt hour	electronvolt	calorie
1 J = 1 kg m ^{2} s $^{-2}$ =	1	2.778×10^{-4}	6.241×10^{18}	0.239
1 W·h =	3600	1	2.247×10^{22}	859.8
1 eV =	1.602×10^{-19}	4.45×10^{-23}	1	3.827×10^{-20}
1 cal =	4.1868	1.163×10^{-3}	2.613×10^{19}	1

Don't confuse power and energy !

POWER AND ENERGY

In domestic and industrial supplies the kilowatt-hour (kWh) is usually used. 1kWh is the energy used when a power of 1kW is supplied for one hour (3600s). Notice that power = energy/time, therefore energy = power × time and since 1kWh is a power multiplied by a time, it is in fact a unit of energy (not power).

Example. How much energy is supplied to a 100Ω resistor which is connected to a 150V supply for 1 hour?

Power: $P = U^2/R = 150^2/100 = 225\ W$

Energy: $W = P{\cdot}t = 225W{\cdot}1h = 225\ Wh = 0.225\ kWh$

$W = P{\cdot}t = 225W{\cdot}1{\cdot}60{\cdot}60s = 810000\ J = 810\ kJ$

The number of joules or kWh supplied is a measure of the amount of electrical energy supplied.

Example. A DC motor takes 15A from a 200V supply. It is used for 40 mins. What will it cost to run if the tariff is 0.13 €/kWh?

Power: $P = U{\cdot}I = 200.15 = 3000\ W = 3\ kW$

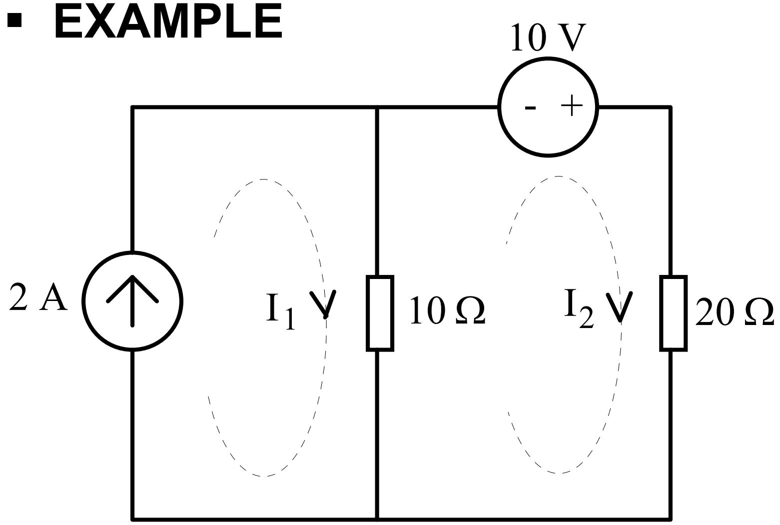
Energy in kWh: $W = P{\cdot}t = 3{\cdot}(40/60) = 2\ kWh$

Cost in €: $C = 2{\cdot}0.13 = 0.26$ €

The mesh-current method

- Also known as the Loop Current Method.
- It uses simultaneous equations, Kirchhoff's Voltage Law, and Ohm's Law to determine unknown currents in a network.
- is a method that is used to solve planar circuits for the voltage and currents at any place in the circuit.

- **EXAMPLE**



Mesh current equations:

$$\begin{cases} \text{loop 1:} & I_1 = 2\text{ A} \\ \text{loop 2:} & 10 = -10\cdot I_1 + 30\cdot I_2 \end{cases}$$

It results: $\mathbf{I_1 = 2\ A,\ I_2 = 1\ A}$

$P_{10V} = -\ 1\cdot 10 = -\ 10$ W (delivers power)

$P_{2A} = -\ 2\cdot 10 = -\ 20$ W (delivers power)

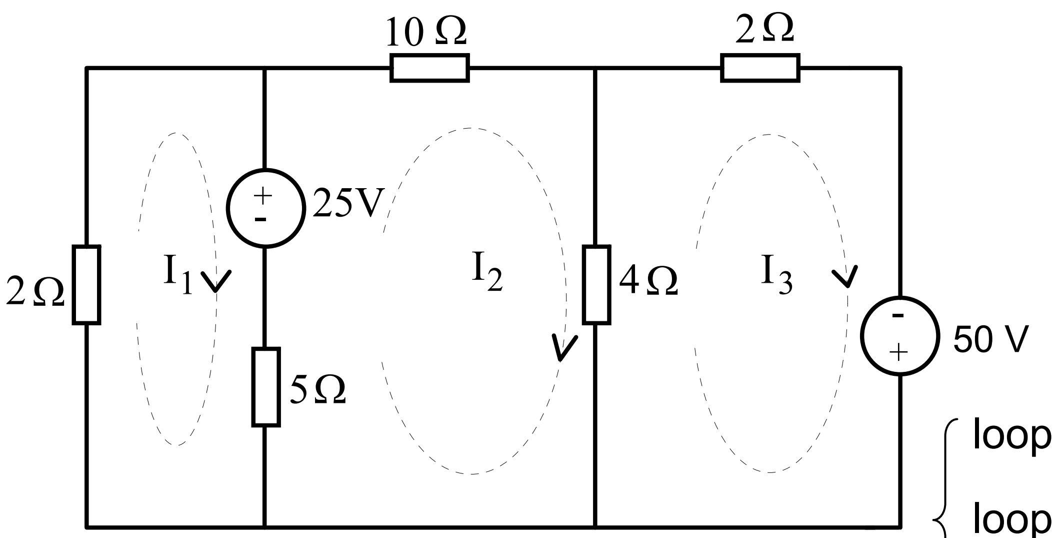
$P_{10\Omega} = +\ 1^2\cdot 10 = +\ 10$ W (absorbs power)

$P_{20\Omega} = +\ 1^2\cdot 20 = +\ 20$ W (absorbs power)

$$\sum_i P_i = 0$$

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

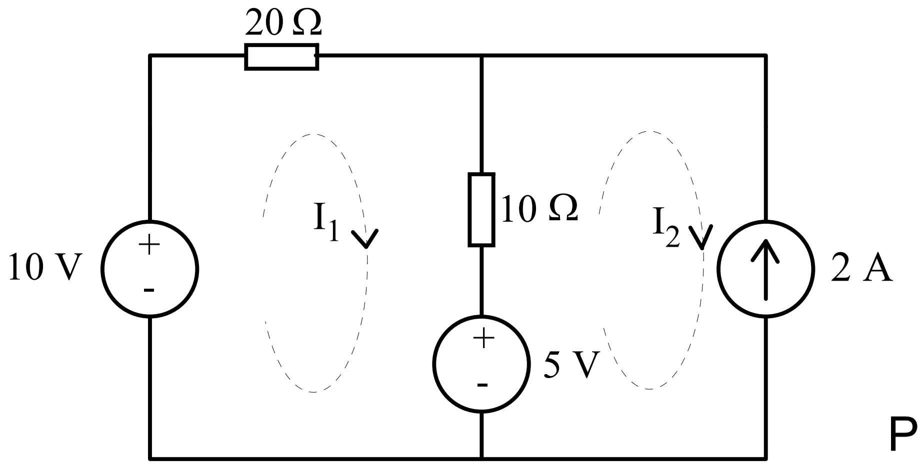


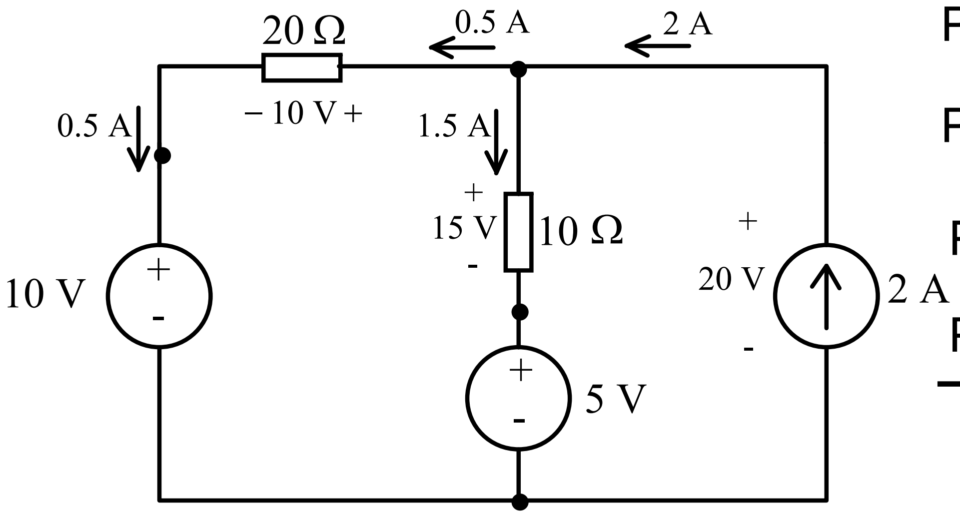
Mesh current equations:

$$\begin{cases} \text{loop 1: } -25 = +I_1\cdot(2+5) - I_2\cdot(5) - I_3\cdot(0) \\ \text{loop 2: } +25 = -I_1\cdot(5) + I_2\cdot(5+10+4) - I_3\cdot(4) \\ \text{loop 3: } +50 = -I_1\cdot(0) - I_2\cdot(4) + I_3\cdot(4+2) \end{cases}$$

$$I_1 = \frac{\begin{vmatrix} -25 & -5 & 0 \\ 25 & 19 & -4 \\ 50 & -4 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = -1'306\text{A} \qquad I_2 = \frac{\begin{vmatrix} 7 & -25 & 0 \\ -5 & 25 & -4 \\ 0 & 50 & 6 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = 3'172\,\text{A} \qquad I_3 = \frac{\begin{vmatrix} 7 & -5 & -25 \\ -5 & 19 & 25 \\ 0 & -4 & 50 \end{vmatrix}}{\begin{vmatrix} 7 & -5 & 0 \\ -5 & 19 & -4 \\ 0 & -4 & 6 \end{vmatrix}} = 10'448\,\text{A}$$

- **EXAMPLE**





Mesh current equations:

$$\begin{cases} \text{loop 1:} \quad 10 - 5 = 30 \cdot I_1 - 10 \cdot I_2 \\ \text{loop 2:} \quad I_2 = -2\text{ A} \end{cases}$$

It results: $I_1 = -0.5$ A, $I_2 = -2$ A.

P_{10V} = + 0'5·10 = + 5 W (absorbs power)

P_{5V} = + 1'5·5 = + 7'5 W (absorbs power)

P_{2A} = - 2·20 = - 40 W (delivers power)

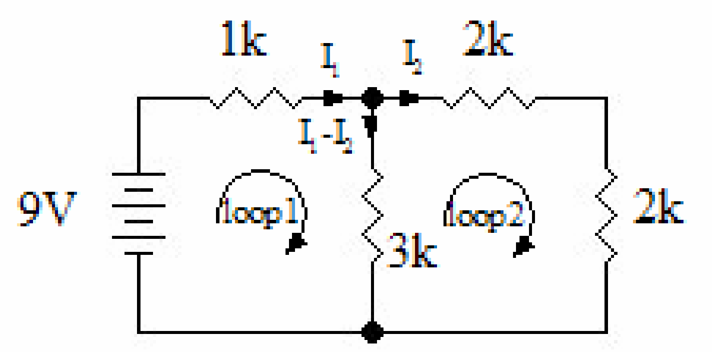
$P_{10\Omega}$ = +10·1'52 = +22'5 W (dissipates power)

$P_{20\Omega}$ = + 20·12 = + 5 W (dissipates power)

$$\sum_i P_i = 0$$

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**



Mesh current equations:

$$\begin{cases} \text{loop 1:} \quad 9 = 400\cdot I_1 - 3000\cdot I_2 \\ \text{loop 2:} \quad 0 = -300\cdot I_1 + 7000\cdot I_2 \end{cases}$$

It results: I_1 = 3.32 mA, I_2 = 1.42 A.

P_{9V} = -3.32·9 mW = -29.88 mW (delivers power)

$P_{1000\Omega}$ = +(3.32)^2·1 mW = 11.02 mW (dissipates power)

$P_{3000\Omega}$ = +(3.32-1.42)^2·3 mW = +10.83 m W (dissipates power)

$P_{2000\Omega}$ = +(1.42)^2·2 mW = 4.03 mW (dissipates power)

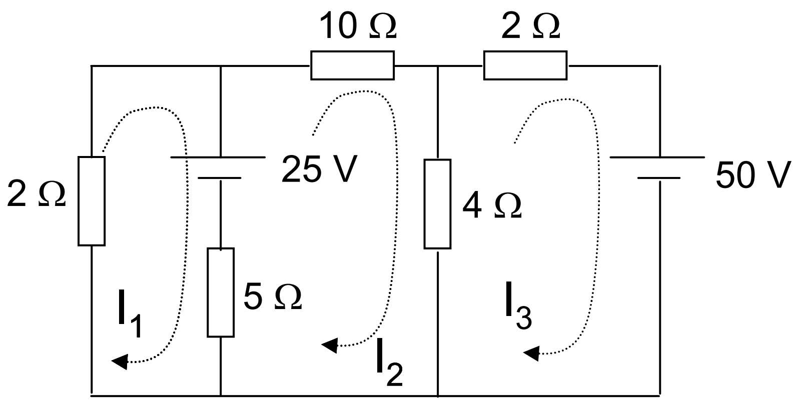
$P_{2000\Omega}$ = +(1.42)^2·2 mW = 4.03 mW (dissipates power)

$$\sum_i P_i = 0$$

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Determine currents in each loop and perform a power balance.

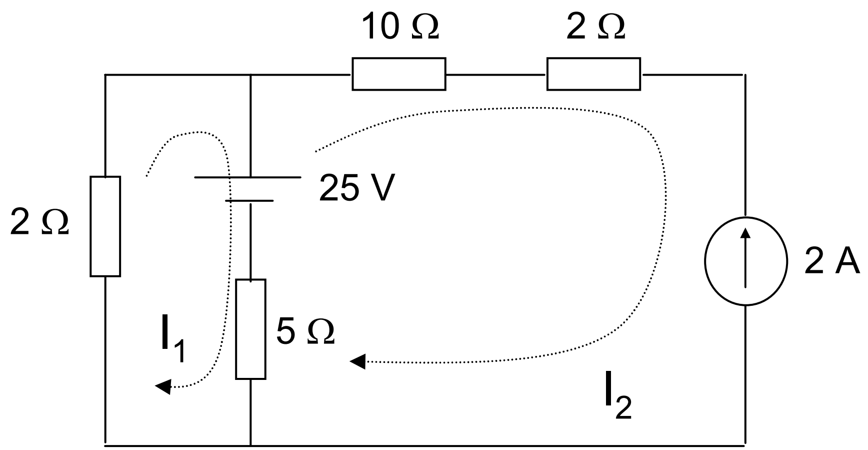


Answer: a) I_1 = -5'037 A, I_2 = -2'052 A, I_3 = -9'701 A b) $\Sigma P_{source} = \Sigma\ P_{resist.}$ = 559'65 W

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Determine currents in each loop and perform a power balance.

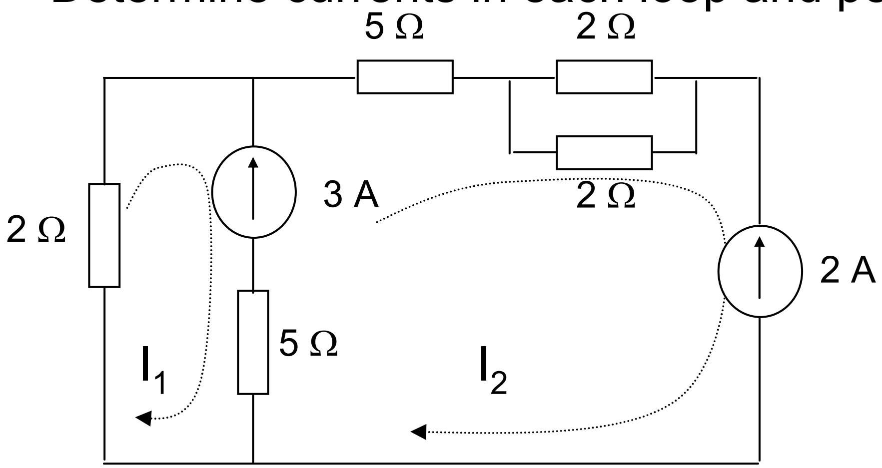


Answer a) I_1 = -5 A, I_2 = -2 A b) $\Sigma P_{source} = \Sigma P_{resist.}$ = 143 W

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Determine currents in each loop and perform a power balance.

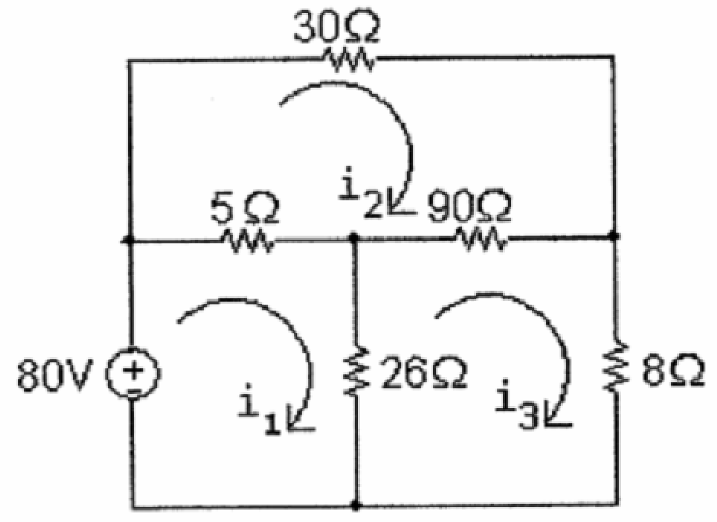


Answer a) I_1 = -5 A, I_2 = -2 A b) $\Sigma P_{source} = \Sigma\ P_{resist.}$ = 119 W

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Determine currents in each loop and perform a power balance.



The mesh current equations are:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

$$\Delta = \begin{vmatrix} 31 & -5 & -26 \\ -5 & 125 & -90 \\ -26 & -90 & 124 \end{vmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 80 & -5 & -26 \\ 0 & 125 & -90 \\ 0 & -90 & 124 \end{vmatrix}}{\Delta} = 5A$$

$$i_2 = \frac{\begin{vmatrix} 31 & 80 & -26 \\ -5 & 0 & -90 \\ -26 & 0 & 124 \end{vmatrix}}{\Delta} = 2A$$

(a) $p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

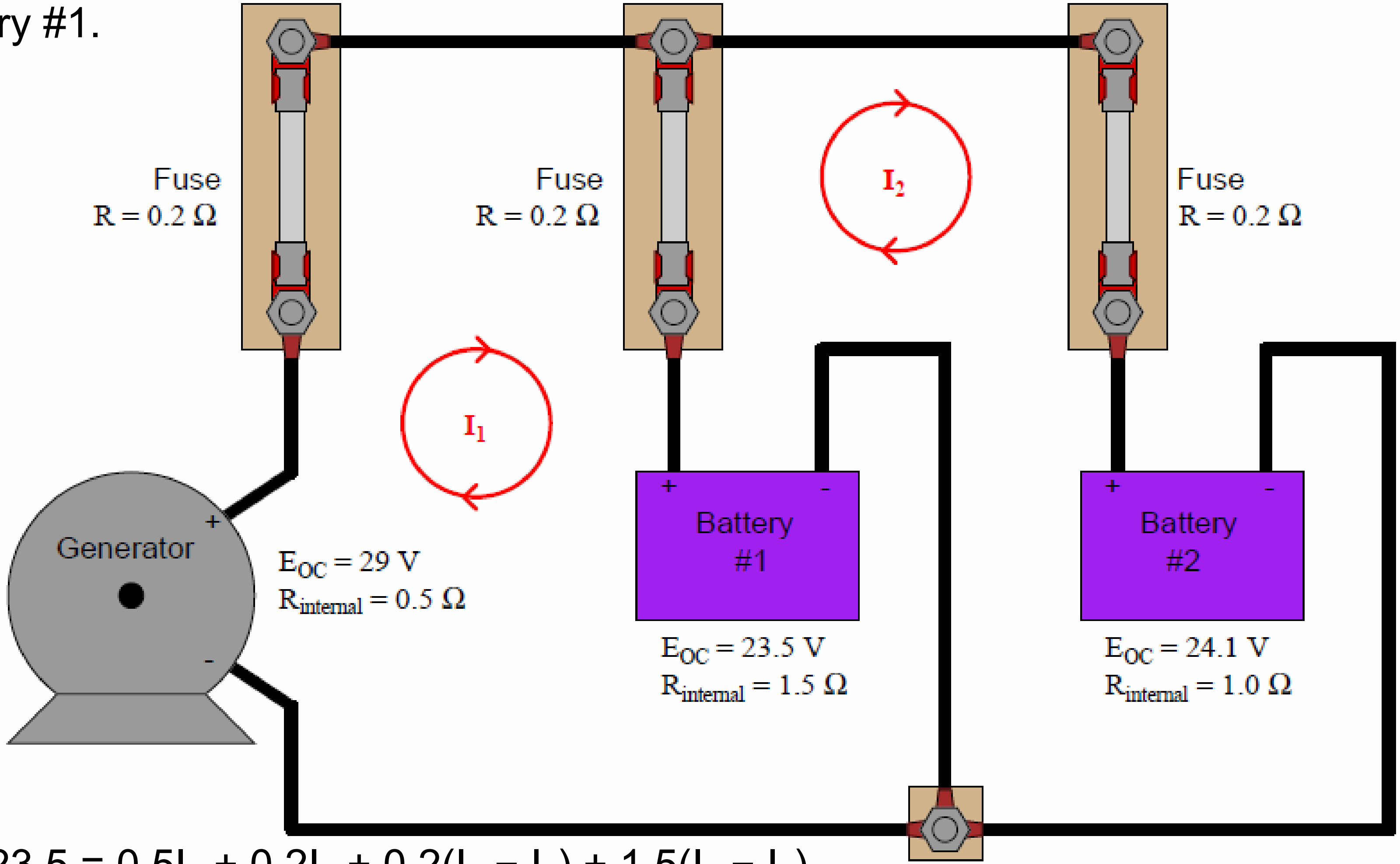
Therefore the 80 V source is delivering 400 W to the circuit.

(b) $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the 8 Ω resistor dissipates 50 W.

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Given the following mesh current directions, solve for the charging current through battery #1.



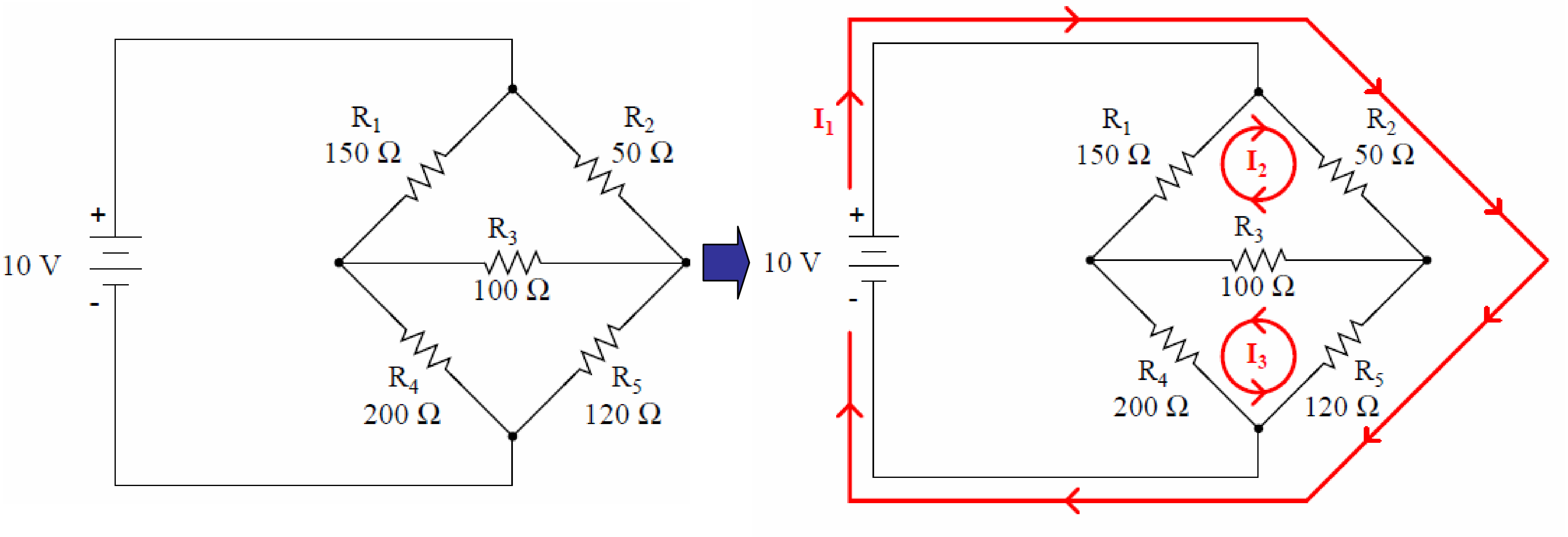
$$\begin{cases} 29 - 23.5 = 0.5I_1 + 0.2I_1 + 0.2(I_1 - I_2) + 1.5(I_1 - I_2) \\ 23.5 - 24.1 = 1.5(I_2 - I_1) + 0.2(I_2 - I_1) + 0.2I_2 + I_2 + 0 \end{cases}$$

The result is $I_{bat1} = 1.7248$ A

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

Given the following mesh current directions, solve for the mesh currents.



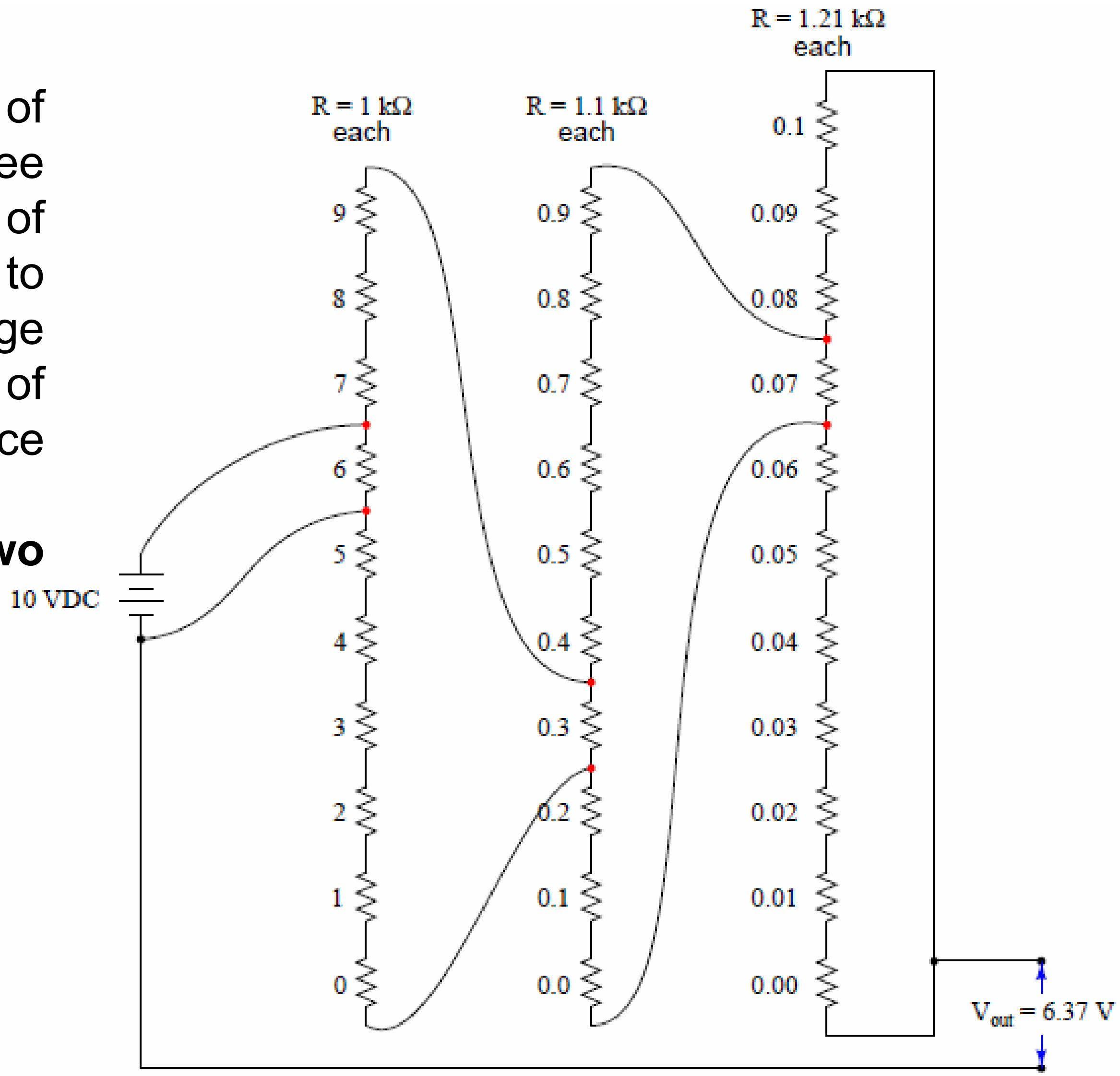
$$
\begin{cases}
\text{Loop 1:} & 10 = 170I_1 + 50I_2 - 120I_3 \\
\text{Loop 2:} & 0 = 50I_1 + 300I_2 + 100I_3 \\
\text{Loop 3:} & 0 = -120I_1 + 100I_2 + 420I_3
\end{cases}
$$

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method

- **EXAMPLE**

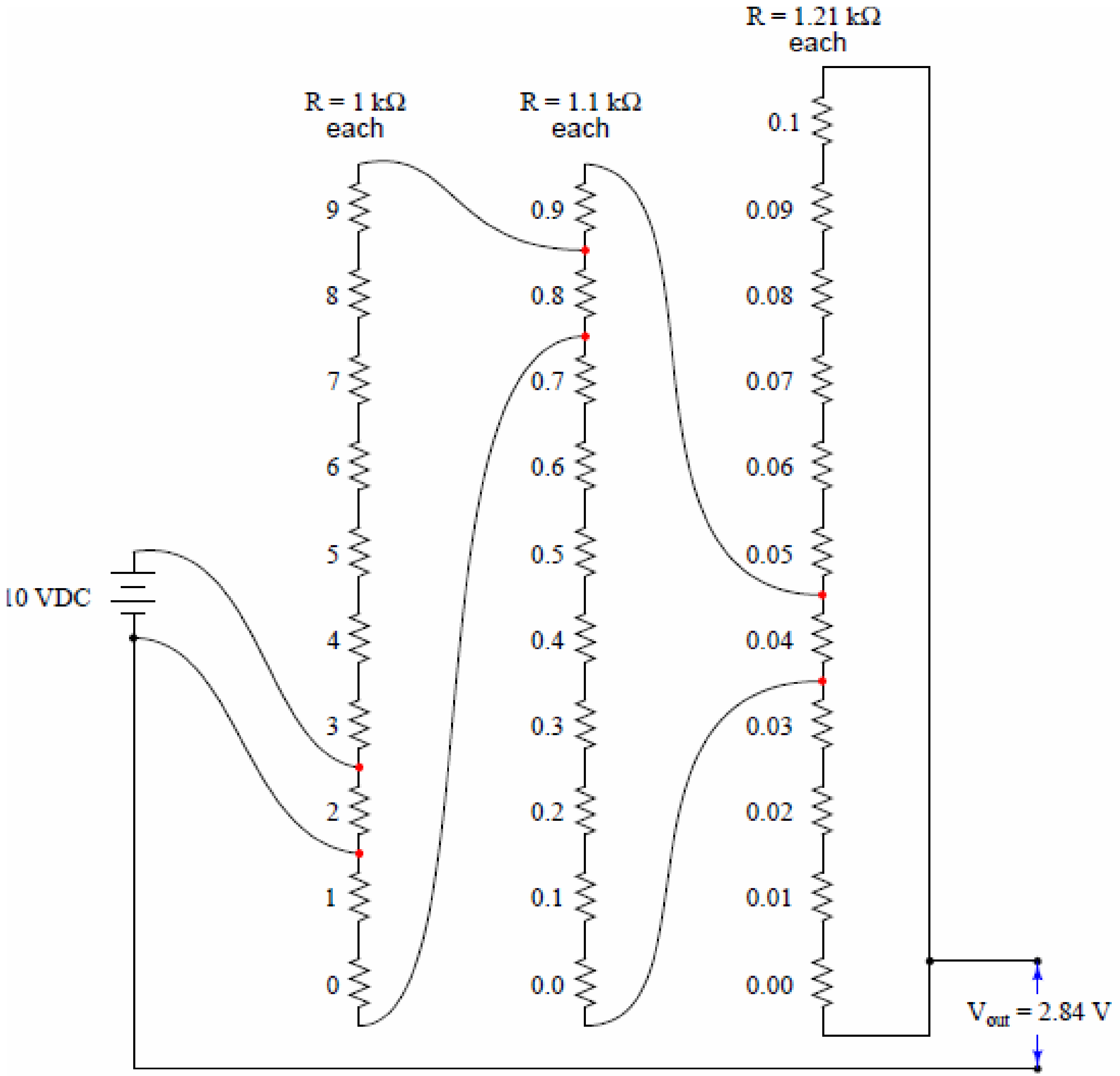
A very interesting style of voltage divider which uses three series-connected strings of resistors and connection clips to provide 1000 steps of voltage division with only 31 resistors, of only 3 different resistance values.

Are you able to solve the two following circuits?



http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

TECHNIQUES OF CIRCUIT ANALYSIS: Mesh-current method



The node-voltage method

- Also known as the Nodal Analysis or Branch Current Method.
- It uses simultaneous equations, Kirchhoff's Voltage Law, and Ohm's Law to determine unknown currents in a network.
- It is a method used to solve planar circuits for the voltage and currents at any place in the circuit.

THÉVENIN'S THEOREM

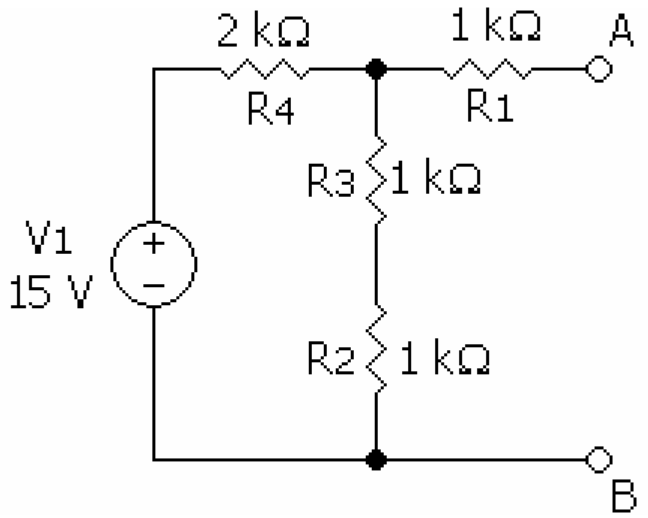
Thévenin's theorem

- Any two terminals circuit composed of a combination of voltage sources, current sources and resistors is electrically equivalent to a single voltage source V in series with a single resistor R.

Steps for calculating the Thévenin's equivalent circuit:

- Calculate the output voltage, V_{AB}, in **open circuit** condition. This is $\mathbf{V_{Th}}$.
- Replace voltage sources with short circuits and current sources with open circuits. Measure the total resistance, R_{AB}. This is $\mathbf{R_{Th}}$

- **EXAMPLE**



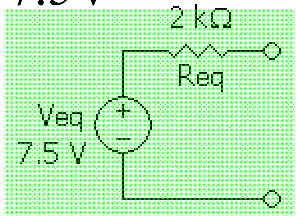
VTH:

$$15 = I\cdot(2000+1000+1000) \rightarrow I = 0.00375A$$

$$V_{AB} = I\cdot(1000+1000) \rightarrow V_{AB} = V_{Th} = 7.5V$$

RTH:

$$R_{Th} = (2000//2000)+1000 = 2000\Omega$$



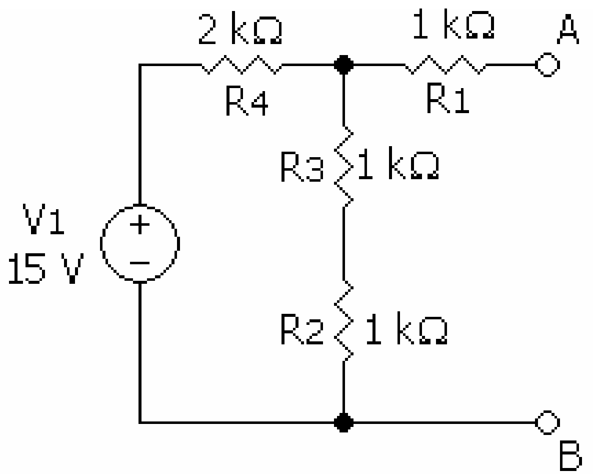
Norton's theorem

- Any two terminals circuit composed of a combination of voltage sources, current sources and resistors is electrically equivalent to a single voltage source V in series with a single resistor R.

Steps for calculating the Thévenin's equivalent circuit:

- Calculate the output current, I_{AB}, with a short circuit as the load This is $\mathbf{I_{No}}$.
- Replace voltage sources with short circuits and current sources with open circuits. Measure the total resistance, R_{AB}. This is $\mathbf{R_{No}}$

- **EXAMPLE**

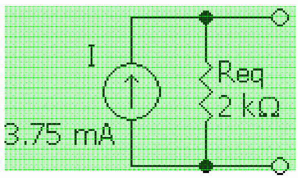


INo: $$I_{tot} = \frac{15}{2000 + 2000//1000} = 0.005625A$$

Current divider: $$I_{No} = I_{tot} \cdot \frac{2000}{3000} = 0.00375A$$

RNo:

$$R_{No} = R_{Th} = (2000//2000) + 1000 = 2000\Omega$$

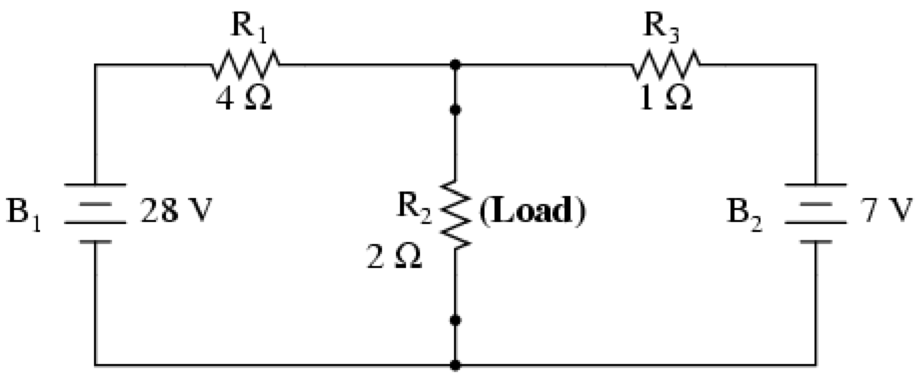


THÉVENIN-NORTON RELATIONSHIP

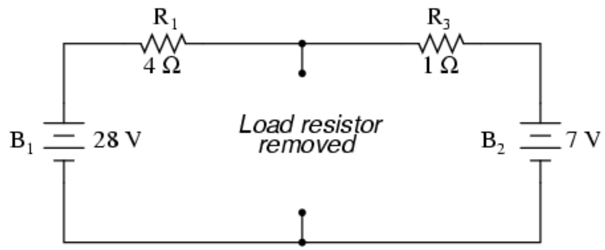
Equivalence between Thévenin and Norton

- $R_{Th} = R_{No}$
- $V_{Th} = R_{No} \cdot I_{No}$
- $I_{No} = V_{Th} / R_{Th}$

- **EXAMPLE**



1) VTH:

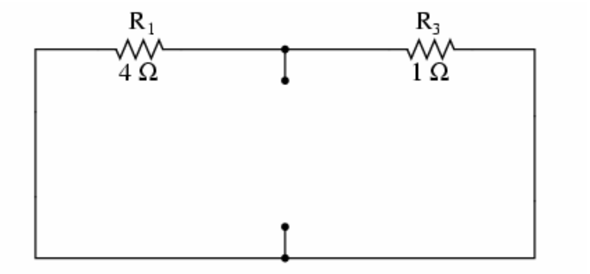


$$28 - 7 = I \cdot (4+1) \;\rightarrow\; I = 4.2\text{A}$$

0.8 Ω

$$V_{AB} = 28 - I \cdot 4 \;\rightarrow\; V_{AB} = V_{Th} = 11.2\text{V}$$

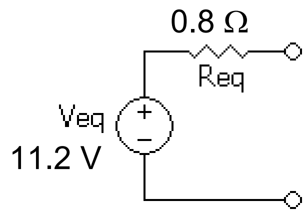
2) RTH, RNo

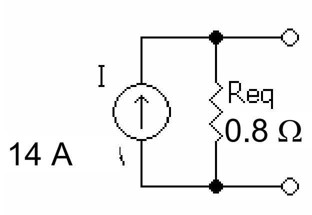


$$R_{Th} = R_{No} = 4 // 1 = 0.8\Omega$$

3) INo:

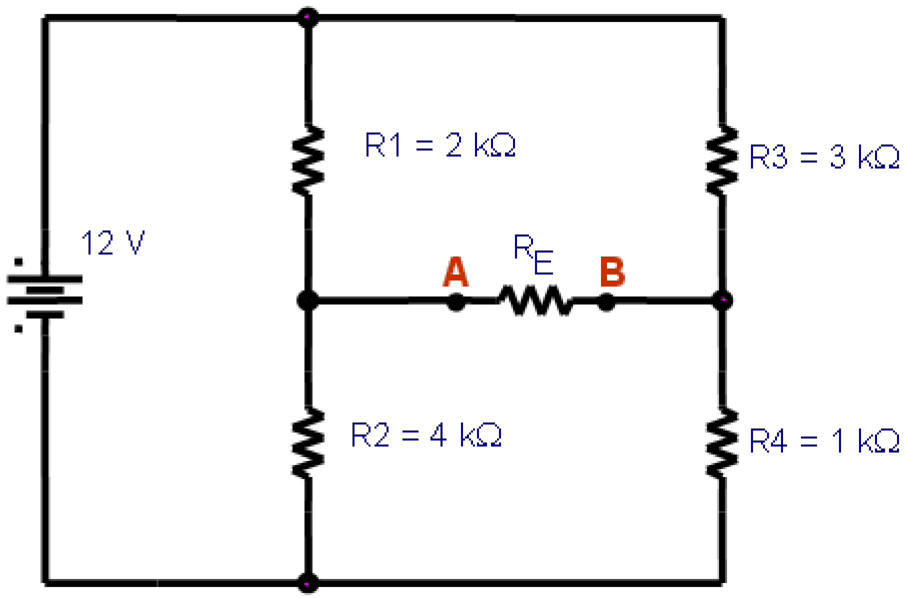
$$I_{No} = V_{Th} / R_{No} = 11.2 / 0.8 = 14\text{A}$$



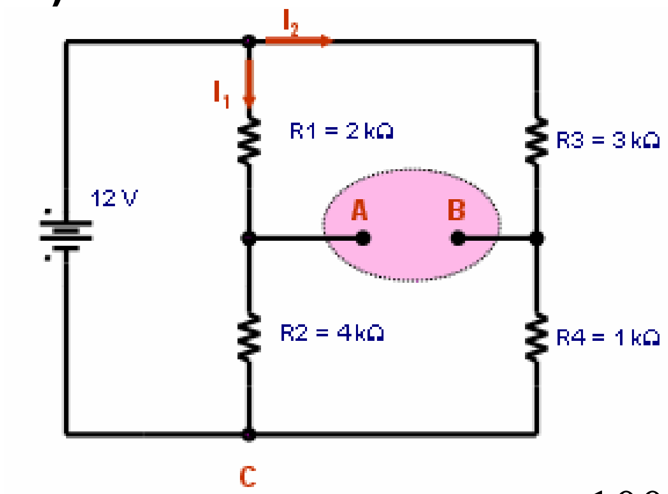


THÉVENIN-NORTON RELATIONSHIP

- **EXAMPLE**



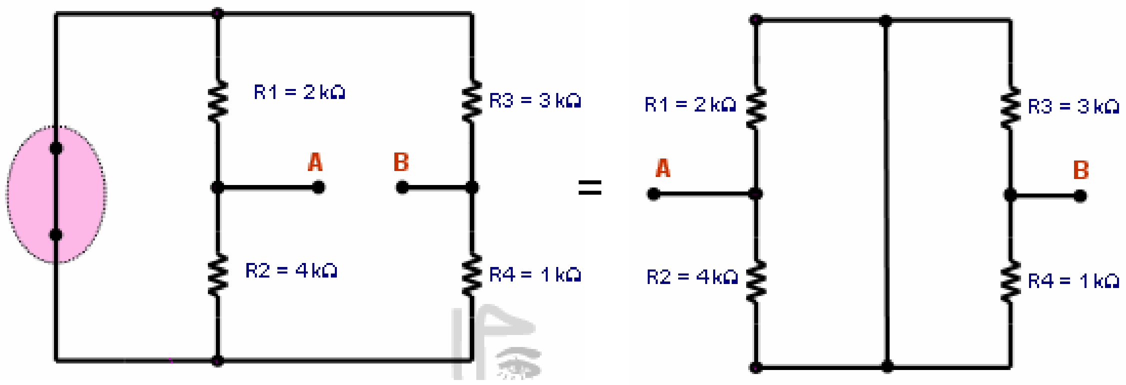
1) VTH:



$$V_{AC} = 12 \cdot \frac{4000}{6000} = 8V \quad V_{BC} = 12 \cdot \frac{1000}{4000} = 3V$$

$$V_{Th} = V_{AB} = V_{AC} - V_{BC} = 8 - 3 = 5V$$

2) RTH, RNo



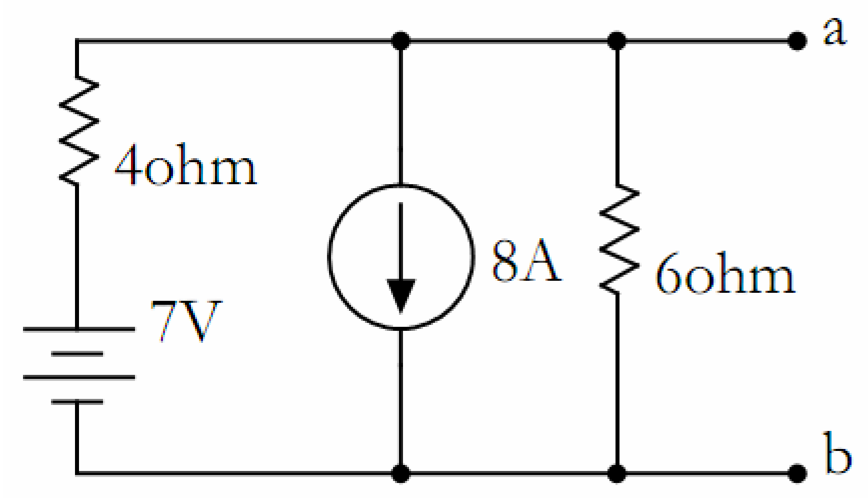
$$R_{Th} = R_{No} = $$
$$= 2 // 4 + 3 // 1 k\Omega = 2.08\hat{3} k\Omega$$

3) INo:

$$I_{No} = V_{Th} / R_{No} = 2.4mA$$

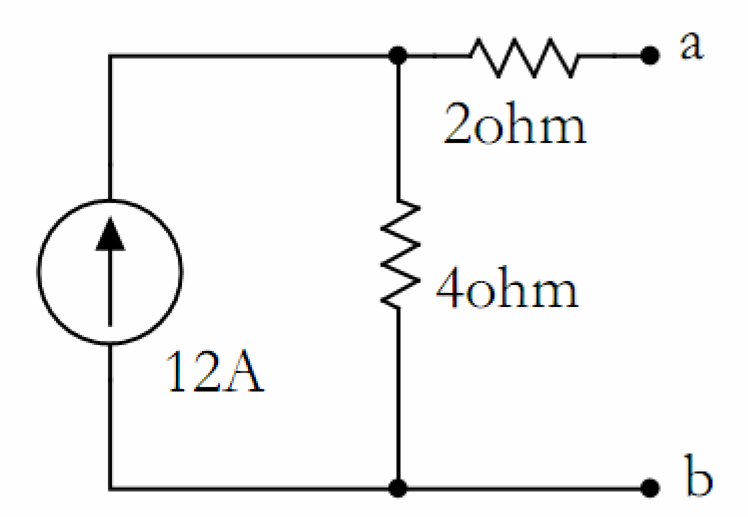
THÉVENIN-NORTON RELATIONSHIP

- **EXAMPLE**



$I_N = 6.25A.,\ R_{Th} = 2.4\ Ohm$

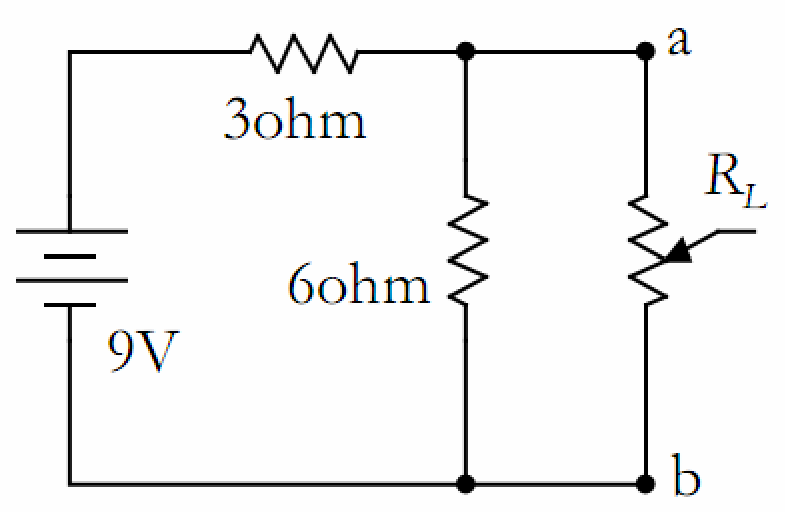
- **EXAMPLE**



$R_{Th} = 6\Omega,\ V_{Th} = 48V$

THÉVENIN-NORTON RELATIONSHIP

- **EXAMPLE**



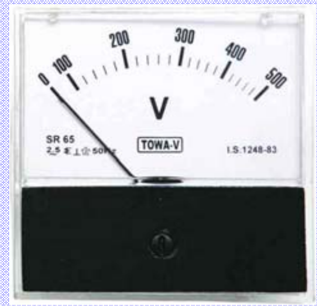
$$R_{Th} = 2\Omega,\ V_{Th} = 6V$$

Online resources:

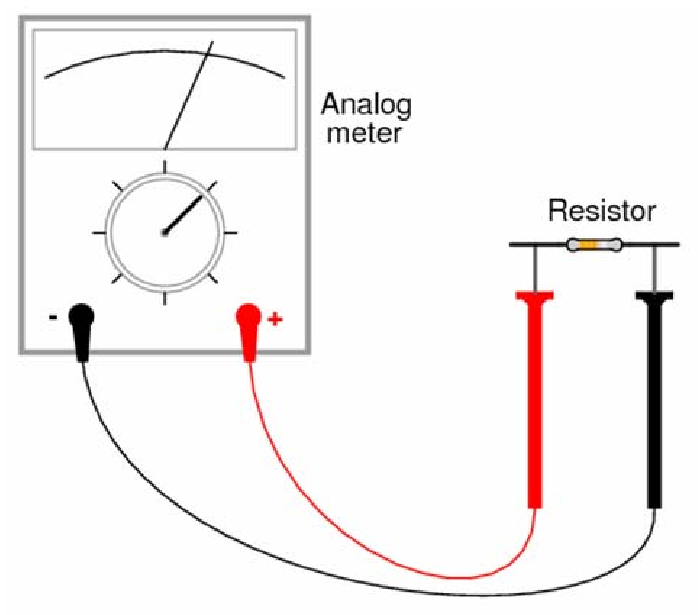
http://utwired.engr.utexas.edu/rgd1/lesson07.cfm

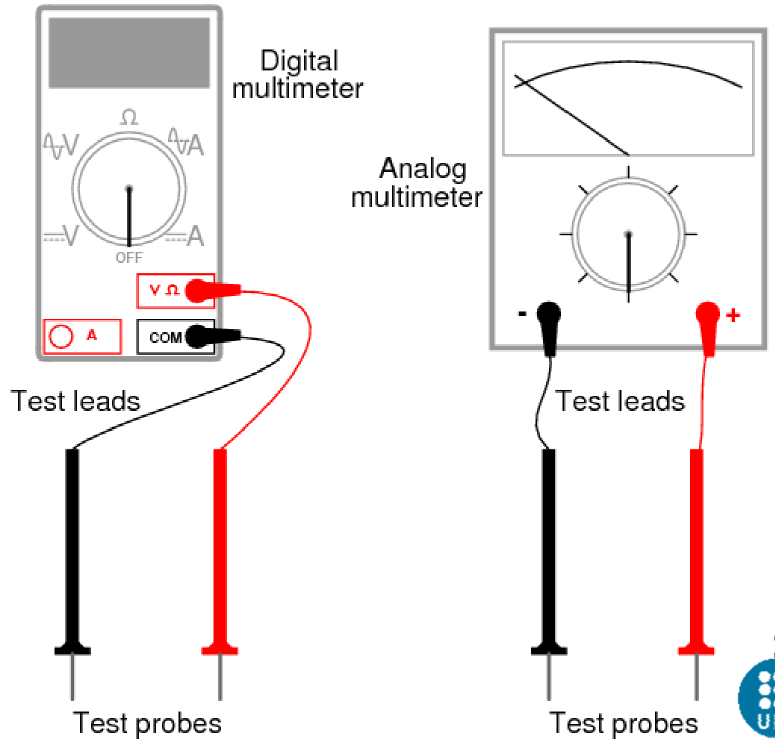
Voltmeters

- Instruments used for measuring the electrical potential difference between two points in an electric circuit.
- Have internal impedance of several MΩ (infinite).
- They are connected in parallel.



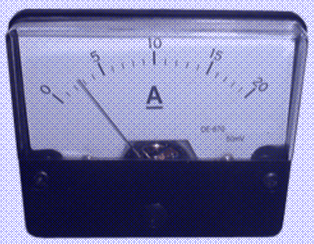
Voltmeter usage

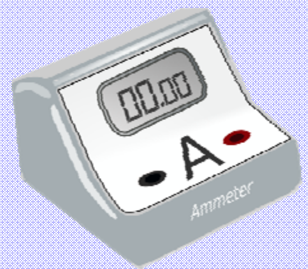




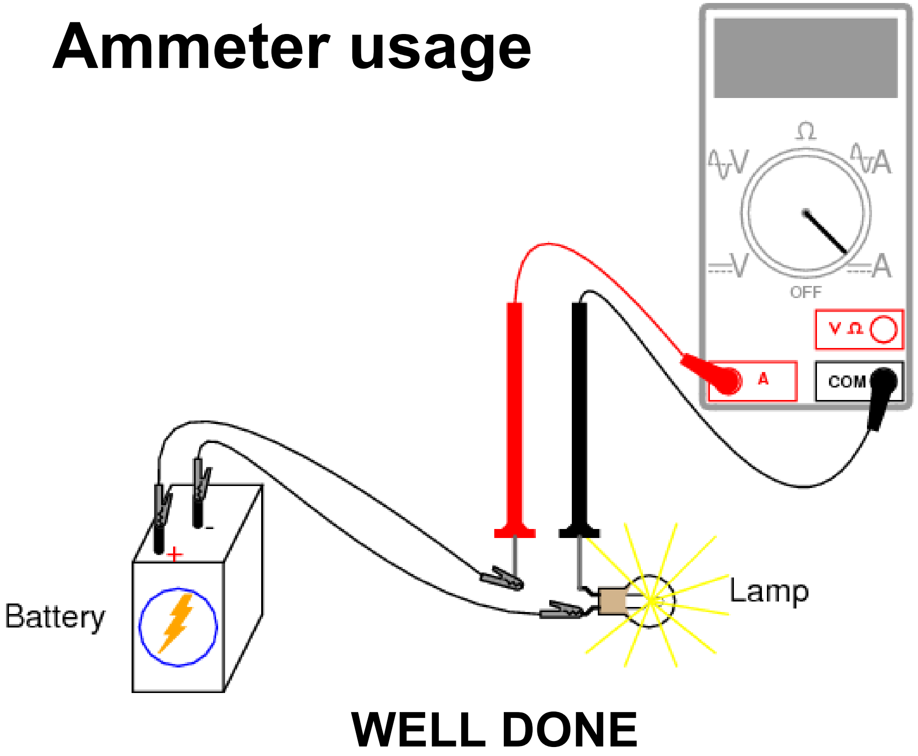
Ammeters

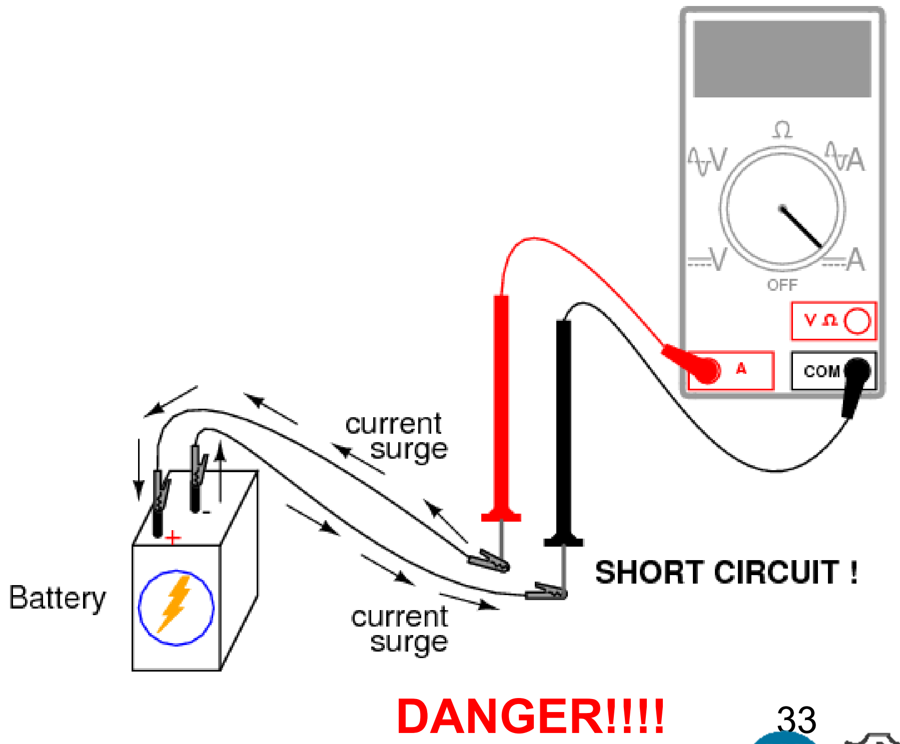
- Instruments used for measuring the electric current in a circuit.
- Have internal impedance close to zero.
- They are connected in series.





Ammeter usage

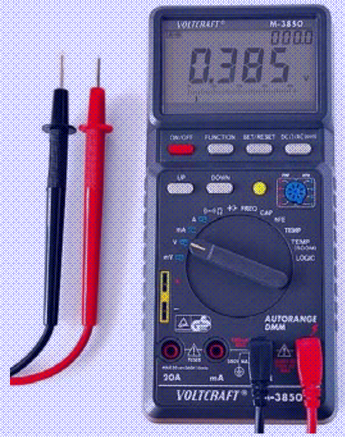




ELECTRICAL MEASUREMENTS

Multimeters

- Electronic measuring instruments that combines several measurement functions in one unit: voltage, current and resistance.



Clamp multimeters

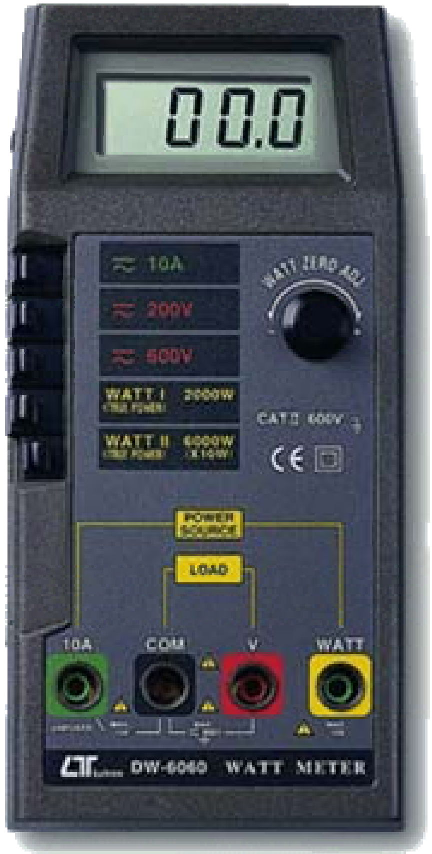
- **Current clamp**: electrical device having two jaws which open to allow clamping around an electrical conductor. This allows the electrical current in the conductor to be measured, without having to make physical contact with it, or to disconnect it for insertion through the probe.
- Also allow measuring voltage, current and resistance.

clamp


Wattmeters

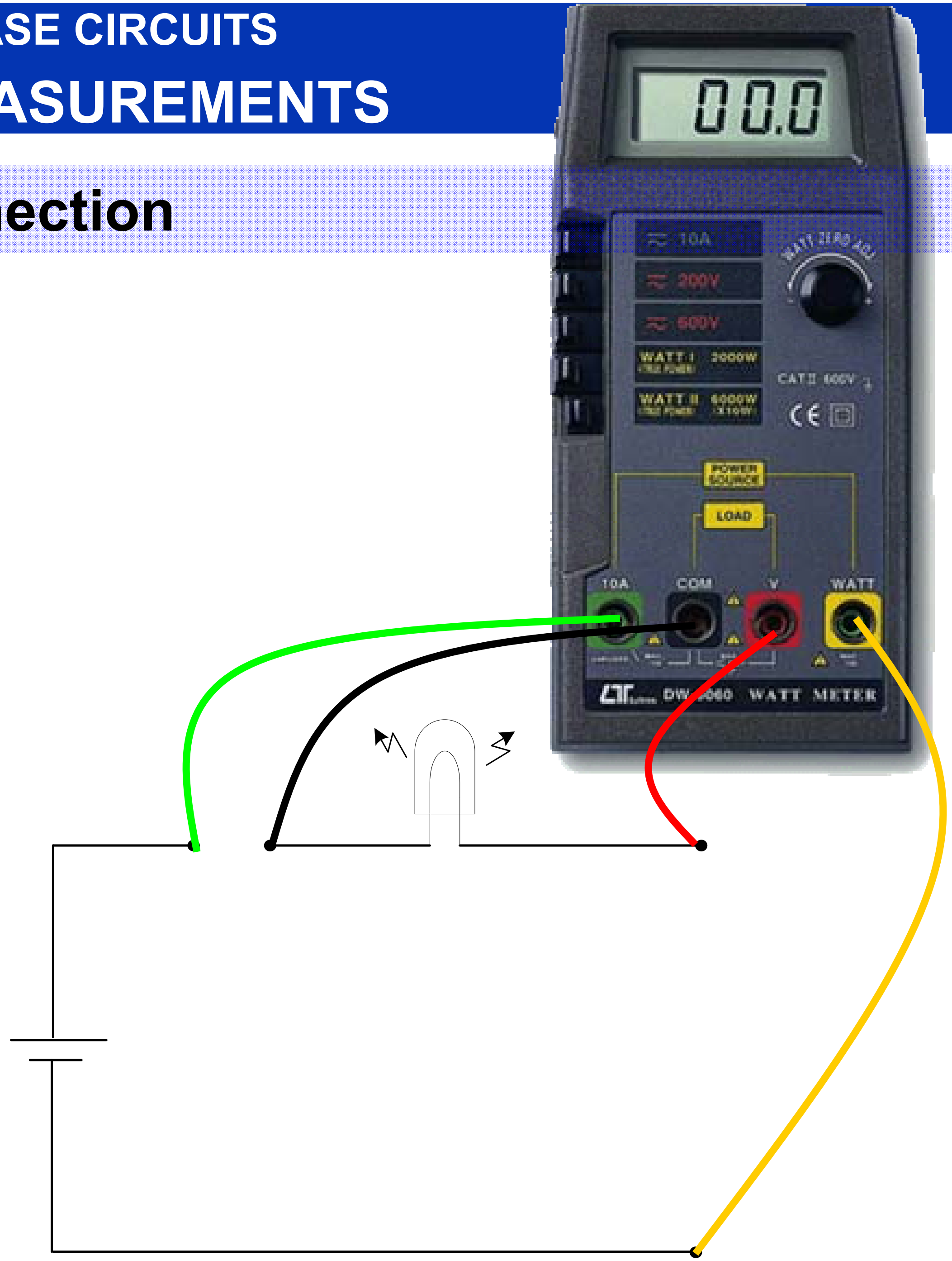
- Instruments for measuring the electric power in watts of any given circuit.
- They have 4 terminals (2 for voltage and 2 for current measurement)

Analog

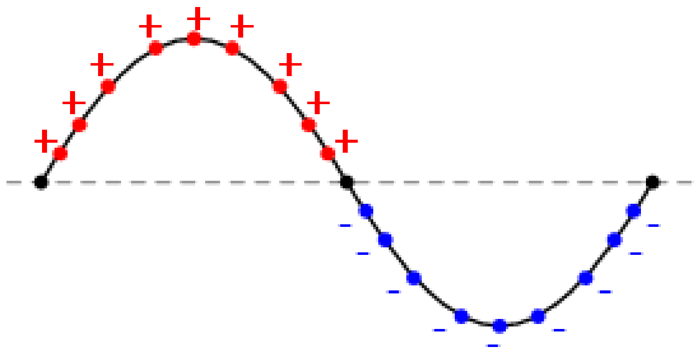


Digital

Wattmeter connection



Alternating Current Single-Phase Circuits



Unit 2. AC SINGLE-PHASE CIRCUITS

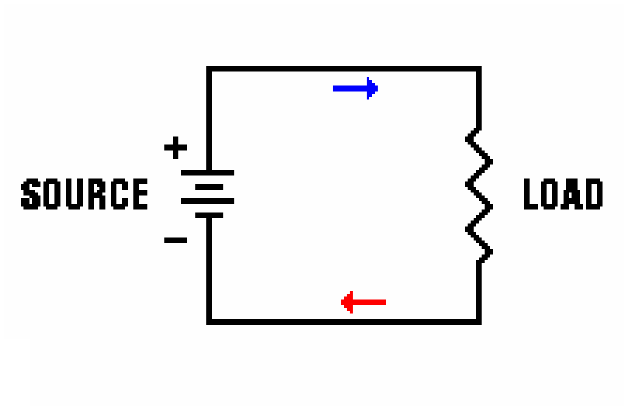
CONTENTS LIST:

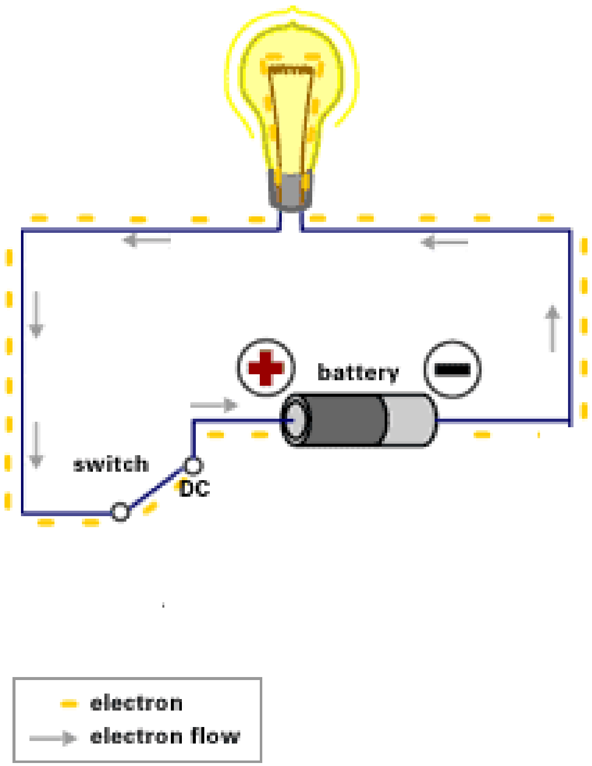
- Basics of AC circuits
- Importance of AC
- RMS and mean values
- Phasorial magnitudes
- Ohm’s law for AC circuits
- AC loads
- Instantaneous power
- Power triangle
- AC power: P, Q and S
- Power factor and cosφ
- Power factor improvement
- Electrical measurements
- Exercises

BASICS OF AC CIRCUITS

Direct Current

- Current flowing in a constant direction
- Voltage with constant polarity.



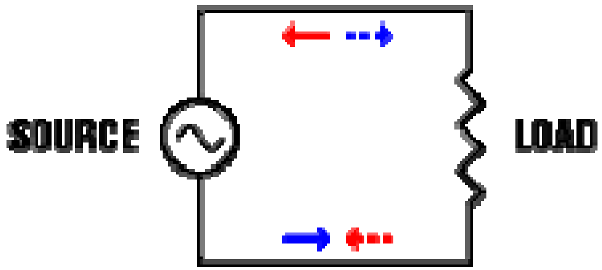


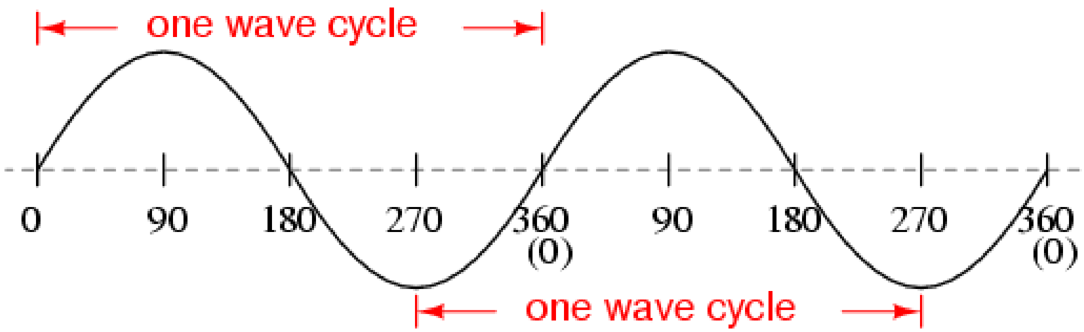
http://www.pbs.org/wgbh/amex/edison/sfeature/acdc.html

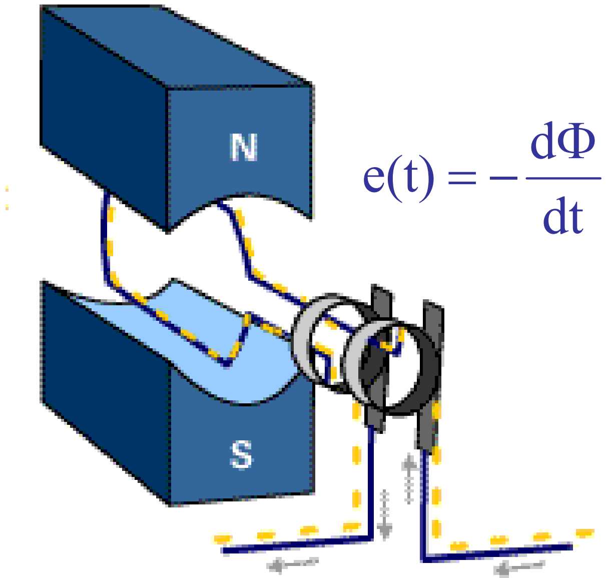
BASICS OF AC CIRCUITS

Alternating Current

- Current flowing with alternating polarity, reversing positive and negative over time.
- Voltages with alternating in polarity.





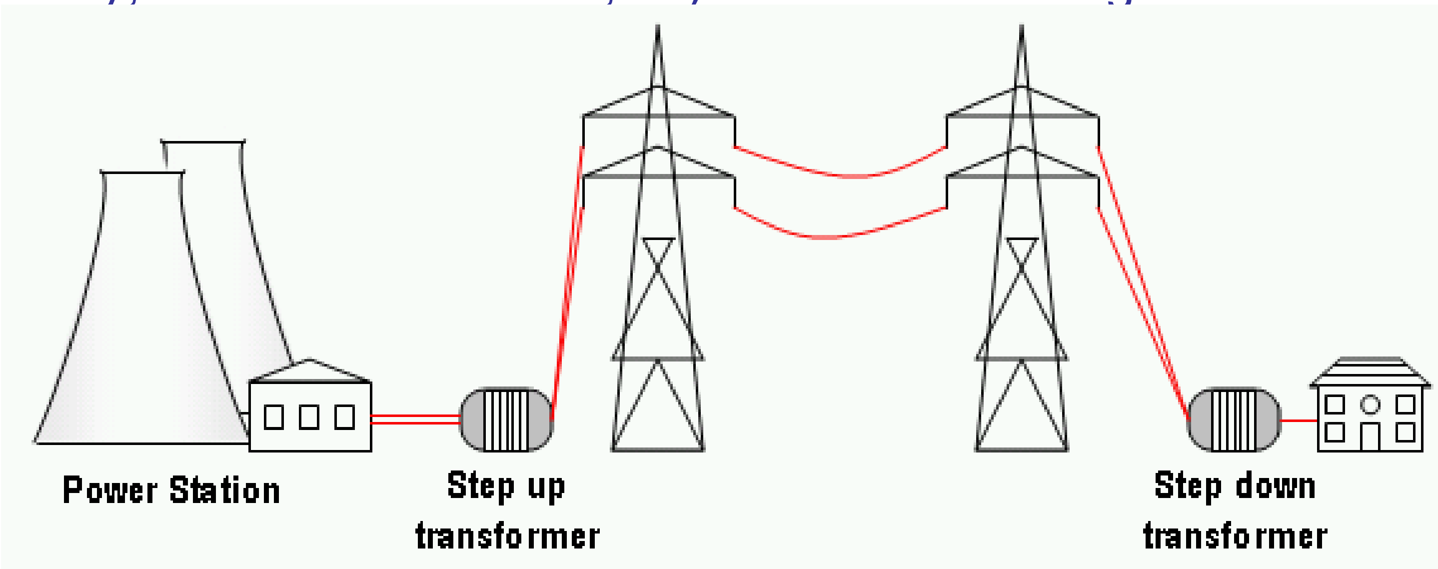


$$e(t) = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\Phi_0 \cos \omega t) = \underbrace{\omega\Phi_0}_{E_0} \sin \omega t$$

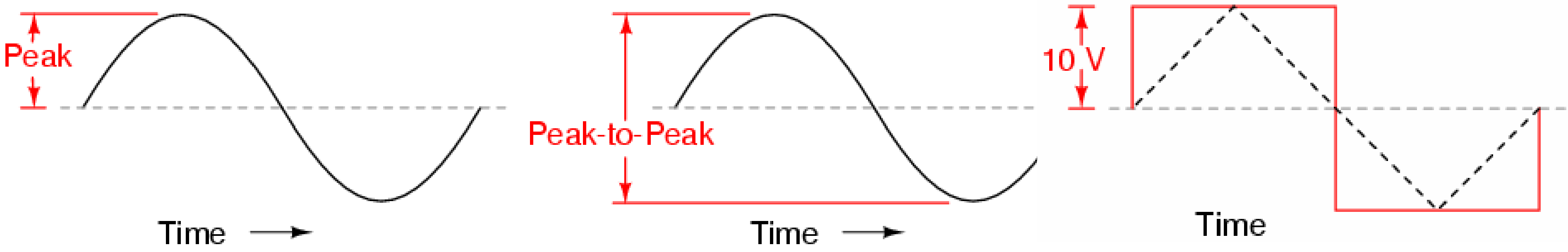
http://www-tc.pbs.org/wgbh/amex/edison/sfeature/images/acdc_inside_generator.gif

WHY AC? Importance of AC

- Electric power is generated, transmitted, distributed and consumed in AC
- ≈ 90% of the total electric power is consumed as AC
- AC amplitude can be easily changed (step up or step down) by means of simple and cost effective electrical machines called **transformers**
- Electric power can be transmitted efficiently and economically as High Voltage AC, minimizing power losses
- Three-phase AC motors/generators have better performance (higher efficiency, lower maintenance, …) than DC motors/generators

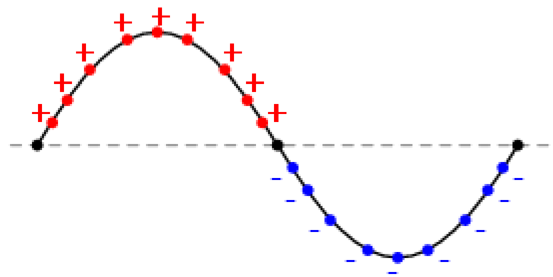


MEASUREMENTS OF AC MAGNITUDES



Peak, Peak-to-Peak and Average Values

- **SAME VALUES** for sinusoidal, rectangular and triangular waveforms
- **Average value = 0**



*True average value of all points (considering their signs) is **zero**!*

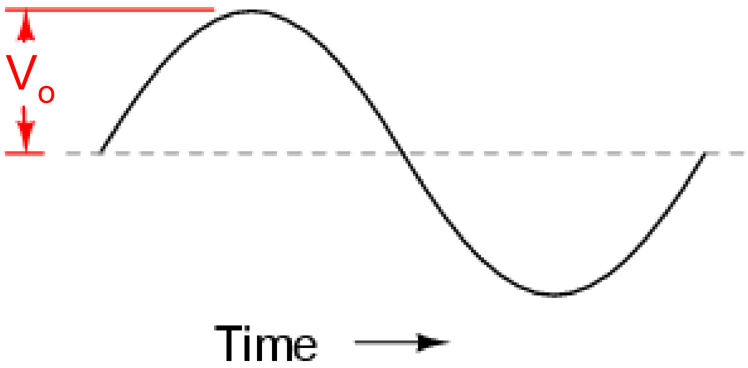
Root Mean Square Value (RMS)

- For sinusoidal waveform **RMS = Amplitude/sqrt(2)**
- T: period of signal (s)

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \frac{V_0}{\sqrt{2}} \qquad I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \frac{I_0}{\sqrt{2}}$$

MATHEMATICAL FORMULATION OF AC MAGNITUDES

A phase vector ("phasor") is a representation of a sine wave whose amplitude (A), phase (φ), and frequency (ω) are time-invariant.

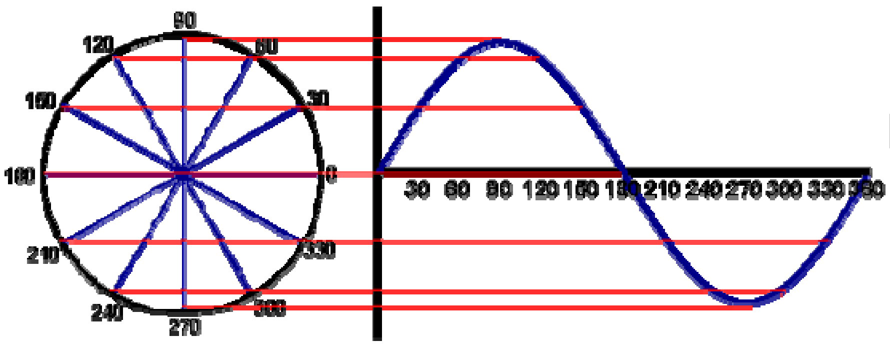


$$v(t) = V_o \cos(\omega t + \varphi_V)$$

$$\omega = 2 \cdot \pi \cdot f \text{ rad/s}$$

Phasorial expression (phasor = phase vector)

$$\overline{V} = V_{RMS}{}^{\varphi_V\,^{o}}$$



Modulus: RMS value of the AC magnitude

Phase: initial phase angle of the AC magnitude

Frequency: does not appear in the phasor

OHM'S LAW FOR AC CIRCUITS

- Ohm's law for DC circuits: $V = I.R$

- Ohm's law for AC circuits: $\bar{V} = \bar{I}\,\bar{Z}$

being $\bar{Z} = R + jX = Z^{\angle\varphi}$

Z: impedance R: resistance X: reactance

All quantities expressed in complex, not scalar form

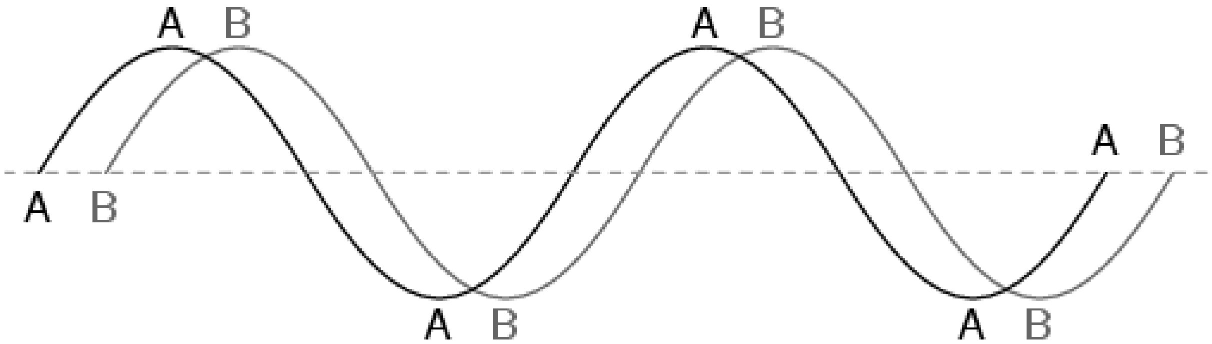
$$\varphi_Z = \varphi = \varphi_V - \varphi_I$$

$v(t) = V_o\cos(\omega t + \varphi_V)$

$i(t) = I_o\cos(\omega t + \varphi_I)$

PHASE IN AC CIRCUITS

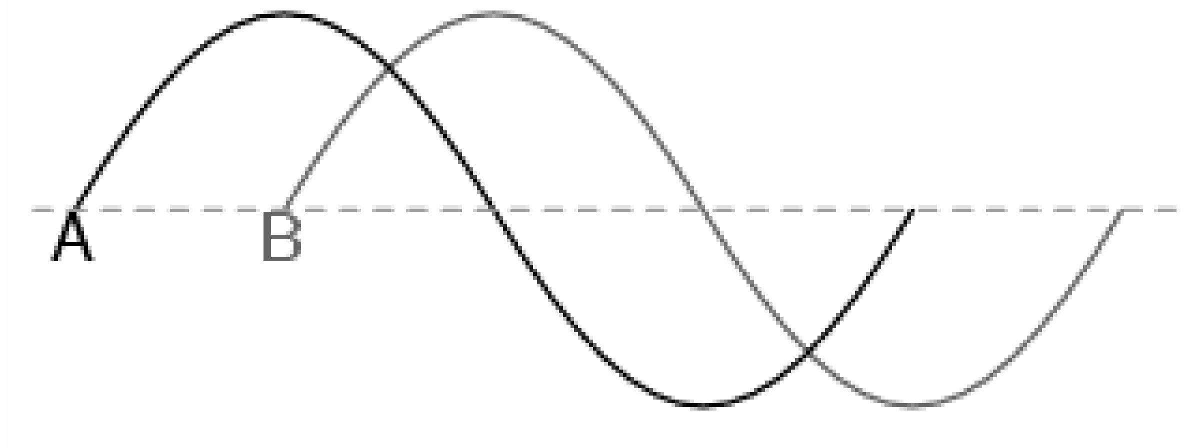
- **Out of phase** waveforms



- Phase shift of 90 degrees:

A leads B

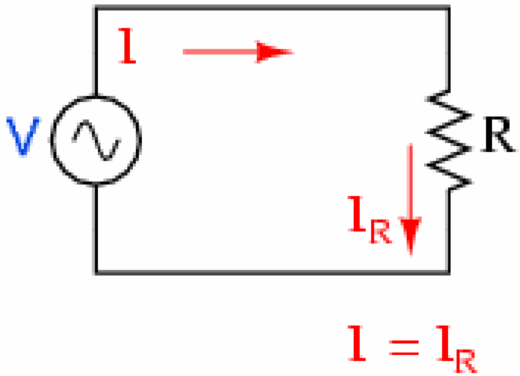
B lags A

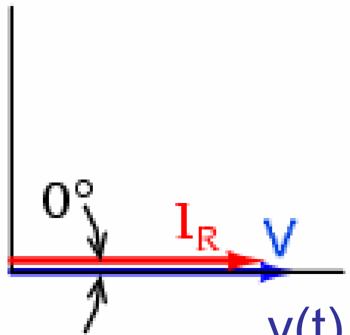


AC PURE RESISTIVE CIRCUITS

- Voltage and current are "***in phase***"

- Instantaneous AC power is always positive.
- φ = 0º phase shift between voltage and current → $\cos\varphi = 1$





$v_R(t) = R \cdot i(t)$

$v(t) = V_o\cos(\omega t + \varphi)$

⬇

$i(t) = (V_0/R)\cos(\omega t + \varphi)$

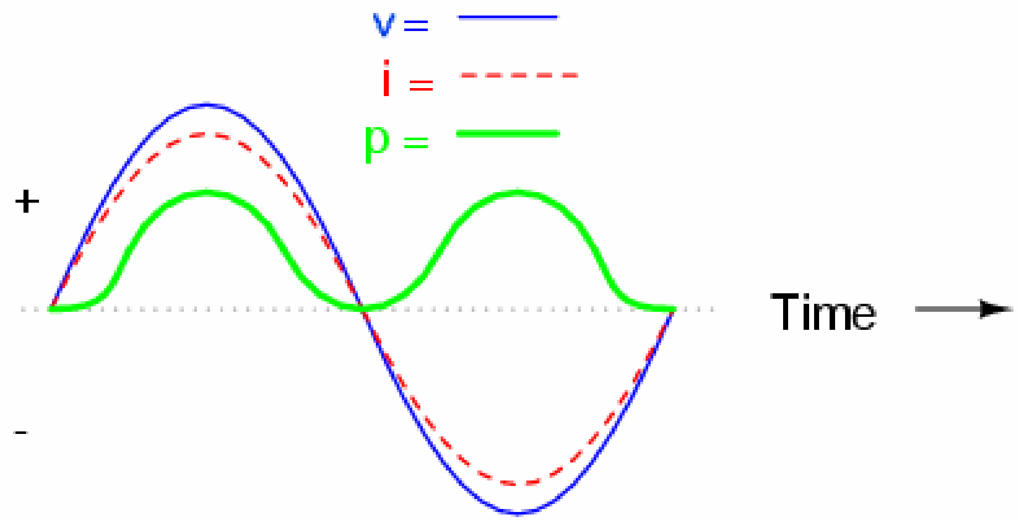
⬇

$i(t) = I_0\cos(\omega t + \varphi)$

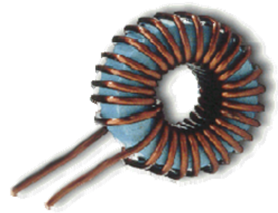
⬇

$\varphi_V = \varphi_I$

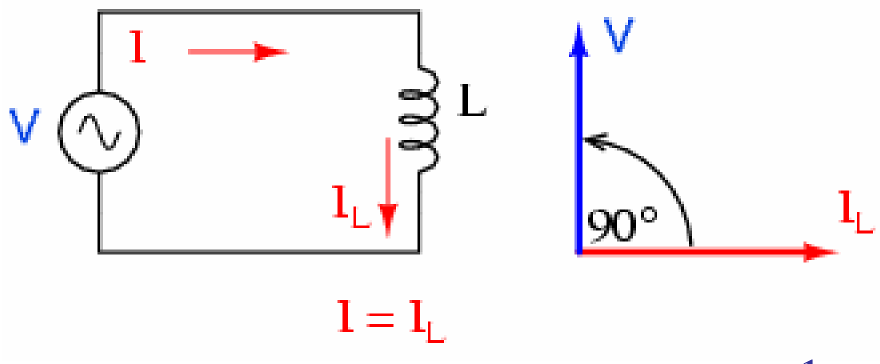
$Z = R + j0$



AC PURE INDUCTIVE CIRCUITS



- Inductor current lags inductor voltage by 90º.
- Instantaneous AC power may be positive or negative
- φ = 90º phase shift between voltage and current $\rightarrow$ $\cos\varphi = 0$



$$v_L(t) = L \cdot di(t)/dt$$

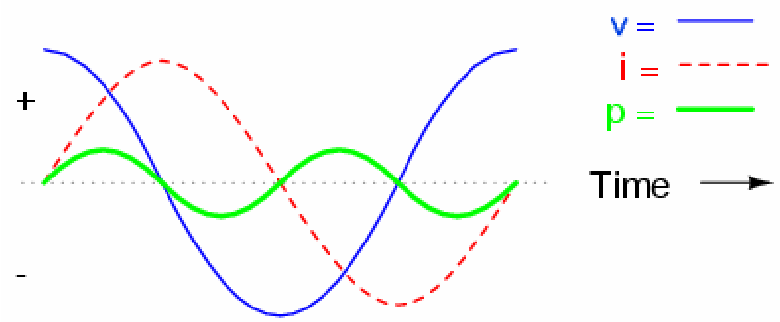
$$v(t) = V_o \cos(\omega t + \varphi)$$

$$i(t) = \frac{1}{L}\int v(t)dt = \frac{V_0}{\omega L}\sin(\omega t + \varphi_V) = \frac{V_0}{\omega L}\cos(\omega t + \varphi_V - 90^o)$$

$$i(t) = I_0 \cos(\omega t + \varphi_I)$$

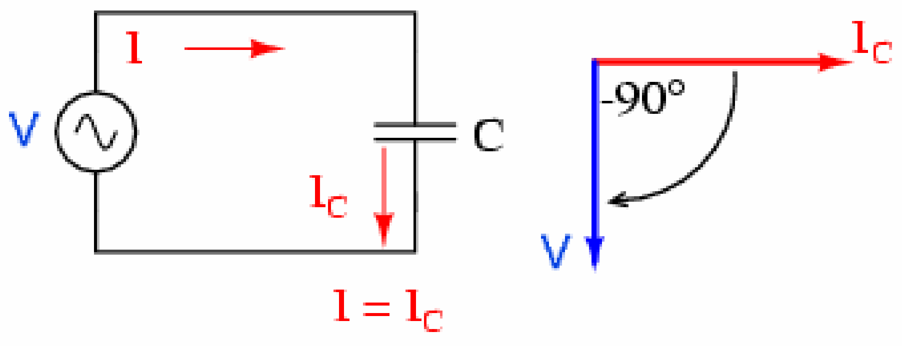
$$\varphi_V = \varphi_I + 90^o$$

$$Z = 0 + j\omega L$$



AC PURE CAPACITIVE CIRCUITS

- Capacitor voltage lags capacitor current by 90º.
- Instantaneous AC power may be positive or negative
- φ = -90º phase shift between voltage and current $\rightarrow$ $\cos\varphi = 0$



$i_C(t) = C \cdot dv(t)/dt$

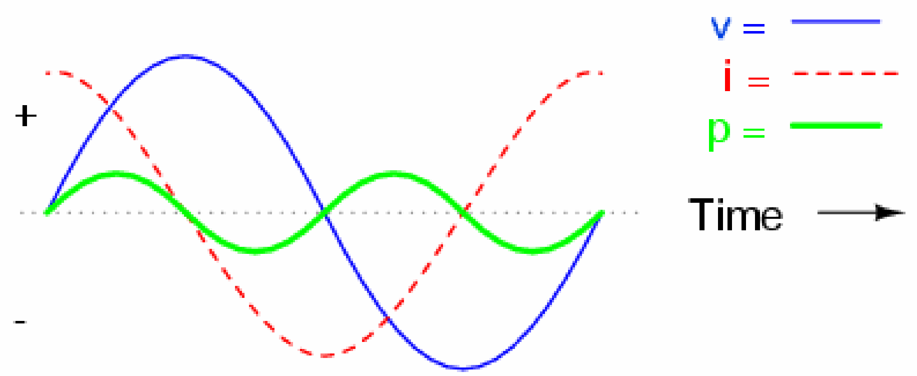
$v(t) = V_o \cos(\omega t + \varphi)$

$$\mathrm{i(t)} = \mathrm{C}\frac{\mathrm{dv(t)}}{\mathrm{dt}} = -\omega C V_0 \sin(\omega t + \varphi_V) = \omega C V_0 \cos(\omega t + \varphi_V + 90^o)$$

$i(t) = I_0 \cos(\omega t + \varphi_I)$

$\varphi_V = \varphi_I - 90º$

$Z = 0 - j/(\omega C)$



AC LOADS SUMMARY

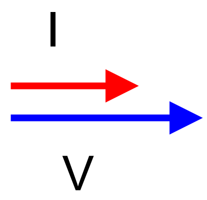
- **PURE RESISTANCE**

  V and I "***in phase***" $\varphi = 0º$

  P consumption but Q = 0

  $\cos\varphi = 1$

  $Z = R + j0$



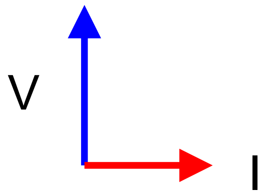
- **PURE INDUCTOR**

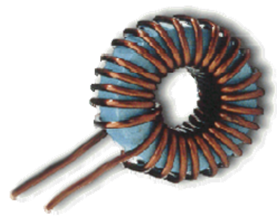
  V ***leads*** I by 90º, $\varphi = 90º$

  P =0 and Q > 0

  $\cos\varphi = 0$

  $Z = 0 + j\omega L$





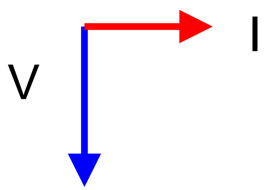
- **PURE CAPACITOR**

  V ***lags*** I by 90º, $\varphi = -90º$

  P =0 and Q < 0

  $\cos\varphi = 0$

  $Z = 0 - j/(\omega C)$



POWER IN THE TIME DOMAIN

- **Instantaneous power:**

$$p(t) = v(t)\cdot i(t) = V_0\cos(wt + \varphi_V)\cdot I_0\cos(wt + \varphi_I)$$

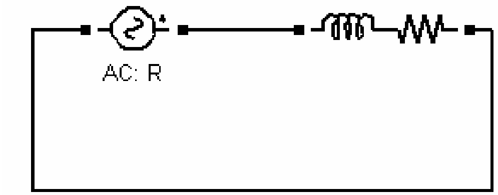
$$\cos A\cdot\cos B = 1/2\cdot[\cos(A+B) + \cos(A - B)]$$

$$p(t) = ½V_0I_0\cos(\varphi_V - \varphi_I) + ½V_0I_0\cos(2wt + \varphi_V + \varphi_I) \text{ watt}$$

- The average value is the **active or true power**:

$$\mathbf{P = ½\cdot V_0\cdot I_0\cdot\cos(\varphi_V - \varphi_I) = V_{RMS}\cdot I_{RMS}\cdot\cos\varphi} \quad \text{watts}$$

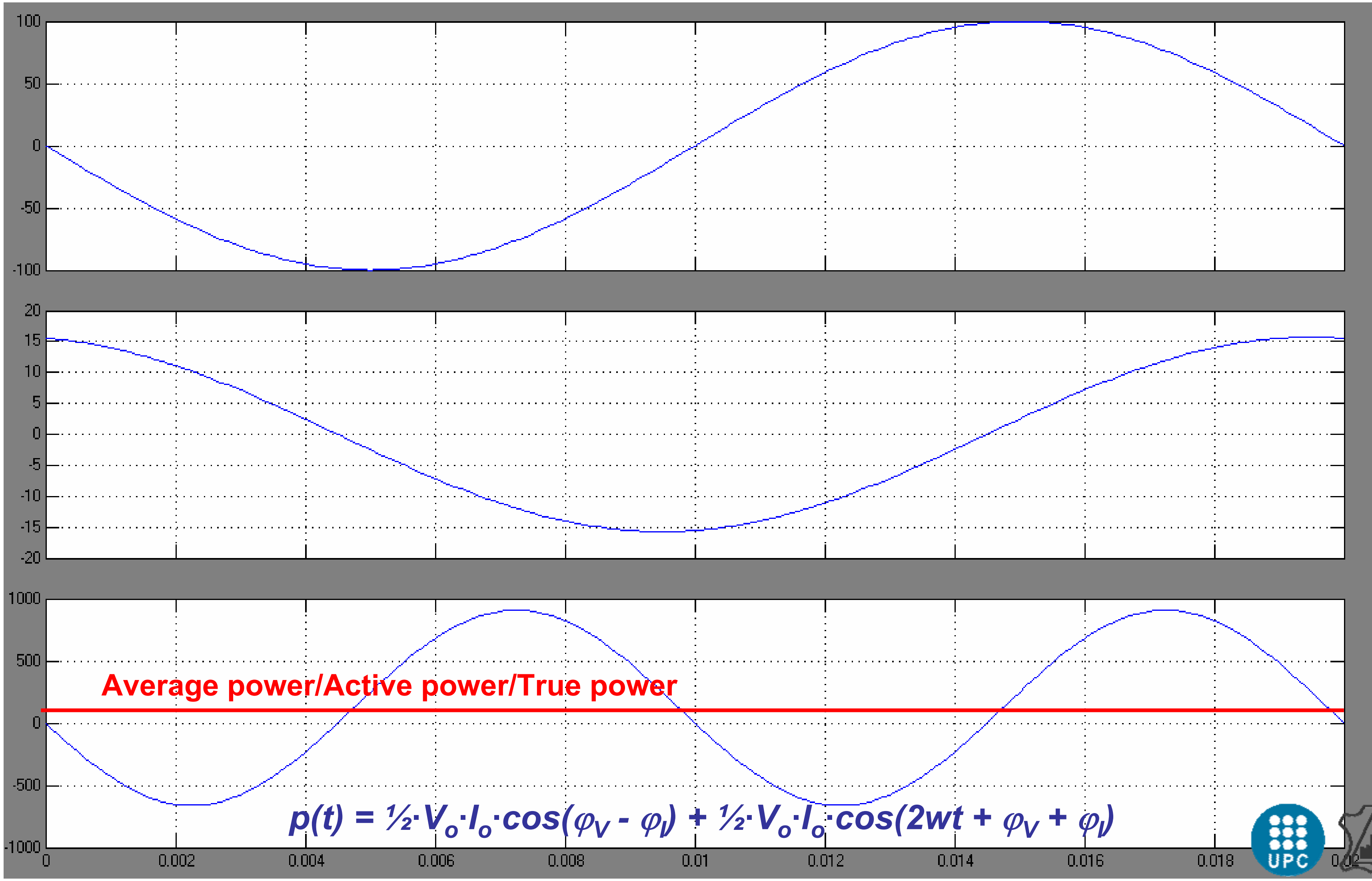
INSTANTANEOUS POWER



v(t)

i(t)

p(t)



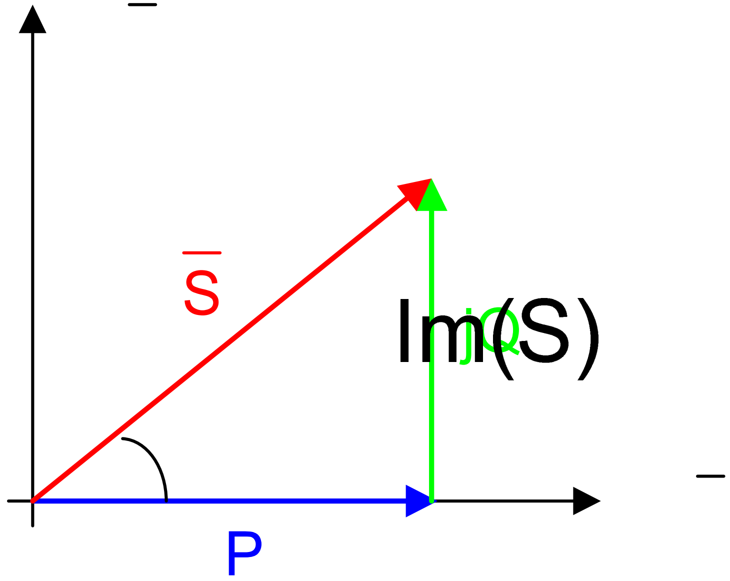
$$p(t) = \tfrac{1}{2} \cdot V_o \cdot I_o \cdot \cos(\varphi_V - \varphi_I) + \tfrac{1}{2} \cdot V_o \cdot I_o \cdot \cos(2wt + \varphi_V + \varphi_I)$$

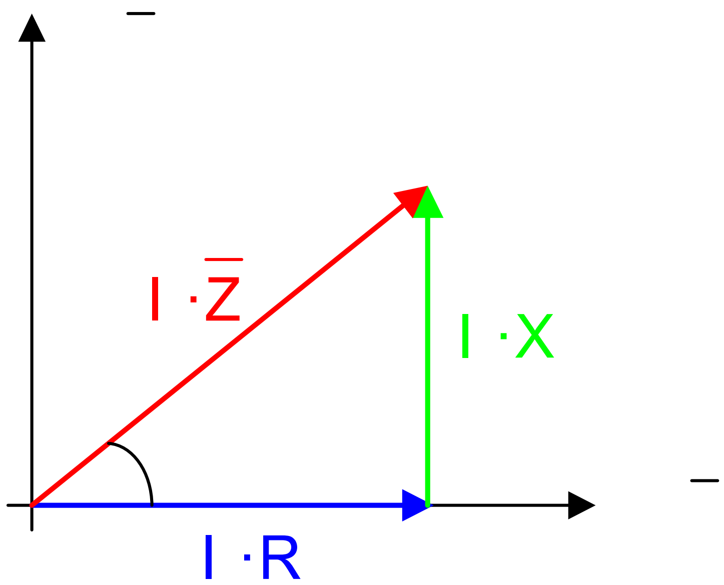
AC POWER

- Active/true power: $P = V \cdot I \cdot \cos\varphi \quad P = I^2 \cdot R$ (Watt)
- Reactive power: $Q = V \cdot I \cdot \sin\varphi \quad Q = I^2 \cdot X$ (VAr)
- Apparent power $S = V \cdot I = \mathrm{sqrt}(P^2+Q^2) \quad S = I^2 \cdot Z$ (VA)
- Complex power $\overline{S} = \overline{V}\,\overline{I}^* = I^2 \cdot \overline{Z} = P + jQ$ (VA)

V, I: RMS values

THE POWER TRIANGLE

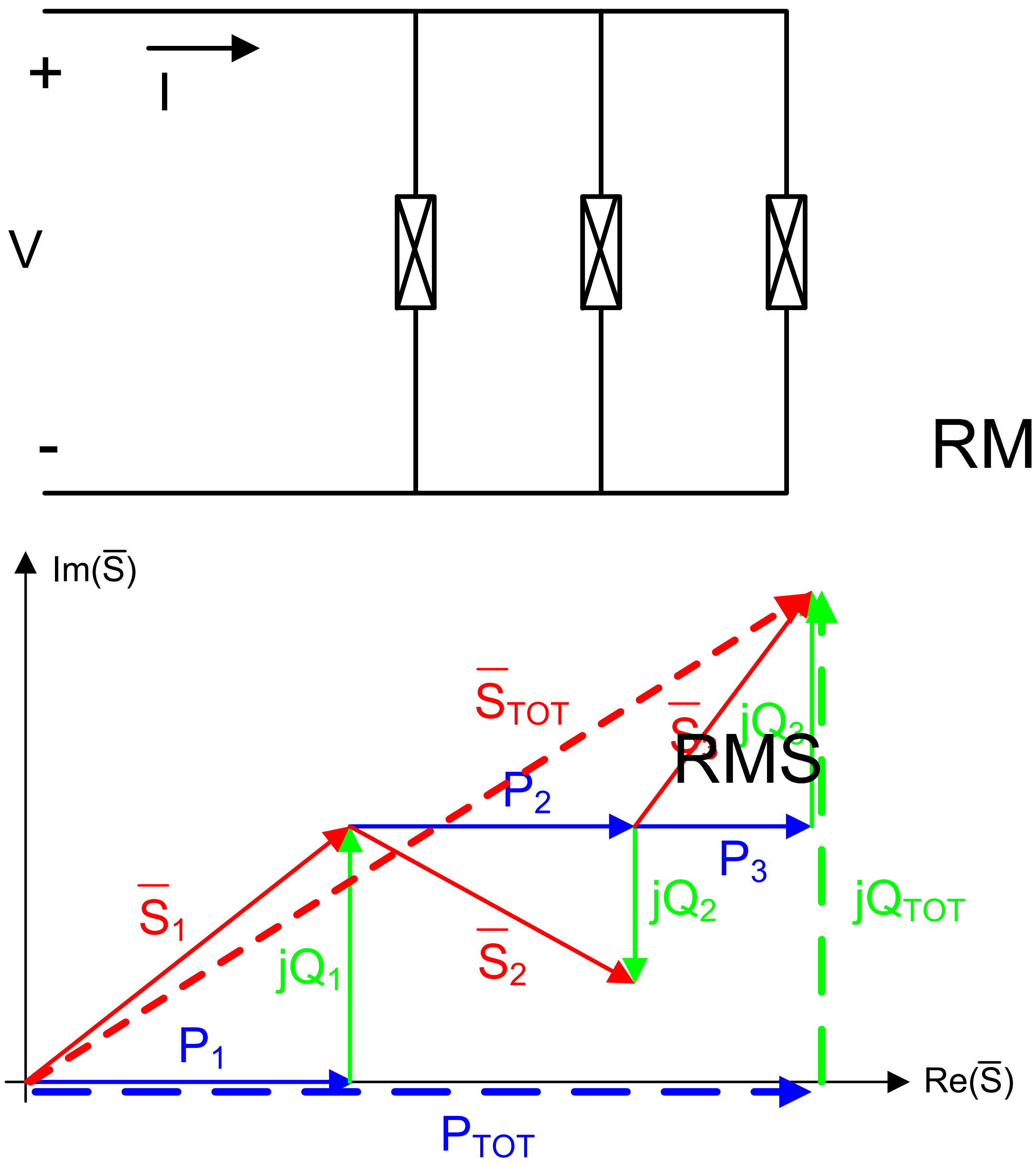




φ

Re(S)

AC POWER: PARALLEL-CONNECTED NETWORKS



RMS

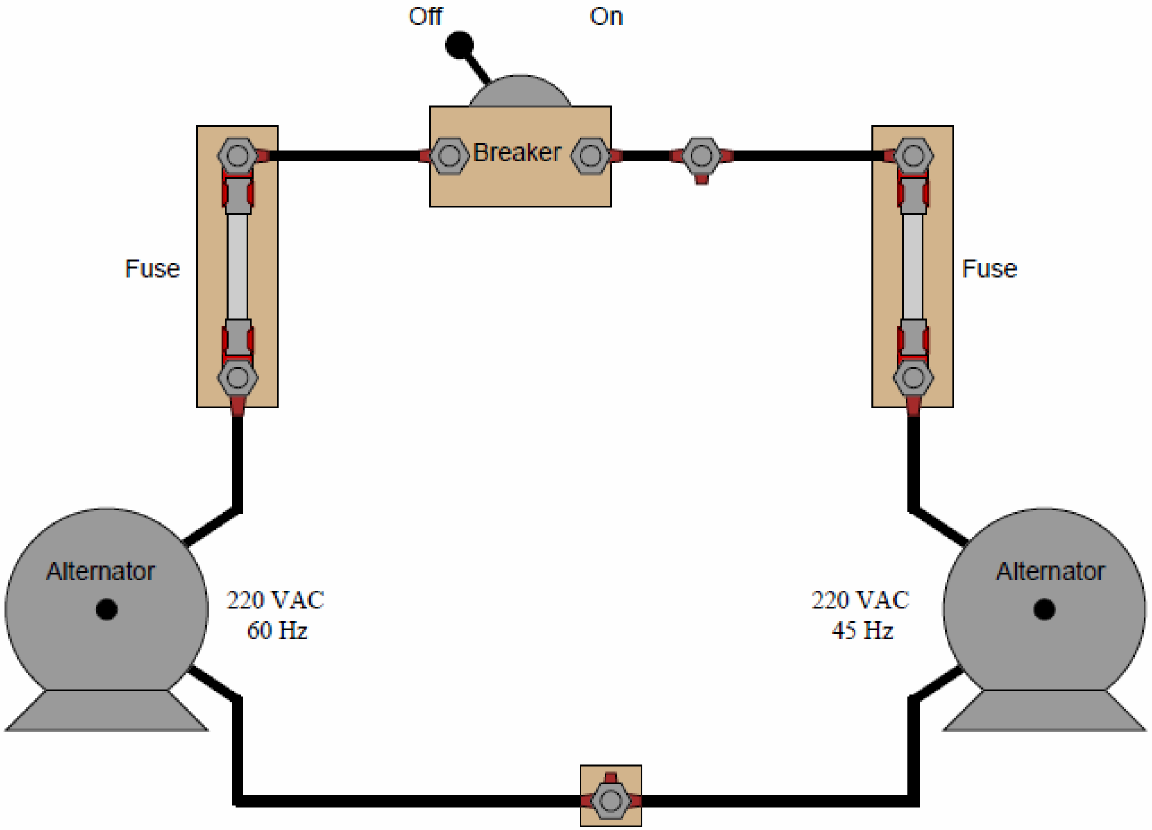
P1

Q1

RMS

QUESTION

Is it safe to close the breaker between these two alternators if their output frequencies are different? Explain why or why not.

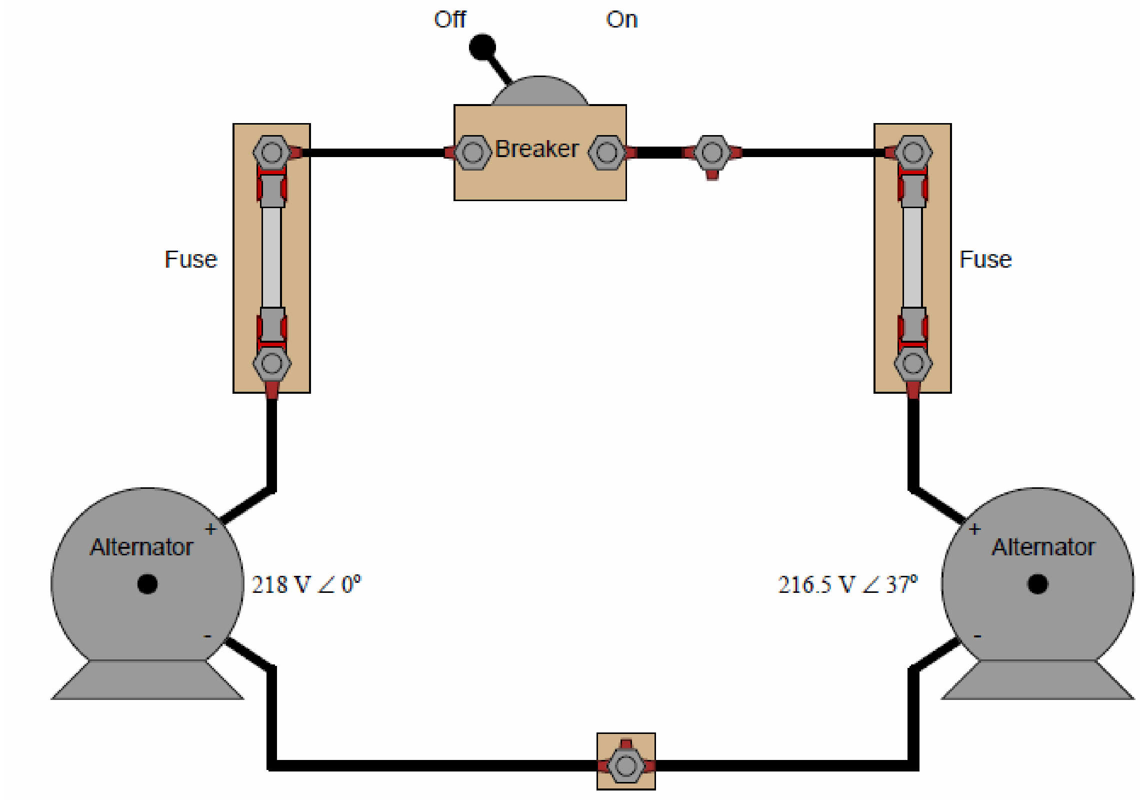


Solution: When the frequencies of two or more AC voltage sources are different, the phase shift(s) between them are constantly changing.

http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

QUESTION

Given the output voltages of the two alternators, it is not safe to close the breaker. Explain why.

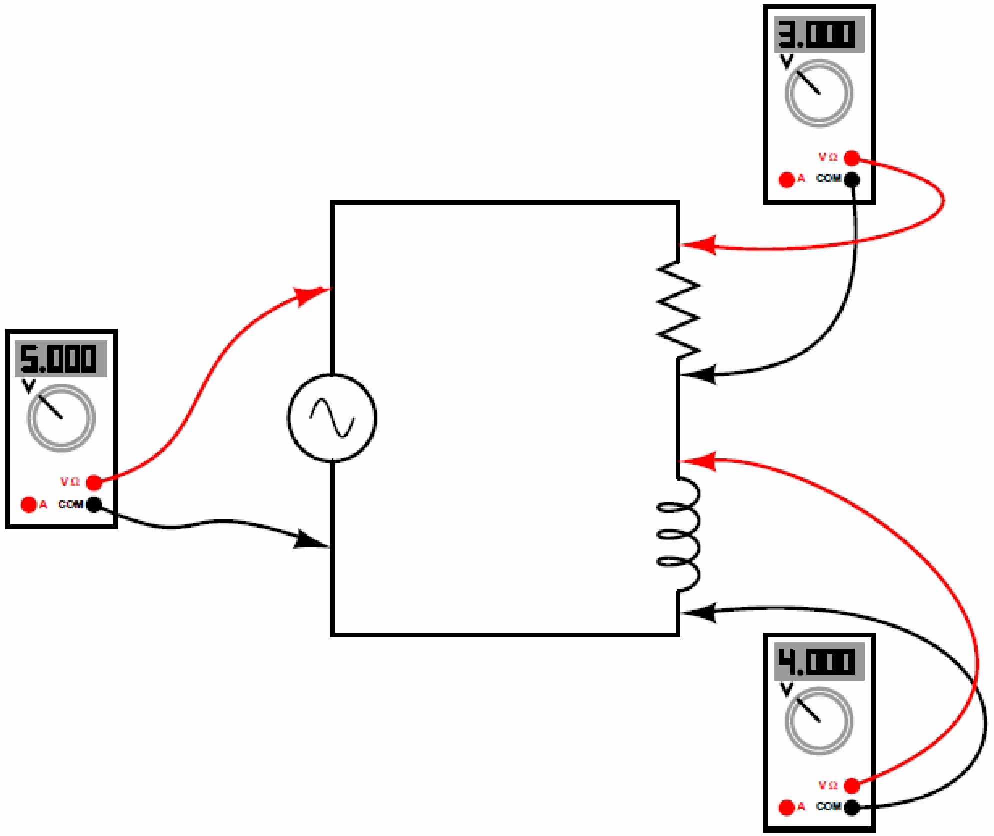


Solution: The greatest problem with closing the breaker is the 37° phase shift between the two alternators’ output voltages.

http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

QUESTION

Are the readings of the voltmeters possible? If true, how would you represent the tree voltages in this circuit in rectangular and polar forms?.



http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

POWER FACTOR AND COSφ

- Power factor definition: **PF = P/S**
- cosφ definition: $\cos\varphi = \cos(\varphi_V - \varphi_I)$ $\begin{cases} \varphi > 0 & \text{lagging} \\ \varphi < 0 & \text{leading} \end{cases}$
- When dealing with single-frequency single-phase circuits:

$$PF = P/S = (V \cdot I \cdot \cos\varphi)/(V \cdot I) = \cos\varphi$$

EXAMPLE

Example. A circuit has equivalent impedance $\bar{Z} = 3 + j\cdot 4\ \Omega$ and an applied voltage v(t) = 42.5·cos(1000t + 30º) (volt). Give complete power information.

$$\bar{V}_{RMS} = \frac{42.5^{\,30^{\circ}}}{\sqrt{2}}\ \text{V}$$

$$\bar{I}_{RMS} = \frac{\bar{V}_{RMS}}{\bar{Z}} = \frac{42.5/\sqrt{2}^{\,30^{\circ}}}{3 + j\cdot 4} = \frac{42.5/\sqrt{2}^{\,30^{\circ}}}{5^{53.13^{\circ}}} = 6.01^{-23.13^{\circ}}\ \text{A}$$

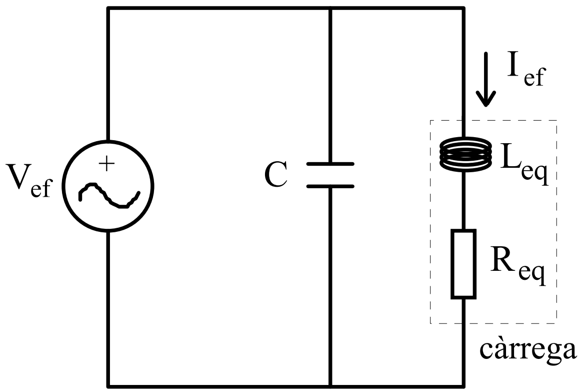
$$\bar{S} = \bar{V}_{RMS}\cdot\bar{I}_{RMS}{}^{*} = 42.5/\sqrt{2}^{\,30^{\circ}}\cdot 6.01^{+23.13^{\circ}} = 180.61^{53.13^{\circ}}\ \text{VA}$$

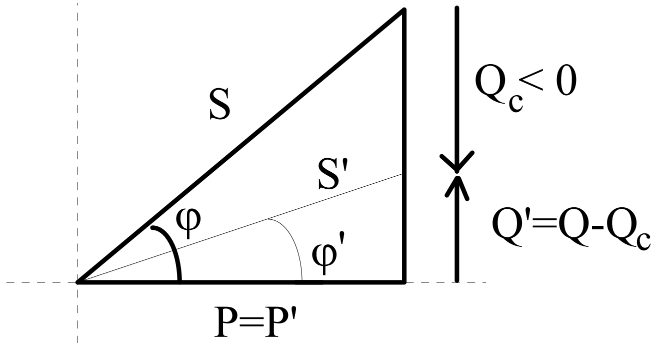
$$\bar{S} = 180.61^{53.13^{\circ}} = 108.4(\text{W}) + j\cdot 144.5(\text{VAr})$$

Hence, P = 108.4 W, Q = 144.5 VAr (i), S = 180.6 VA, and PF = cos 53.13º = 0.6 (i).

POWER FACTOR IMPROVEMENT

- Inductive circuits: **C in parallel**





$$tg\varphi' = \frac{Q - |Q_C|}{P} = \frac{Q - \left|\frac{V^2}{1/(2\pi fC)}\right|}{P} = \frac{Q - |V^2 2\pi fC|}{P}$$

Capacitive circuits: **L in parallel**

QUESTIONS REGARDING PF IMPROVEMENT

- Transformers, distribution systems, and utility company alternators are all rated in kVA or MVA.
- Consequently, an improvement in the power factor, with its corresponding reduction in kVA, releases some of this generation and transmission capability so that it can be used to serve other customers.
- This is the reason that **make it more costly for an industrial customer to operate with a lower power factor**.

Example. A load of P = 23 kW with PF = 0.5 (i) is fed by a 230 V source. A capacitor is added in parallel such that the power factor is improved to 1. Find the reduction in current drawn from the generator.

Before improvement:

$$P = 23000\ W = 230 \cdot I \cdot 0.5 \quad \rightarrow \quad I = \mathbf{200\ A}$$

After improvement:

$$P = 23000\ W = 230 \cdot I \cdot 1 \quad \rightarrow \quad I = \mathbf{100\ A}$$

PF IMPROVEMENT BENEFITS

- Reduces reactive power and amps absorbed by the load.
- Active power driven by transformers is optimized
- Reduce voltage drop in the conductors
- Reduces power losses in the conductors of the consumer.
- Reduces power losses in the conductors during transmission.
- Conductor section can be minimized (**capital savings**).
- Base price of electrical energy (kWh) increases if PF is low.
- Installation is used more efficiently.
- Energy is used more efficiently
- Less power has to be generated (**environmental benefits**).

Electric bill reduction

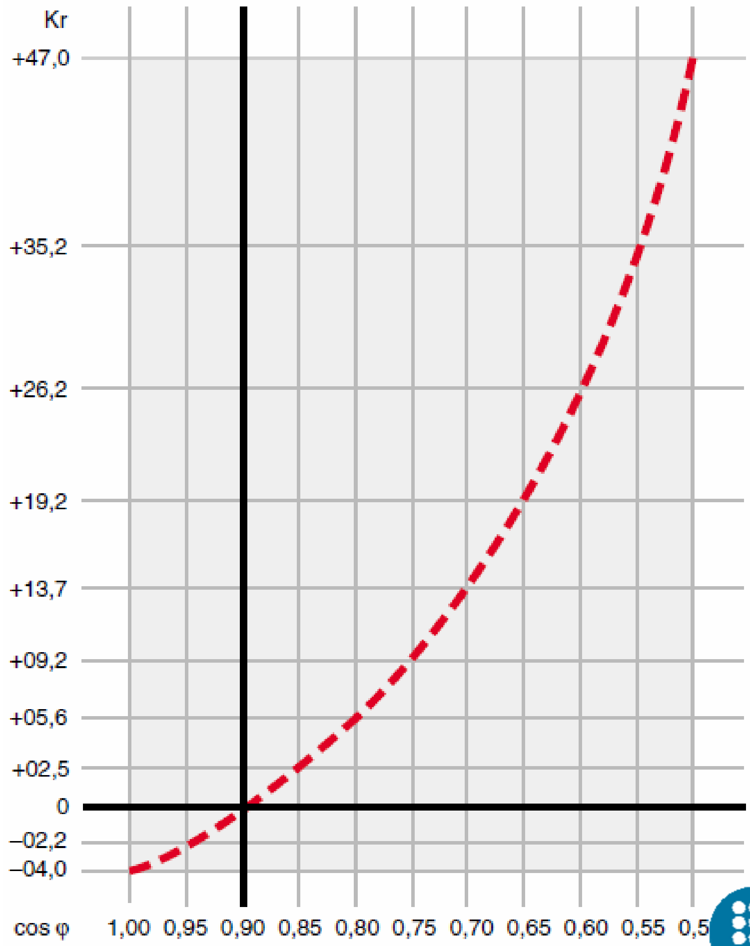
The improvement of the PF of an existing electrical installation has many economical advantages and allows to reduce the base cost of the kWh.

$Kr = 17/\cos^2\varphi - 21$ %

Kr is limited within:

+ 47 % (PF = 0.5)

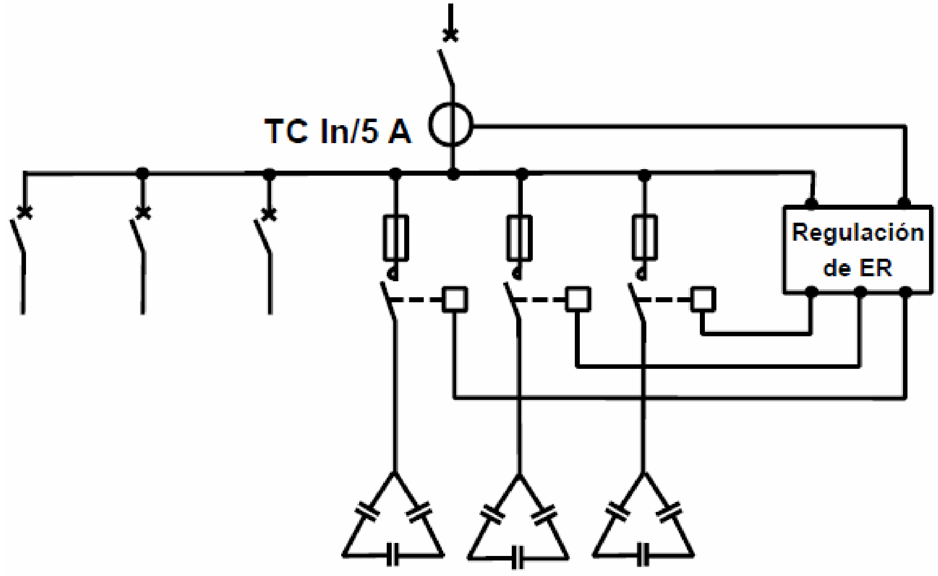
– 4 % (PF = 1)



PF IMPROVEMENT

- Capacitor banks with automatic regulation allow to adaptate the compensation to variable load.
- Capacitor banks with automatic regulation are placed at the end of the LV installation or in the distributing switch-board (*cuadro de distribución*) with an important consumption of reactive power.
- Capacitor banks with automatic regulation consist of several steps of reactive power.

Automatic capacitor bank for PF correction



Automatic capacitor bank with several steps

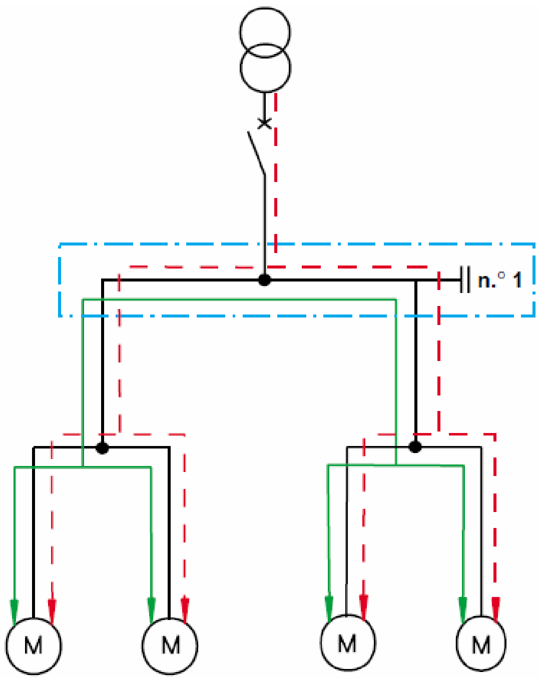
PF IMPROVEMENT

Types of power faction correction:

- Global
- Partial
- Individual.

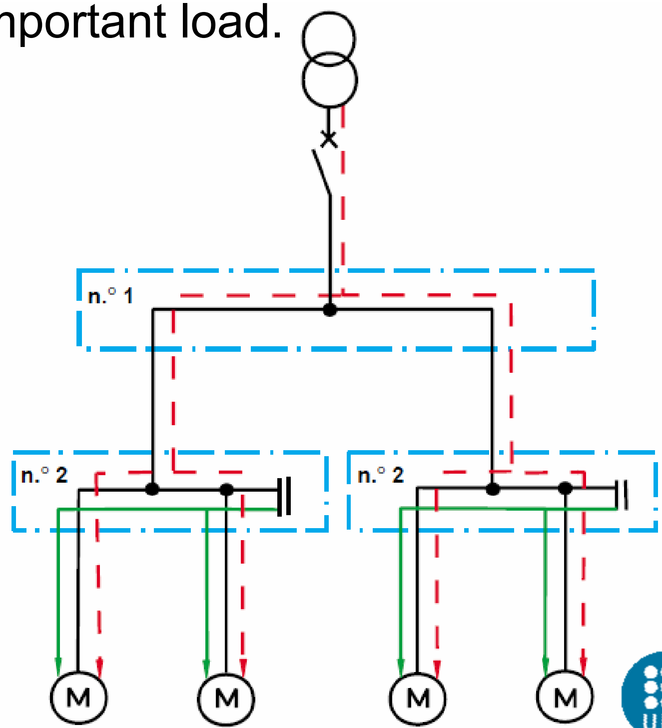
Global correction

This method is suitable for stable and continuous operated loads.



Partial correction

This method is suitable when distribution of loads is unbalanced and when a distributing switch-board (quadre de distribució) feeds an important load.

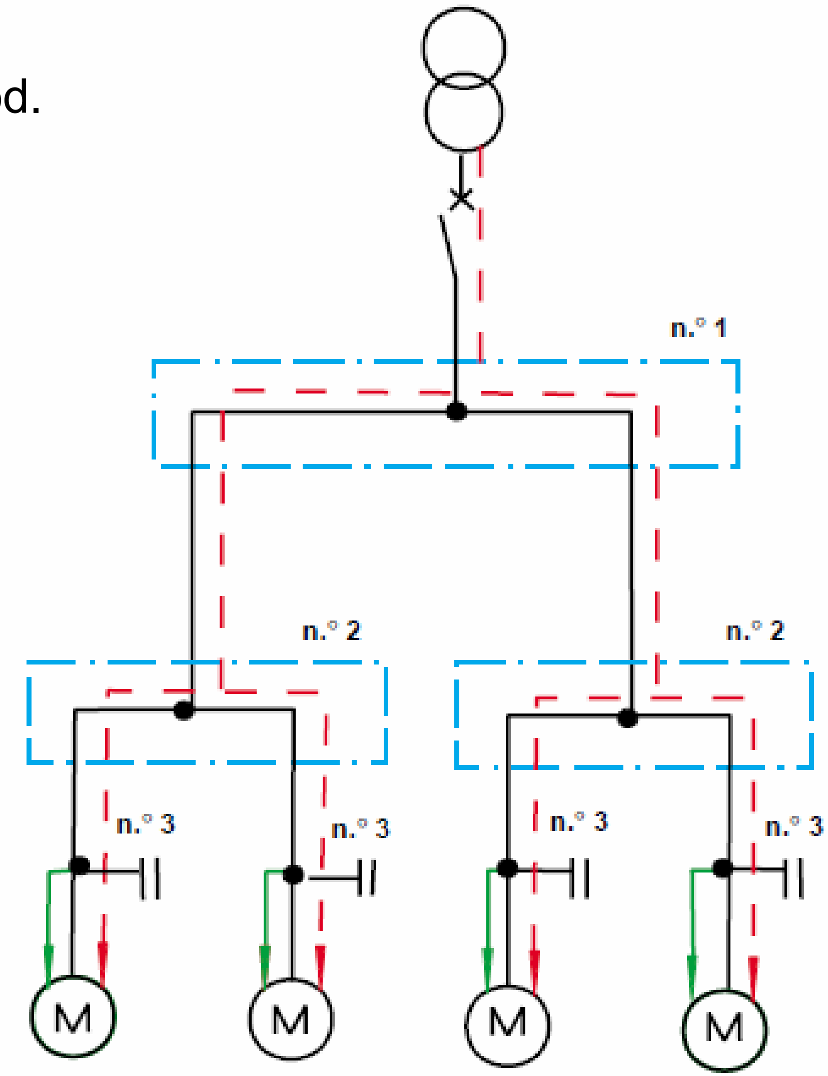


PF IMPROVEMENT

Individual correction

This method is suitable when exist loads which are very heavy in relation to the total load.

It is the most advantageous method.



QUESTIONS REGARDING PF IMPROVEMENT

Example. A sinusoidal source (230 V, 50 Hz) feeds an inductive load which absorbs 8 A and 1000 W. Calculate the PF, Q and S in the load. Improve the PF to unit and calculate the associated expenses savings.

$P = V \cdot I \cdot \cos\varphi$; $1000 = 230 \cdot 8 \cdot \cos\varphi \Rightarrow$ **$\cos\varphi = 0.54$ (i)**

$Q = V \cdot I \cdot \sin\varphi = 230 \cdot 8 \cdot 0.84 \Rightarrow$ **$Q = 1545$ VAr**

$S = V \cdot I = 230.8 \Rightarrow S = 1840$ VA $\Rightarrow$ **PF = P/S = 0.54 (i) = $\cos\varphi$**

Capacitor needed to improve the PF to unit: $Q_c = -1545$ VAr.

It results $C = 93\ \mu F$

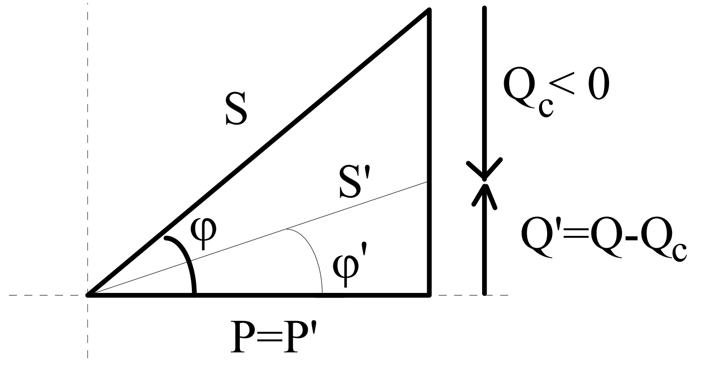
Increase in basis price due to PF: **$Kr = 17/\cos^2\varphi - 21$ %**

When PF = 0.54 $\Rightarrow Kr = 17/0{,}54^2 - 21 = +\ 37{,}3$ % (increase)

When PF = 1.0 $\Rightarrow Kr = 17/1{,}0 - 21 = -\ 4$ % (discount)

EXAMPLE 1

A 230 V and 50 Hz AC power supply feeds a 5 KVA single-phase load that presents a PF = 0,6 lagging. Improve the PF to: a) 0,9(i) b) 0,9(c).



$$tg\varphi' = \frac{Q - |Q_C|}{P}$$

lagging = (i)

leading = (c)

Before improving the PF: P = S·cosφ = 3 kW, Q = S·sinφ = 4 kVAr

a) cosφ' = 0.9 → φ' = 25.84º

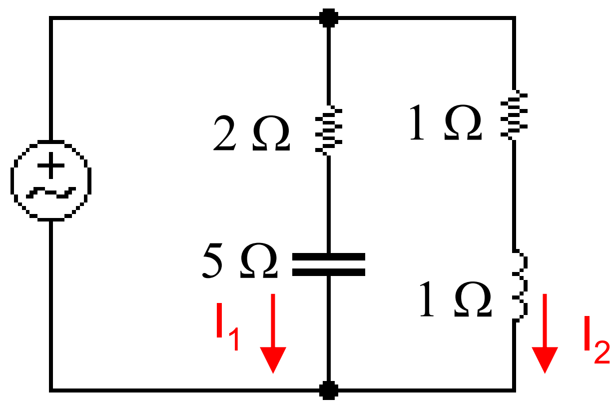
$$tg\varphi' = \frac{Q - |Q_C|}{P} \rightarrow tg25.84 = \frac{4000 - |Q_C|}{3000} \rightarrow |Q_C| = 2547.16\,\mathrm{VAr} = 230^2(2\pi 50C)$$

It results C = 153.26 μF

b) Operating in a similar manner as in a) the result is: C = 328.1 μF

EXAMPLE 2

The consumption of the 2 Ω resistance is 20 W. Represent the power triangle of the figure whole circuit.



$P_{2\Omega} = 20 = I_1^2 2 \rightarrow I_1 = 3.162\ A$

$V_{TOT} = I_1 Z_1 = 3.162 \cdot (2^2+5^2)^{1/2} = 17.03\ V$

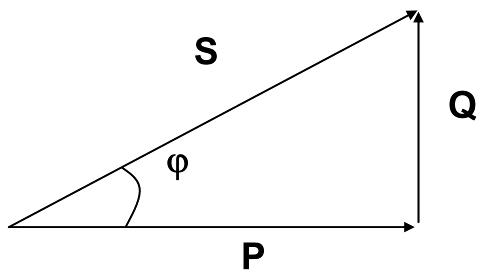
$I_2 = V_{TOT}/Z_2 = 17.03/(1^2+1^2)^{1/2} = 12.04\ A$

$P_{TOT} = P_{1\Omega} + P_{2\Omega} = 20 + 12.04^2 \cdot 1 = 165\ W$

$Q_{TOT} = Q_{5\Omega} + Q_{1\Omega} = -3.162^2 \cdot 5 + 12.04^2 \cdot 1 = 95\ VAr$

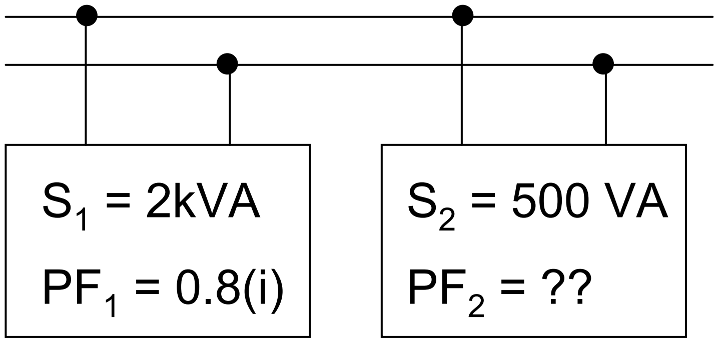
$S_{TOT} = (P_{TOT}^2 + Q_{TOT}^2)^{1/2} = 190.4\ VA$

$\varphi_{TOT} = \mathrm{arctg}(Q_{TOT}/P_{TOT}) = 29.93^o$



EXAMPLE 3

Determine the PF_2 when the measured PF_{TOTAL} = 0.90 (i).



$\cos\varphi_{TOTAL} = 0.90\ (i) \rightarrow \varphi_{TOTAL} = +25.84^{o}$

$P_1 = 2000 \cdot 0.8 = 1600\ W$ $\qquad Q_1 = 2000 \cdot 0.6 = 1200\ VAr$

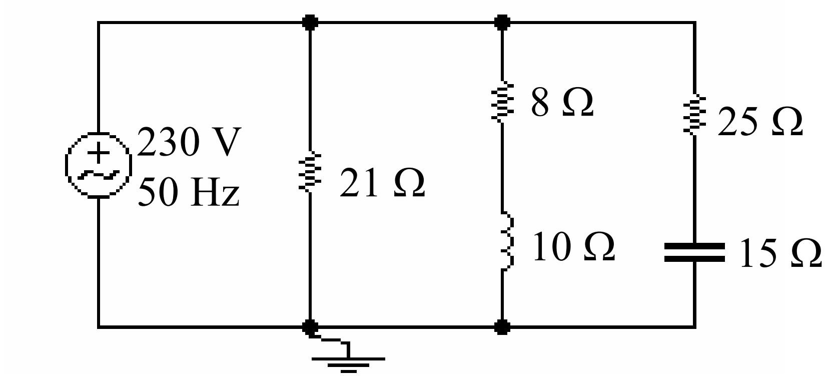
$P_2 = 500 \cdot \cos\varphi_2\ W$ $\qquad Q_2 = 500 \cdot \sin\varphi_2\ VAr$

$$\mathrm{tg}\,\varphi_{TOT} = \frac{Q_1 + Q_2}{P_1 + P_2} \rightarrow \mathrm{tg}\,25.84^{o} = \frac{1200 + 500\sin\varphi_2}{1600 + 500\cos\varphi_2}$$

$-425.08 = 500\sin\varphi_2 - 242.16\cos\varphi_2$ resulting in: $\varphi_2 = -24.07^{o} \rightarrow PF_2 = 0.913(c)$

EXAMPLE 4

From the following circuit, determine P, Q, S and PF in the load.



$$\frac{1}{\overline{Z}_{eq}} = \frac{1}{21+j0} + \frac{1}{8+j10} + \frac{1}{25-j15} = \frac{1}{21}^{0^o} + 0.0781^{-51.34^o} + 0.0343^{30.96^o} = 0.1331^{-19^o}\ \Omega^{-1}$$

The equivalent impedance of the circuit is: $\overline{Z}_{eq} = 7.52^{19^o}\ \Omega$

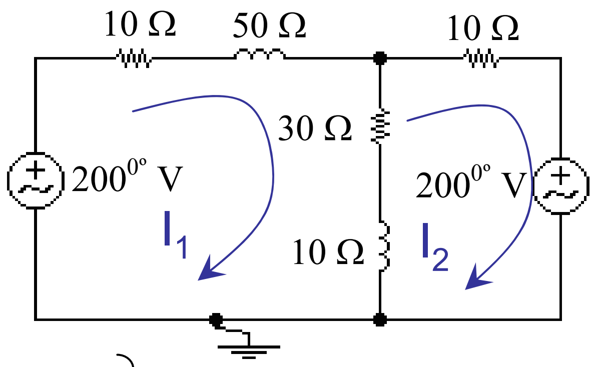
PF = cos(19º) =0.945 (i)

$\overline{I} = \overline{V}/\overline{Z}_{eq} = 230^{0^o} / 7.52^{19^o} = 30.61^{-19^o}$ A

$\overline{S} = \overline{V} \cdot \overline{I}^* = 230^{0^o} \cdot 30.61^{+19^o} = 7039.25^{19^o}$ VA = 6655.75 (W) + j.2291.76 (VAr)

EXAMPLE 5. The mesh method (malles)

From the following circuit, determine the current in the 200 V voltage sources.



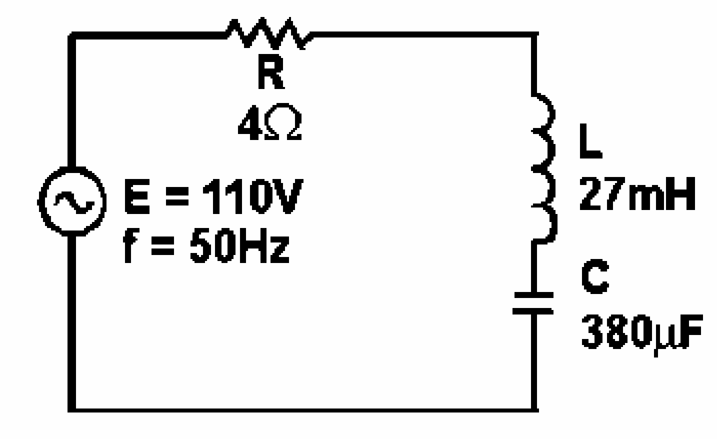
$$\left.\begin{aligned}&\text{Mesh 1: } \Sigma V_i = \Sigma I_i Z_i \qquad 200^{0°} = I_1(10+j50+30+j10) - I_2(30+j10)\\ &\text{Mesh 1: } \Sigma V_i = \Sigma I_i Z_i \qquad -200^{0°} = -I_1(30+j10) + I_2(10+30+j10)\end{aligned}\right\}$$

$$\bar{I}_1 = \frac{\begin{vmatrix} 200 & -30-j10 \\ -200 & 40+j10 \end{vmatrix}}{\begin{vmatrix} 40+j60 & -30-j10 \\ -30-j10 & 40+j10 \end{vmatrix}} = \frac{2000}{200+j2000} = 0.995^{-84.29°}\ A$$

$$\bar{I}_2 = \frac{\begin{vmatrix} 40+j60 & 200 \\ -30-j10 & -200 \end{vmatrix}}{\begin{vmatrix} 40+j60 & -30-j10 \\ -30-j10 & 40+j10 \end{vmatrix}} = \frac{-2000-j10000}{200+j2000} = 5.704^{-185.60°}\ A$$

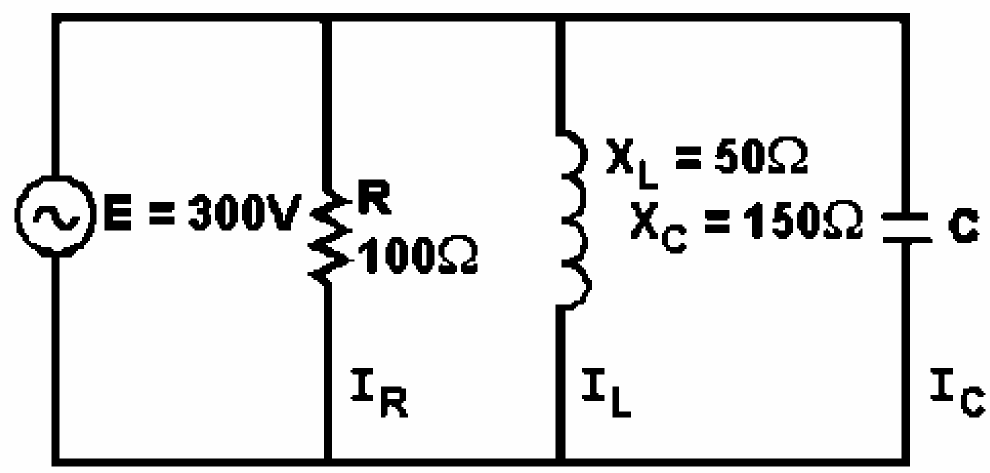
EXAMPLE 6. Series RLC circuit

Determine a) Voltage and current in each element. c) P, Q and S in each element and the overall power factor. c) The resonance frequency (frequency at which the imaginary part of the equivalent impedance is null).



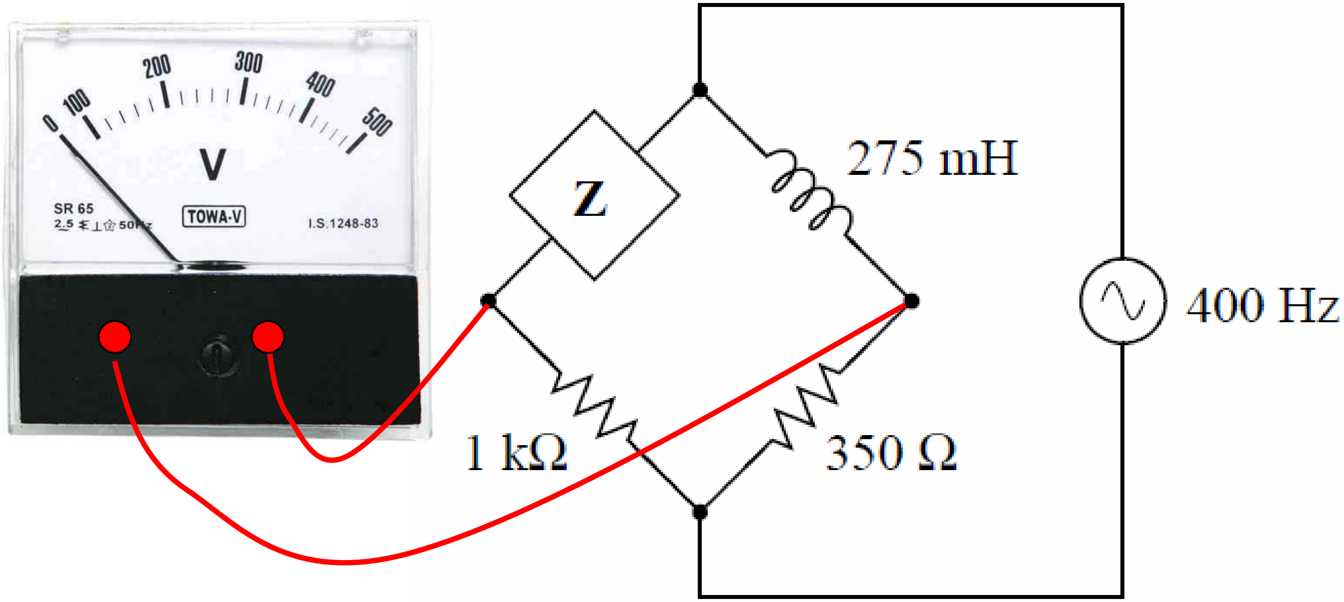
EXAMPLE 7. Parallel RLC circuit

Determine a) Voltage and current in each element. c) P, Q and S in each element and the overall power factor. c) The resonance frequency of this circuit.



EXAMPLE 8. The mesh method (malles)

Calculate the impedance value necessary to balance this AC bridge, expressing your answer in both polar and rectangular forms. What type and size of component will provide this exact amount of impedance at 400 Hz?



http://powerelectrical.blogspot.com/2007/04/short-questions-and-solved-problems-in.html

$Z = 1.975\ k\Omega^{90°}$ (polar form)

$Z = 0 + j1.975\ k\Omega$ (rectangular form)

INSTRUMENTS FOR INDUSTRIAL MAINTENANCE

Multimeters

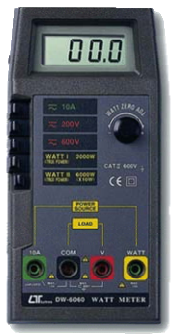
- Voltage AC/DC
- Current AC/DC
- Resistance
- Others: capacitance, frequency, temperature, …

Current probes

- Voltage AC/DC (depending on models)
- Current AC/DC

Wattmeters

- Voltage AC
- Current AC
- Power : S, P, Q AC
- PF
- Frequency
- Single-phase or three-phase

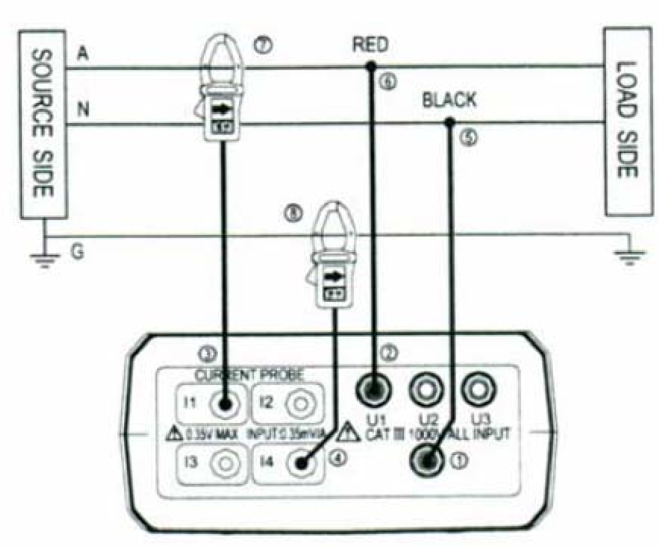


Single-phase

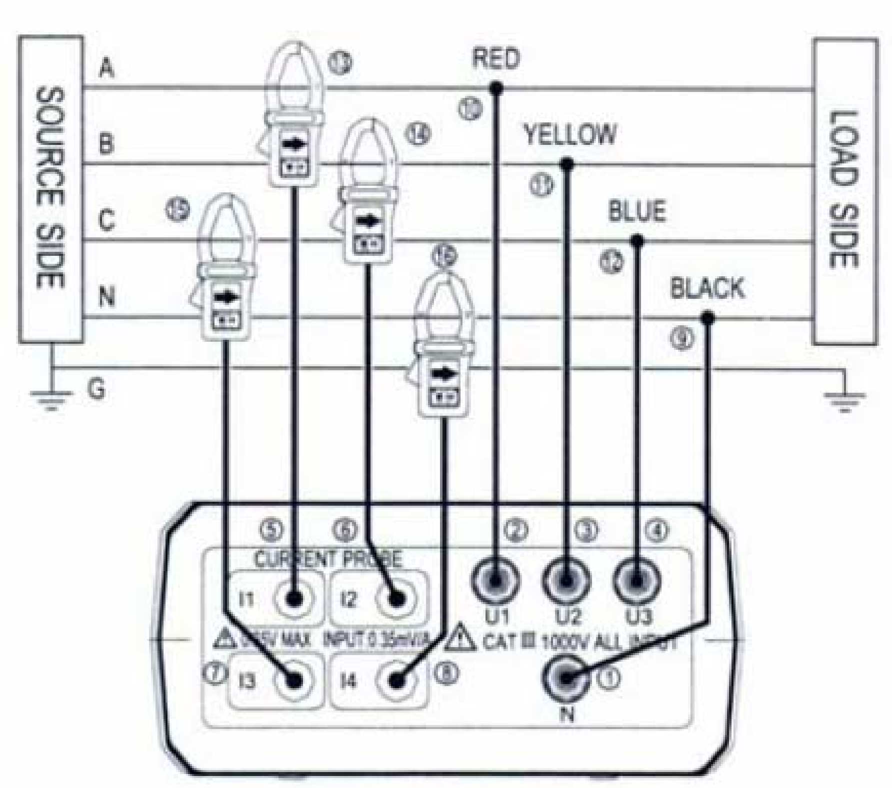
Three-phase

ELECTRICAL MEASUREMENTS

Three-phase



1P2W Wiring Connection Diagram



3P4W Wiring Connection Diagram

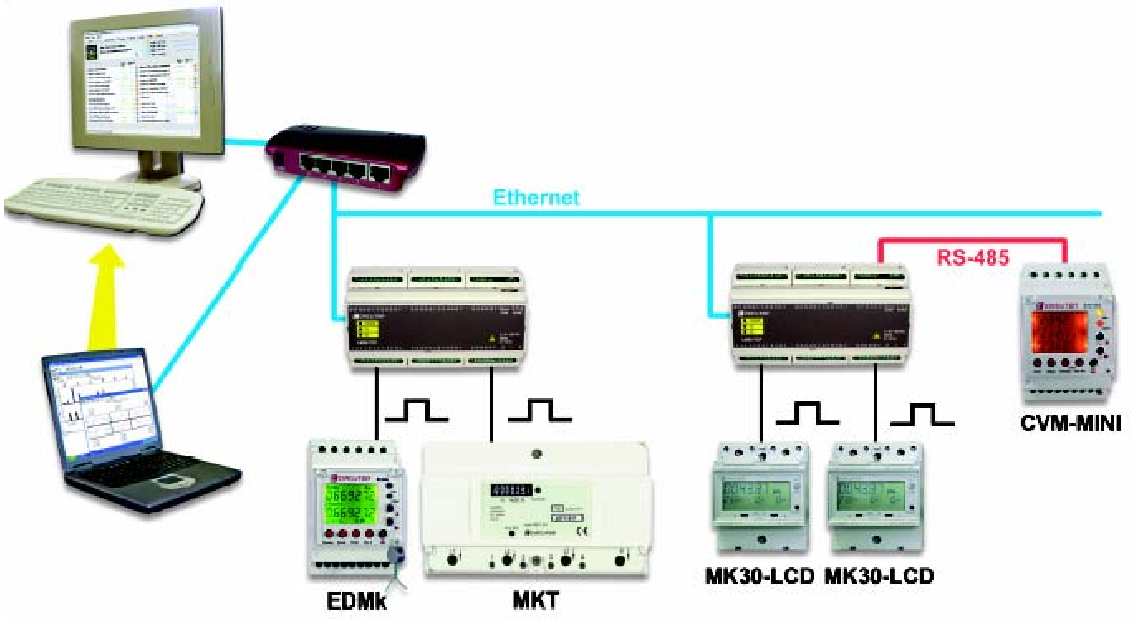
ELECTRICAL MEASUREMENTS

INSTRUMENTS FOR INDUSTRIAL MEASUREMENT AND CONTROL

- Useful for carrying out low cost energy control checks on electrical consumption
- Can also be used to carry out an accurate control of the consumption of any other physical unit which has a meter with a digital impulse output.

Possible measurements:

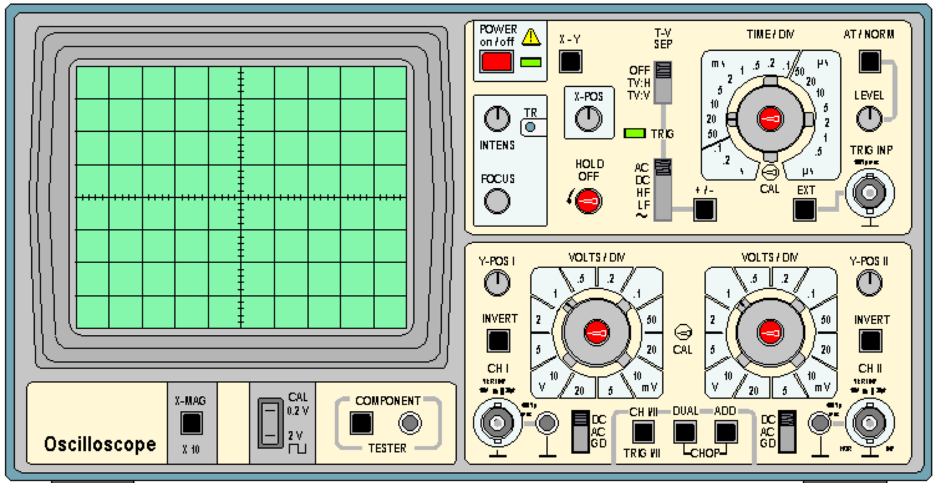
- Voltage
- Current
- Hz
- Power (S, P, Q, PF)
- Energy
- Single-phase or three-phase

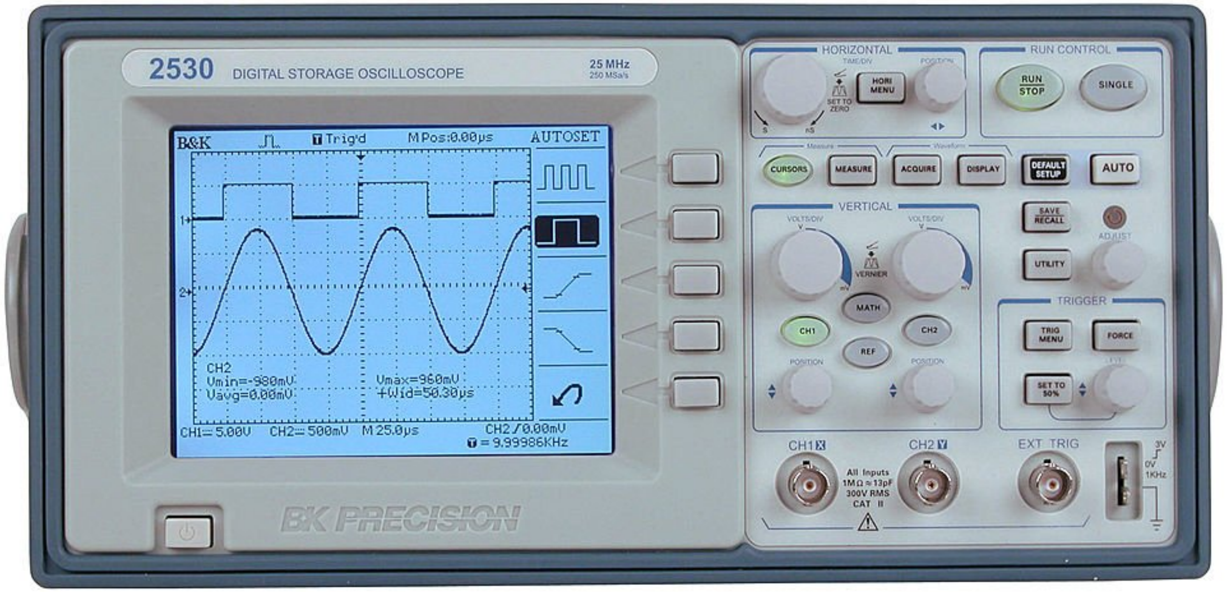


Circutor

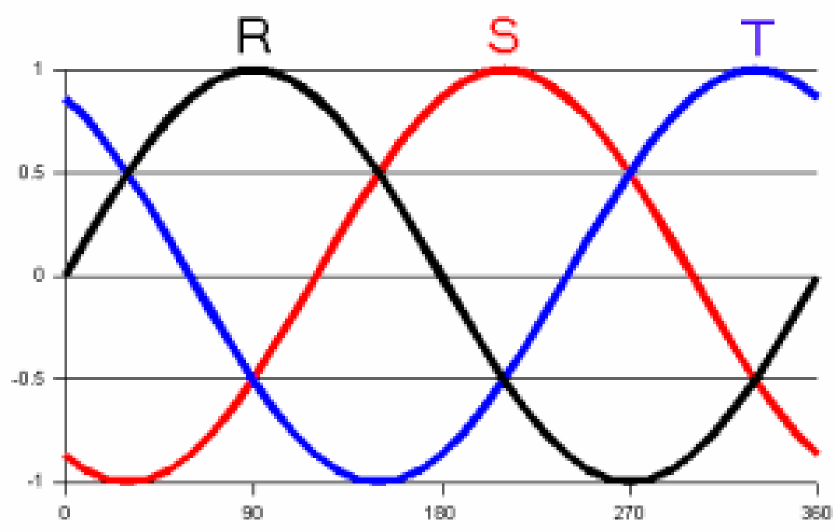
ELECTRICAL MEASUREMENTS

LAB MEASURES: OSCILLOSCOPE





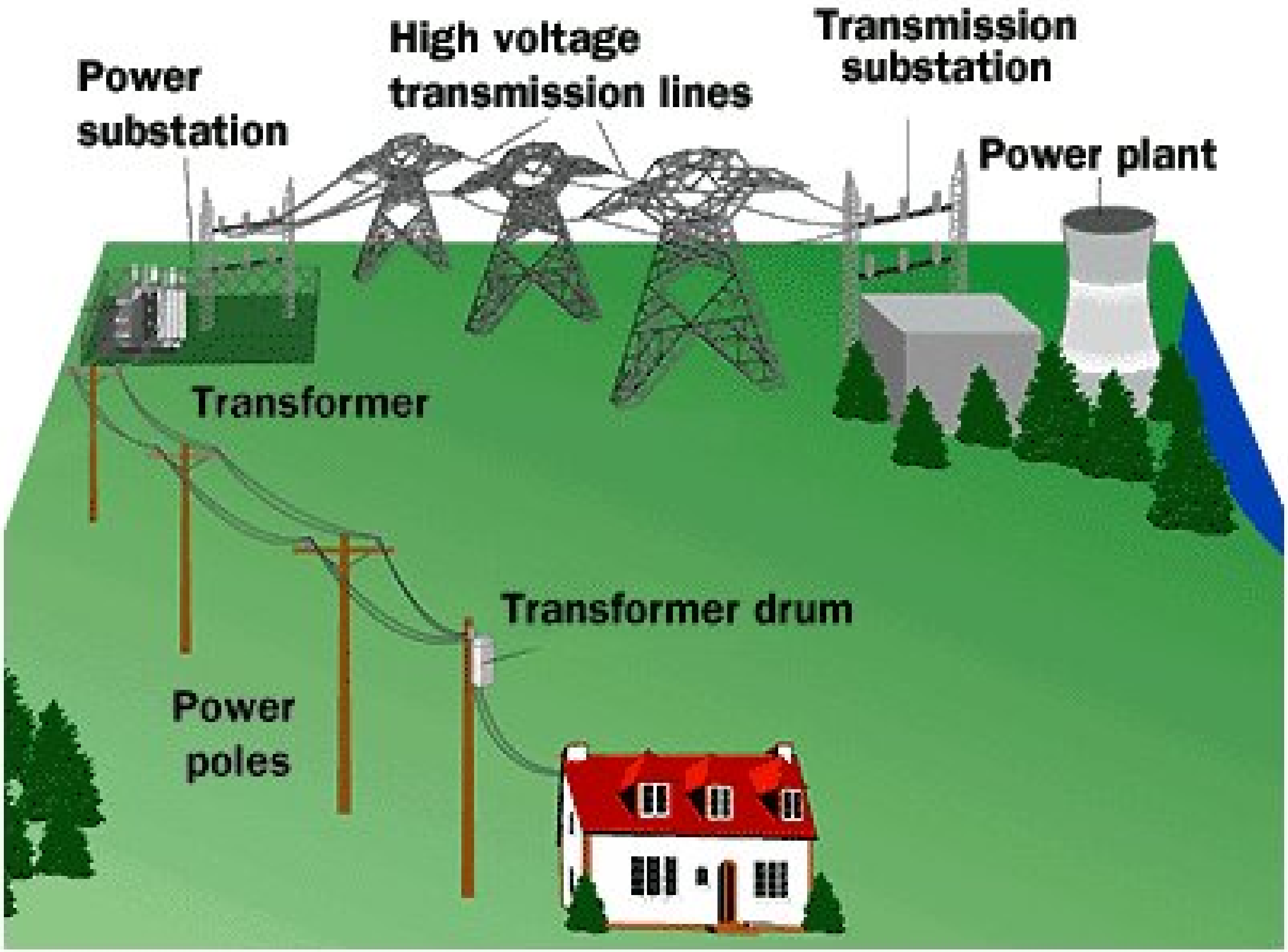
Alternating Current Three-Phase Circuits



Unit 3. AC THREE-PHASE CIRCUITS

CONTENTS LIST:

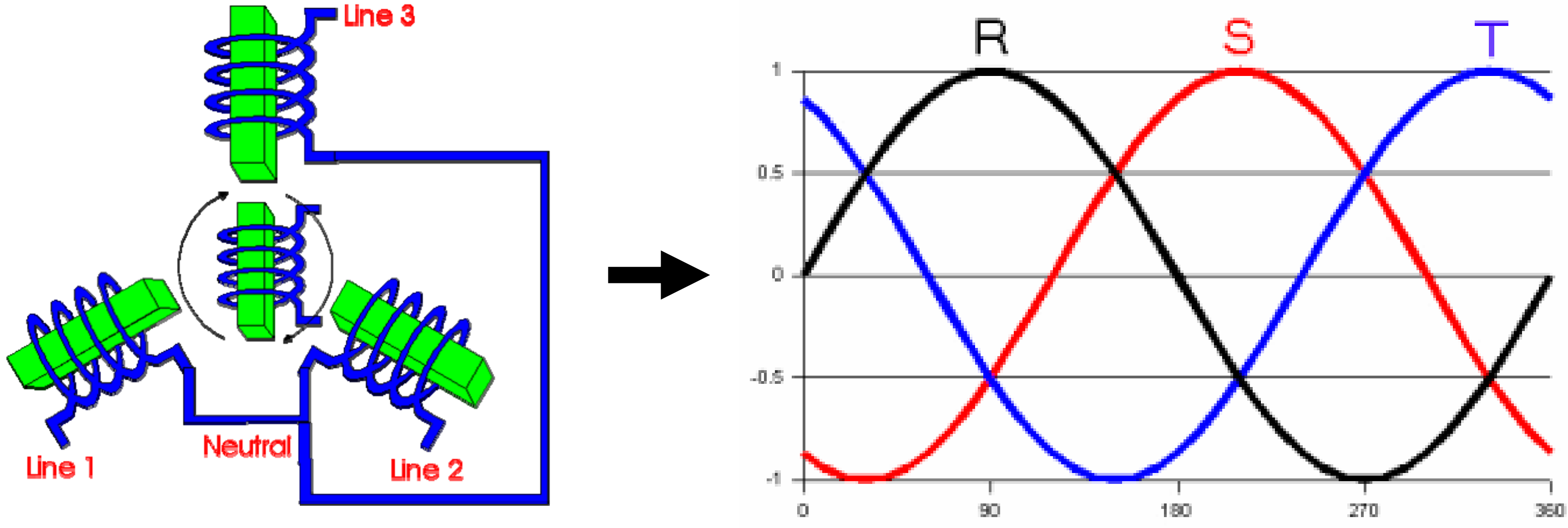
THREE-PHASE SYSTEMS CHARACTERISTICS



THREE-PHASE SYSTEMS CHARACTERISTICS

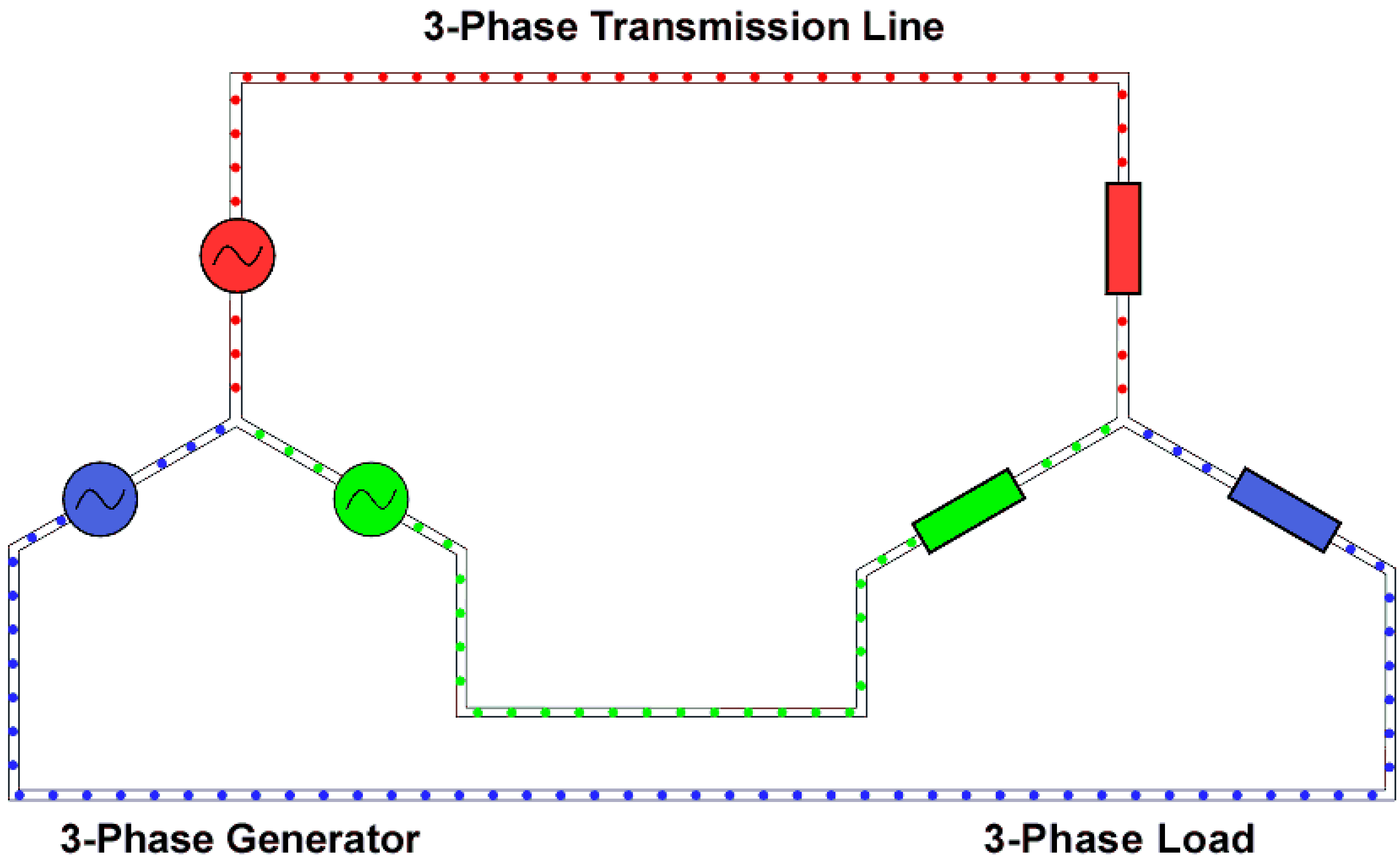
- Instantaneous electric power has sinusoidal shape with double frequency than voltage or current
- SINGLE-PHASE AC CIRCUITS: instantaneous electric power is negative twice a period (power flows from load to generator) and positive twice a period, falling to zero.
- BALANCED THREE-PHASE AC CIRCUITS: instantaneous electric power is constant. **Three-phase power never falls to zero**
- Three-phase electric motors have better performance than single-phase AC motors
- Three-phase power systems allow two voltage levels (L-L, L-N)
- When transmitting electric power, three-phase AC systems require 25% less of Cu/Al than single-phase AC systems

GENERATION OF THREE-PHASE VOLTAGES

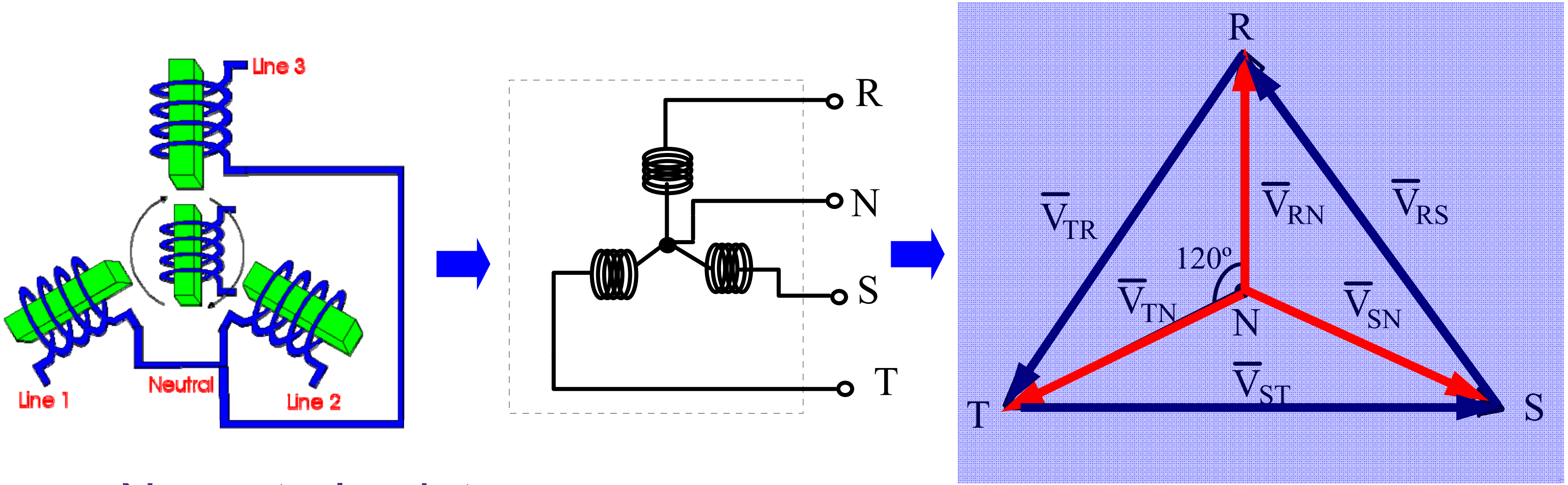


- Three-phase generators contain three sinusoidal voltage sources with voltages of the same frequency but a 120º-phase shift with respect to each other
- This is realized by positioning three coils at 120º electrical angle separations on the same rotor
- Amplitudes of the three phases are also equal
- The generator is then balanced

INTRODUCTION



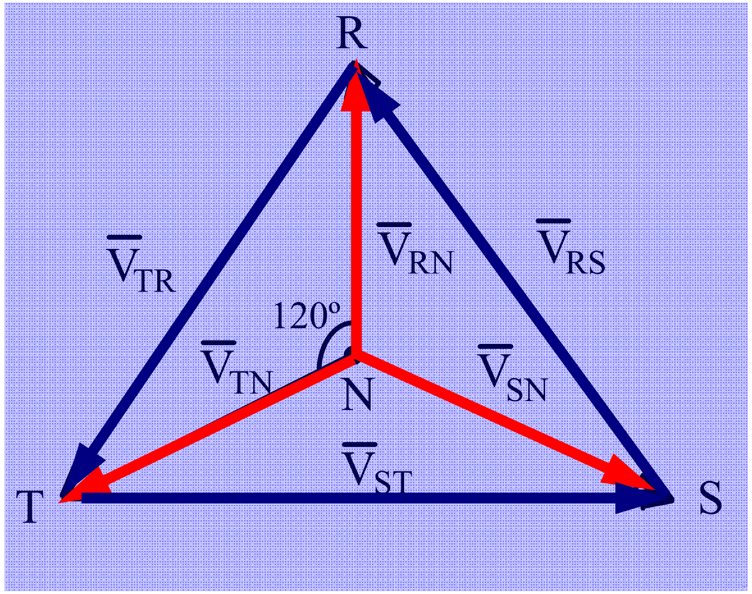
INTRODUCTION

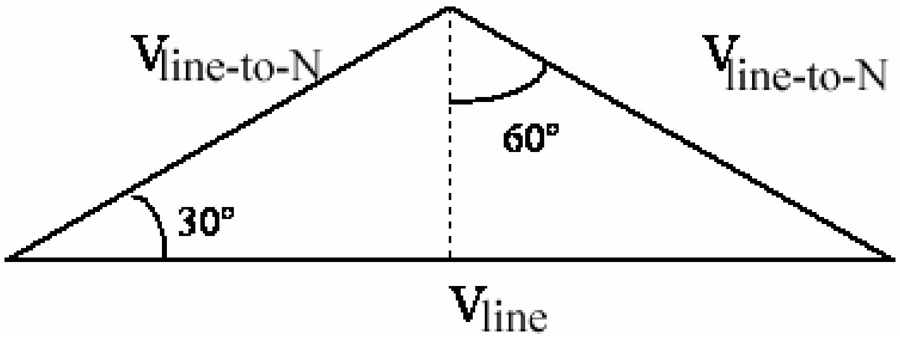


- N: neutral point
- R S T (or A B C) direct sequence or sequence RST
- V_{RS}, V_{ST}, V_{TR}: line voltages or line-to-line voltages
- V_{RN}, V_{SN}, V_{TN}: line-to-neutral voltages
- $V_{line} = \sqrt{3}\, V_{line\text{-}to\text{-}neutral}$

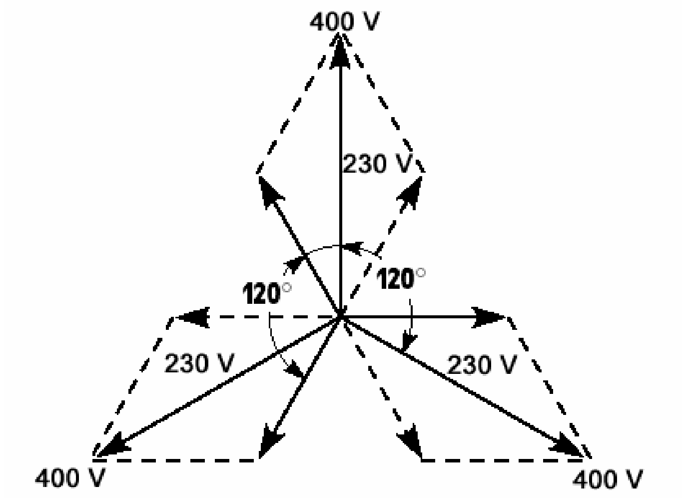
$$v_{RN}(t) = V_0 \cdot \cos(\omega \cdot t + 90º) \quad V$$
$$v_{SN}(t) = V_0 \cdot \cos(\omega \cdot t - 30º) \quad V$$
$$v_{TN}(t) = V_0 \cdot \cos(\omega \cdot t + 210º) \quad V$$

INTRODUCTION





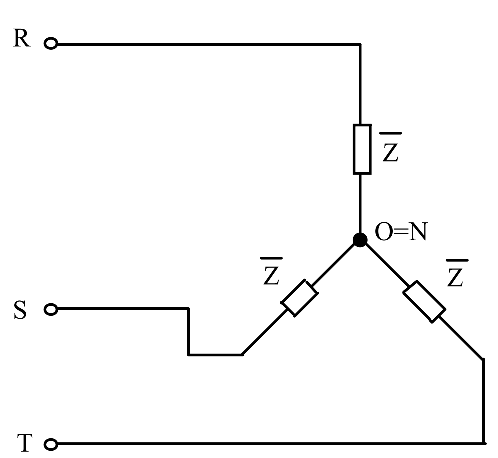
$$V_{line} = \sqrt{3}.V_{\underbrace{line-to-N}_{phase}}$$

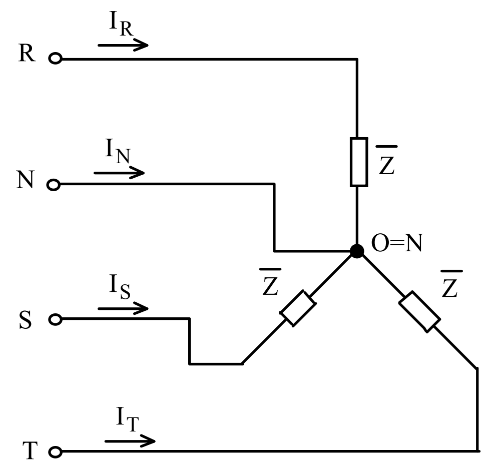


50 Hz	Usual system
$V_{phase} = V_{line\text{-}to\text{-}neutral}$	**230 volt**
V_{line}	**400 volt**
Frequency	**50 Hz**

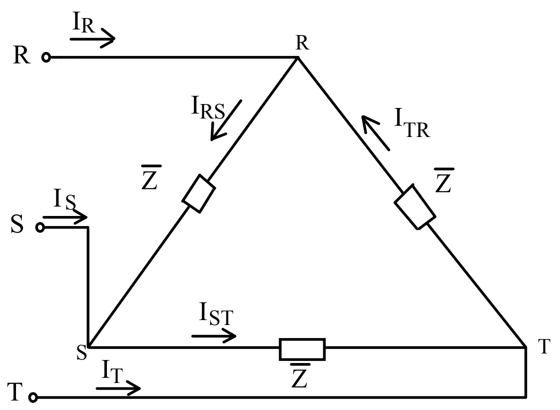
THREE-PHASE LOADS CLASSIFICATION

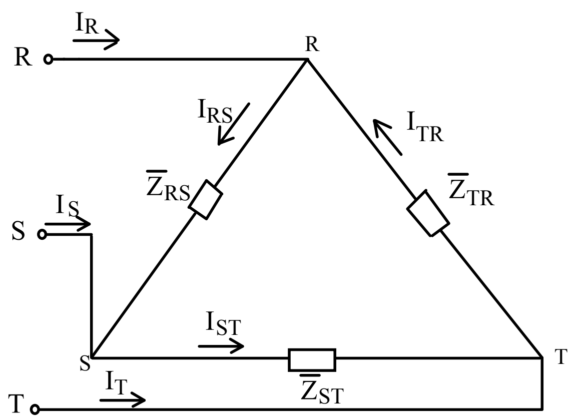
- **WYE** (two voltages)
  - **Balanced**
    - 3-wires
    - 4-wires
  - **Unbalanced**
    - 3-wires
    - 4-wires





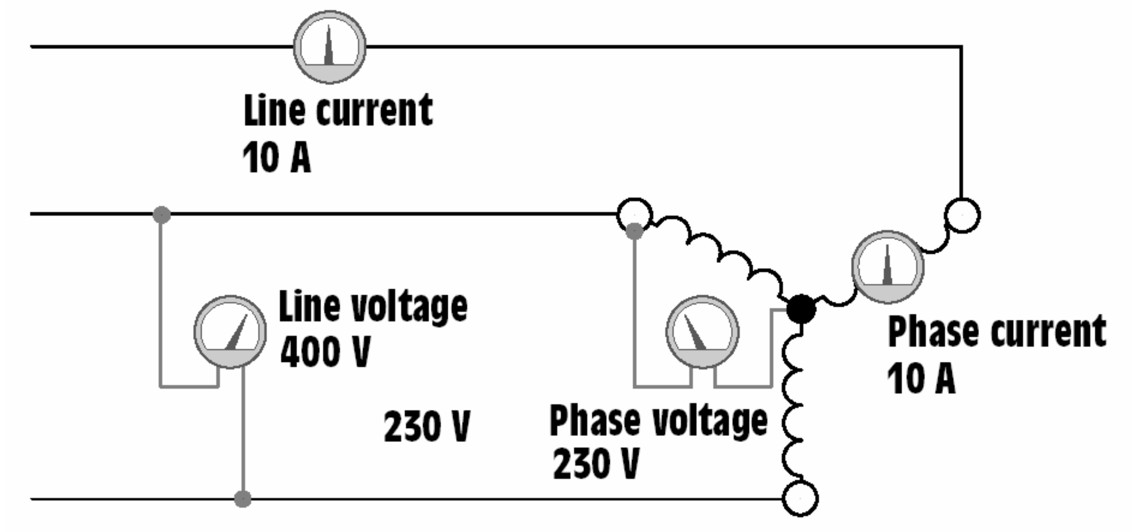
- **DELTA** (one voltage)
  - **Balanced**
  - **Unbalanced**





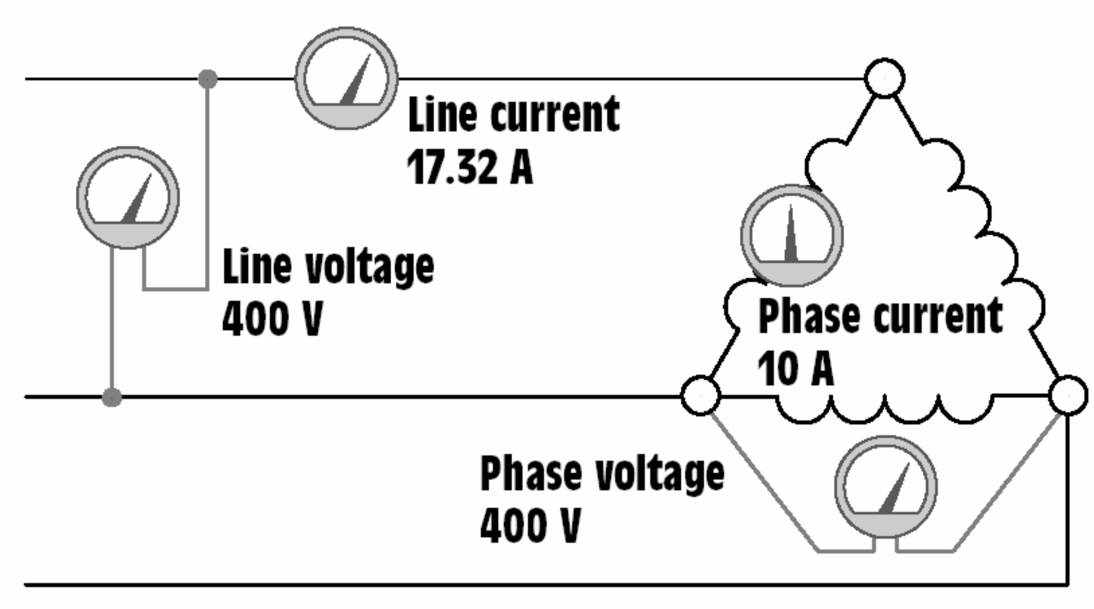
BALANCED WYE-CONNECTED LOAD

- The wye or star connection is made by connecting one end of each of the three-phase loads together
- The voltage measured across a single load or phase is known as the phase voltage
- The voltage measured between the lines is known as the line-to-line voltage or simply as the line voltage
- In a wye-connected system, the line voltage is higher than the load phase voltage by a factor of the square root of 3
- In a wye-connected system, phase current and line current are the same



BALANCED DELTA-CONNECTED LOAD

- This connection receives its name from the fact that a schematic diagram of this connection resembles the Greek letter delta (Δ)
- In the delta connection, line voltage and phase voltage in the load are the same
- The line current of a delta connection is higher than the phase current by a factor of the square root of 3



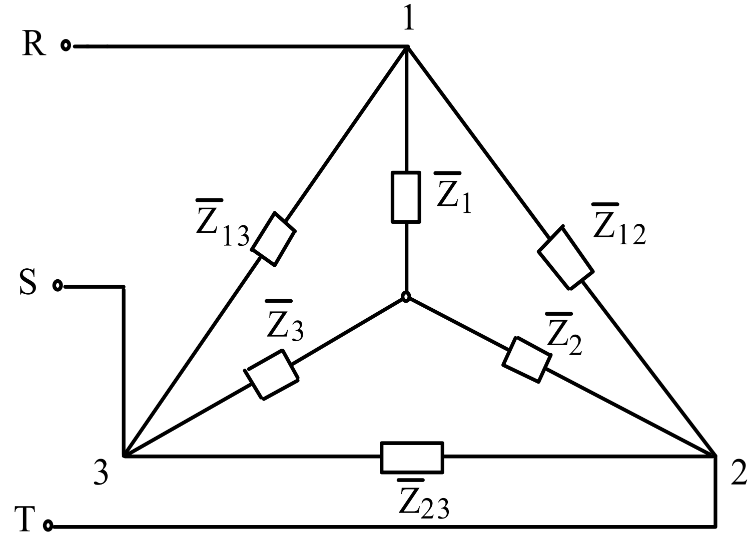
Δ-Y TRANSFORMATION

Z between nodes 1 and 2: Δ: $\overline{Z}_{\Delta 1,2} = \dfrac{\overline{Z}_{12}\cdot(\overline{Z}_{13}+\overline{Z}_{23})}{\overline{Z}_{12}+(\overline{Z}_{13}+\overline{Z}_{23})}$ Y: $\overline{Z}_{Y1,2} = \overline{Z}_1 + \overline{Z}_2$

Z nodes 1-2: (1) $\overline{Z}_1 + \overline{Z}_2 = \dfrac{\overline{Z}_{12}\cdot(\overline{Z}_{13}+\overline{Z}_{23})}{\overline{Z}_{12}+(\overline{Z}_{13}+\overline{Z}_{23})}$

Z nodes 1-3: (2) $\overline{Z}_1 + \overline{Z}_3 = \dfrac{\overline{Z}_{13}\cdot(\overline{Z}_{12}+\overline{Z}_{23})}{\overline{Z}_{12}+(\overline{Z}_{13}+\overline{Z}_{23})}$

Z nodes 2-3: (3) $\overline{Z}_2 + \overline{Z}_3 = \dfrac{\overline{Z}_{23}\cdot(\overline{Z}_{12}+\overline{Z}_{13})}{\overline{Z}_{12}+(\overline{Z}_{13}+\overline{Z}_{23})}$



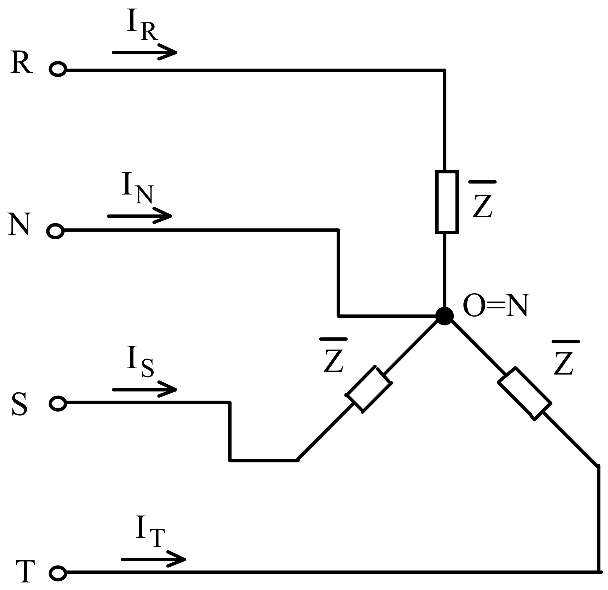
From expressions (1), (2) and (3) it results:

$$\overline{Z}_1 = \frac{\overline{Z}_{12}\cdot\overline{Z}_{13}}{\overline{Z}_{12}+\overline{Z}_{13}+\overline{Z}_{23}} \qquad \overline{Z}_2 = \frac{\overline{Z}_{12}\cdot\overline{Z}_{23}}{\overline{Z}_{12}+\overline{Z}_{13}+\overline{Z}_{23}} \qquad \overline{Z}_3 = \frac{\overline{Z}_{13}\cdot\overline{Z}_{23}}{\overline{Z}_{12}+\overline{Z}_{13}+\overline{Z}_{23}}$$

(1) + (2) - (3) (1) - (2) + (3) -(1) + (2) + (3)

Balanced loads: $Z_Y = Z_\Delta/3$

BALANCED THREE/FOUR-WIRE, WYE-CONNECTED LOAD

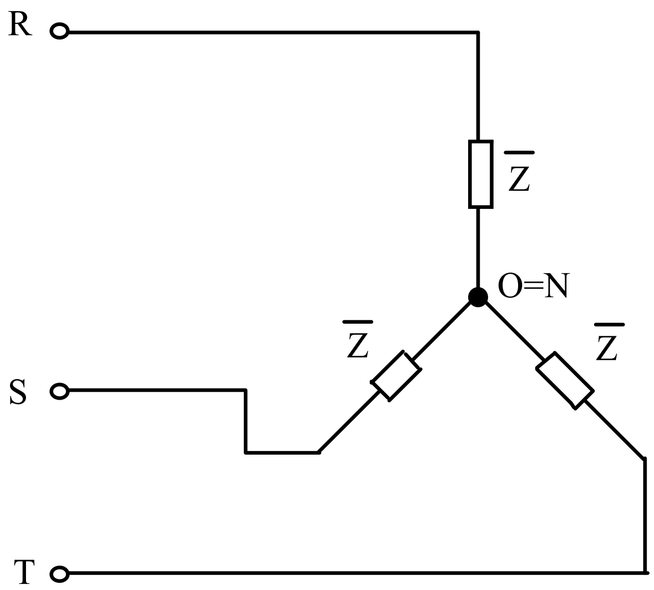


$$\bar{I}_R = \frac{\overline{V}_{RN}}{\overline{Z}} \qquad \bar{I}_S = \frac{\overline{V}_{SN}}{\overline{Z}} \qquad \bar{I}_T = \frac{\overline{V}_{TN}}{\overline{Z}}$$

$$\bar{I}_N = -(\bar{I}_R + \bar{I}_S + \bar{I}_T) = 0$$

$$\overline{S}_{total} = 3.\overline{S}_{fase} = 3.\overline{V}_{RN}.\bar{I}^*_R = \sqrt{3}.V_{línia}.I_{línia}{}^{\varphi^o}$$

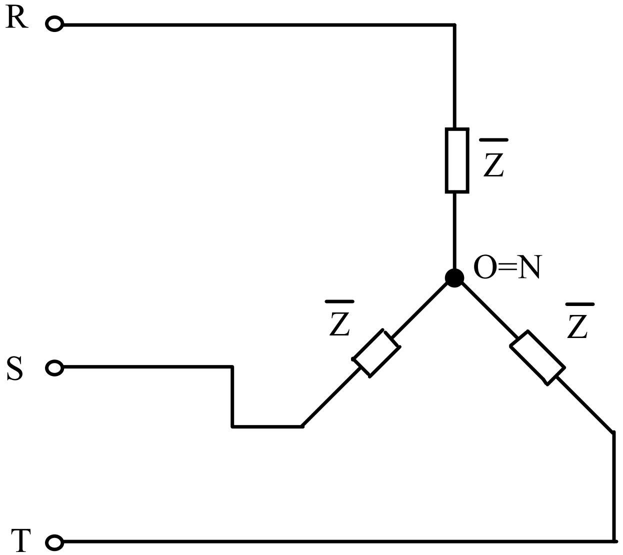
$$= \underbrace{\sqrt{3}.V_{línia}.I_{línia}.\cos\varphi}_{P} + j\underbrace{\sqrt{3}.V_{línia}.I_{línia}.\sin\varphi}_{Q}$$



- The three currents are balanced
- Thus the sum of them is always zero
- Since the neutral current of a balanced, Y-connected, three-phase load is always zero, the neutral conductor may, for computational purposes, be removed, with no change in the results

BALANCED THREE/FOUR-WIRE, WYE-CONNECTED LOAD

Example A three-phase, RST system (400 V, 50 Hz), has a three-wire Y-connected load for which $Z = 10^{30^\circ}\ \Omega$· Obtain the line currents and the complex power consumption·



$$\bar{I}_R = \frac{\bar{V}_{RN}}{\bar{Z}} = \frac{400/\sqrt{3}^{\,90^\circ}}{10^{30^\circ}} = \frac{40}{\sqrt{3}}^{60^\circ}\ \text{A}$$

$$\bar{I}_S = \frac{\bar{V}_{SN}}{\bar{Z}} = \frac{400/\sqrt{3}^{\,-30^\circ}}{10^{30^\circ}} = \frac{40}{\sqrt{3}}^{-60^\circ}\ \text{A}$$

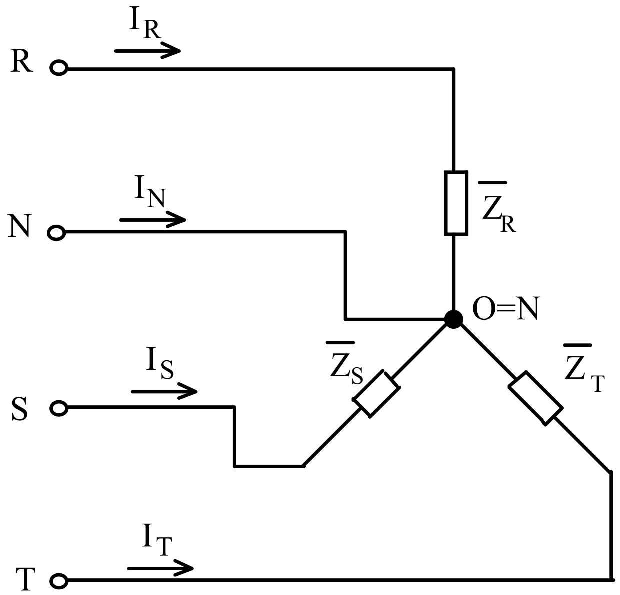
$$\bar{I}_T = \frac{\bar{V}_{TN}}{\bar{Z}} = \frac{400/\sqrt{3}^{\,210^\circ}}{10^{30^\circ}} = \frac{40}{\sqrt{3}}^{180^\circ}\ \text{A}$$

$$\bar{S}_{total} = 3\cdot\bar{S}_{phase} = 3\cdot\bar{V}_{RN}\cdot\bar{I}_R^{*} = 3\cdot(400/\sqrt{3}^{\,90^\circ})\cdot(40/\sqrt{3}^{\,60^\circ})^{*} =$$

$$= 16000^{30^\circ}\ \text{VA} = 13856.41\ (\text{watt}) + j8000\ (\text{VAr})$$

$$\begin{cases} S_{total} = \sqrt{3}\cdot V_l\cdot I_l = \sqrt{3}\cdot 400\cdot\dfrac{40}{\sqrt{3}} = 16000\text{VA} \\ P_{total} = \sqrt{3}\cdot V_l\cdot I_l\cdot\cos\varphi = \sqrt{3}\cdot 400\cdot\dfrac{40}{\sqrt{3}}\cdot\cos 30^\circ = 13856.41\text{watt} \\ Q_{total} = \sqrt{3}\cdot V_l\cdot I_l\cdot\sin\varphi = \sqrt{3}\cdot 400\cdot\dfrac{40}{\sqrt{3}}\cdot\sin 30^\circ = 8000\text{VAr} \end{cases}$$

UNBALANCED FOUR-WIRE, WYE-CONNECTED LOAD



$$\bar{I}_R = \frac{\bar{V}_{RN}}{\bar{Z}_R}$$

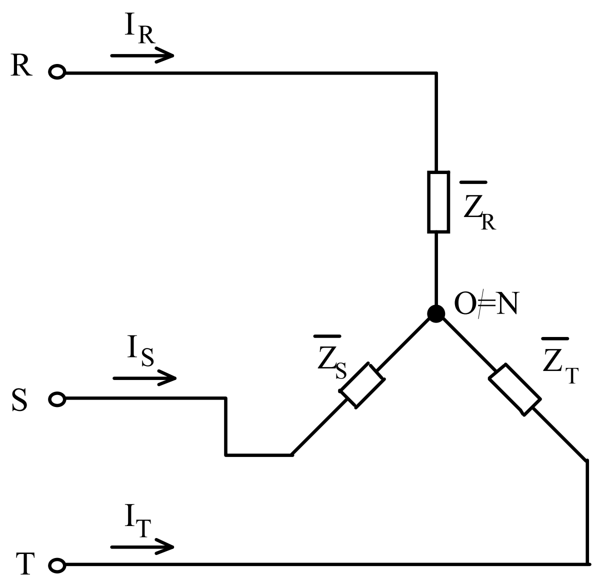
$$\bar{I}_S = \frac{\bar{V}_{SN}}{\bar{Z}_S}$$

$$\bar{I}_T = \frac{\bar{V}_{TN}}{\bar{Z}_T}$$

$$\bar{I}_N = -(\bar{I}_R + \bar{I}_S + \bar{I}_T) \neq 0$$

$$\bar{S}_{total} = \bar{V}_{RN} \cdot \bar{I}_R^* + \bar{V}_{SN} \cdot \bar{I}_S^* + \bar{V}_{TN} \cdot \bar{I}_T^*$$

UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD



$$\bar{I}_R + \bar{I}_S + \bar{I}_T = 0$$

$$\Downarrow$$

$$\frac{\bar{V}_{RO}}{\bar{Z}_R} + \frac{\bar{V}_{SO}}{\bar{Z}_S} + \frac{\bar{V}_{TO}}{\bar{Z}_T} = 0$$

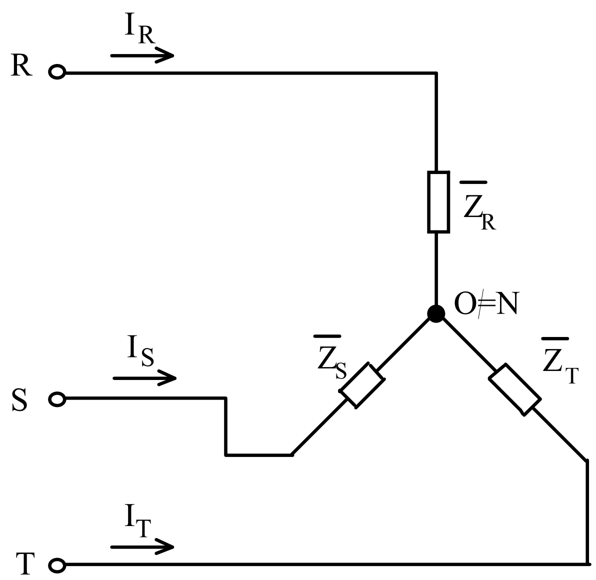
$$\Downarrow$$

$$\frac{\bar{V}_{RN} + \bar{V}_{NO}}{\bar{Z}_R} + \frac{\bar{V}_{SN} + \bar{V}_{NO}}{\bar{Z}_S} + \frac{\bar{V}_{TN} + \bar{V}_{NO}}{\bar{Z}_T} = 0$$

$$\Downarrow$$

$$\bar{V}_{ON} = \frac{\dfrac{\bar{V}_{RN}}{\bar{Z}_R} + \dfrac{\bar{V}_{SN}}{\bar{Z}_S} + \dfrac{\bar{V}_{TN}}{\bar{Z}_T}}{\dfrac{1}{\bar{Z}_R} + \dfrac{1}{\bar{Z}_S} + \dfrac{1}{\bar{Z}_T}} = \frac{\bar{V}_{RN} \cdot \bar{Y}_R + \bar{V}_{SN} \cdot \bar{Y}_S + \bar{V}_{TN} \cdot \bar{Y}_T}{\bar{Y}_R + \bar{Y}_S + \bar{Y}_T}$$

UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD



$$1)\quad \overline{V}_{ON} = \frac{\overline{V}_{RN}\cdot\overline{Y}_R + \overline{V}_{SN}\cdot\overline{Y}_S + \overline{V}_{TN}\cdot\overline{Y}_T}{\overline{Y}_R + \overline{Y}_S + \overline{Y}_T}$$

$$2)\quad \overline{I}_R = \frac{\overline{V}_{RO}}{\overline{Z}_R} = \frac{\overline{V}_{RN} - \overline{V}_{ON}}{\overline{Z}_R}$$

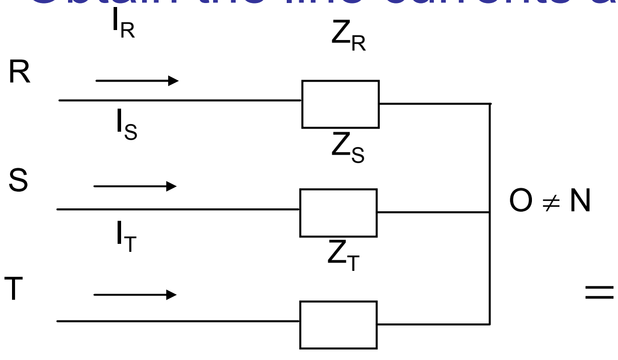
$$\overline{I}_S = \frac{\overline{V}_{SO}}{\overline{Z}_S} = \frac{\overline{V}_{SN} - \overline{V}_{ON}}{\overline{Z}_S}$$

$$\overline{I}_T = \frac{\overline{V}_{TO}}{\overline{Z}_T} = \frac{\overline{V}_{TN} - \overline{V}_{ON}}{\overline{Z}_T}$$

$$3)\quad \overline{S}_{total} = \overline{V}_{RO}\cdot\overline{I}_R^* + \overline{V}_{SO}\cdot\overline{I}_S^* + \overline{V}_{TO}\cdot\overline{I}_T^*$$

UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD

Example· A three-phase, RST system (400 V, 50 Hz), has a three-wire unbalanced Y-connected load for which $Z_R = 10_{0^o}\ \Omega$, $Z_S = 10_{0^o}\ \Omega$ i $Z_T = 10_{30^o}\ \Omega$· Obtain the line currents and the total complex power consumption.



$$\overline{V}_{ON} = \frac{\overline{V}_{RN}\cdot\overline{Y}_R + \overline{V}_{SN}\cdot\overline{Y}_S + \overline{V}_{TN}\cdot\overline{Y}_T}{\overline{Y}_R + \overline{Y}_S + \overline{Y}_T}$$

$$= \frac{230_{90^o}\cdot 0.1_{0^o} + 230_{-30^o}\cdot 0.1_{0^o} + 230_{210^o}\cdot 0.1_{-30^o}}{0.1_{0^o} + 0.1_{0^o} + 0.1_{-30^o}} = 40.93_{114.89^o}\ \text{V}$$

$$\overline{V}_{RO} = \overline{V}_{RN} - \overline{V}_{ON} = 230_{90^o} - 40.93_{114.89^o} = 193.64_{84.90^o}\ \text{V}$$

$$\overline{V}_{SO} = \overline{V}_{SN} - \overline{V}_{ON} = 230_{-30^o} - 40.93_{114.89^o} = 264{,}54_{-35.10^o}\ \text{V}$$

$$\overline{V}_{TO} = \overline{V}_{TN} - \overline{V}_{ON} = 230_{210^o} - 40.93_{114.89^o} = 237{,}18_{-140.10^o}\ \text{V}$$

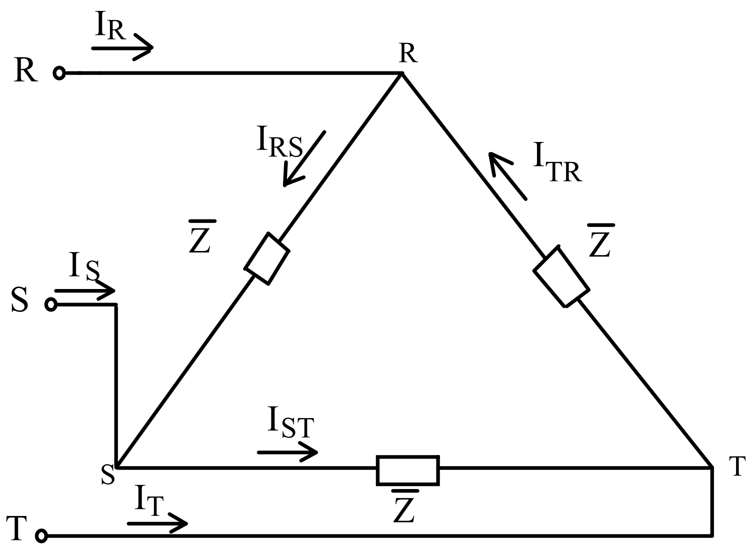
$$\overline{I}_R = \overline{V}_{RO}/\overline{Z}_R = 193.64_{84.90^o}/10_{0^o} = 19{,}36_{84.90^o}\ \text{A}$$

$$\overline{I}_S = \overline{V}_{SO}/\overline{Z}_S = 264.54_{-35.10^o}/10_{0^o} = 26{,}45_{-35.10^o}\ \text{A}$$

$$\overline{I}_T = \overline{V}_{TO}/\overline{Z}_T = 237.18_{-140.10^o}/10_{30^o} = 23.72_{-170.10^o}\ \text{A}$$

$$\overline{S}_{tot} = \overline{V}_{RO}\cdot\overline{I}_R^* + \overline{V}_{SO}\cdot\overline{I}_S^* + \overline{V}_{TO}\cdot\overline{I}_T^* = 15619.56\ \text{W} + j2812.72\ \text{VAr}$$

BALANCED DELTA-CONNECTED LOAD



$$\bar{I}_{RS} = \frac{\bar{V}_{RS}}{\bar{Z}} \qquad \bar{I}_{ST} = \frac{\bar{V}_{ST}}{\bar{Z}} \qquad \bar{I}_{TR} = \frac{\bar{V}_{TR}}{\bar{Z}}$$

$$\bar{I}_R = \bar{I}_{RS} - \bar{I}_{TR}$$

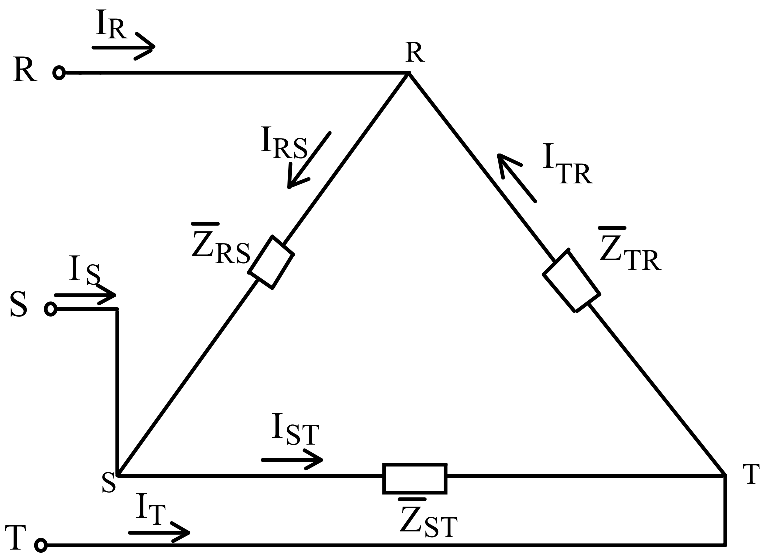
$$\bar{I}_S = \bar{I}_{ST} - \bar{I}_{RS}$$

$$\bar{I}_T = \bar{I}_{TR} - \bar{I}_{ST}$$

$$\bar{S}_{total} = 3 \cdot \bar{S}_{fase} = 3 \cdot \bar{V}_{RS} \cdot \bar{I}^*_{RS} = \sqrt{3} \cdot V_{línia} \cdot I_{línia} \, \underline{\varphi^{o}}$$

$$= \underbrace{\sqrt{3} \cdot V_{línia} \cdot I_{línia} \cdot \cos\varphi}_{P} + j \underbrace{\sqrt{3} \cdot V_{línia} \cdot I_{línia} \cdot \sin\varphi}_{Q}$$

UNBALANCED DELTA-CONNECTED LOAD



$$\overline{I}_{RS} = \frac{\overline{V}_{RS}}{\overline{Z}_{RS}} \qquad \overline{I}_{R} = \overline{I}_{RS} - \overline{I}_{TR}$$

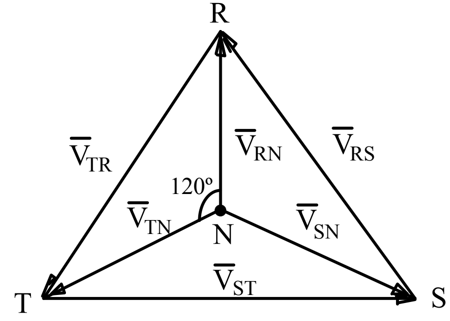
$$\overline{I}_{ST} = \frac{\overline{V}_{ST}}{\overline{Z}_{ST}} \qquad \overline{I}_{S} = \overline{I}_{ST} - \overline{I}_{RS}$$

$$\overline{I}_{TR} = \frac{\overline{V}_{TR}}{\overline{Z}_{TR}} \qquad \overline{I}_{T} = \overline{I}_{TR} - \overline{I}_{ST}$$

$$\overline{S}_{total} = \overline{V}_{RS} \cdot \overline{I}^{*}_{RS} + \overline{V}_{ST} \cdot \overline{I}^{*}_{ST} + \overline{V}_{TR} \cdot \overline{I}^{*}_{TR}$$

UNBALANCED THREE-WIRE, Δ-CONNECTED LOAD

Example· A three-phase, RST system (400 V, 50 Hz), has an unbalanced Δ-connected load for which $Z_{RS} = 10^{0^\circ}\ \Omega$, $Z_{ST} = 10^{30^\circ}\ \Omega$ i $Z_{TR} = 10^{-30^\circ}\ \Omega$· Obtain the line currents and the complex power consumption·



$$\bar{I}_{RS} = \frac{\bar{V}_{RS}}{\bar{\bar{Z}}_{RS}} = \frac{400^{120^\circ}}{10^{0^\circ}} = 40^{120^\circ}\ \mathrm{A}$$

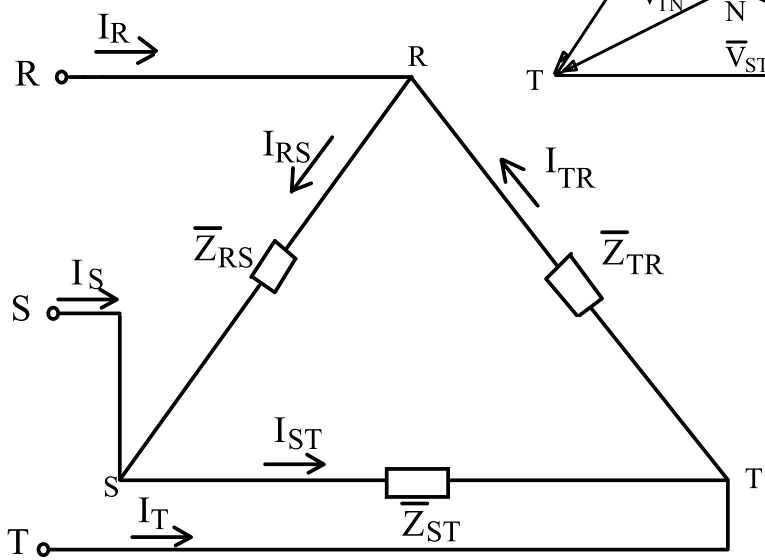
$$\bar{I}_{ST} = \frac{\bar{V}_{ST}}{\bar{\bar{Z}}_{ST}} = \frac{400^{0^\circ}}{10^{30^\circ}} = 40^{-30^\circ}\ \mathrm{A}$$

$$\bar{I}_{TR} = \frac{\bar{V}_{TR}}{\bar{\bar{Z}}_{TR}} = \frac{400^{-120^\circ}}{10^{-30^\circ}} = 40^{-90^\circ}\ \mathrm{A}$$

$$\bar{I}_{R} = \bar{I}_{RS} - \bar{I}_{TR} = 77.29^{105^\circ}\ \mathrm{A}$$

$$\bar{I}_{S} = \bar{I}_{ST} - \bar{I}_{RS} = 77.29^{-45^\circ}\ \mathrm{A}$$

$$\bar{I}_{T} = \bar{I}_{TR} - \bar{I}_{ST} = 40.00^{-150^\circ}\ \mathrm{A}$$

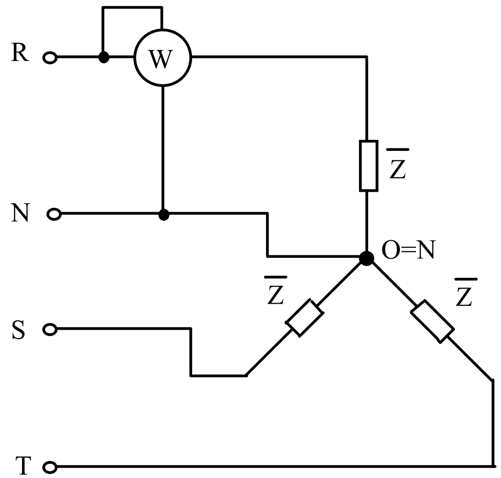


$$\bar{S}_{total} = \bar{V}_{RS}\cdot\bar{I}^*_{RS} + \bar{V}_{ST}\cdot\bar{I}^*_{ST} + \bar{V}_{TR}\cdot\bar{I}^*_{TR} = 43712.81(\mathrm{W}) + j0\,(\mathrm{VAr}) =$$

$$= 43712.81\mathrm{VA}$$

POWER MEASURMENT. Four-wires load

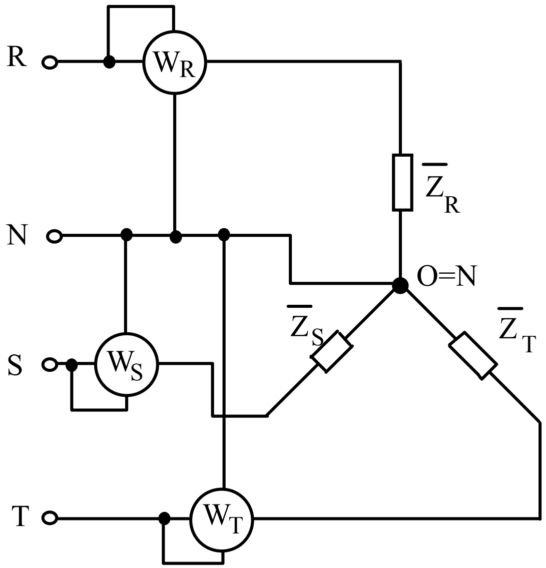
Balanced wye-connected, four-wires load



$$W = V_{RN} \cdot I_R \cdot \cos(V_{RN} - I_R)$$

$$P_{total} = 3W$$

Unbalanced wye-connected, four-wires load



$$W_R = V_{RN} \cdot I_R \cdot \cos(V_{RN} - I_R)$$

$$W_S = V_{SN} \cdot I_S \cdot \cos(V_{SN} - I_S)$$

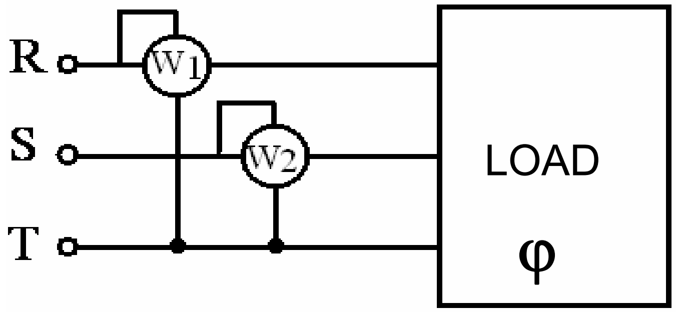
$$W_T = V_{TN} \cdot I_T \cdot \cos(V_{TN} - I_T)$$

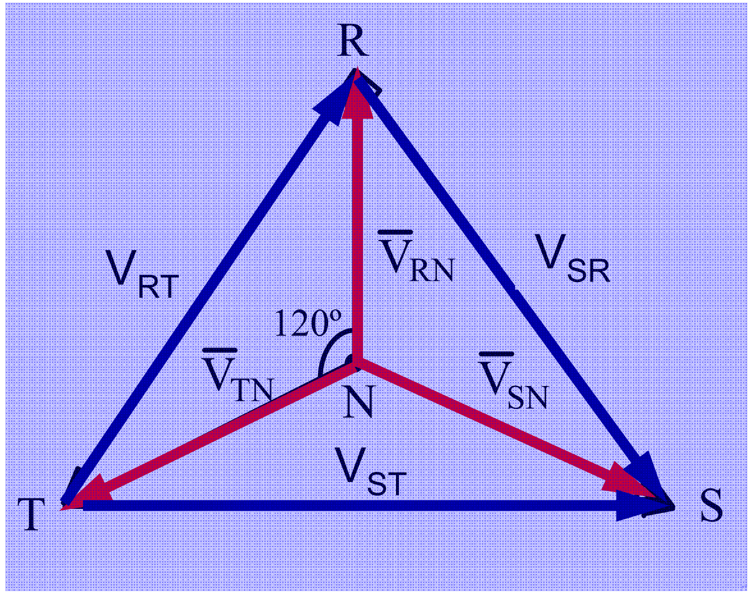
$$P_{total} = W_R + W_S + W_T$$

POWER MEASURMENT. ARON CONNECTION

General 3-wires load. Two-wattmeter method (ARON connection)

Demonstration done for a balanced 3-wires load





$$W_1 = V_{RT}\cdot I_R \cdot \cos(\overbrace{\varphi_{V_{RT}}}^{=60^o} - \overbrace{\varphi_{I_R}}^{=90^o - \varphi}) = V\cdot I\cdot \cos(-30^o + \varphi)$$

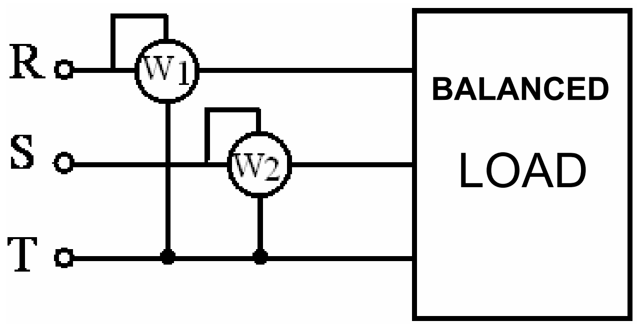
$$W_2 = V_{ST}\cdot I_S \cdot \cos(\underbrace{\varphi_{V_{ST}}}_{=0^o} - \underbrace{\varphi_{I_S}}_{=-30^o - \varphi}) = V\cdot I\cdot \cos(30^o + \varphi)$$

$$P_{TOTAL} = W_1 + W_2 = V\cdot I\cdot[\cos(-30^o+\varphi) + \cos(30^o+\varphi)] = \sqrt{3}\cdot V\cdot I\cdot \cos\varphi$$

$$Q_{TOTAL} = \sqrt{3}\cdot[W_1 + W_2] = \sqrt{3}\cdot V\cdot I\cdot[\cos(-30^o+\varphi) - \cos(30^o+\varphi)] = \sqrt{3}\cdot V\cdot I\cdot \sin\varphi$$

POWER MEASURMENT. BALANCED LOAD

Balanced load, general (Y/D, 3/4 wires). Two-wattmeter method (ARON connection)

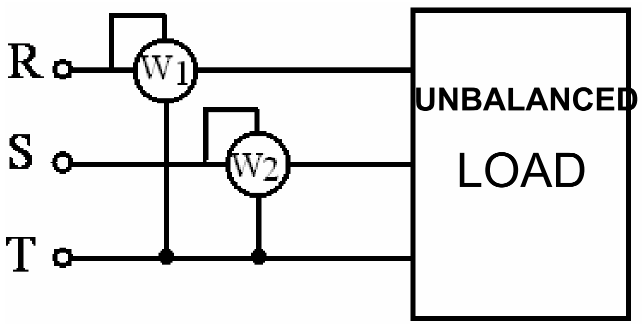


$$P_{total} = W_1 + W_2$$

$$Q_{total} = \sqrt{3} \cdot (W_1 - W_2)$$

$$\varphi = \text{arctg}\left(\sqrt{3} \cdot \frac{W_1 - W_2}{W_1 + W_2}\right)$$

Unbalanced wye/delta-connected, three-wires load

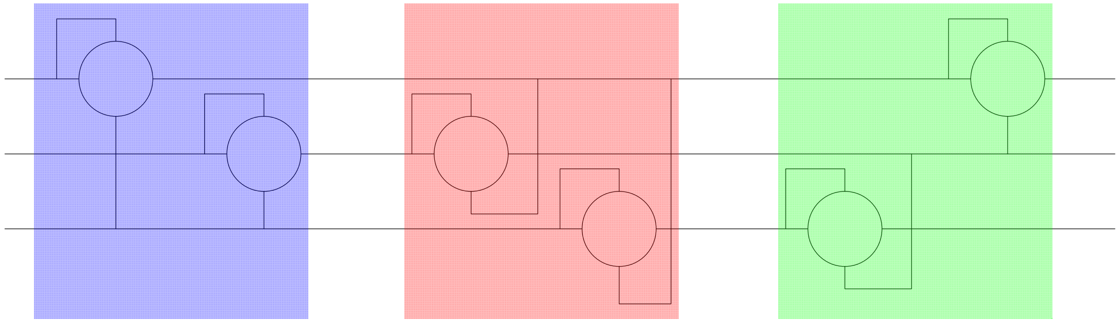


$$P_{total} = W_1 + W_2$$

POWER MEASURMENT; THE TWO-WATTMETER METHOD

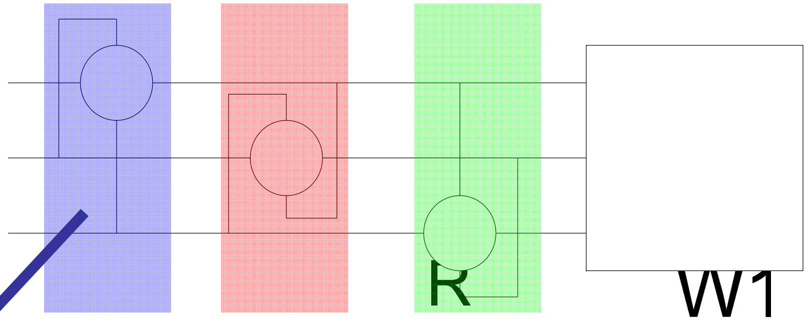
Aron Cyclic Permutations

W1		W2	
V	I	V	I
RT	R	ST	S
SR	S	TR	T
TS	T	RS	R



Q measurement: Cyclic Permutations

W	
V	I
ST	R
TR	S
RS	T



$$Q_{TOT} = \sqrt{3}W$$

$$W = V_{ST} \cdot I_{R} \cdot \cos(\cancel{\varphi_{VST}} - \varphi_{IR}) = V_{line} \cdot I_{line} \cdot \cos(90º\text{-}\varphi) = V_{line} \cdot I_{line} \cdot \sin\varphi = \frac{Q_{TOTAL}}{\sqrt{3}}$$

S W2 W1

INSTANTANEOUS THREE-PHASE POWER

- **Single-phase load:** $\cos A \cdot \cos B = 0{\cdot}5 \cdot [\cos(A+B) + \cos(A-B)]$

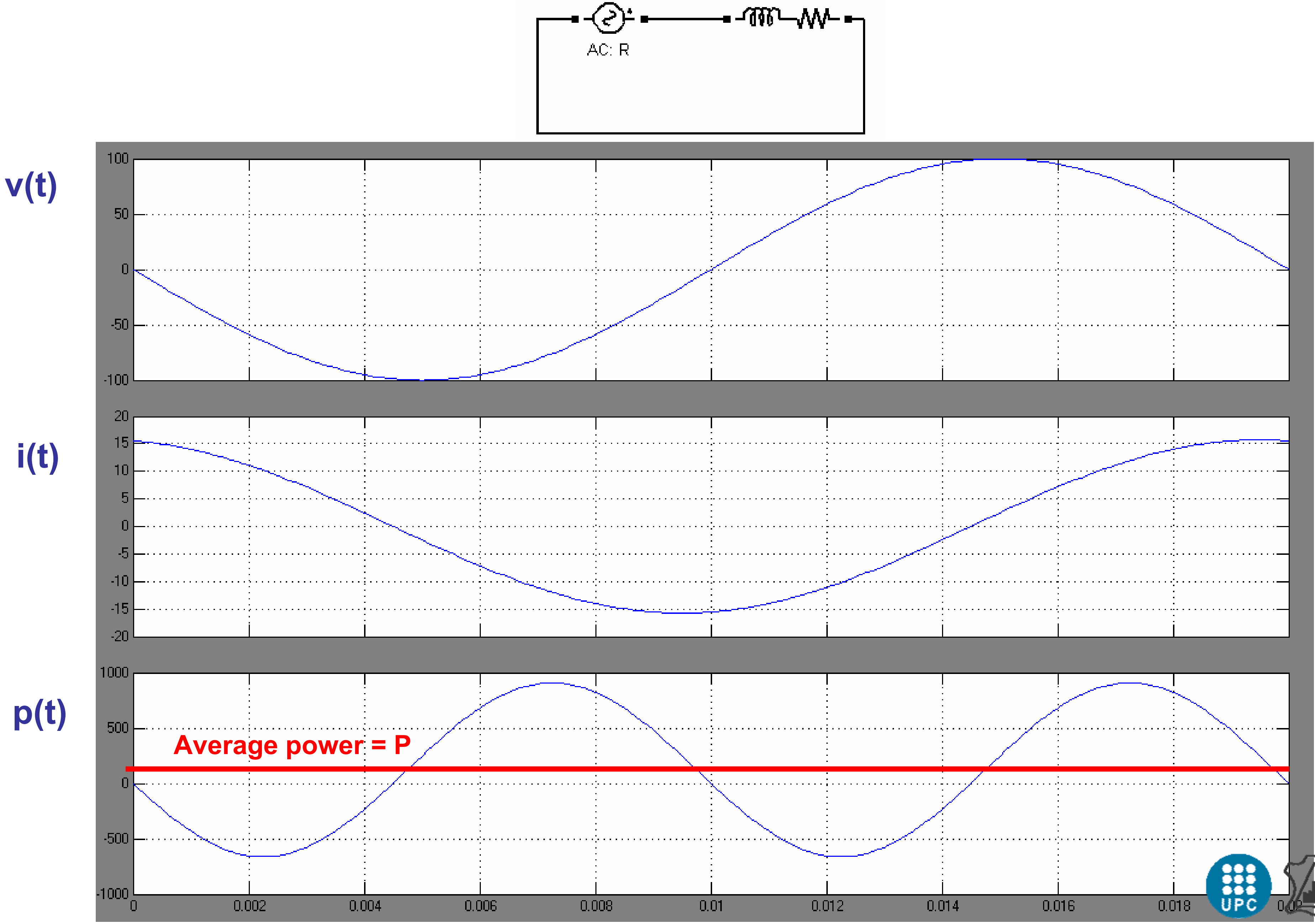
$$p(t) = v(t) \cdot i(t) = V_0 \cdot \cos(w \cdot t + \varphi_V) \cdot I_0 \cdot \cos(wt + \varphi_I)$$

$$p(t) = \underbrace{1/2 \cdot V_0 \cdot I_0 \cdot \cos(\varphi_V - \varphi_I)}_{\textbf{Constant}} + \underbrace{1/2 \cdot V_0 \cdot I_0 \cdot \cos(\mathbf{2w}t + \varphi_V + \varphi_I)}_{\textbf{Oscillates twice mains frequency!!!!}} \text{ watt}$$

- **Three-phase wye balanced load:**

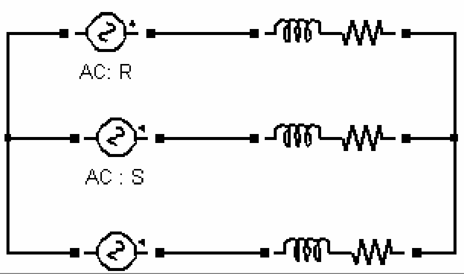
$$\begin{aligned}
\mathbf{p(t)} &= v_{RN}(t) \cdot i_R(t) + v_{SN}(t) \cdot i_S(t) + v_{TN}(t) \cdot i_T(t) = \\
&= \sqrt{2}V_p \cdot \cos(wt + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + \varphi_I) \\
&+ \sqrt{2}V_p \cdot \cos(wt - 120^o + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt - 120^o + \varphi_I) \\
&+ \sqrt{2}V_p \cdot \cos(wt + 120^o + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + 120^o + \varphi_I) \\
&= \\
&\quad V_p \cdot I_p \cdot \cos(\varphi_V - \varphi_I) + \cancel{V_p \cdot I_p \cdot \cos(2wt + \varphi_V + \varphi_I)} \\
&+ V_p \cdot I_p \cdot \cos(\varphi_V - \varphi_I) + \cancel{V_p \cdot I_p \cdot \cos(2wt - 240^o + \varphi_V + \varphi_I)} \\
&+ V_p \cdot I_p \cdot \cos(\varphi_V - \varphi_I) + \cancel{V_p \cdot I_p \cdot \cos(2wt + 240^o + \varphi_V + \varphi_I)} \\
&= 3/2 \cdot V_p \cdot I_p \cdot \cos(\varphi_V - \varphi_I) = \mathbf{3/2 \cdot V_p \cdot I_p \cdot \cos\varphi = constant!!!}
\end{aligned}$$

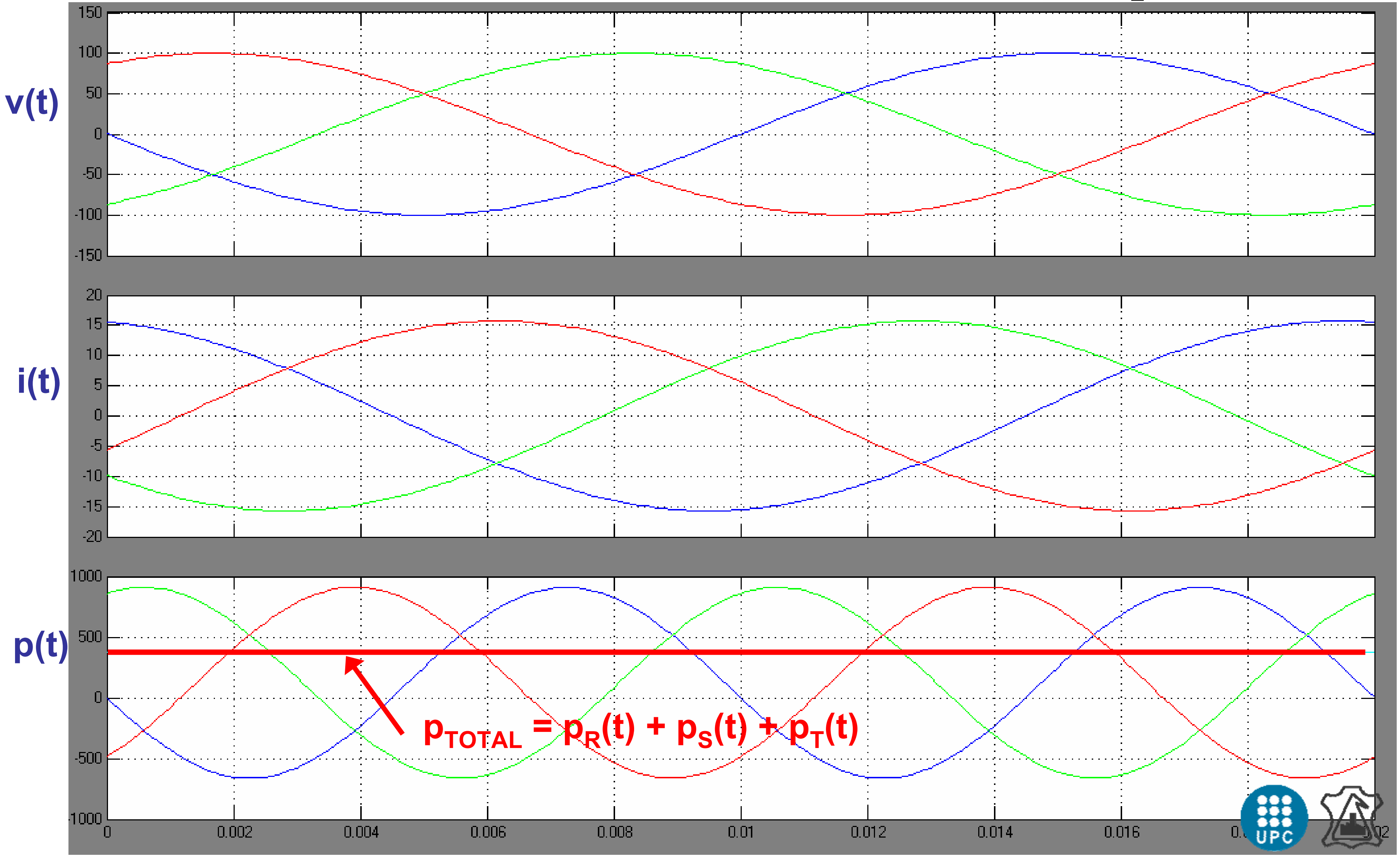
INSTANTANEOUS POWER: SINGLE PHASE LOAD



Unit 3. AC THREE-PHASE CIRCUITS

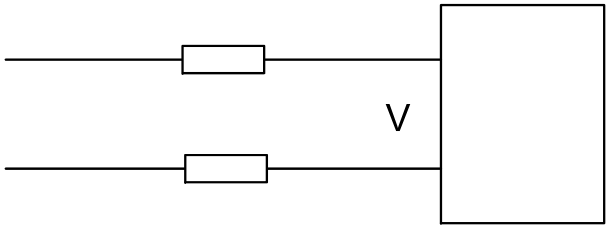
INSTANTANEOUS THREE-PHASE POWER





POWER LOSSES THREE-PHASE/SINGLE PHASE

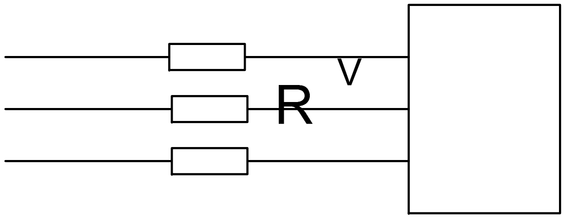
Single-phase line



$$I = \frac{P_{load}}{V \cdot \cos\varphi}$$

$$P_{losses} = 2 \cdot R_1 \cdot I^2 = 2 \cdot R_1 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Three-phase line



$$I = \frac{P_{load}}{\sqrt{3} \cdot V \cdot \cos\varphi}$$

$$P_{losses} = 3 \cdot R_2 \cdot I^2 = 3 \cdot R_2 \cdot \frac{P_{load}^2}{(\sqrt{3})^2 . V^2 \cdot \cos^2\varphi} = R_2 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Supposing same losses

R₁

$$2R_1 = R_2 \rightarrow 2\rho \frac{l}{S_{1p}} = \rho \frac{l}{S_{3p}} \rightarrow S_{3p} = \frac{1}{2} S_{1p}$$

Single-phase line: 2 conductors of length l and section S_{1p}

Three-phase line: 3 conductors of length l and section $S_{3p} = 1/2S_{1p}$

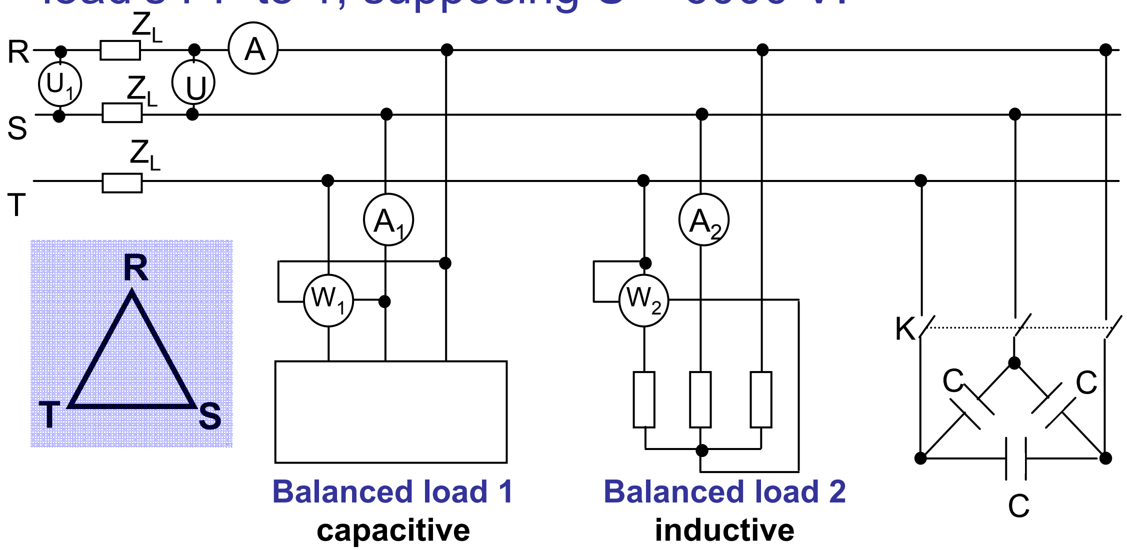
As a result: $weight_{3p\text{-}cables} = 3/4 weight_{1p\text{-}cables}$

R₂

R

Example 1

Three-phase, balanced RST system for which A_1 = 1.633 A, A_2 = 5.773 A, W_1 = 6928.2 W, W_2 = 12000 W, U = 6000 V, Z_{line}=4+j3 Ω· a) Obtain the complex power in the loads, the ammeter A and the voltmeter U_1 readings b) Obtain the value of C_Δ to improve the load's PF to 1, supposing U = 6000 V.



$$Q_1 = -\sqrt{3}W_1 = -12000\,\mathrm{VAr}$$

$$Q_1 = -12000\,\mathrm{VAr} = \sqrt{3}\underbrace{U}_{=6000}\underbrace{I_1}_{=1.633}\sin\varphi_1$$

$$\varphi_1 = -45^\circ$$

$$P_1 = \sqrt{3}UI_1\cos\varphi_1 = 12000\,\mathrm{W}$$

$$P_2 = 3W_2 = 36000\,\mathrm{W} = \sqrt{3}UI_2\cos\varphi_2$$

$$\varphi_2 = 53.12^\circ$$

$$Q_2 = \sqrt{3}UI_2\sin\varphi_2 = 48000\,\mathrm{VAr}$$

$$\overline{S}_{1+2} = (P_1 + P_2) + j(Q_1 + Q_2) = 48000 + j36000 = 60000^{36.87^\circ}\,\mathrm{VA}$$

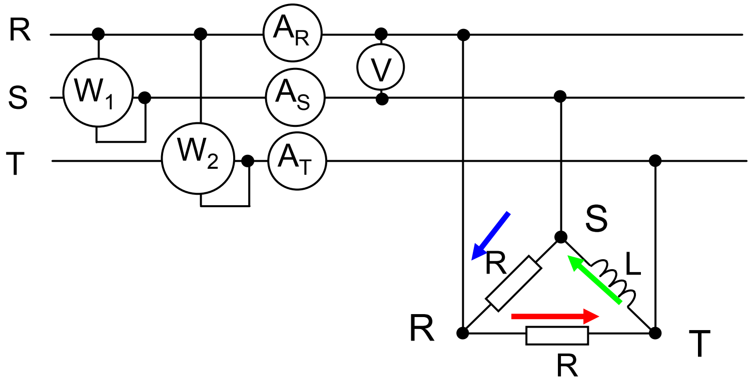
$$\overline{I}_{TOTAL} = \overline{I}_1 + \overline{I}_2 = 1.633^{90^\circ+45^\circ} + 5.773^{90^\circ-53.12^\circ} = 5.774^{90^\circ-36.85^\circ}\,\mathrm{A}$$

$$\overline{U}_{1,phase} = \overline{I}.\overline{Z}_L + \overline{U}_{\mathrm{phase}} = 5.774^{90^\circ-36.85^\circ}(4+j3) + \frac{6000}{\sqrt{3}}^{90^\circ} = 3492.97^{90^\circ}\,\mathrm{V} \qquad U_1 = \sqrt{3}U_{1,\mathrm{phase}} = 6050\,\mathrm{V}$$

$$Q_{1C,\Delta} = -Q/3 = -36000/3\,\mathrm{VAr} = -\frac{(6000)^2}{1/(2\pi.50.C_\Delta)} \qquad C_\Delta = 1.06\,\mu\mathrm{F}$$

Example 2

Three-phase 50 Hz system, for which V = 400 V, W_1 = -8569.24 W, W_2 = -5286.36 W, A_S = 21.56 A. Obtain a) the value of R b) the reading of A_R c) the value of the inductance L

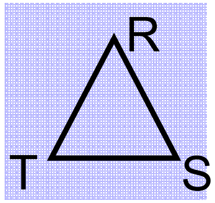


$$W_1' = -W_1$$

and

$$W_2' = -W_2$$

W_1		W_2	
V	**I**	**V**	**I**
RT	R	ST	S
SR	S	TR	T
TS	T	RS	R



$$\text{a)}\ P_{total} = W_1' + W_2' = 13855.6\ \text{W} = 2 \cdot \frac{400^2}{R} \quad \text{resulting in:} \quad R = 23.095\,\Omega$$

$$\text{b)}\ \bar{I}_{SR} = \frac{\bar{V}_{SR}}{R} = \frac{400^{-60^\circ}}{23.095} = 17.32^{-60^\circ}\ \text{A} \qquad \bar{I}_{RT} = \frac{\bar{V}_{RT}}{R} = \frac{400^{+60^\circ}}{23.095} = 17.32^{+60^\circ}\ \text{A}$$

$$\text{It results:} \quad \bar{I}_R = \bar{I}_{RT} - \bar{I}_{SR} = 30^{90^\circ}\ \text{A}$$

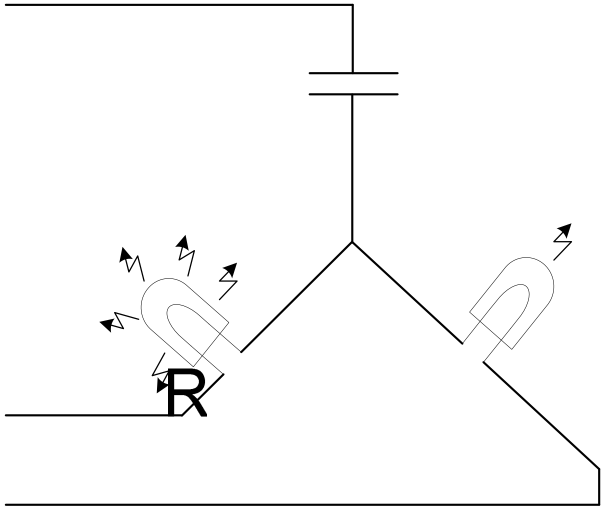
$$\text{c)}\ \bar{I}_S = \bar{I}_{SR} - \bar{I}_{TS} = 17.32^{-60^\circ} - \frac{\bar{V}_{TS}}{jX_L} = 17.32^{-60^\circ} - \frac{400^{180^\circ}}{X_L{}^{90^\circ}} = 8.66 - j15 - j\frac{400}{X_L}$$

$$I_S = 21.56\ \text{A} = \sqrt{8.66^2 + (-15 - \frac{400}{X_L})^2} \rightarrow X_L = 84.308\,\Omega = 2\pi 50 L$$

$$\text{The result is:} \quad L = 0.2684\ \text{H}$$

Example 3

Varley phase-sequence indicator· Calculate the voltage in each element and deduce practical consequences· Three-phase 400 V/50 Hz system·



$C = 1\mu F \rightarrow X_C = 3183\ \Omega$

$R_{2bulbs} = 2 \cdot V^2/P = 2 \cdot 230^2/20 = 5290\ \Omega$

$$\overline{V}_{ON} = \frac{\overline{V}_{RN} \cdot \overline{Y}_R + \overline{V}_{SN} \cdot \overline{Y}_S + \overline{V}_{TN} \cdot \overline{Y}_T}{\overline{Y}_R + \overline{Y}_S + \overline{Y}_T}$$

$$= \frac{230^{90^\circ}/3183^{-90^\circ} + 230^{-30^\circ}/5290^{0^\circ} + 230^{210^\circ}/5290^{0^\circ}}{1/3183^{-90^\circ} + 1/5290^{0^\circ} + 1/5290^{0^\circ}}$$

$$= 171.55^{171.31^\circ}\ V$$

1uF, 400 V

$\overline{V}_{RO} = \overline{V}_{RN} - \overline{V}_{ON} = 230^{90^\circ} - 171.55^{171.31^\circ} = 265.34^{50.28^\circ}\ V$

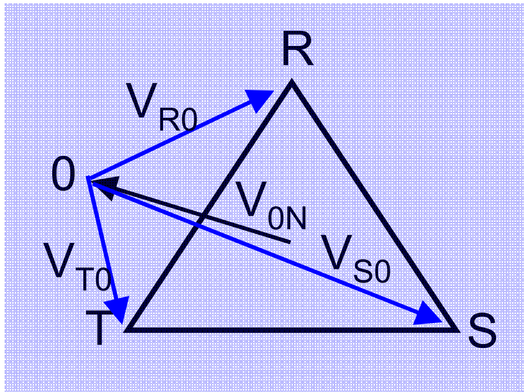
$\overline{V}_{SO} = \overline{V}_{SN} - \overline{V}_{ON} = 230^{-30^\circ} - 171.55^{171.31^\circ} = \mathbf{394.78^{-20.91^\circ}\ V}$

2 bulbs in series

$\overline{V}_{TO} = \overline{V}_{TN} - \overline{V}_{ON} = 230^{210^\circ} - 171.55^{171.31^\circ} = 144.00^{-101.86^a}\ V$

230 V, 20 W

0



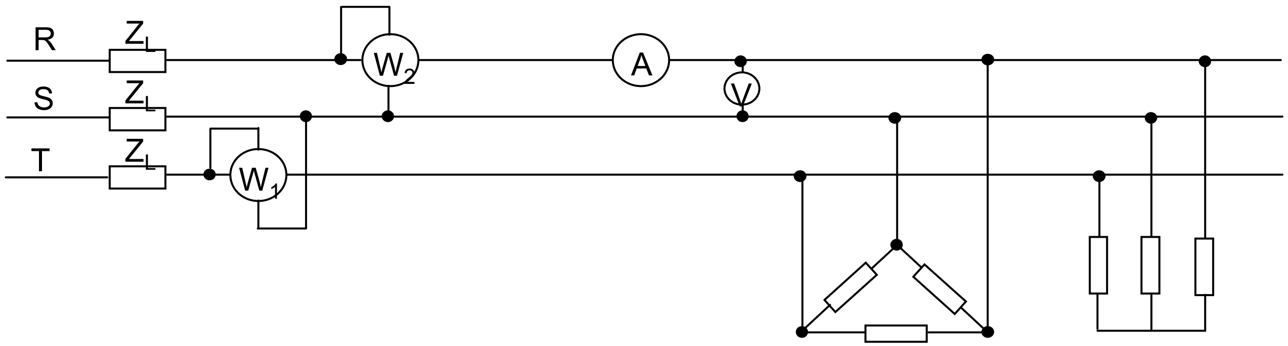
Conductor R is that where the capacitor is placed, conductor S is that where placed the brighter bulb and the remaining one is T.

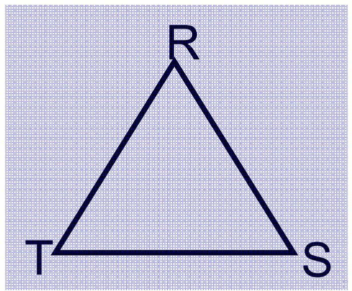
2 bulbs in series

230 V, 20 W

Example 4

A 400 V and 50 Hz three-phase line feeds two balanced loads through a line which has an internal impedance Z_L=0.5 + j·1 Ω· The Δ-connected load has phase impedances which values are 45+j·30 Ω, whereas the Y-connected load has phase impedances of 15–j·30 Ω· Determine: a) The reading of the ammeter A b) The reading of the voltmeter V c) The readings of watt-meters W_1 and W_2





$$\overline{Z}_{TOT} = \overline{Z}_L + (\overline{Z}_{\Delta\rightarrow Y} // \overline{Z}_Y) = \overline{Z}_L + \overline{Z}_{//} = (0.5 + j\cdot 1) + (16.731 + j\cdot 1.154) = 17.365^{\,7.125^\circ}\ \Omega$$

a) $$\overline{I}_R = \frac{\overline{V}_{RN}}{\overline{Z}_{TOT}} = \frac{400/\sqrt{3}^{\,+90^\circ}}{17.365^{\,7.125^\circ}} = 13.30^{\,82.875^\circ}\ \text{A}$$

b) $$V = \sqrt{3}\cdot(I\cdot Z_{//}) = \sqrt{3}\cdot(13.30\cdot 16.7706) = 386.33\text{V}$$

W1		W2	
V	**I**	**V**	**I**
RT	R	ST	S
SR	S	TR	T
TS	T	RS	R

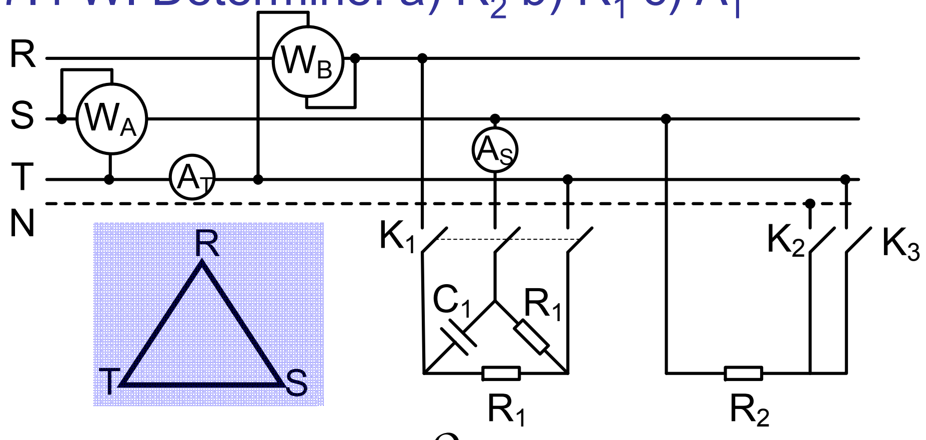
c) $$\left.\begin{aligned} P_{LOAD} &= W_1 + W_2 = 3\cdot 13.30^2\cdot 16.731 \\ Q_{LOAD} &= \sqrt{3}\cdot(W_1 - W_2) = 3\cdot 13.30^2\cdot 1.154 \end{aligned}\right\}$$

From the Aron connection it results:

$W_1 = 4616{,}1$ W, $W_2 = 4262{,}5$ W

Example 5

Three phase 400 V-50 Hz line. When switch K_2 is closed W_A = 4000 W. When K_1 and K_3 are closed, W_A = 28352.6 and W_B = -11647.4 W. Determine: a) R_2 b) R_1 c) A_T



a) **K_2 closed**: $W_A = V_{ST} \cdot I_S \cdot \cos(\varphi_{VST} - \varphi_{IS})$

$4000 = 400 \cdot I_S \cdot \cos(0^\circ + 30^\circ) \rightarrow I_S = 11.55$ A

$\mathbf{R_2} = V_{SN}/I_S = (400/\sqrt{3})/11.55 = \mathbf{20\ \Omega}$

b) **K_1 and K_3 closed**:

$P_{TOT} = W_1 + W_2 = -W_B + W_A = 40000\text{ W} = 2 \cdot 400^2/R_1 + 400^2/\overset{=20\,\Omega}{R_2} \rightarrow \mathbf{R_1 = 10\ \Omega}$

W1		W2	
V	**I**	**V**	**I**
RT	R	ST	S
SR	S	TR	T
TS	T	RS	R

c) **K_1 and K_3 closed**:

$$\bar{I}_{T1} = \bar{I}_{TS} - \bar{I}_{RT} = \frac{\bar{V}_{TS}}{R_1} - \frac{\bar{V}_{RT}}{R_1} = \frac{400^{180^\circ}}{10} - \frac{400^{60^\circ}}{10} = -60 - j \cdot 34.64\text{A}$$

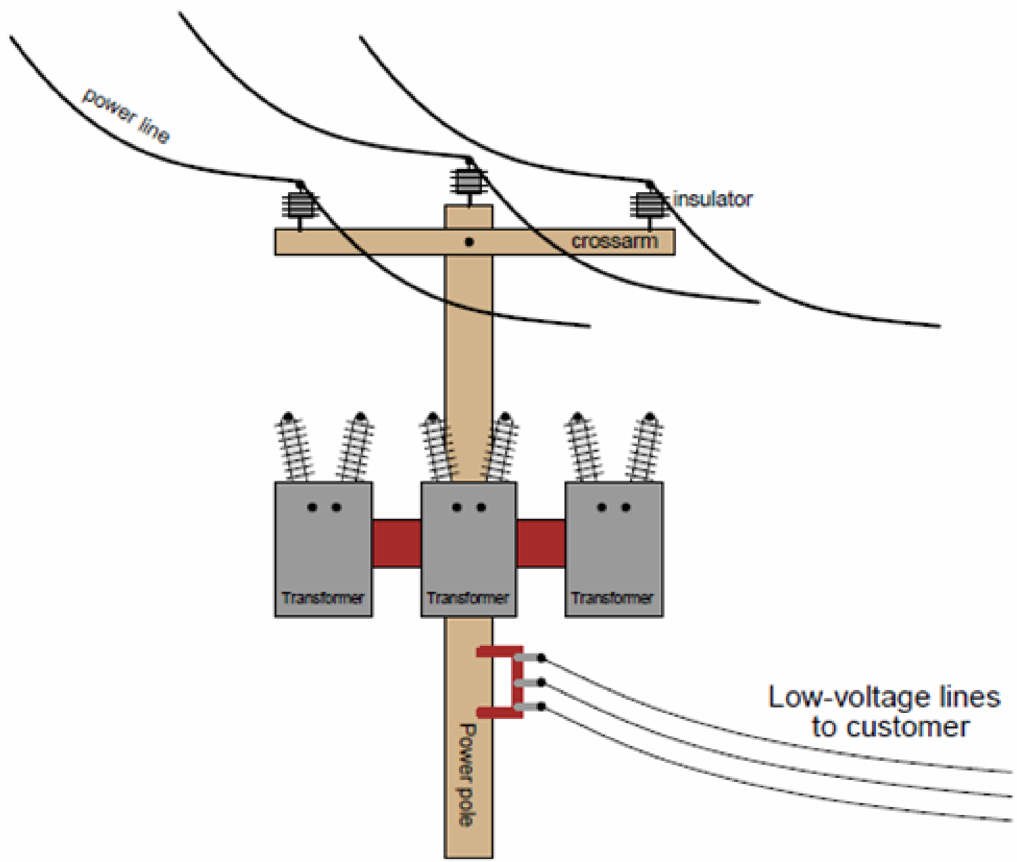
$$\bar{I}_{T2} = \frac{\bar{V}_{TS}}{R_2} = \frac{400^{180^\circ}}{20} = -20 + j \cdot 0\text{A}$$

$$\bar{I}_{Ttotal} = \bar{I}_{T1} + \bar{I}_{T2} = (-60 - j \cdot 34.64) + (-20 + j \cdot 0) = -80 - j \cdot 34.64 = 87.18^{87.18^\circ}\text{ A}$$

It results, $\mathbf{A_T = 87.18\ A}$

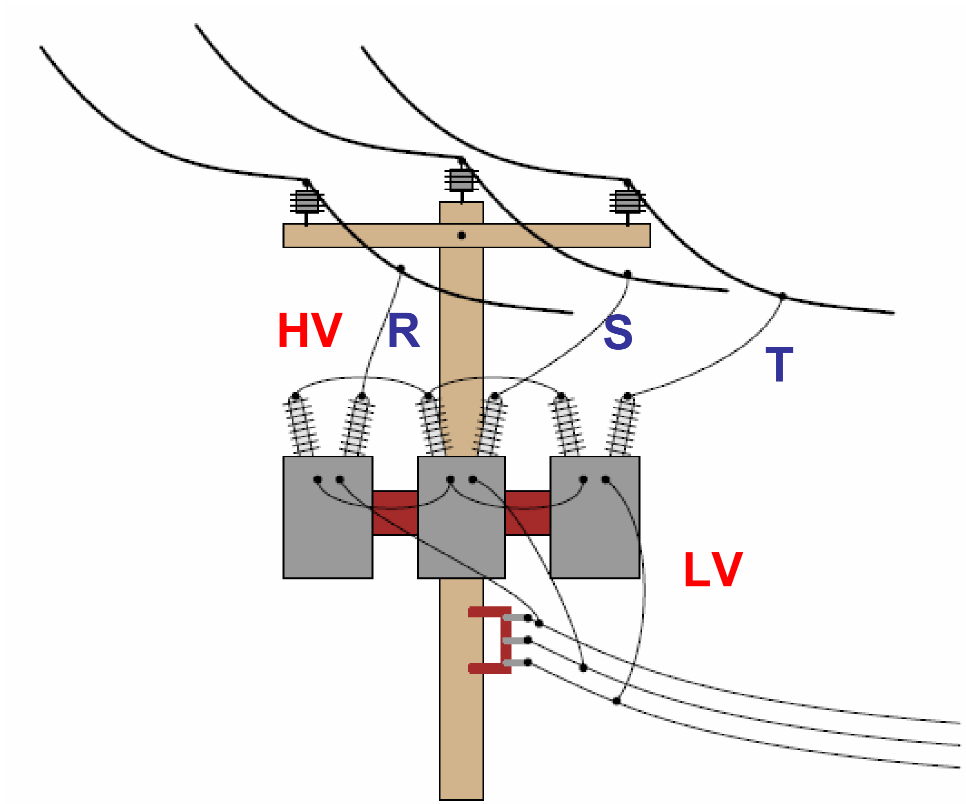
Question 1

An electrical lineman is connecting three single-phase transformers in a Y(primary)-Y(secondary) configuration, for power service to a business. Draw the connecting wires necessary between the transformer windings, and between the transformer terminals and the lines. Note: fuses have been omitted from this illustration, for simplicity.



Question 2

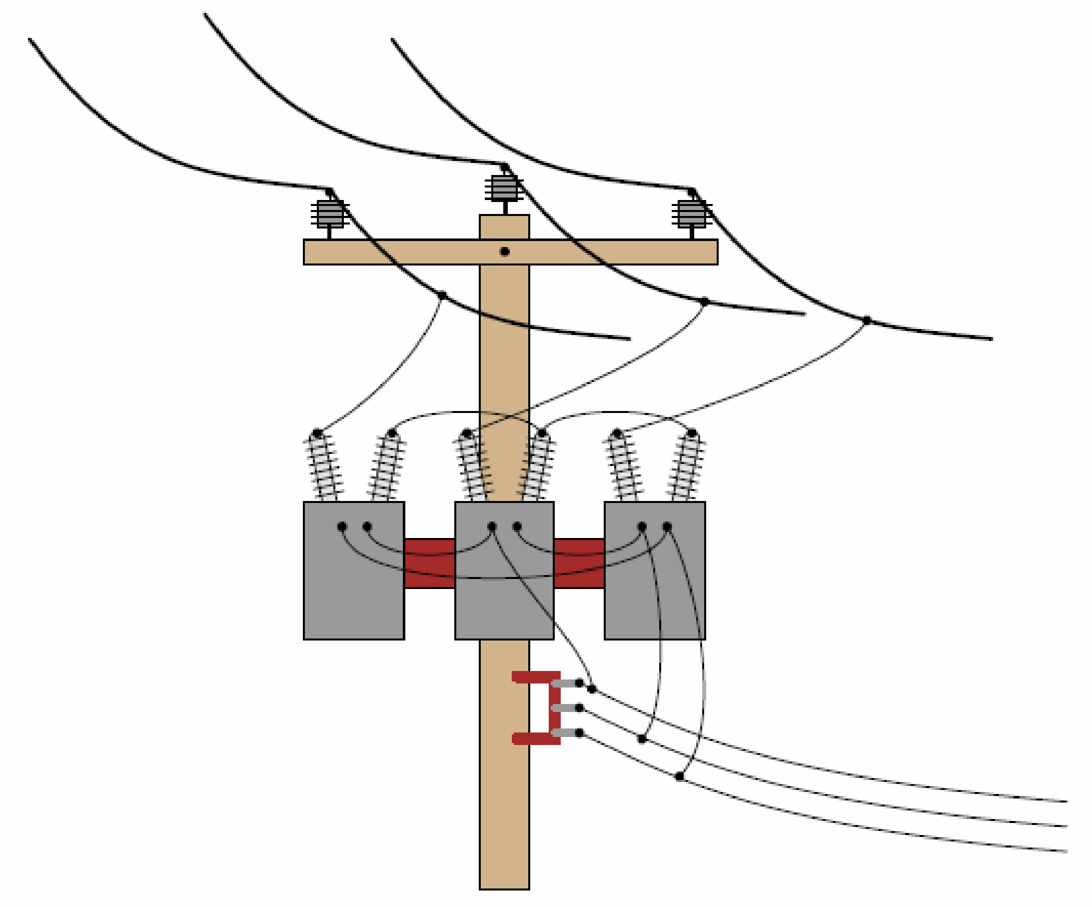
Identify the primary-secondary connection configuration of these pole-mounted power transformers (i.e. Y-Y, Y-Delta, Delta-Y, etc.).



These transformers are connected in a **Yy** configuration.

Question 3

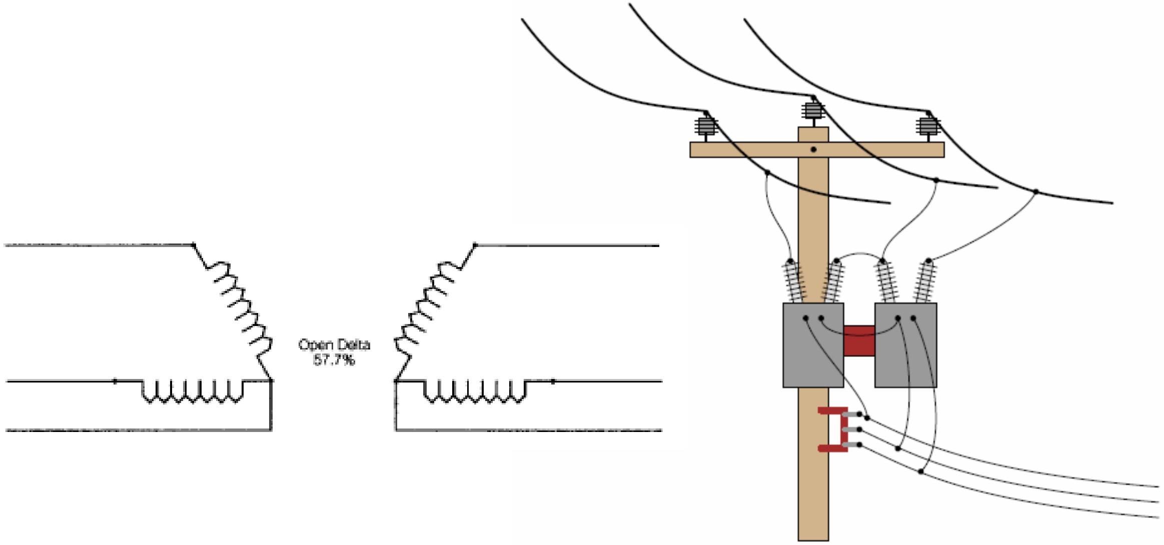
Identify the primary-secondary connection configuration of these pole-mounted power transformers (i.e. Y-Y, Y-Delta, Delta-Y, etc.).



These transformers are connected in a **Yd** configuration.

Question 3

Identify the primary-secondary connection configuration of these pole-mounted power transformers (i.e. Y-Y, Y-Delta, Delta-Y, etc.).

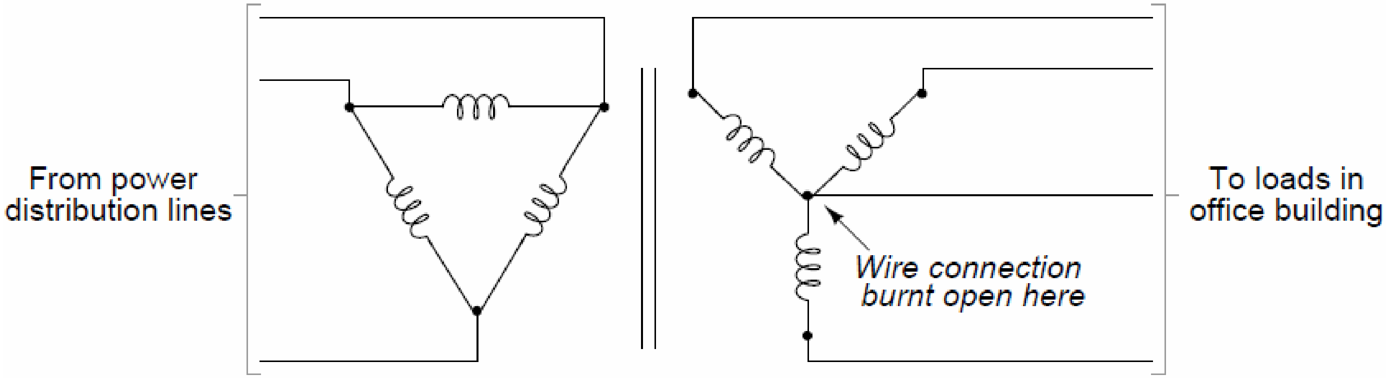


These transformers are connected in **open-delta** configuration.

- Utilizing 3 single-phase transformers is normally not done because it is more expensive than utilizing 1 three-phase transformer.
- However, there is an advantage which is called the **open-Delta** or **V-Connection**
- It functions as follows: a defective single-phase transformer in a Dd three-phase bank can removed for repair. Partial service can be restored using the open-Delta configuration until a replacement transformer is obtained.
- With two transformers three-phase is still obtained, but at reduced power 57.7% of original power.
- It is a very practical transformer application for emergency conditions.

Question 4

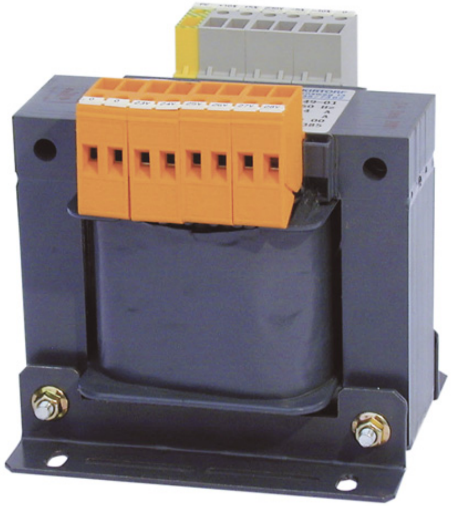
One of the conductors connecting the secondary of a three-phase power distribution transformer to a large office building fails open. Upon inspection, the source of the failure is obvious: the wire overheated at a point of contact with a terminal block, until it physically separated from the terminal. What is strange, though, is that the overheated wire is the neutral conductor, not any one of the ”line” conductors. Based on this observation, what do you think caused the failure?
After repairing the wire, what would you do to verify the cause of the failure?



- Here’s a hint (“pista”): if you were to repair the neutral wire and take current measurements with a digital instrument (using a clamp-on current probe, for safety), you would find that the predominant frequency of the current is 150 Hz, rather than 50 Hz.
- This scenario is all too common in modern power systems, as non-linear loads such as switching power supplies and electronic power controls become more prevalent. Special instruments exist to measure harmonics in power systems, but a simple DMM (digital multimeter) may be used as well to make crude assessments.

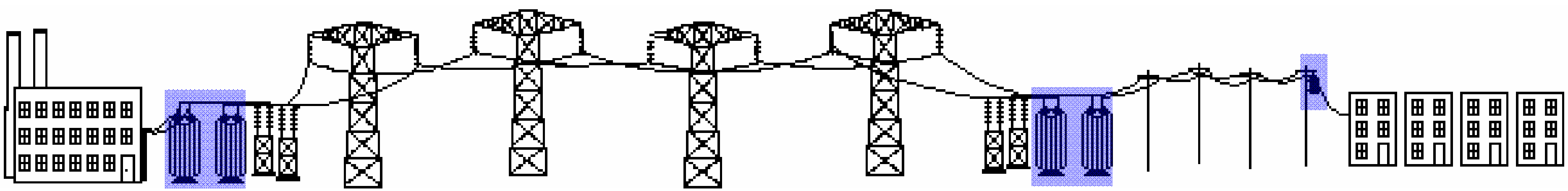
TRANSFORMERS

SINGLE-PHASE AND THREE-PHASE TRANSFORMERS



Miracle transformers

Overhead power lines



Unit 4. TRANSFORMERS

CONTENTS LIST:

- Introduction
- Core designs
- Energy loss
- Electrical steel
- The Ideal transformer
- Quantities referred to the primary/secondary side
- The practical transformer
- Equivalent circuit
- Standard tests
- Efficiency
- Parallel connected transformers
- Three-phase bank
- Three-phase connections. Vector/phasor group
- Preferred connections
- Exercises

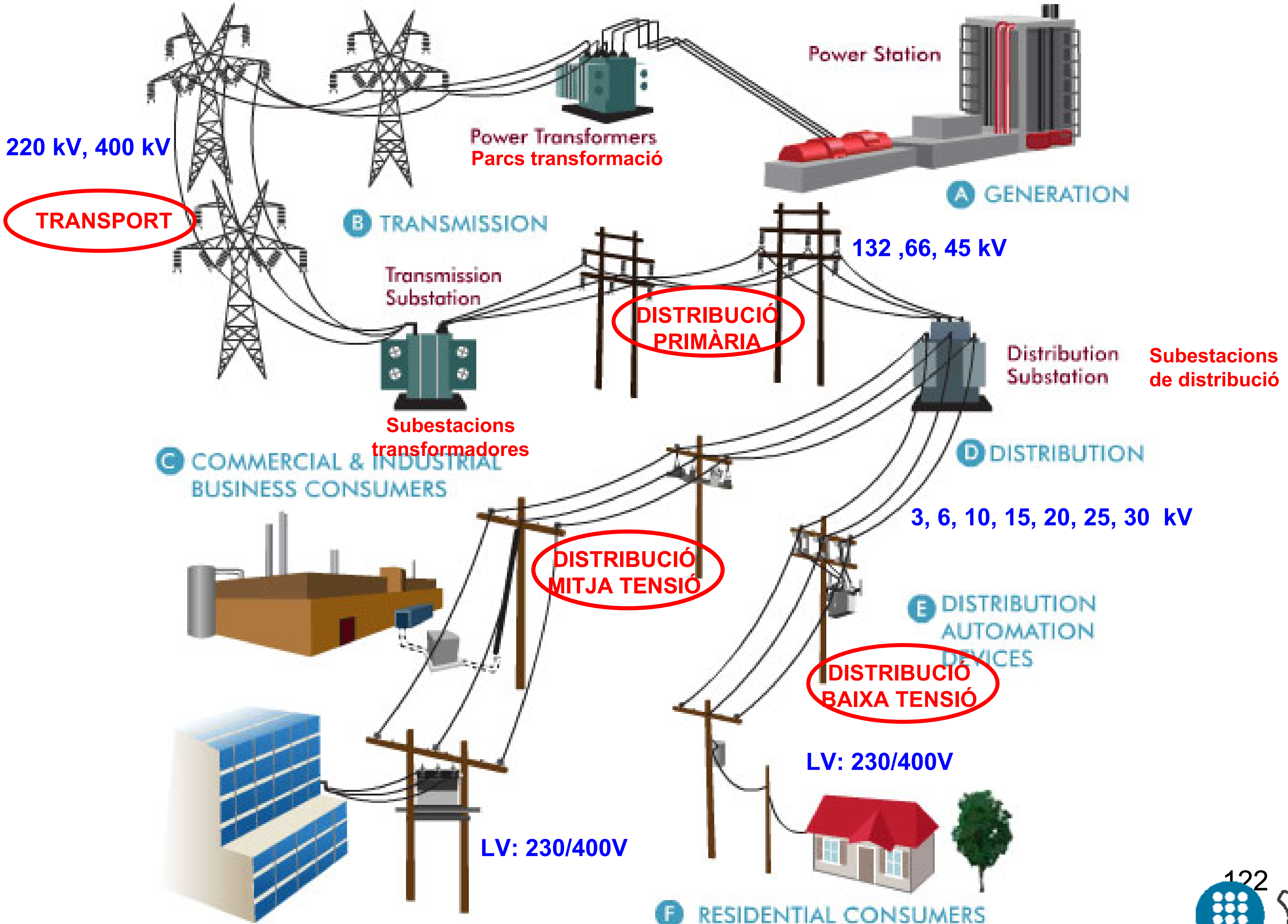
POWER GRID

- Utility frequency: 50 Hz (60 Hz in some countries)
- Use of transformers made it possible to transmit the power economically hundreds of kilometers away from the generating station.
- The success of the current AC power system is in grand extend due to power, transmission and distribution transformers.
- Generators voltages: from 6 to 25 kV.
- Transmission voltages(>100 km) : 220 and 400 kV
- Primary distribution voltages(<100 km): 132, 66, 45 kV
- Medium voltage distribution(<10 km): 3, 6, 10, 15, 20 kV

RATED VOLTAGE (kV) Overhead power lines	
3rd category	3
	6
	10
	15
	20*
	25
	30
2nd Category	45
	66*
1st category	110
	132*
	150
special category	220*
	400*

Reglamento sobre condiciones técnicas y garantías de seguridad en líneas eléctricas de alta tensión 2008
ITC-LAT 07
* Suggested

POWER GRID



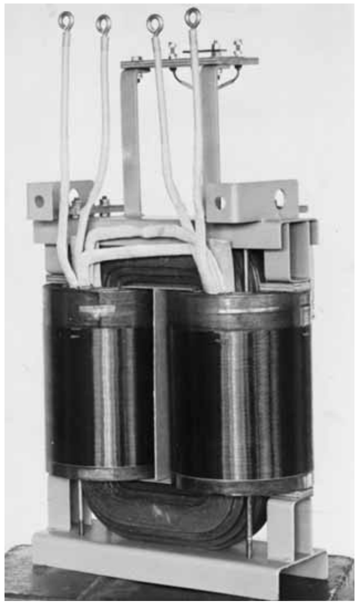
http://www.oncor.com/images/content/grid.jpg

INTRODUCTION

- Transformers were invented towards the end of the nineteenth century
- Transformers make possible the development of the modern constant voltage AC supply system, with power stations often located many miles from centers of electrical load.
- The transformer is an electromagnetic conversion device which has a primary and a secondary windings. It accomplishes: $V_1I_1 \approx V_2I_2$.
- The primary and secondary windings are not connected electrically, but coupled magnetically.
- A transformer is termed as either a **step-up or step-down** transformer.
- Machines with higher efficiency, **up to 99.7%**

INTRODUCTION

- High power ≈ three-phase transformers
- Low power ≈ single-phase transformers
- *General Electric* makes single-phase distribution transformers up to 167 kVA for residential power distribution.
- Single-phase power distribution is used especially in rural areas, where the cost of a three-phase distribution network is high and motor loads are small and uncommon.
- Small single-phase transformers are also used in electronic appliances such as TV, video, …



Single-phase rural-type transformer with C-type core, rated at 16 kVA 11 000/200 250 V (Allenwest Brentford)

USA

DEFINITIONS

- **Rating:** The nominal value of any electrical, thermal, mechanical, or environmental quantity assigned to define the operating conditions under which a component, machine, apparatus, electronic device, etc., is expected to give satisfactory service.
- **Rated frequency (f_n)**: The frequency of the alternating current for which a device is designed.
- **Rated continuous current (I_n)**: The current expressed in amperes, root mean square, that the device can carry continuously under specified service conditions without exceeding the allowable temperature rise.
- **Rated voltage**: The RMS voltage, at rated frequency, which may be impressed between the terminals of the device under standard operating conditions.

Rated value: nominal o assignat

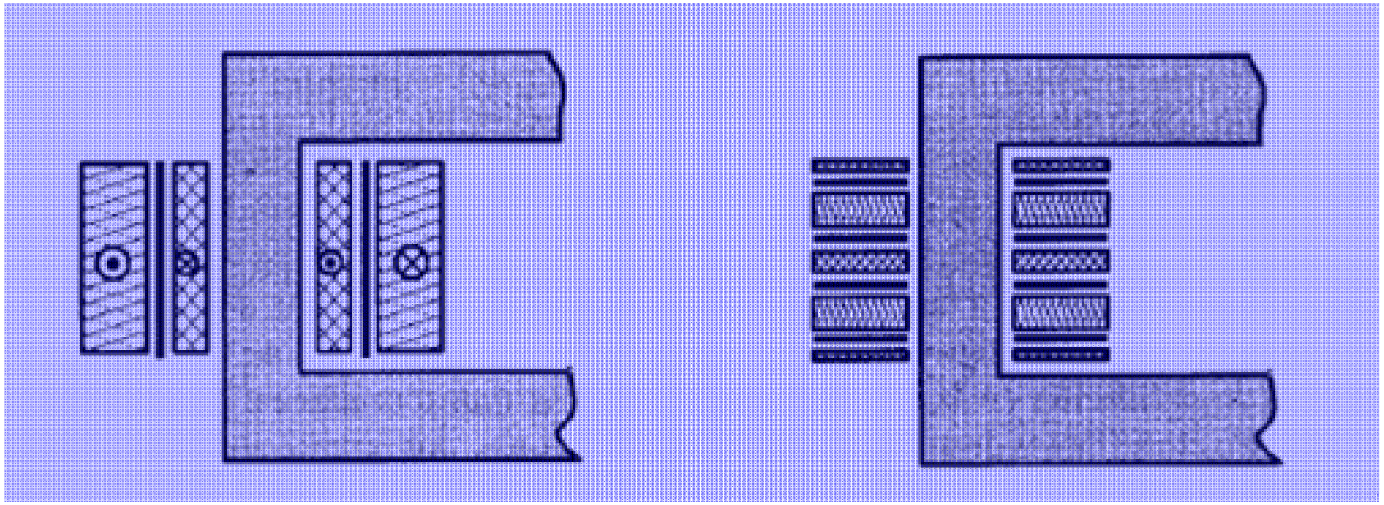
CORE DESIGNS

Type of windings

Low-current: circular-section wires with varnish insulation

High-current: flat bars with paper insulation

- **Concentric**: windings with shape of coaxial cylinders
- **Alternate**: windings are divided into different sections, placed consecutively as Low-Voltage/High-Voltage/Low-Voltage/ High-Voltage/Low-Voltage …



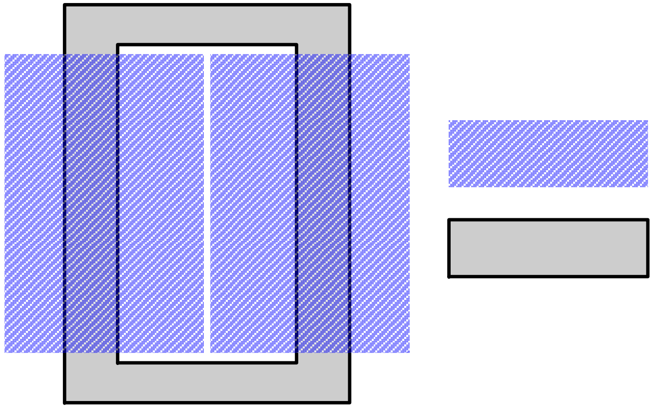
Concentric winding

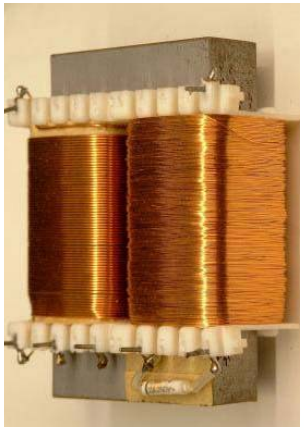
Alternate winding

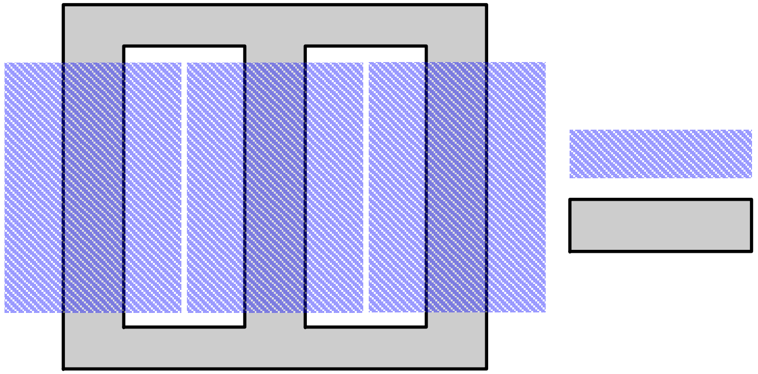
CORE DESIGNS

Core-type designs (de columnes)

- Predominate throughout most of the world
- The coil surrounds the core



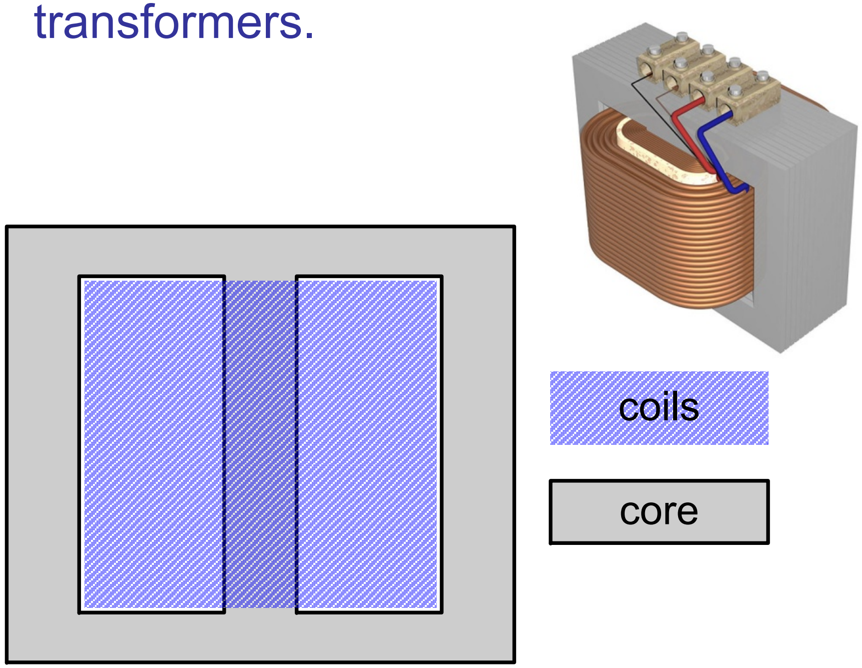




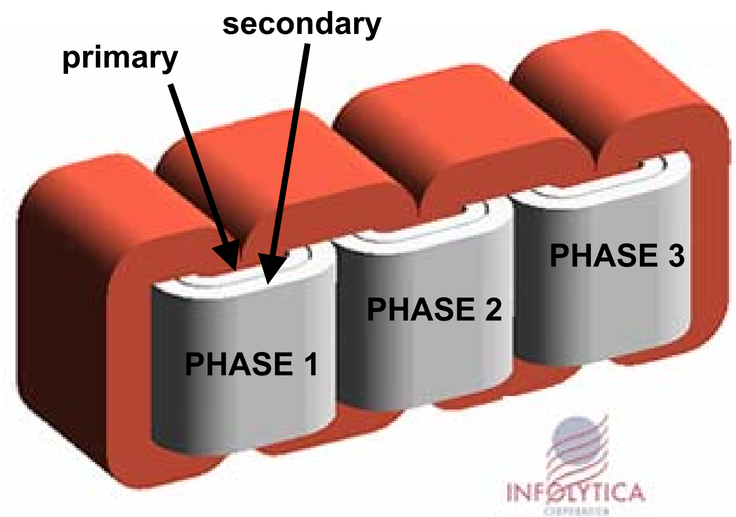
CORE DESIGNS

Shell-type designs (acuirassat)

- The core surrounds the coil
- The flux-return paths of the core are external to and enclose the windings.
- This design provides better magnetic shielding → suitable for supplying power at low voltage and heavy current, as, for example, in the case of arc furnace transformers.



Single-phase shell-type core

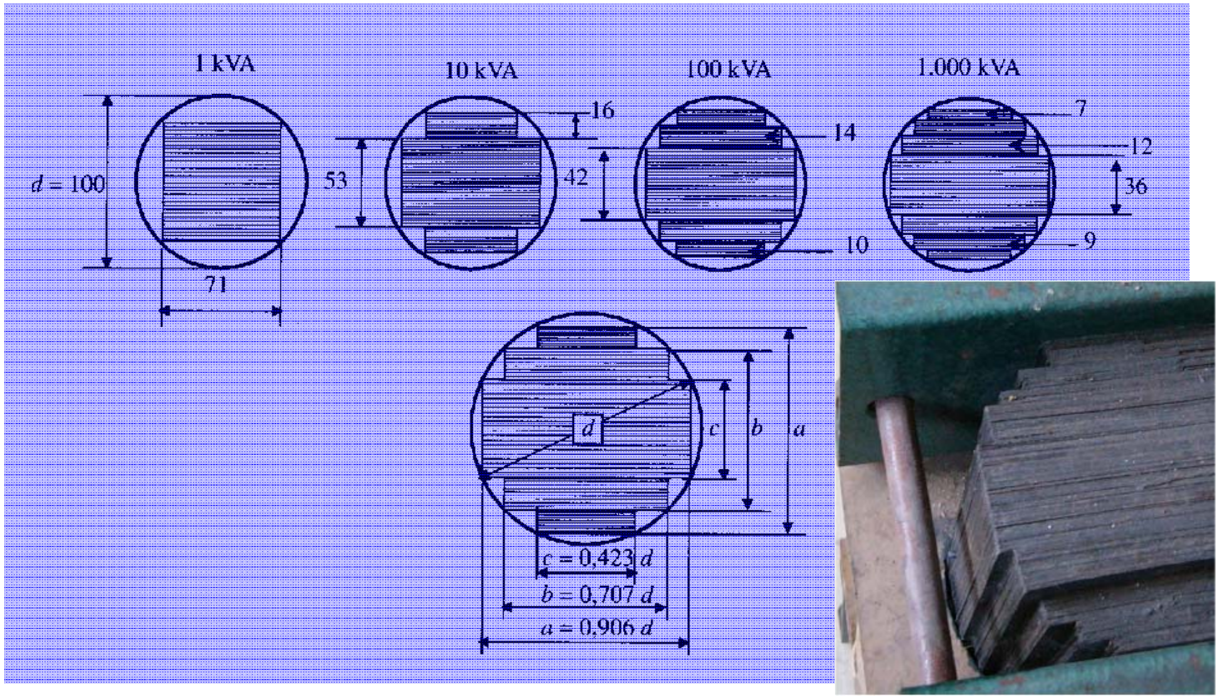


Three-pahse shell-type core

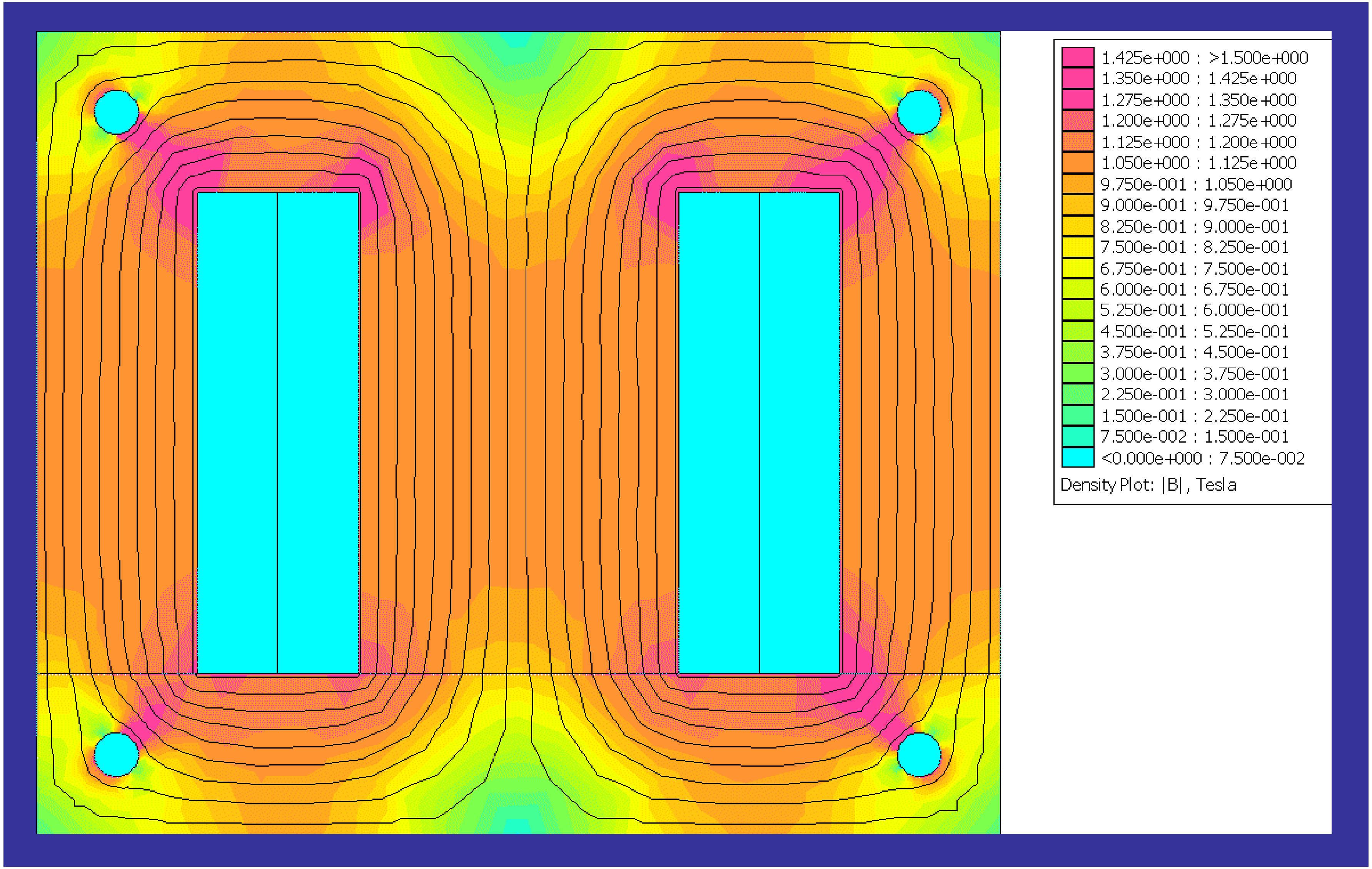
CORE DESIGNS

Core sections

- Core laminations are built up to form a limb or leg having as near as possible a circular cross-section.
- Circular cross-sections allow to obtain optimum use of space within the cylindrical windings.
- The stepped cross-section approximates to a circular shape depending only on how many different widths of strip there are.



MAGNETIC FLUX



CORE DESIGNS

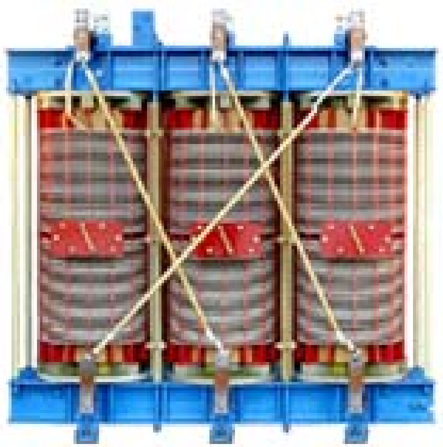
Refrigeration systems

- Power loss generate heat
- **Dry transformers**: for low power
- **Oil-filled transformers**: for large power, the transformer is completely submerged in an oil tank. The oil actuates as a coolant and an insulator. Mineral oil (from distillation of petrol) and silicone oil (most actual)
- **Dry-type transformers encapsulated in epoxy resin:** for installations that require high security, specially in indoor. They don't spread fire. High voltage windings are completely encapsulate in epoxy resin.

Kotsons Catalogue

Oil insulated type transformer

- 33 kV to 400 V
- 25 KVA - 20 MVA
- Refrigeration: mineral oil a low flammability synthetic fluid
- Ventilation (possibility)



Kotsons Catalogue

Dry type transformer

- 33 kV to 400 V
- 40 KVA - 5 MVA
- Insulation: epoxy resin
- Ventilation
- Indoor
- Very safe

CORE DESIGNS

Refrigeration systems

Insulation	Symbol	Type of circulation	Symbol
Mineral oil	O	Natural	N
Pyralene	L	Forced	F
Gas	G		
Water	W		
Air	A		
Solid insulation	S		

ONAN: Oil **N**atural (transformer submerged in an oil tank; oil with natural convection)
It is refrigerated by **A**ir with **N**atural convection

ONAF: In this case the **A**ir is **F**orced by means of a fan

TRANSFORMER ASSEMBLY

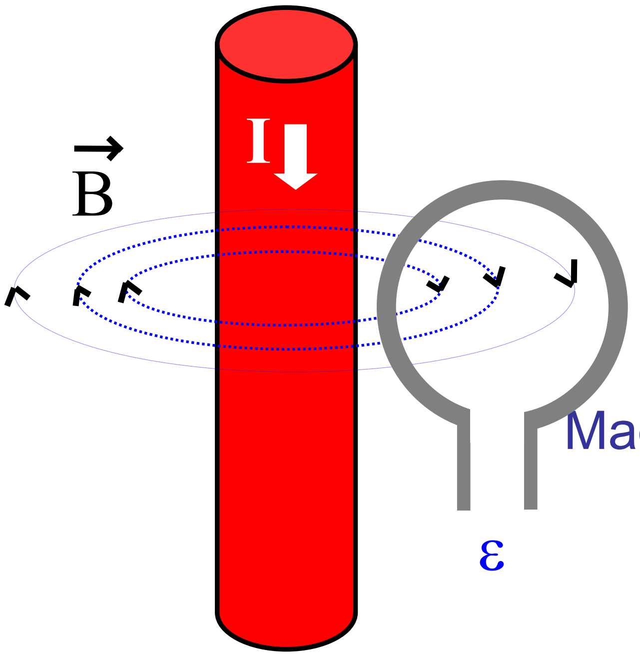
USA

Europe

FARADAY'S LAW OF INDUCTION

Faraday's Law

The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

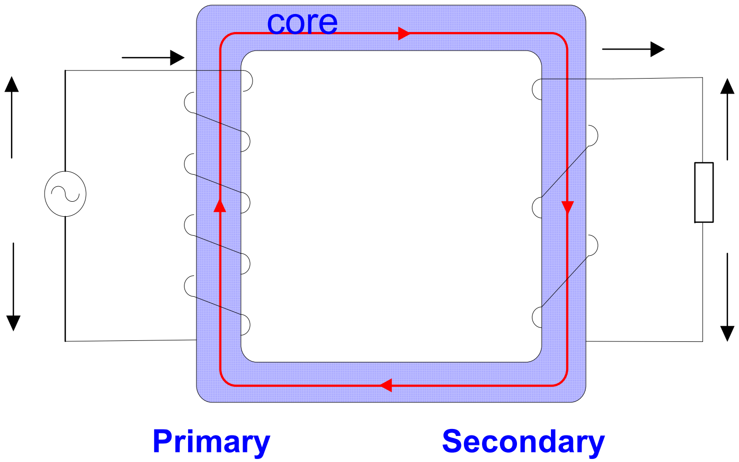


EMF $\varepsilon = -d\Phi/dt$ volt

Magnetic flux: $\Phi = \oint \vec{B}\,d\vec{S}$

IDEAL TRANSFORMER

- **Transformer:** static device that transfers electrical energy from one circuit to another by electromagnetic induction without the change in frequency. It changes the relationship between voltage and current in primary and secondary sides.
- It is made of steel laminations wrapped with two coils of wire.
- The coil-ratio or turns-ratio derives the voltage change.



Turns-ratio $r_t = \dfrac{N_1}{N_2}$

$$r_t = \left|\frac{V_1}{V_2}\right| = \frac{N_1 d\Phi_m/dt}{N_2 d\Phi_m/dt} = \frac{N_1}{N_2}$$

Operating principle of a zero-loss transformer:

$S_1 \approx S_2 \Rightarrow V_1 I_1 \approx V_2 I_2 \Rightarrow r_t = \dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$

Φ_m I_2

The transformer alters the relationship between voltages and currents, but does not change the power transferred

V_1 V_2

ENERGY LOSS. GENERAL

IDEAL TRANSFORMER

Zero-energy loss

Efficiency=100%

PRACTICAL TRANSFORMER

WINDINGS LOSS (resistive)

IRON LOSS

- Hysteresis
- Eddy currents
- Magnetostriction

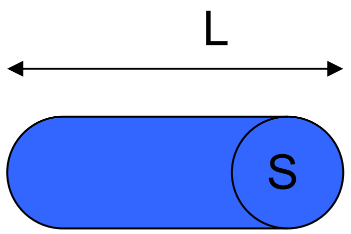
Efficiency: < 100%

- It has been estimated that some 5% of all electricity generated is dissipated as iron losses in electrical equipment.
- Larger transformers are generally more efficient, and those rated for electricity distribution usually perform better than 98%.

ENERGY LOSS. WINDING RESISTANCE

Winding resistance

Current flowing through the windings causes resistive heating of the conductors. At higher frequencies, skin effect and proximity effect create additional winding resistance and losses.



$$R = \rho \cdot L/S$$

$$P_{TOTAL} = P_1 + P_2 = I_1^2 R_1 + I_2^2 R_2$$

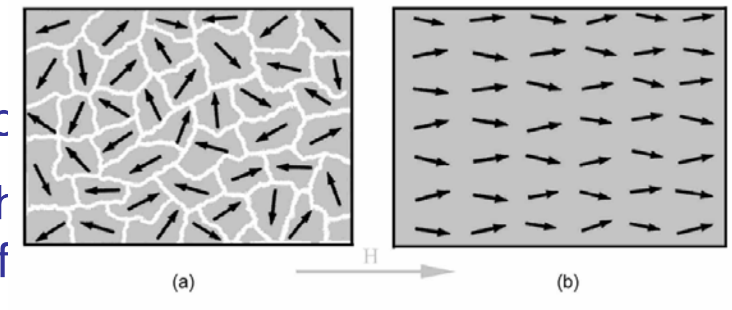
$$\rho_{Cu} = 1.72 \cdot 10^{-8}\ \Omega m\ (20ºC)$$

$$\rho_{Al} = 2.82 \cdot 10^{-8}\ \Omega m\ (20ºC)$$

ENERGY LOSS. HYSTERESIS LOSS

Hysteresis loop

- Ferromagnetic materials have microscopic ferro
- In the presence of an exterior magnetic field, th c domains become faced towards the direction of



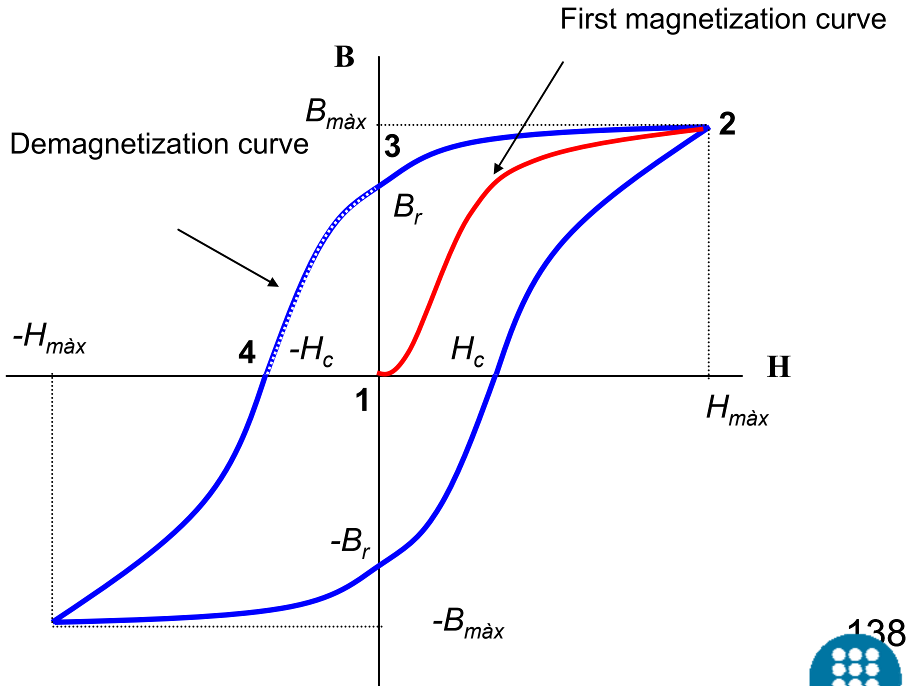
(a) a piece of ferromagnetic material which is not magnetised, where the domain poles are not aligned; (b) the domain poles aligned with an external magnetising force (H).

- Each time the magnetic field is reversed, a sm o hysteresis within the core due to a friction process.
- Hysteresis losses are minimized by use of a better grade of core material.

$$P_{hist} = k_2 f B^n_{peak-value}$$

W/kg

$n \in (1.5, 2.5)$
Steinmetz coef.

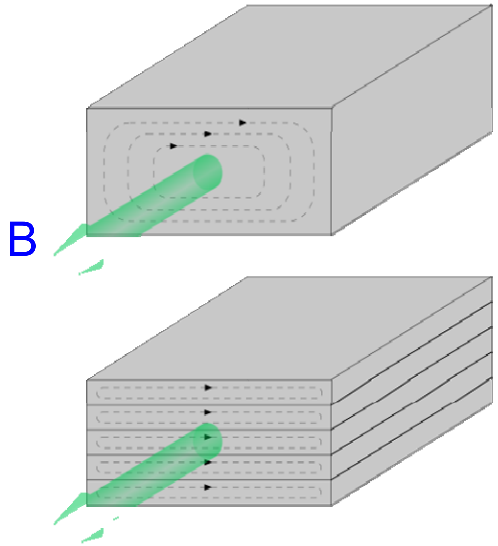


Ferromagnetic domain structure on oriented steel

ENERGY LOSS. EDDY CURRENTS

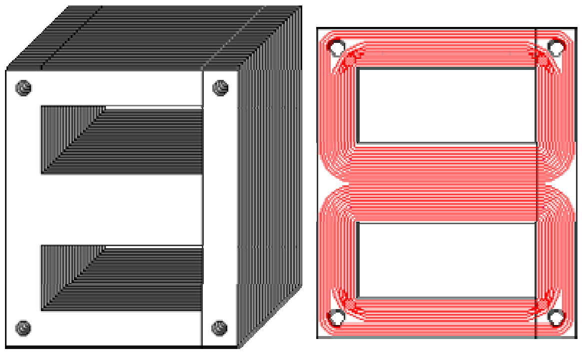
Eddy currents

- Ferromagnetic materials are good conductors.
- A solid core made from such a material also constitutes a single short-circuited turn throughout its entire length.
- Eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material.
- Eddy losses are minimized by use of **thinner laminations**.
- Currently the lowest thickness available is 0.23 mm, and the popular thickness range is 0.23 mm to **0.35** mm for power transformers.



$$P_{eddy} = k_1 \sigma f^2 thickness^2 B^2_{peak-value}$$

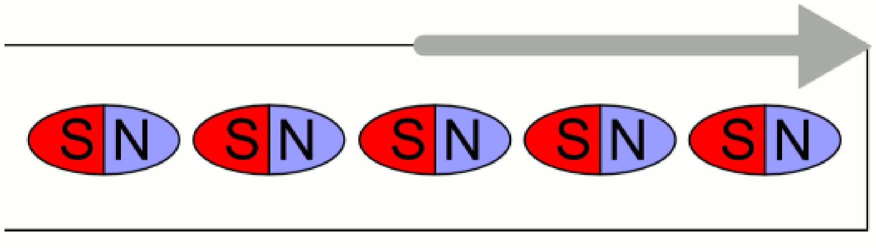
W/kg



ENERGY LOSSES. MAGNETOSTRICTION

Magnetostriction (Transformer noise)

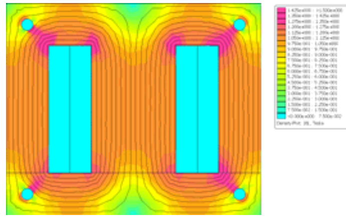
- Magnetic flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle of the magnetic field, an effect known as magnetostriction.
- This produces the ***buzzing sound*** commonly associated with transformers, and in turn causes losses due to frictional heating in susceptible cores.
- Transformers located near a residential area should have sound level as low as possible.



ELECTRICAL STEEL XAPA MAGNÈTICA

- Electrical steel is designed to produce small hysteresis area (low core loss) and high permeability.
- Electrical steel is an iron alloy which may have from zero to 6.5% silicon.
- Silicon significantly increases the electrical resistivity of the steel, which decreases the induced eddy currents and thus reduces core losses.
- The presence of silicon has the disadvantage that the steel becomes brittle and hard. For workability reasons and ease of manufacture, the quantity must be limited.
- The elimination of impurities, including carbon, also has a significant effect in the reduction of losses.
- The material is usually manufactured in the form of cold-rolled strips.
- These strips are called laminations when stacked together to form a core.
- Silicon steel laminations of thickness from 0.2 mm to 0.5 mm (predominantly **0.35 mm**) are used in transformers.

ELECTRICAL STEEL



Grain-oriented electrical steel

- Usually has a silicon level of 3%.
- Optimum properties are developed in the rolling direction, due to a tight control of the crystal orientation relative to the sheet.
- The magnetic flux density is increased by 30% in the coil rolling direction, although its magnetic saturation is decreased by 5%.
- Used for the cores of high-efficiency transformers, electric motor and generators.

Non-oriented electrical steel

- Usually has a silicon level of 2 to 3.5%.
- Similar magnetic properties in all directions, which makes it isotropic.
- Less expensive. Used in applications where the direction of magnetic flux changes.
- It is also used when efficiency is less important or when there is insufficient space to correctly orient components to take advantage of the anisotropic properties of grain-oriented electrical steel.

ELECTRICAL STEEL

Quality	Nominal thickness (mm)	Maximum specific total loss ($W\,kg^{-1}$) at peak induction		Minimum magnetic flux density (T) in direct or alternating field at field strength		
		1.5 T	1.0 T	2500 Am^{-1}	5000 Am^{-1}	10 000 Am^{-1}
250-35-A5	0.35	2.50	1.00	1.49	1.60	1.71
270-35-A5	0.35	2.70	1.10	1.49	1.60	1.71
300-35-A5	0.35	3.00	1.20	1.49	1.60	1.71
330-35-A5	0.35	3.30	1.30	1.49	1.60	1.71
270-50-A5	0.50	2.70	1.10	1.49	1.60	1.71
290-50-A5	0.50	2.90	1.15	1.49	1.60	1.71
310-50-A5	0.50	3.10	1.25	1.49	1.60	1.71
330-50-A5	0.50	3.30	1.35	1.49	1.60	1.71
350-50-A5	0.50	3.50	1.50	1.50	1.60	1.71
400-50-A5	0.50	4.00	1.70	1.51	1.61	1.72
470-50-A5	0.50	4.70	2.00	1.52	1.62	1.73
530-50-A5	0.50	5.30	2.30	1.54	1.64	1.75
600-50-A5	0.50	6.00	2.60	1.55	1.65	1.76
700-50-A5	0.50	7.00	3.00	1.58	1.68	1.76
800-50-A5	0.50	8.00	3.60	1.58	1.68	1.78
350-65-A5	0.65	3.50	1.50	1.49	1.60	1.71
400-65-A5	0.65	4.00	1.70	1.50	1.60	1.71
470-65-A5	0.65	4.70	2.00	1.51	1.61	1.72
530-65-A5	0.65	5.30	2.30	1.52	1.62	1.73
600-65-A5	0.65	6.00	2.60	1.54	1.64	1.75
700-65-A5	0.65	7.00	3.00	1.55	1.65	1.76
800-65-A5	0.65	8.00	3.60	1.58	1.68	1.76
1000-65-A5	0.65	10.00	4.40	1.58	1.68	1.78

Standard IEC specification for nonoriented magnetic steel sheet

ELECTRICAL STEEL

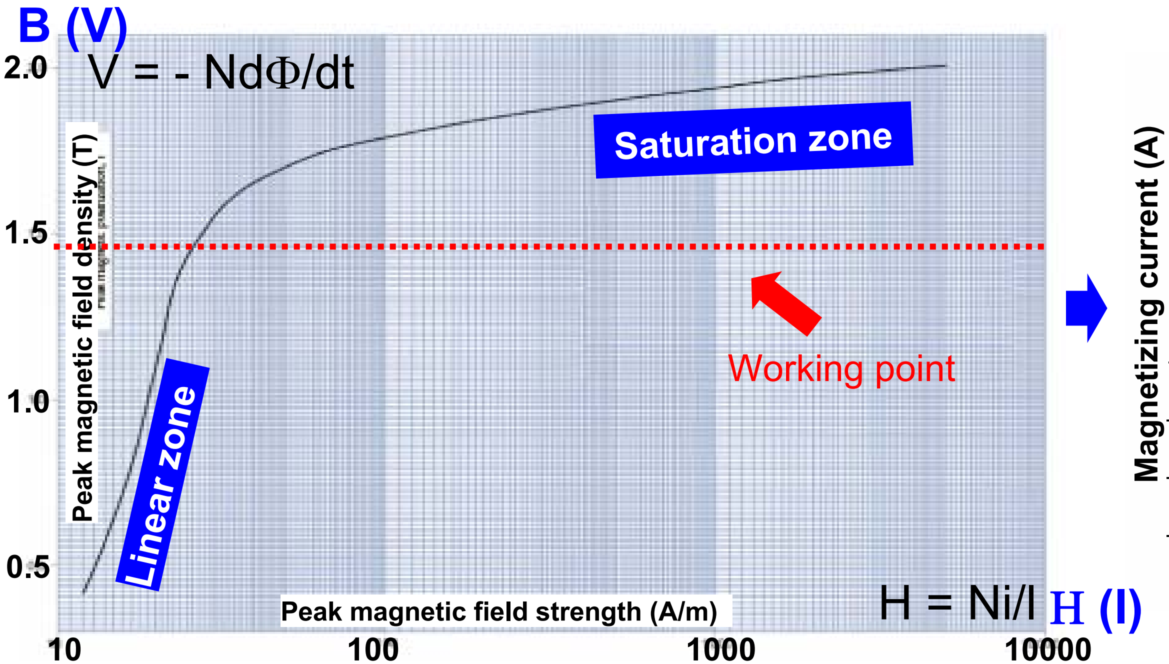
a) Grade	Thickness (mm)	Maximum specific total loss ($W\,kg^{-1}$) at peak induction 1.5 T	1.7 T	Minimum magnetic flux density (T) for $H = 800\ Am^{-1}$	Minimum stacking factor
089-27-N 5	0.27	0.89	1.40	1.75	0.950
097-30-N 5	0.30	0.97	1.50	1.75	0.955
111-35-N 5	0.35	1.11	1.65	1.75	0.960

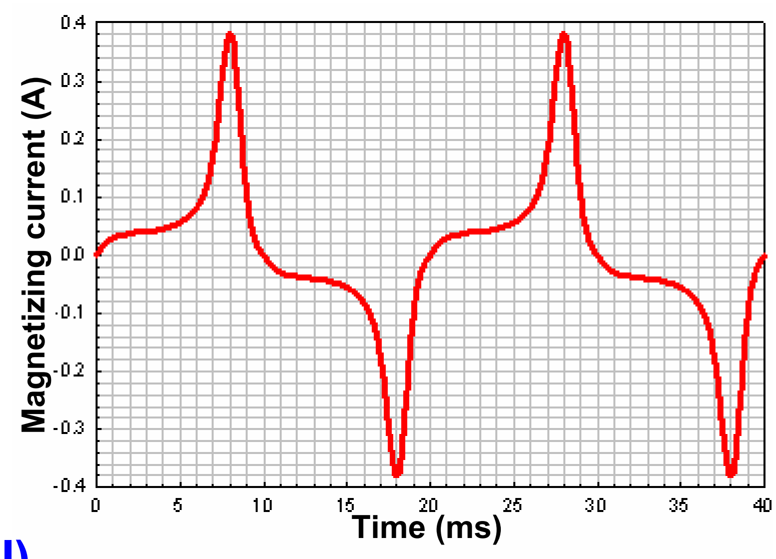
b) Grade	Thickness (mm)	Maximum specific total loss ($W\,kg^{-1}$) at 1.7 T peak induction	Minimum magnetic flux density (T) for $H = 800\ Am^{-1}$	Minimum stacking factor
130-27-S 5	0.27	1.30	1.78	0.950
140-30-S 5	0.30	1.40	1.78	0.955
155-35-S 5	0.35	1.55	1.78	0.960

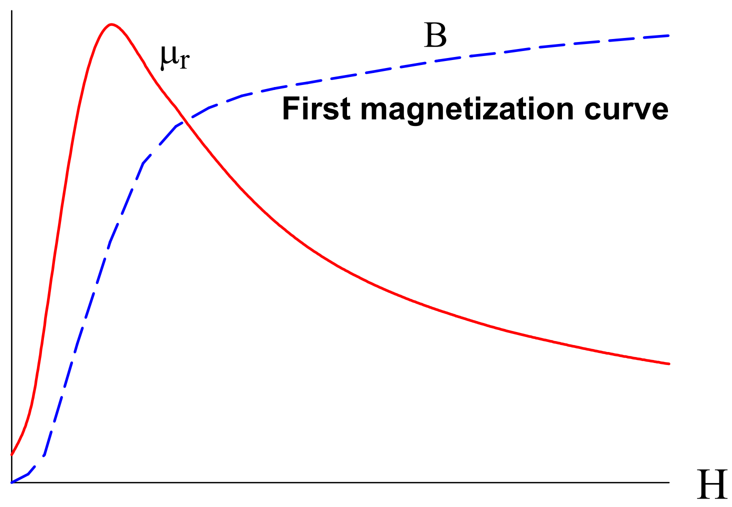
c) Grade	Thickness (mm)	Maximum specific total loss ($W\,kg^{-1}$) at 1.7 T peak induction	Minimum magnetic flux density (T) for $H = 800\ Am^{-1}$	Minimum stacking factor
111-30-P 5	0.30	1.11	1.85	0.955
117-30-P 5	0.30	1.17	1.85	0.955
125-35-P 5	0.35	1.25	1.85	0.960
135-35-P 5	0.35	1.35	1.85	0.960

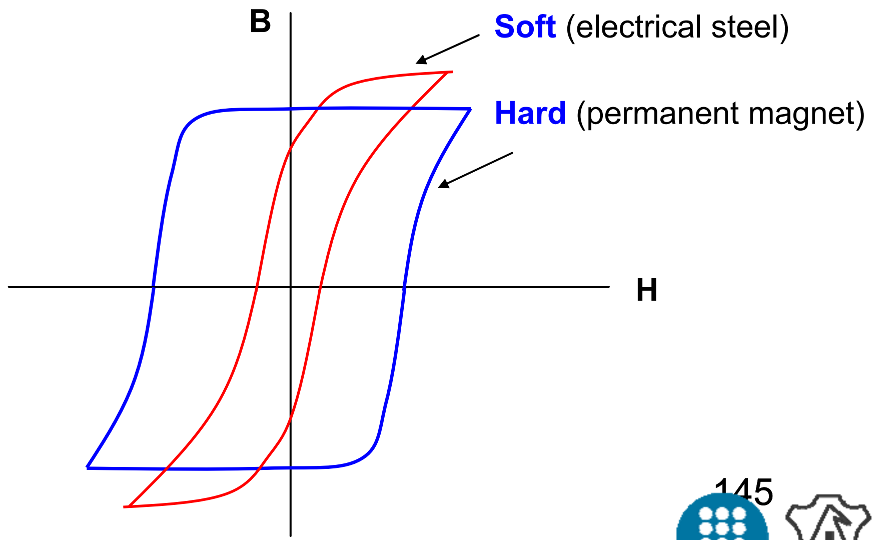
Standard IEC speci.cation for grain-oriented magnetic steel sheet. (a) normal material; (b) material with reduced loss; (c) high-permeability material.

ELECTRICAL STEEL



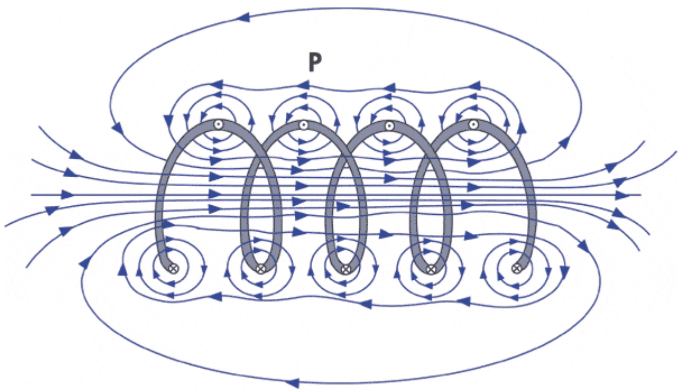


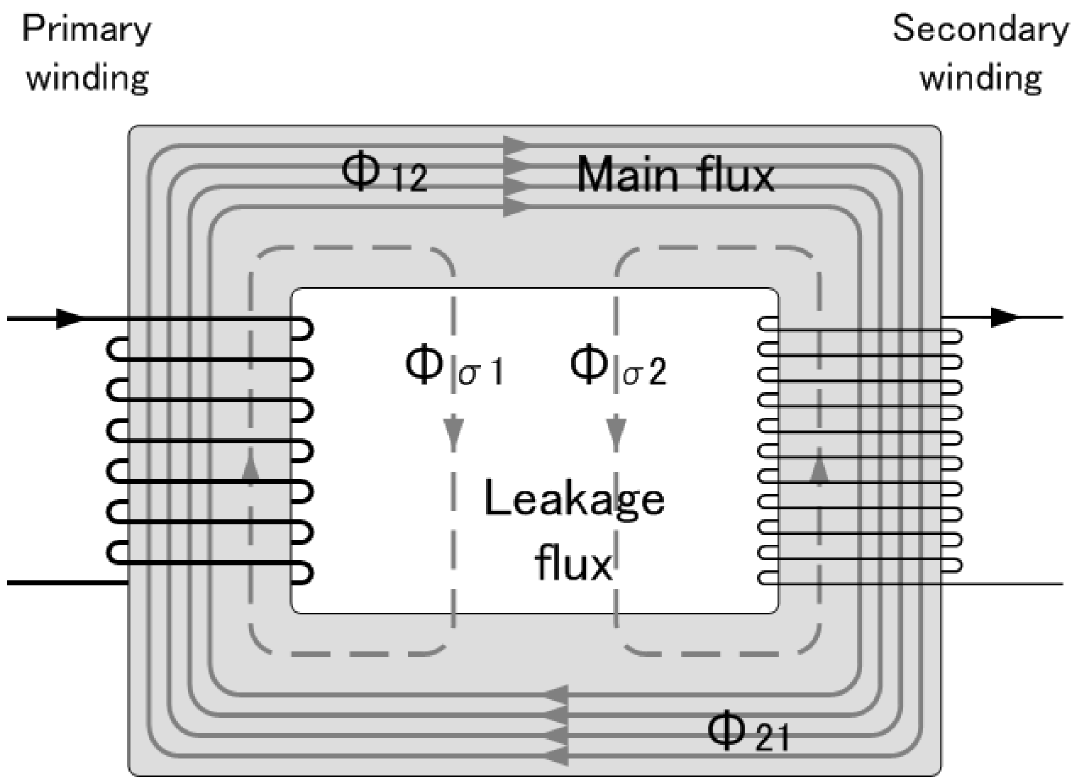




LEAKAGE FLUX

In a practical transformer, some part of the flux linking primary winding does not link the secondary.

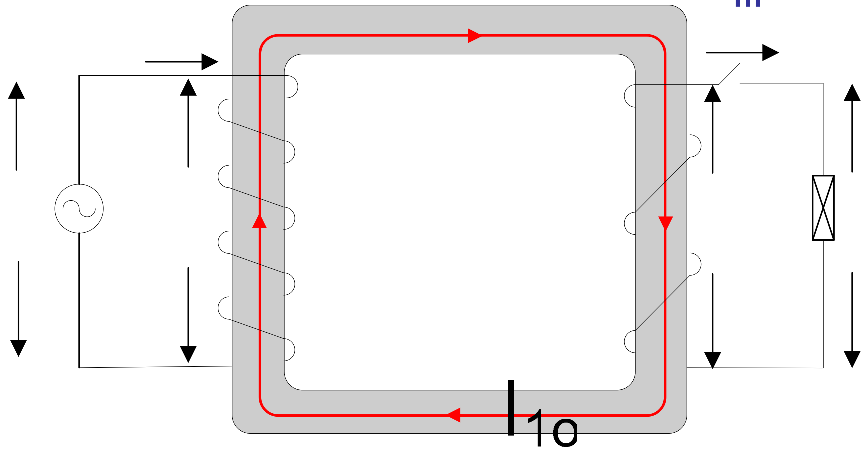


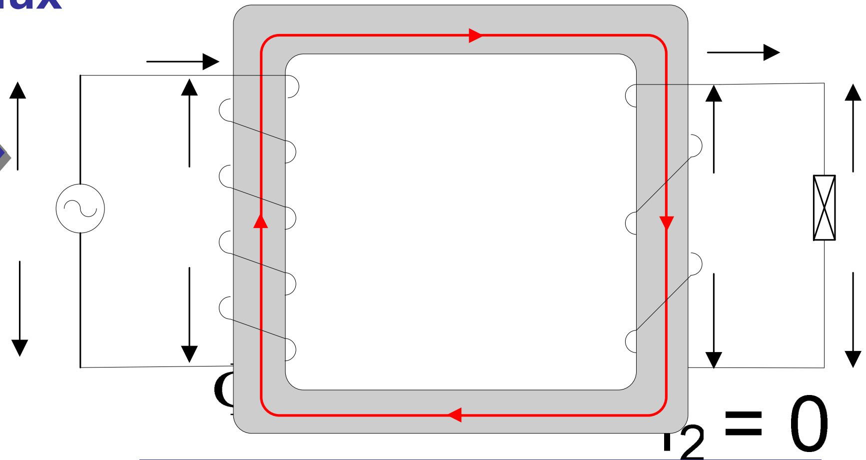


THE IDEAL TRANSFORMER

The ideal transformer has not energy loss

Φ_m: mutual flux



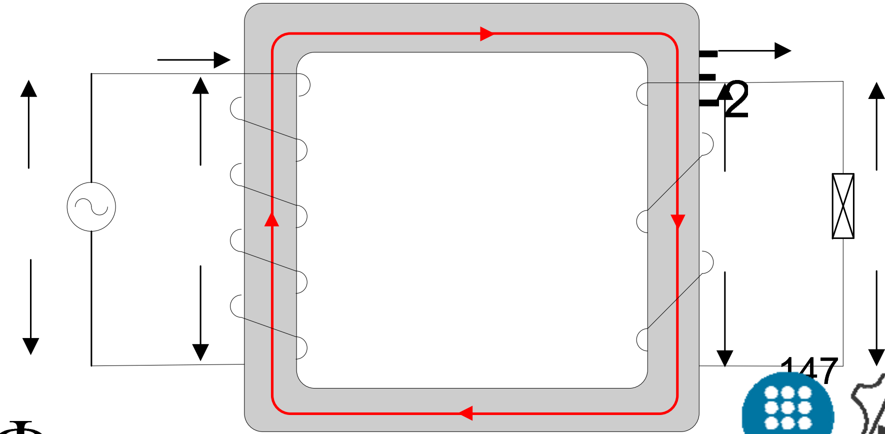


$$\Phi_1 = \Phi_2 = \Phi_m$$

$$e_1 = N_1 \cdot \frac{d\Phi_m}{dt} \qquad e_2 = N_2 \cdot \frac{d\Phi_m}{dt}$$

$$S_1 = S_2 \rightarrow E_1 \cdot I_1 = E_2 \cdot I_2 \rightarrow \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Under **load conditions** Φ_m results unaltered because E_1 is imposed by the mains



$$\begin{cases} E_1 = r_t \cdot E_2 \\ I_1 = I_2 / r_t \end{cases}$$

N_1 Φ_m N_2

QUANTITIES REFERED TO THE PRIMARY/SECONDARY SIDE

- Quantities referred to the **primary side**:

$$V_2' = V_2 \cdot r_t$$

$$I_2' = I_2 / r_t$$

$$Z_2' = V_2'/I_2' = Z_1 \cdot r_t^2$$

$$S_2' = V_2' I_2' = V_1 I_1 = S_1$$

r_t = turns ratio

- Quantities referred to the **secondary side**:

$$V_1' = V_2 / r_t$$

$$I_1' = I_2 \cdot r_t$$

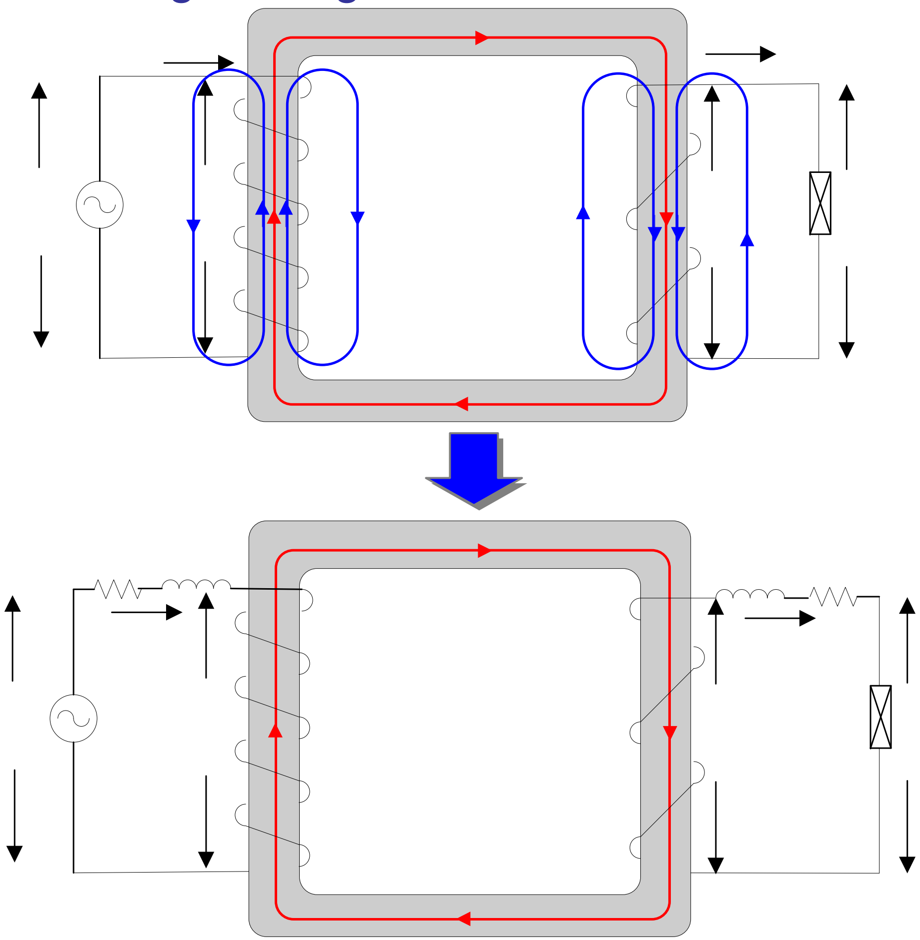
$$Z_1' = V_1'/I_1' = Z_2 / r_t^2$$

$$S_1' = V_1' I_1' = V_2 I_2 = S_2$$

THE PRACTICAL TRANSFORMER

The practical transformer experiments power loss

- R_1, R_2: resistances of windings 1 and 2, respectively.
- X_1, X_2: inductive reactances due to leakage flux
- R_m, X_m: resistance of iron and magnetizing reactance

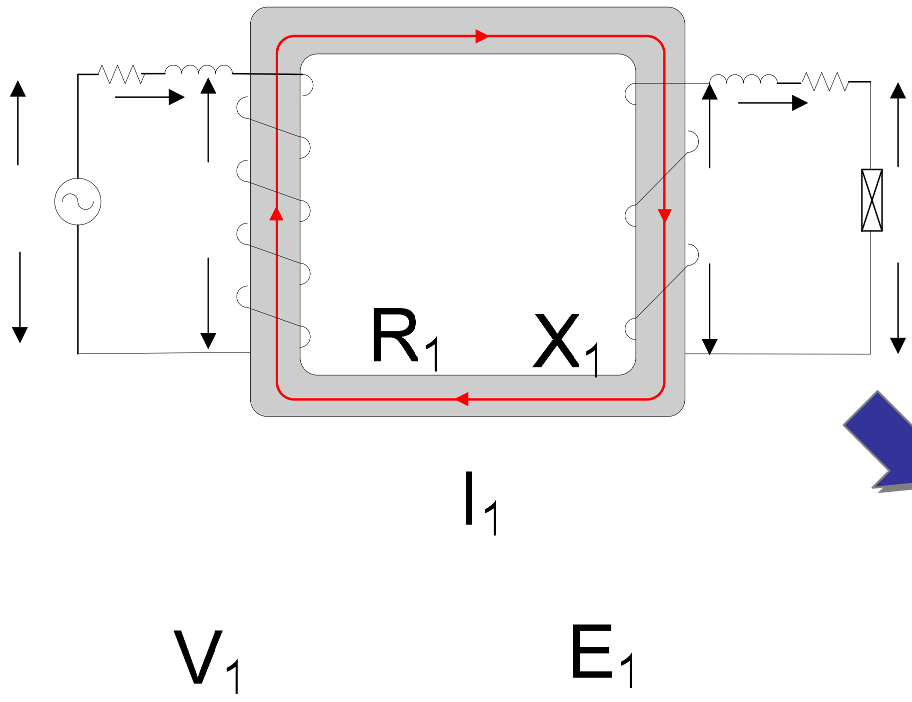


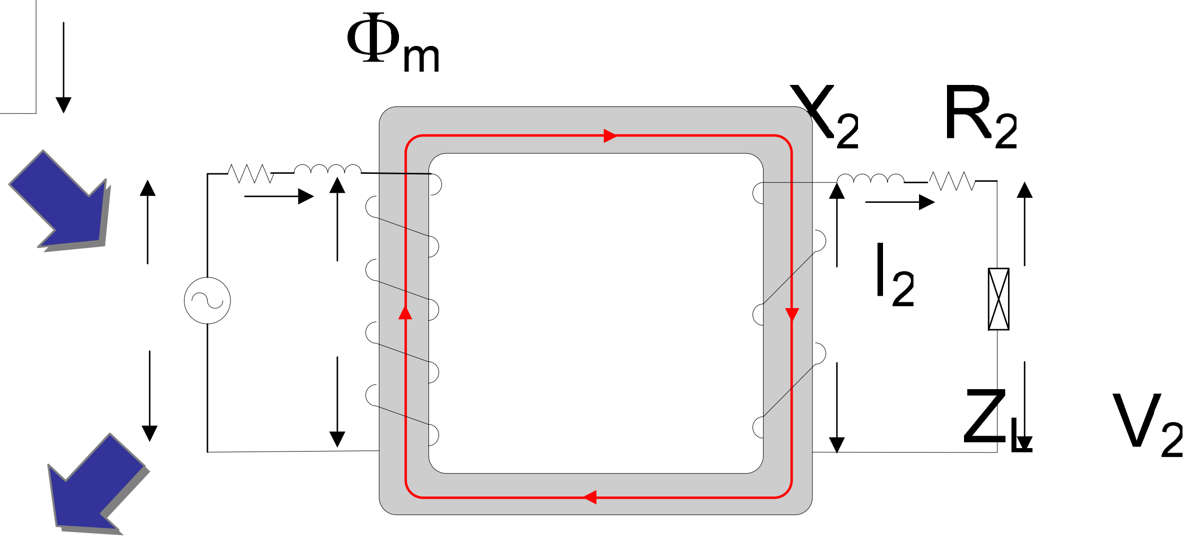
I_1

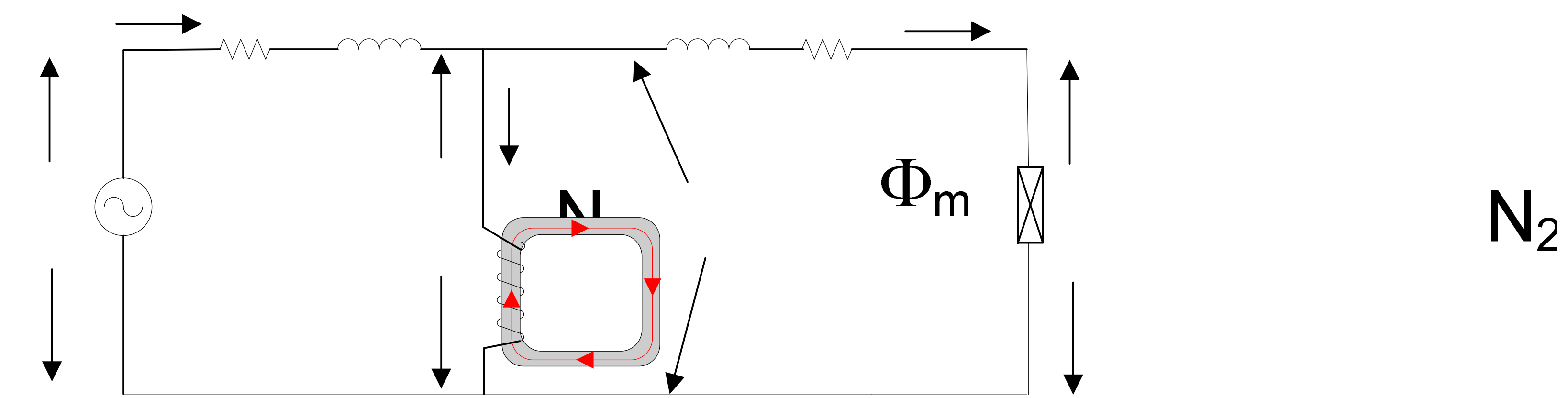
V_1 E_1

THE PRACTICAL TRANSFORMER

- Two electrically isolated circuits
- Tight magnetic coupling

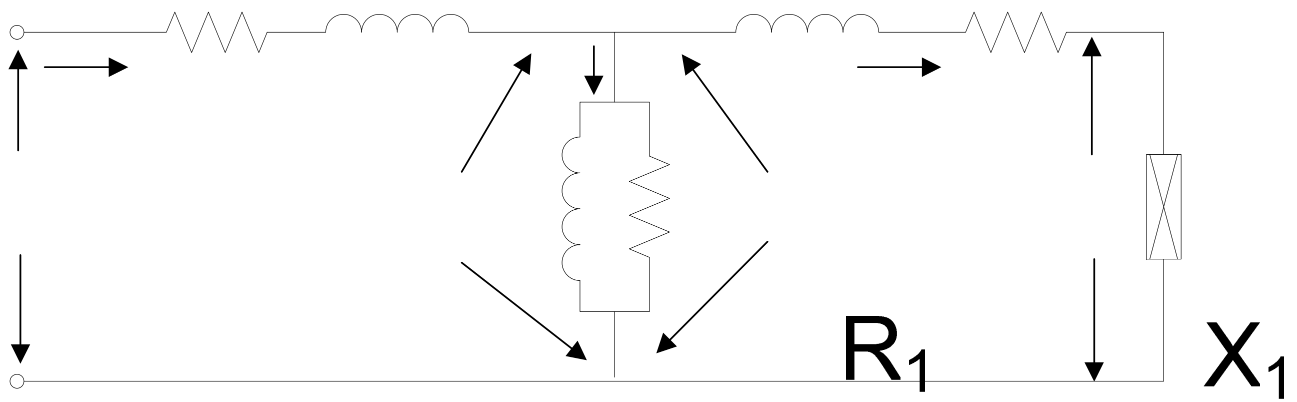






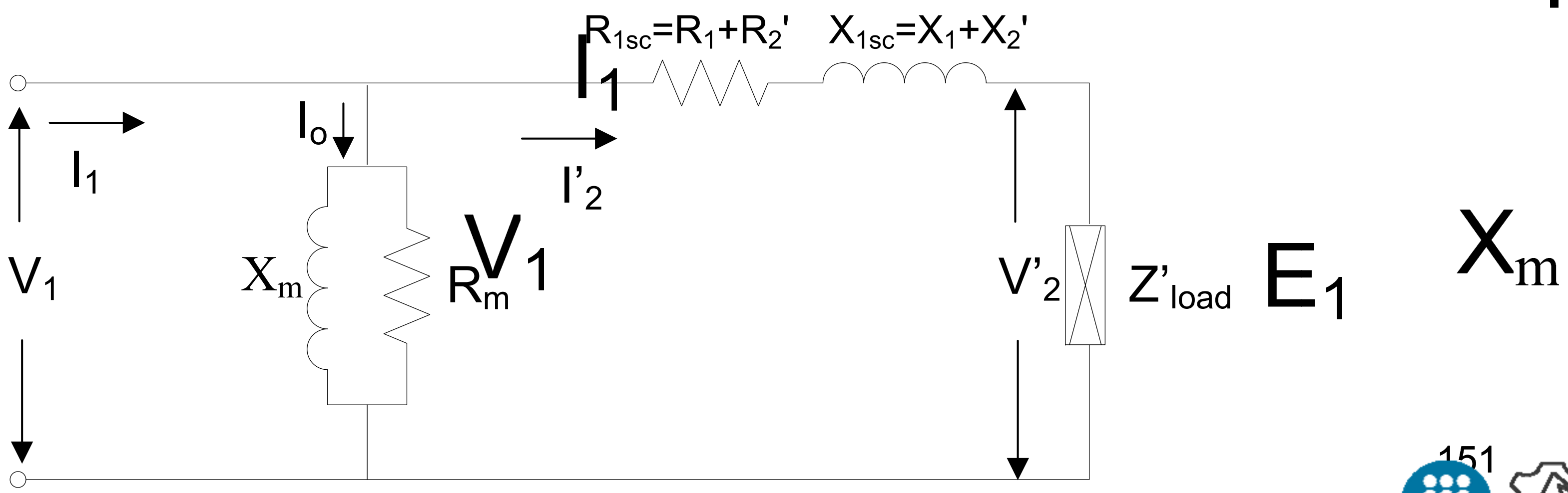
THE PRACTICAL TRANSFORMER. EQUIVALENT CIRCUIT

Exact equivalent circuit



Simplified equivalent circuit (R_m >> R_1,R_2' X_m >> X_{L1},X_{L2}')

I_o



STANDARD TESTS

Two standard tests to measure power loss and the parameters of the equivalent circuit:

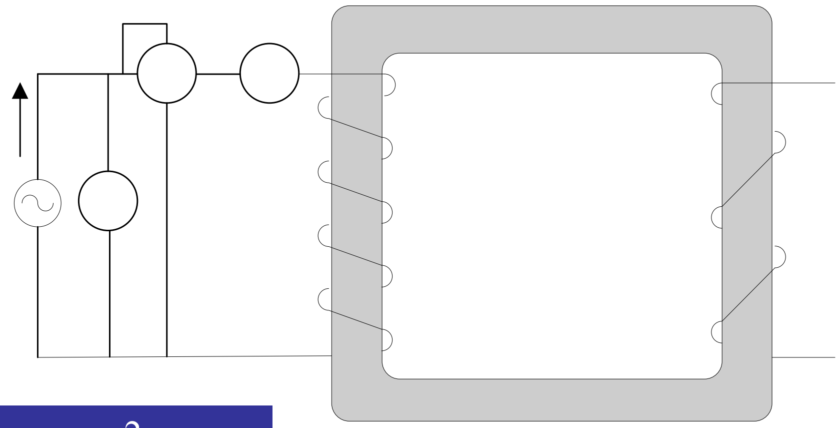
- **STANDARD NO LOAD TEST**
- **STANDARD SHORT-CIRCUIT TEST**

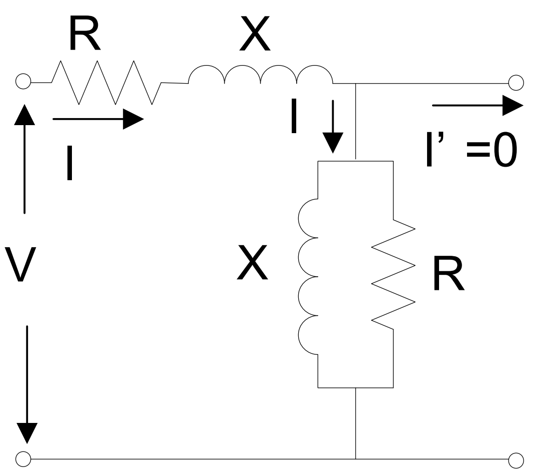
In both tests the following variables are measured:

- Applied voltage
- Input current
- Input electrical power

STANDARD NO LOAD TEST (Low current, rated voltage)

- No load conditions = **open circuit** $I_0 << I_{Full\text{-}load}$
- The rated voltage is applied to one winding and other winding is kept open (usually LV winding is supplied, while HV is kept open).
- P_o, $V_{1o} = V_{1n}$ and I_{1o} are measured
- Q_o is deduced from P_o





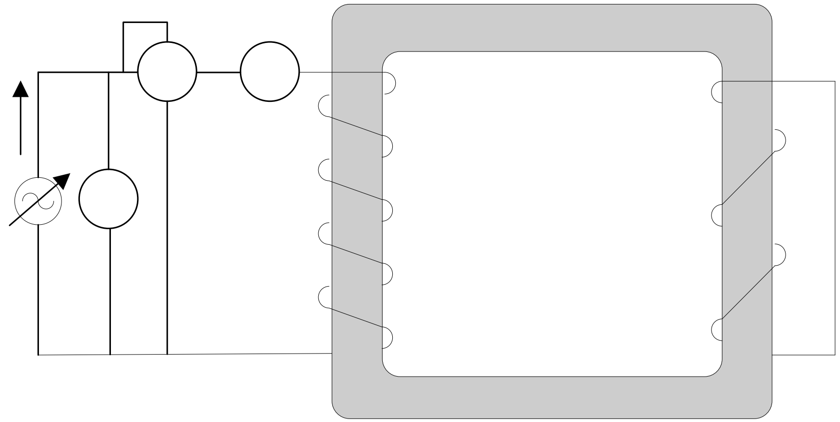
$$R_m = \frac{V_{1O}^2}{P_O - \cancel{I_{1O}^2 R_1}} \approx \frac{V_{1O}^2}{P_O}$$

$$X_m = \frac{V_{1O}^2}{Q_O - \cancel{I_{1O}^2 X_1}} \approx \frac{V_{1O}^2}{P_O \operatorname{tg}\varphi_O}$$

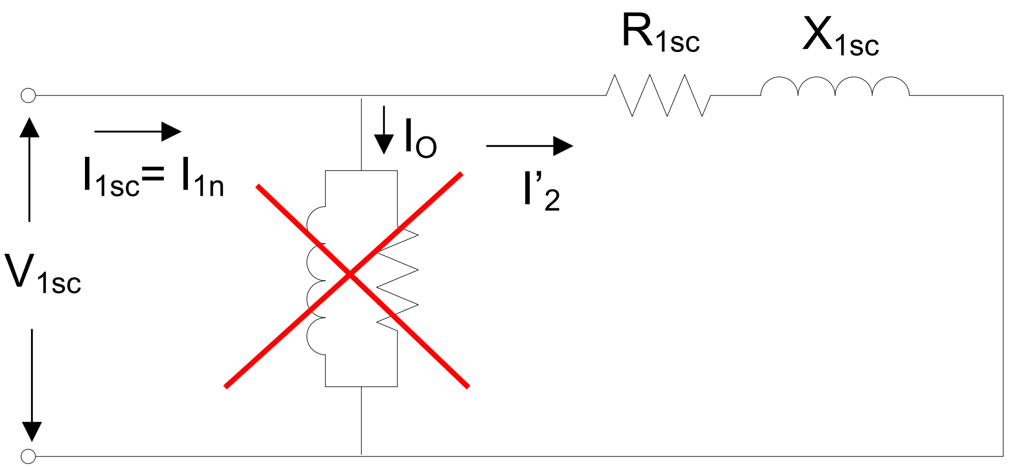
$$P_{iron} = \left(\frac{V_1}{V_{1n}}\right)^2 P_O$$

STANDARD SHORT-CIRCUIT TEST (Low voltage, rated current)

- $I_1 = I_{1rated} = I_{1n}$
- $I_O << I_1 \rightarrow I_2' = I_1 = I_{1n}$
- Secondary winding short-circuited.
- Primary winding is fed with an increasing variable voltage, starting from 0 volt, such that $I_1 = I_{1n}$



- P_{sc}, V_{1sc} and $I_{1sc} = I_{1n}$ are measured (Q_{sc} is deduced)



$$R_{1sc} = \frac{P_{sc} - \cancel{V_{1sc}^2/R_m}}{I_{1n}^2} \approx \frac{P_{sc}}{I_{1n}^2}$$

$$X_{1sc} \approx \frac{Q_{sc}}{I_{1n}^2} = \frac{P_{sc}\,tg\varphi_{sc}}{I_{1n}^2}$$

$$P_{Cu} = \left(\frac{I_1}{I_{1n}}\right)^2 P_{sc} = c^2 P_{sc}$$

EFFICIENCY

- The efficiency of a transformer is defined as the ratio of useful output power to input power

$$\eta = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_2}{P_1} = \frac{V_2 I_2 \cos\varphi}{V_2 I_2 \cos\varphi + P_{iron} + P_{Cu}} = \frac{V_2 I_2 \cos\varphi}{V_2 I_2 \cos\varphi + (V_1/V_{1n})^2 P_O + c^2 P_{sc}}$$

Being c the **load index**, defined as: $c = \frac{I}{I_n}$

The efficiency can be rewritten as:

$$\eta = \frac{c V_2 I_{2n} \cos\varphi}{c V_2 I_{2n} \cos\varphi + P_{iron} + c^2 P_{sc}} = \frac{V_2 I_{2n} \cos\varphi}{V_2 I_{2n} \cos\varphi + P_{iron}/c + c P_{sc}}$$

Maximizing the efficiency with respect to c it results:

$$c_{\eta max} = \sqrt{\frac{P_{iron}}{P_{sc}}}$$

VOLTAGE DROP IN THE WINDINGS

Per-unit drop in the winding resistances: $$\varepsilon_{Rsc} = \frac{I_{1n}R_{1sc}}{V_{1n}} = \frac{I_{2n}R_{2sc}}{V_{2n}}$$

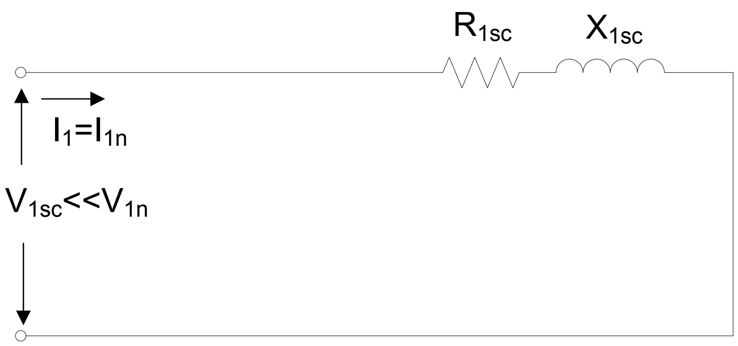
Per-unit drop in the winding reactances: $$\varepsilon_{Xsc} = \frac{I_{1n}X_{1sc}}{V_{1n}} = \frac{I_{2n}X_{2sc}}{V_{2n}}$$

Per-unit drop in the winding impedances: $$\varepsilon_{sc} = \frac{I_{1n}Z_{1sc}}{V_{1n}} = \frac{I_{2n}Z_{2sc}}{V_{2n}}$$

- ε_{sc} increases with the power of the transformer to limit the short circuit current
- Distribution transformers (S_n < 1000 kVA): $\varepsilon_{sc} \approx$ 1-6%
- Power transformers: $\varepsilon_{sc} \approx$ 6-13%.

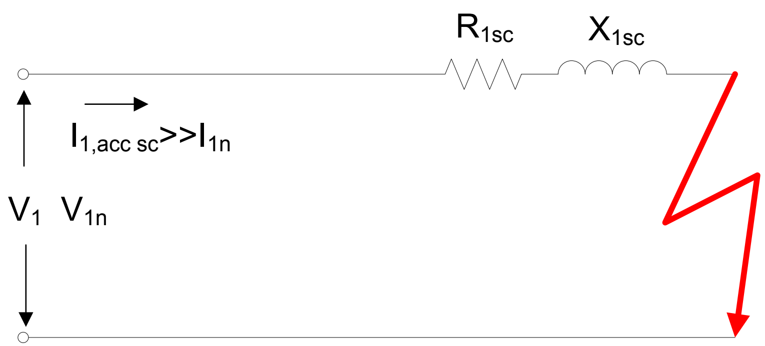
SHORT CIRCUIT CURRENT

Standard short-circuit test



$$I_{1n} = \frac{V_{1sc}}{Z_{1sc}}$$

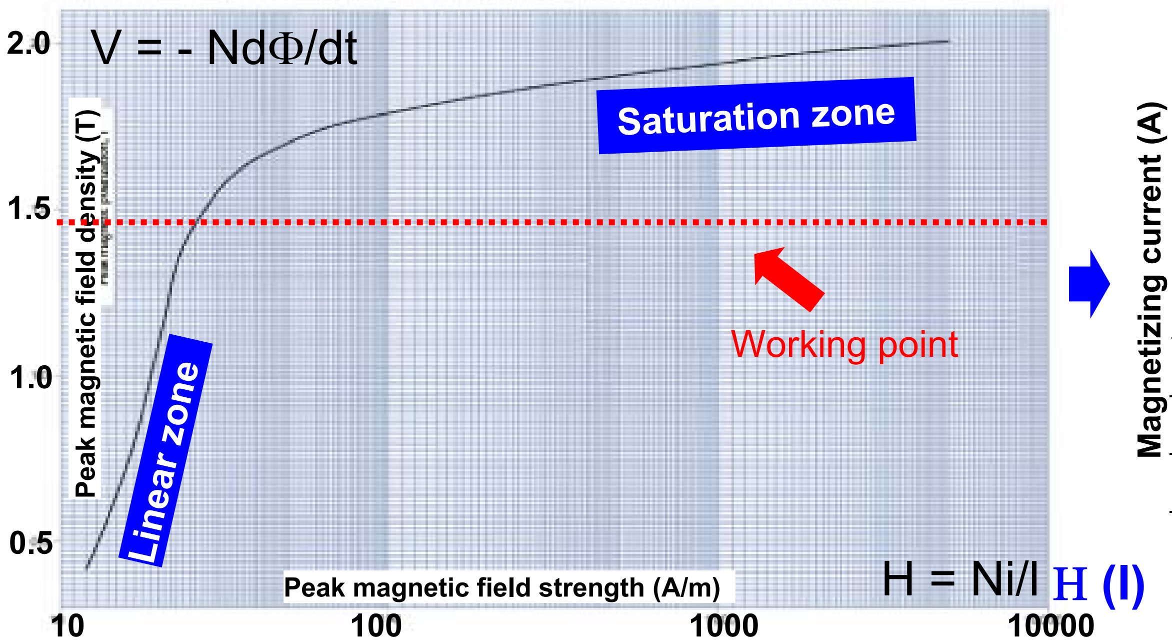
Accidental short-circuit

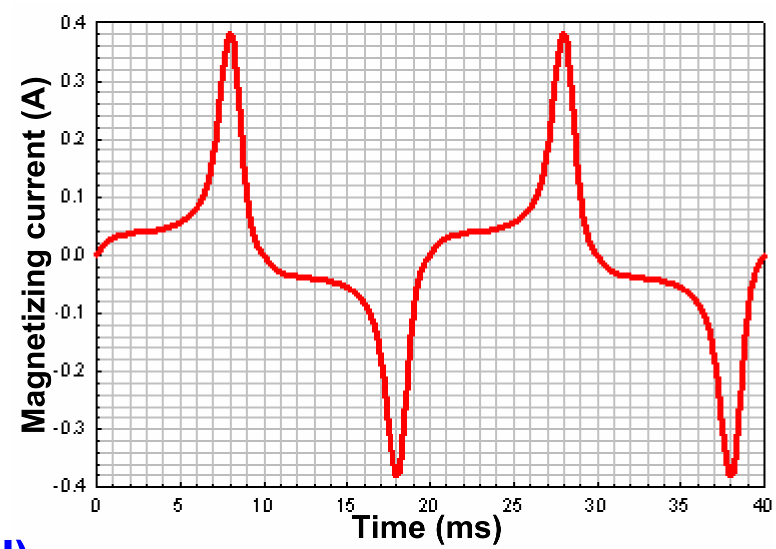


$$I_{1\,acc\,sc} = \frac{V_{1n}}{Z_{1sc}} = \frac{V_{1n}}{\varepsilon_{sc} V_{1n}/I_{1n}} = \frac{I_{1n}}{\varepsilon_{sc}}$$

For usual values of ε_{sc} (5-10%) it results $I_{1acc\,sc}$ = 10-20 I_{1n}

NO LOAD CURRENT





- V is sinusoidal (imposed by the power supply)
- The relationship between V and I is not linear
- Thus, the current in the transformer under no-load conditions is not perfectly sinusoidal and has some harmonic content (specially 3rd harmonic)
- It generates harmonic distortion in line voltages (harmonics causes voltages drops in lines and loads)
- Solution: working point under the saturation threshold by increasing the core section, but Cost increase!

PARALLEL CONNECTED TRANSFORMERS

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Operating conditions that suggest paralleling of transformers:

- The load has grown beyond the capacity of the existing transformer kVA rating.
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- Stiffen an existing bus. It can be accomplished through the parallel operation of transformers.

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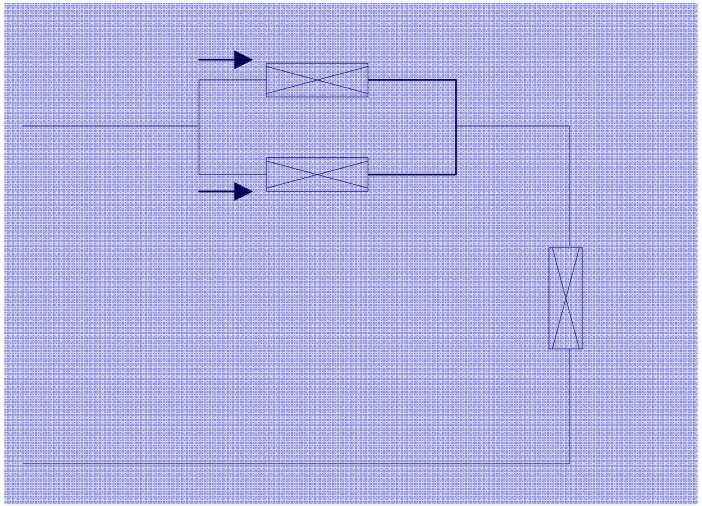
CONDITIONS FOR CONNECTING 2 TRANSFORMERS A AND B IN PARALLEL

1. Same turns-ratio: $r_{tA} = r_{tB}$ ($V_{1n,A} = V_{1n,B}$ and $V_{2n,A} = V_{2n,B}$)

2. Similar ε_{sc}: $\varepsilon_{scA} \approx \varepsilon_{scB}$ ($c_A\varepsilon_{scA} = c_B\varepsilon_{scB}$)

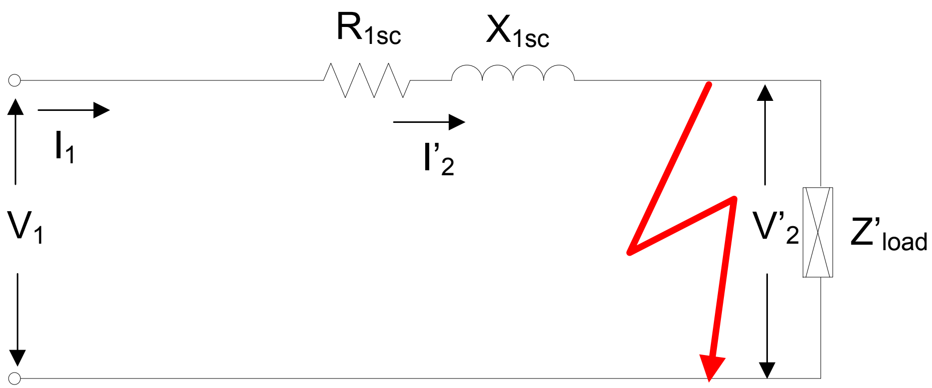


$$I_{2A}Z_{2scA} = I_{2B}Z_{2scB} \rightarrow c_A I_{2An}Z_{2scA} = c_B I_{2Bn}Z_{2scB}$$

$$c_A \frac{I_{2An}Z_{2scA}}{V_{2n}} = c_B \frac{I_{2Bn}Z_{2scB}}{V_{2n}} \rightarrow c_A\varepsilon_{scA} = c_B\varepsilon_{scB}$$

EXAMPLE 1

A 230/6-V, single-phase transformer is tested with its secondary winding short circuited. a) A low voltage (20V) is applied to the primary terminals and it then takes an input current of 1 A; the power supplied being 10 watts. Calculate the values of R_{1sc}, X_{1sc}, R_{2sc} and X_{2sc}. b) If the magnetizing impedance is neglected, calculate the secondary terminal voltage when a $Z_{load} = 0.12 + j0.09\ \Omega$ is connected.



a) $r_t = V_{1n}/V_{2n} = 230/6 = 38.33$

$V_{1sc} = 20 = 1.Z_{1sc} \rightarrow Z_{1sc} = 20\ \Omega$

$P_{sc} = 10 = 1^2.R_{1sc} \rightarrow R_{1sc} = 10\ \Omega$

$X_{1sc} = 17.32\ \Omega$

$R_{2sc} = R_{1sc}/r_t^2 = 0.0068\ \Omega$

$X_{2sc} = X_{1sc}/r_t^2 = 0.0118\ \Omega$

b) $\overline{Z}_{total} = \overline{Z}_{1sc} + \overline{Z'}_{load} = 10 + j17.32 + (0.12 + j0.09)\, r_t^2 = 238.94^{38.75^\circ}\ \Omega$

$V_1 = I_1 Z_{total} \rightarrow 230 = I_1 238.94 \rightarrow I_1 = I_2' = 0.963\ \text{A} \rightarrow V'_2 = I_2' \cdot Z'_{load} = 212.15\ \text{V}$

$V_2 = V'_2/r_t = 5.53\ \text{V}$

EXAMPLE 2

A 44kVA 2200/220-V, single-phase transformer takes an input power of 176 W when feed at 2200V and with its secondary winding in open circuit. Values ε_{Rsc}=1.4% and ε_{Xsc}=2.1% are known. Calculate the primary terminal voltage (V_1) and the power taken by the transformer when a 30 kW load with PF=0.8 (i) is connected at 220 V.

From the statement it results: I_{1n} = 20 A I_{2n} = 200 A r_t=10

$$\varepsilon_{Rsc} = \frac{I_{1n} R_{1sc}}{V_{1n}} \rightarrow R_{1sc} = 1.54\Omega \qquad \varepsilon_{Xsc} = \frac{I_{1n} X_{1sc}}{V_{1n}} \rightarrow X_{1sc} = 2.31\Omega$$

$$\overline{Z}_{1sc} = R_{1sc} + jX_{1sc} = 1.54 + j2.31 = 2.78^{56.31^\circ}\,\Omega$$

$$I_2 = P_2/(V_2 \cos\varphi_2) = 170.45\text{A} \qquad I_2' = I_2/r_t = 17.045\text{A} \quad \cos\varphi_2 = 0.8 \rightarrow \varphi_2 = 36.87^\circ$$

$$\overline{V}_1 = \overline{V}_2' + \overline{I}_2'\overline{Z}_{1sc} = 2200^{0^\circ} + 17.045^{-36.87^\circ} 2.78^{56.31^\circ} = 2244.74^{0.40^\circ}\,\text{V}$$

$$P_1 = P_2 + P_{iron} + P_{Cu} = 30000 + 176\left(\frac{2244.74}{2200}\right)^2 + 17.045^2 1.54 = 30630.65\text{W}$$

$$\eta_{\%} = 100\frac{P_2}{P_1} = 100\frac{30000}{30630.65} = 97.94\%$$

EXAMPLE 3

A 10kVA 220/380-V, single-phase transformer takes an input power of 150 W and a current of 2 A when feed at 220V and with its secondary winding in open circuit. From the short circuit test it results 10V, 26.32A and 75W (HV side). a) Calculate R_{1sc}, X_{1sc}, R_{1m} and X_{1m}. b) Being V_1=220V calculate V_2 when the transformer operates at full load and FP_{load} = 0.8(i).

From the statement it results: $I_{1n} = 45.45$ A $\quad I_{2n} = 26.32$ A $\quad r_t = 0.57895$

a) No-load test: $150 = 220^2/R_{1m} \rightarrow R_{1m} = 322.67\ \Omega$

$$150 = 220 \cdot 2 \cdot \cos\varphi_0 \rightarrow \varphi_0 = 70.06^\circ \rightarrow Q_0 = P_0 . tg\varphi_0 = 413.64 \text{ Var} = 220^2/X_{1m}$$

It results $X_{1m} = 117.02\ \Omega$

Short-circuit test: $V_{2sc} = I_{2n} \cdot Z_{2sc} \rightarrow 10 = 26.32 \cdot Z_{2sc} \rightarrow Z_{2sc} = 0.38\ \Omega$

$$P_{sc} = I_{2n}{}^2 \cdot R_{2sc} \rightarrow 75 = 26.32^2 \cdot R_{2sc} \rightarrow R_{2sc} = 0.108\ \Omega$$

□ $$X_{2sc} = (Z_{2sc}{}^2 - R_{2sc}{}^2)^{1/2} = 0.364\ \Omega$$

$$\overline{Z}_{1sc} = r_t^2 (R_{2sc} + jX_{2sc}) = 0.036 + j0.122 = 0.127^{73.44^\circ}\ \Omega$$

□b) $\overline{V}_1 = \overline{V}_2' + \overline{I}_2' \overline{Z}_{1sc} \rightarrow 220^{\varphi_{V1}} = V_2'^{0^\circ} + 45.45^{-36.87^\circ} 0.127^{73,44^\circ}$

□ It results: $\varphi_{V1} = 0.895^\circ$ $\quad V_2' = 215.34$ V $\rightarrow V_2 = V_2'/r_t = 371.9$ V

EXAMPLE 4

Single-phase transformer with features: 10kVA, 3300/220-V, 50 Hz, R_1 = 5Ω, R_2= 0.02 Ω. Short-circuit test: V_{1sc} = 165 V, I_{1sc} = 3.03 A. Determine: a) Z_{1sc} and Z_{2sc} b) Voltage applied in the HV side when the LV side feeds a pure resistive load which consumes 7 kW at 220 V.

From the statement it results: $I_{1n} = 3.03$ A $I_{2n} = 45.45$ A $r_t = 15$ $R_{1sc} = R_1 + r_t^2 \cdot R_2 = 9.5\ \Omega$

a) **Short-circuit test:** $165 = 3.03 \cdot Z_{1sc} \rightarrow Z_{1sc} = 54.45\ \Omega$

$$X_{1sc} = \sqrt{Z_{1sc}^{\ 2} - R_{1sc}^{\ 2}} = 53.61\,\Omega$$

$$\overline{Z}_{1sc} = 9.5 + j \cdot 53.61 = 54.45^{79.95^\circ}\ \Omega \quad \rightarrow \quad \overline{Z}_{2sc} = \overline{Z}_{1sc}/r_t^{\ 2} = 0.242^{79.95^\circ}\ \Omega$$

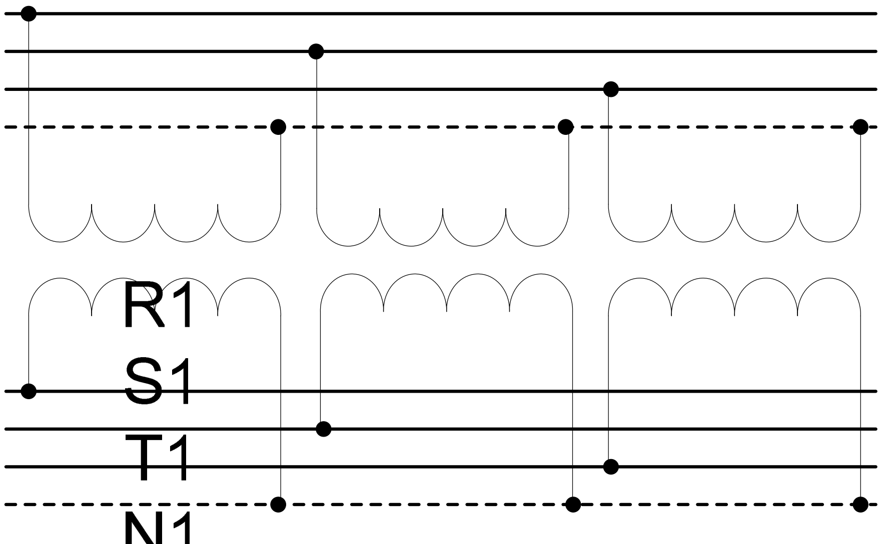
b) **The transformer operates under a load** rated: 7 kW, 220 V and PF = 1

$$P_{load} = V_2^2/R_{load} \rightarrow 7000 = 220^2/R_{load} \rightarrow R_{load} = 6.914\ \Omega$$

$$V_2 = I_2 \cdot R_{load} \rightarrow 220 = I_2 \cdot 6.914 \rightarrow I_2 = 31.82\ \text{A}$$

$$\overline{V}_1 = \overline{V}_2' + \overline{I}_2' \overline{Z}_{1sc} \rightarrow \overline{V}_1 = (220 \cdot 15)^{0^\circ} + (31.82/15)^{0^\circ} \cdot 54.45^{79.95^\circ} = 3322.12^{1.96^\circ}\ \text{V}$$

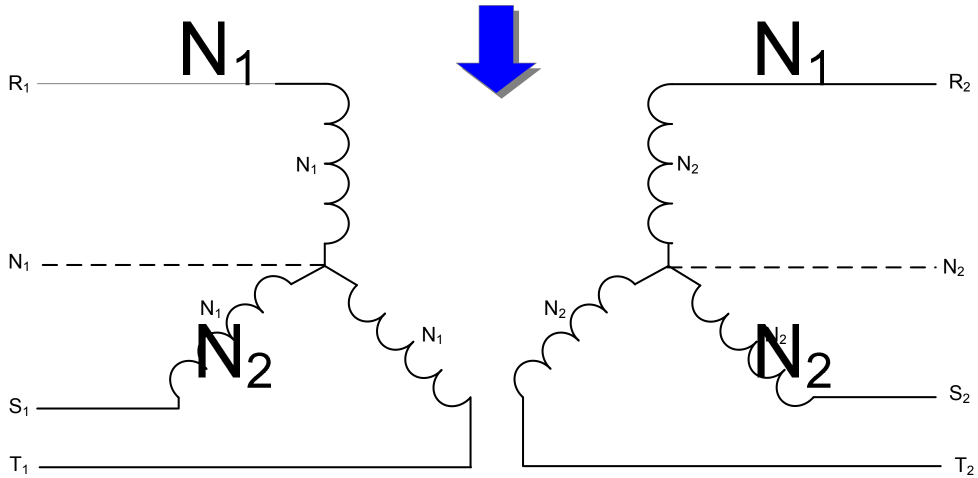
THREE-PHASE BANK OF SINGLE-PHASE TRANSFORMERS



Three-phase bank of single-phase transformers

N_1

The most elemental three-phase transformer consists on an arrangement made of 3 single-phase transformers



Drawbacks of 3-phase banks:

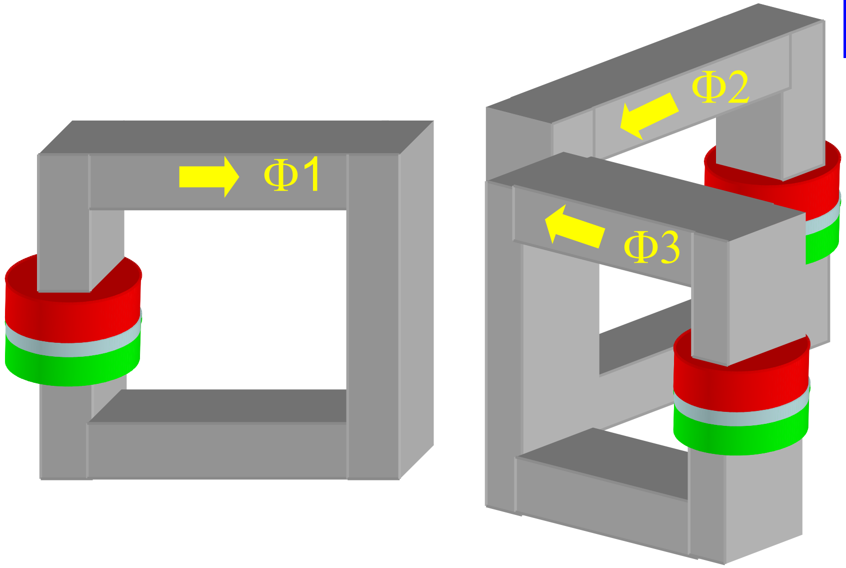
- **Three units are needed**
- **Much space is required**
- **Much iron weight is required**

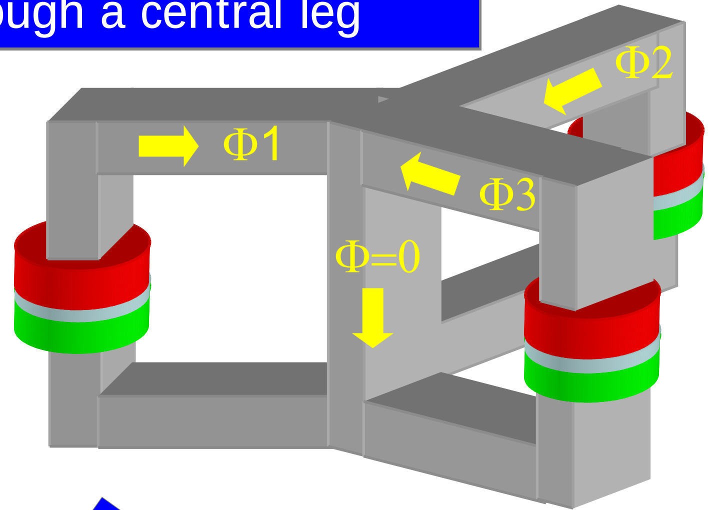
R2
S2
T2
N2

THREE-PHASE TRANSFORMERS

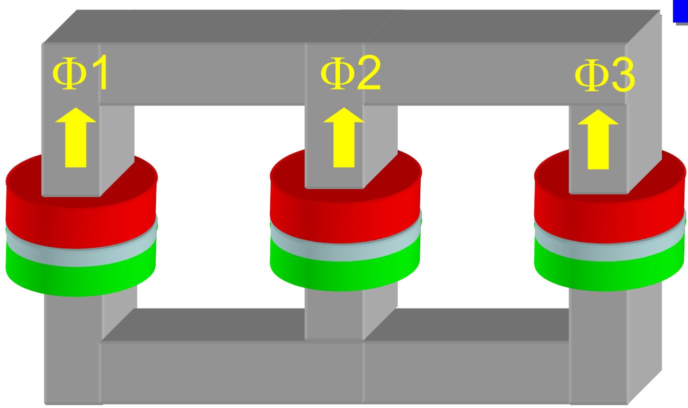
$\Phi_1+\Phi_2+\Phi_3 = 0$

The 3 legs can be joined through a central leg





The central leg can be removed because the flux is null

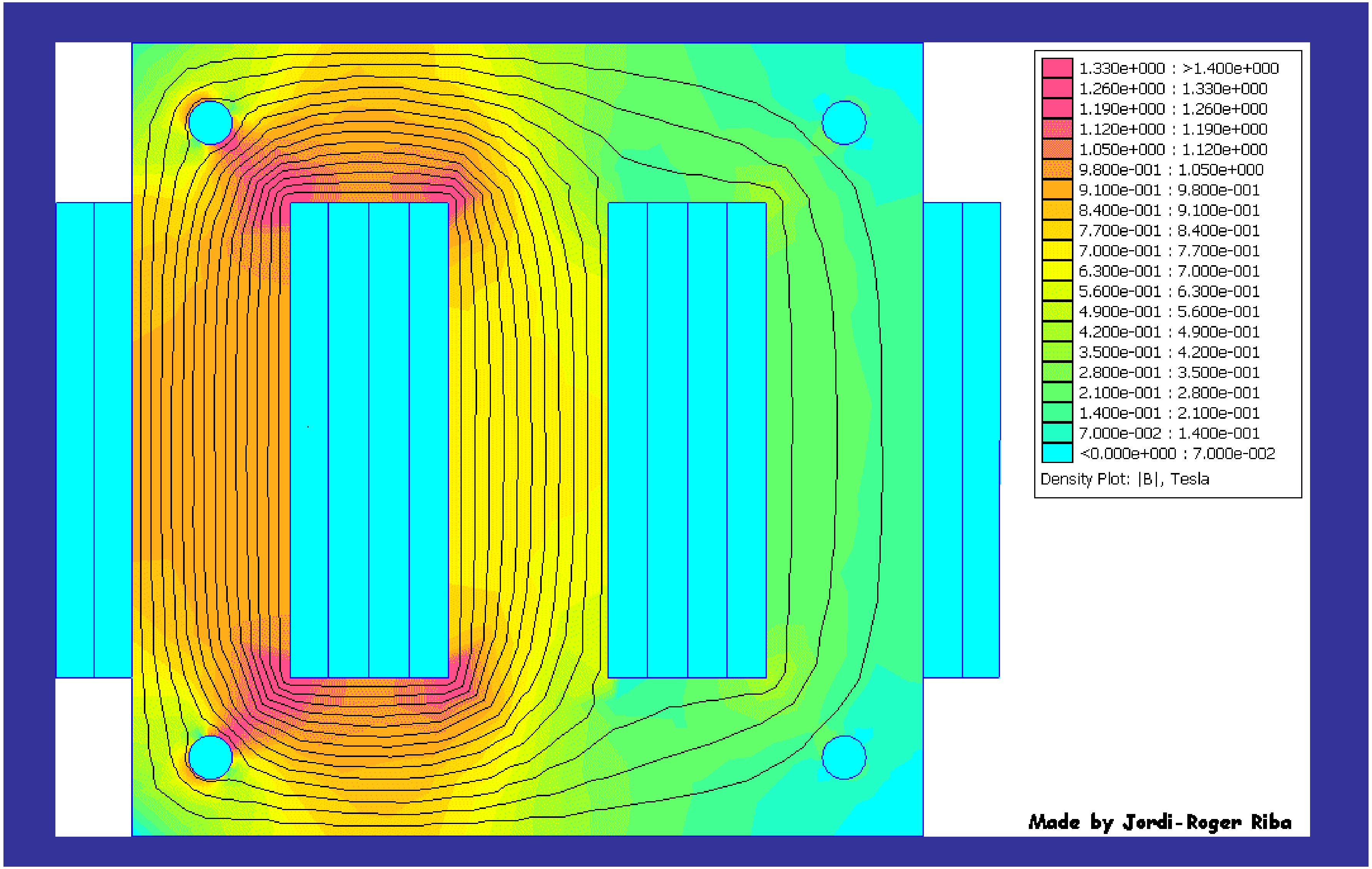


By removing the central leg, material, weight and space is saved

Each leg can be considered as a single-phase transformer

Small asymmetry of the no load currents. Not significant when load

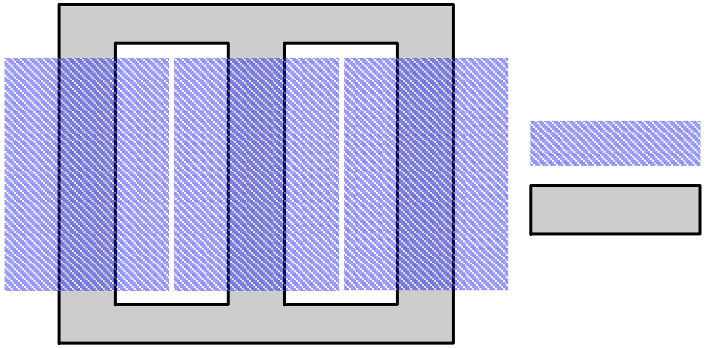
MAGNETIC FLUX



CORE DESIGNS

Core-type designs (de columnes)

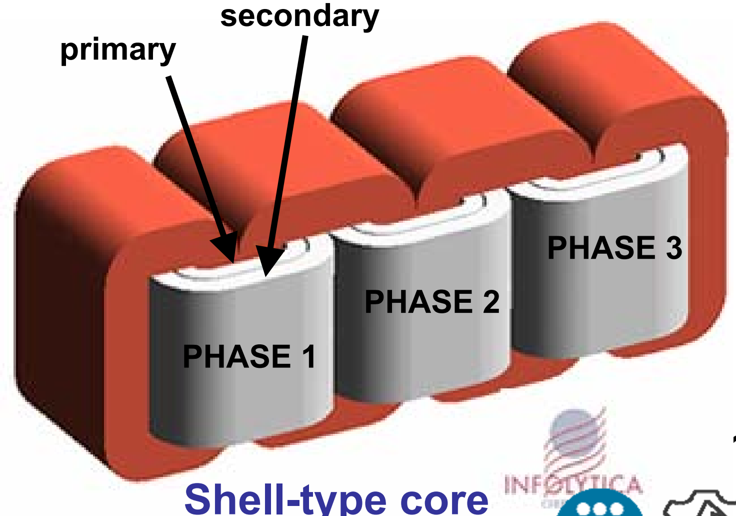
- Predominate throughout most of the world
- The coil surrounds the core



Core-type

Shell-type designs (acuirassat)

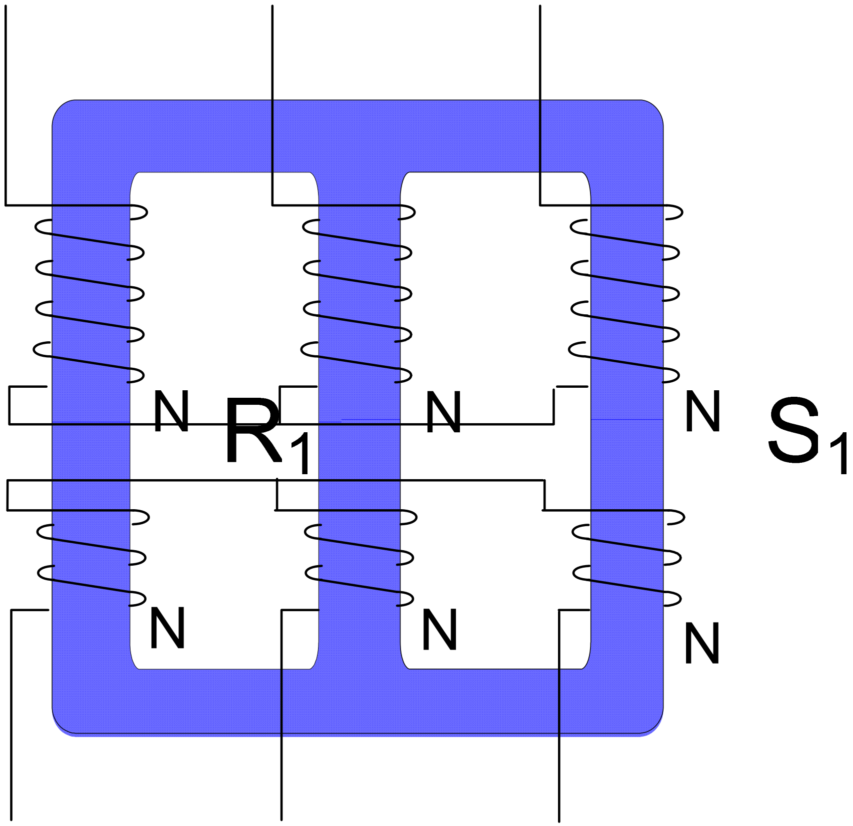
- The core surrounds the coil
- The flux-return paths of the core are external to and enclose the windings.
- This design provides better magnetic shielding → suitable for supplying power at low voltage and heavy current, as, for example, in the case of arc furnace transformers.



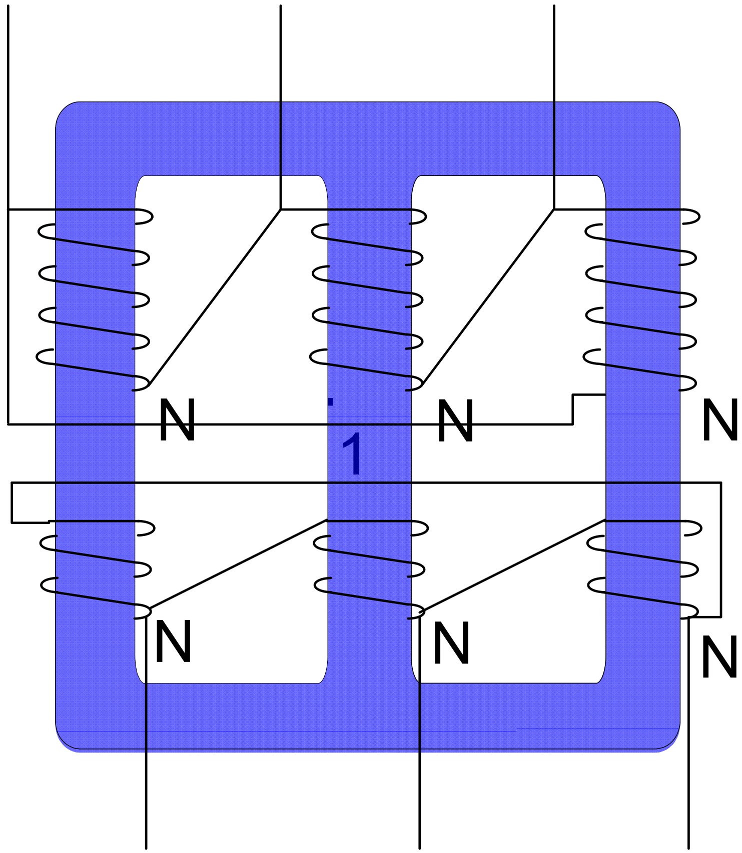
Shell-type core

THREE-PHASE CONNECTIONS

Wye-wye connection: Yy



Delta-delta connection: Dd



THREE-PHASE CONNECTIONS

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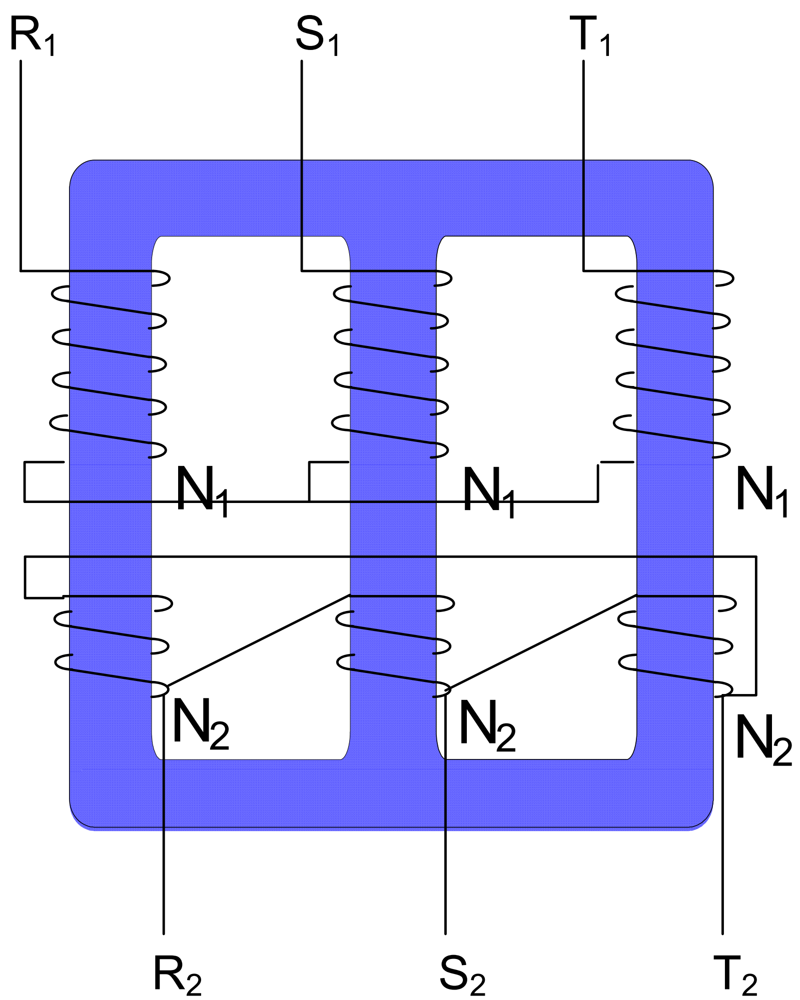
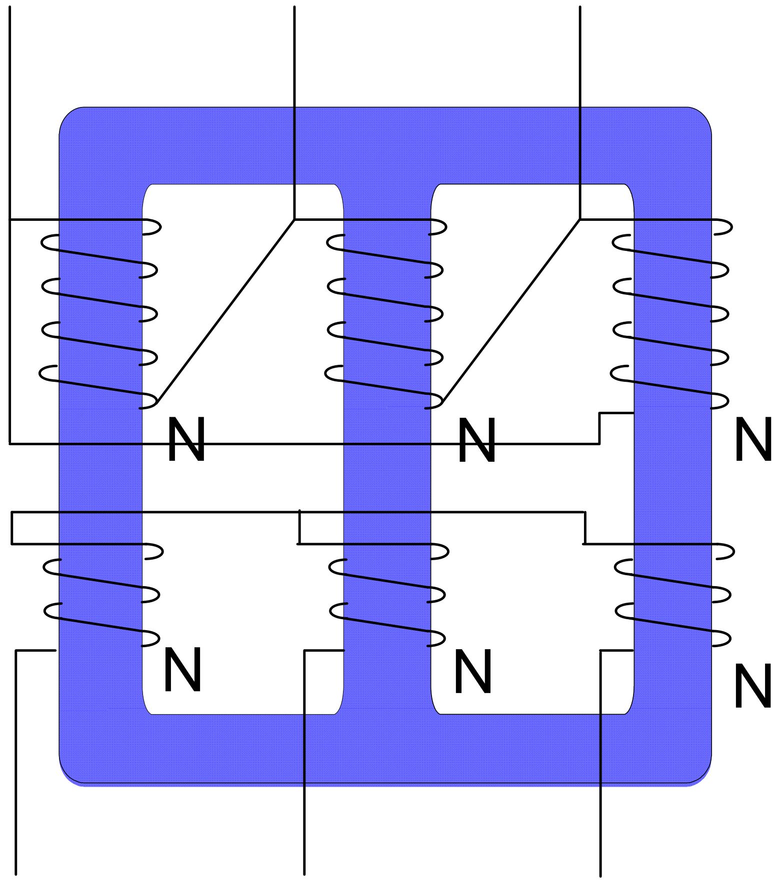
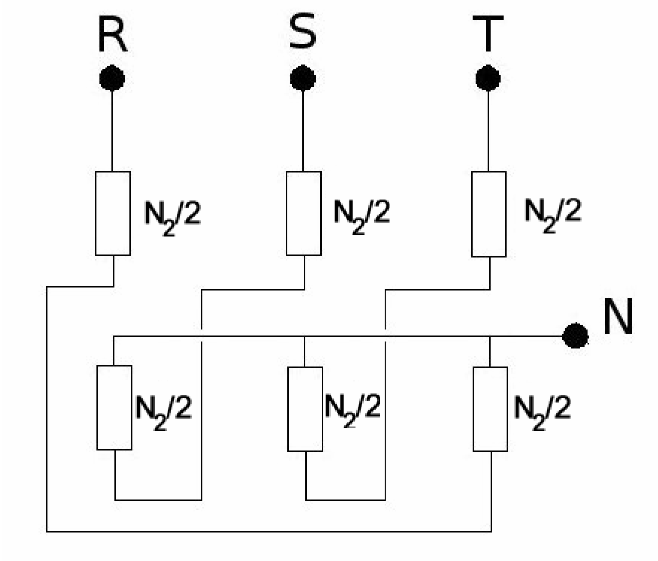


Delta-wye connection: Dy



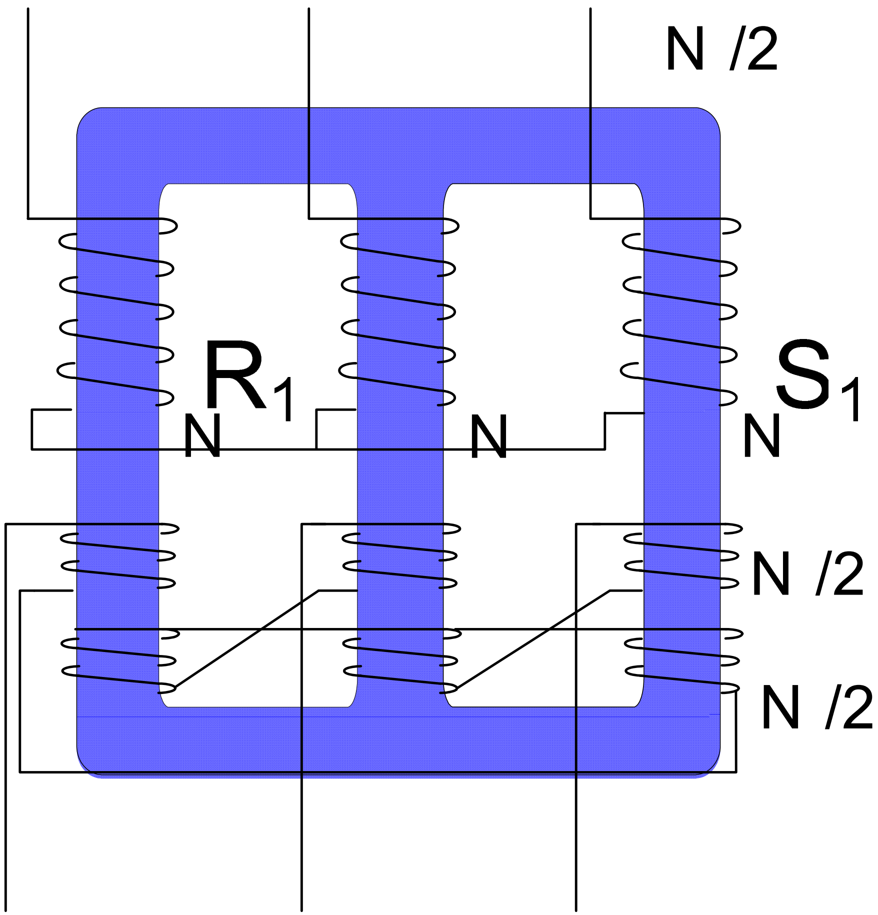
ZIGZAG OR INTERCONNECTED-STAR

- The interconnected-star (Zigzag) connection is obtained by subdividing the transformer windings into halves and then interconnecting these between phases.
- The interconnected-star arrangement is used to provide a neutral for connection to earth.
- The zigzag connection in power systems allows trapping multiples of third harmonics currents (3rd, 9th, 15th, etc.).
- Zigzag units are installed near loads that produce large third harmonic currents, preventing them from traveling upstream.
- The zigzag winding produces the same angular displacement as a delta winding, and at the same time provides a neutral for earthing purposes

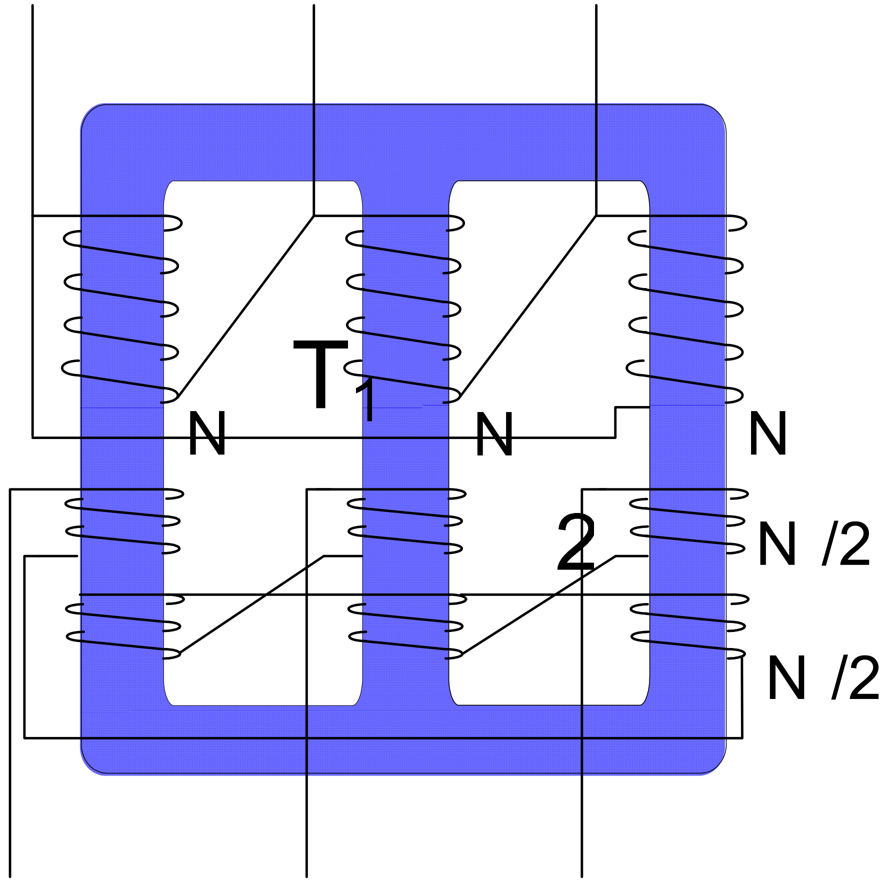


THREE-PHASE CONNECTIONS

Wye-zigzag connection: Yz



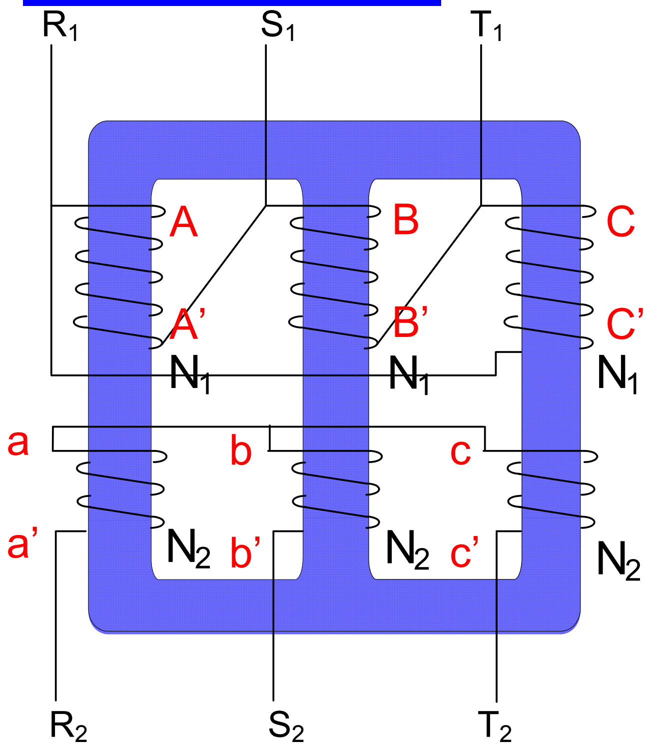
Delta-zigzag connection: Dz

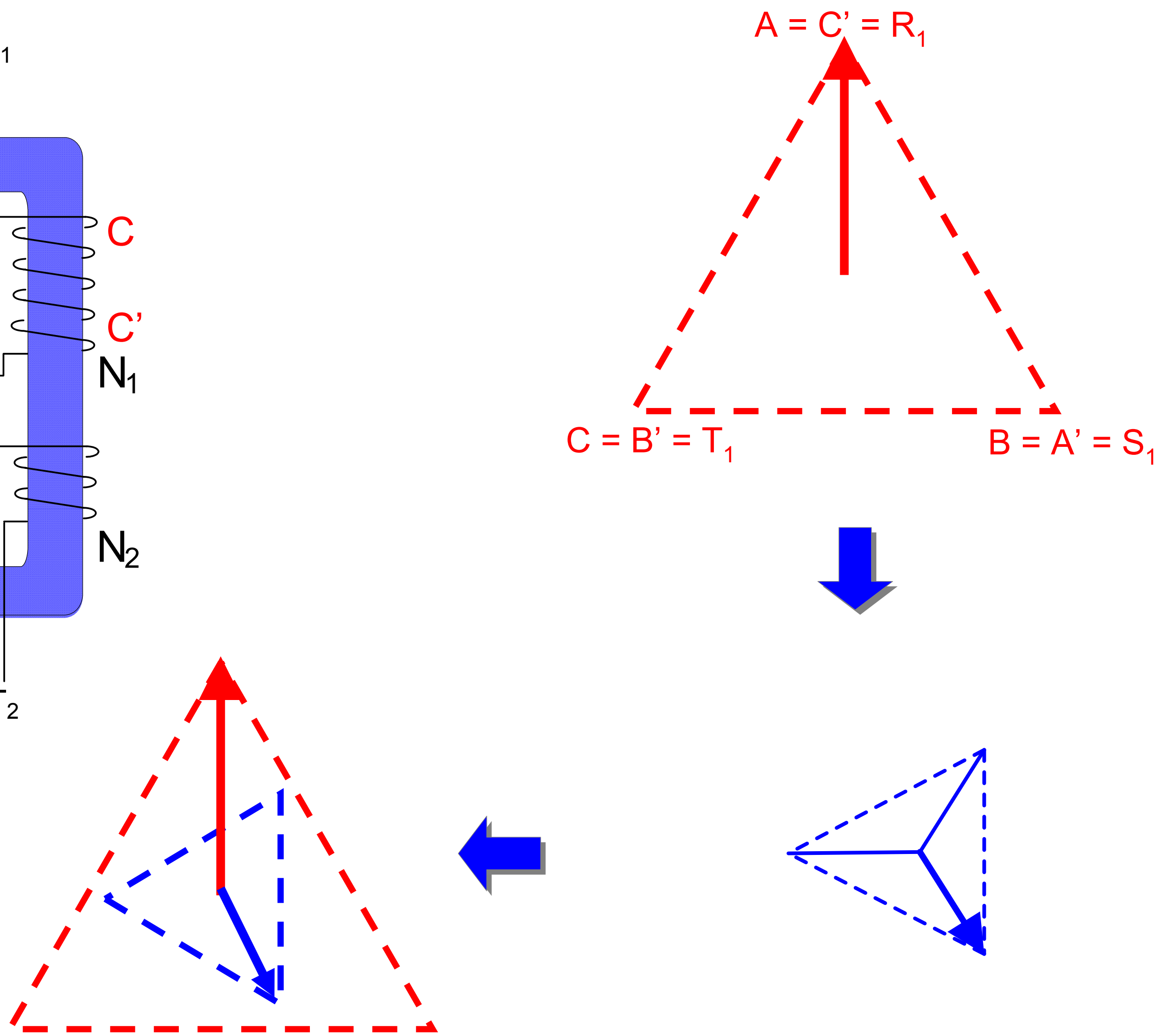


Zigzag = star-interconnected

VECTOR/PHASOR GROUP (angular displacement between primary and secondary)

Connection Dy?



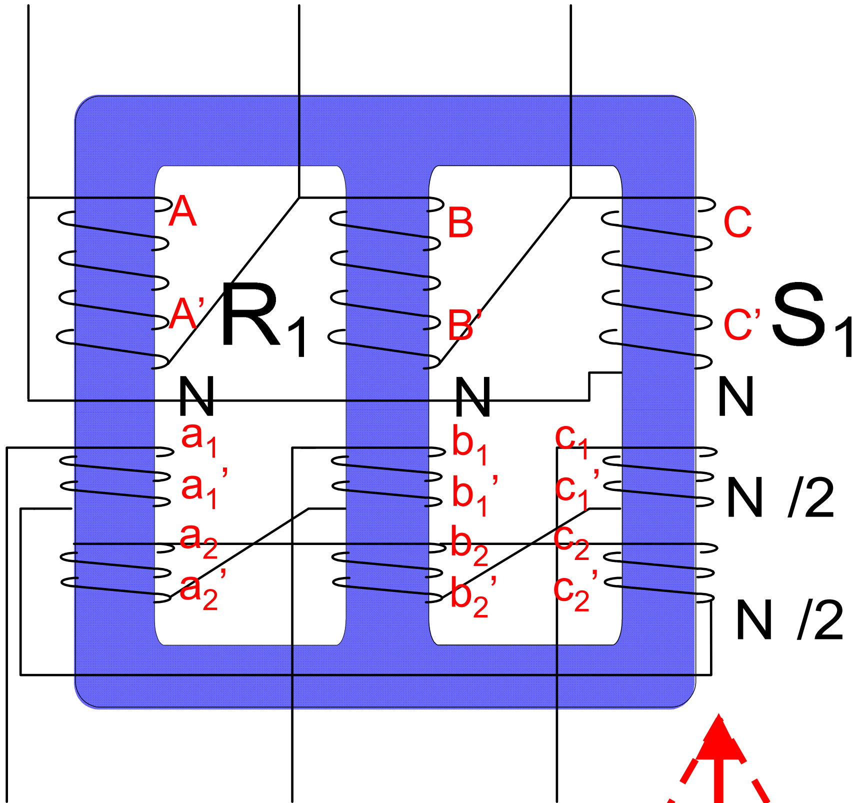


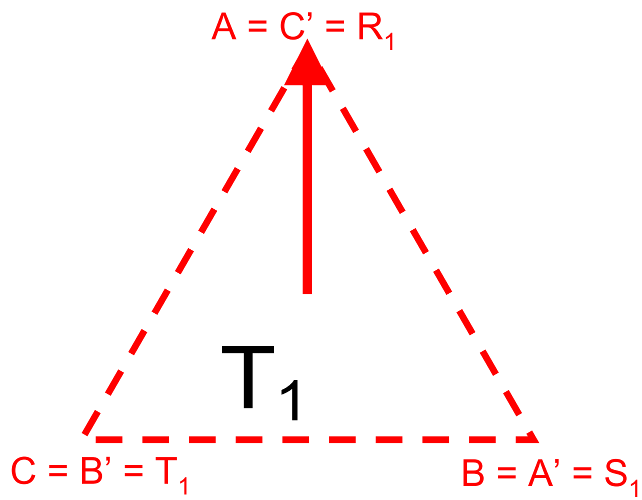
Dy 5

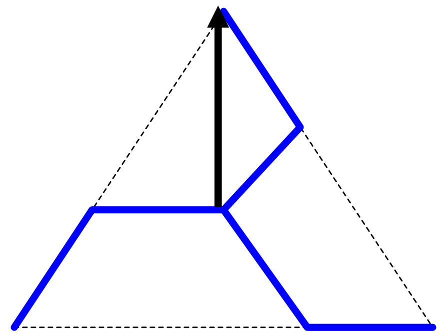
Clock number

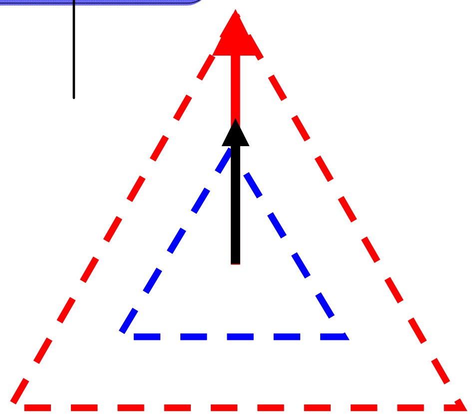
VECTOR/PHASOR GROUP (angular displacement between primary and secondary)

Connection Dz?







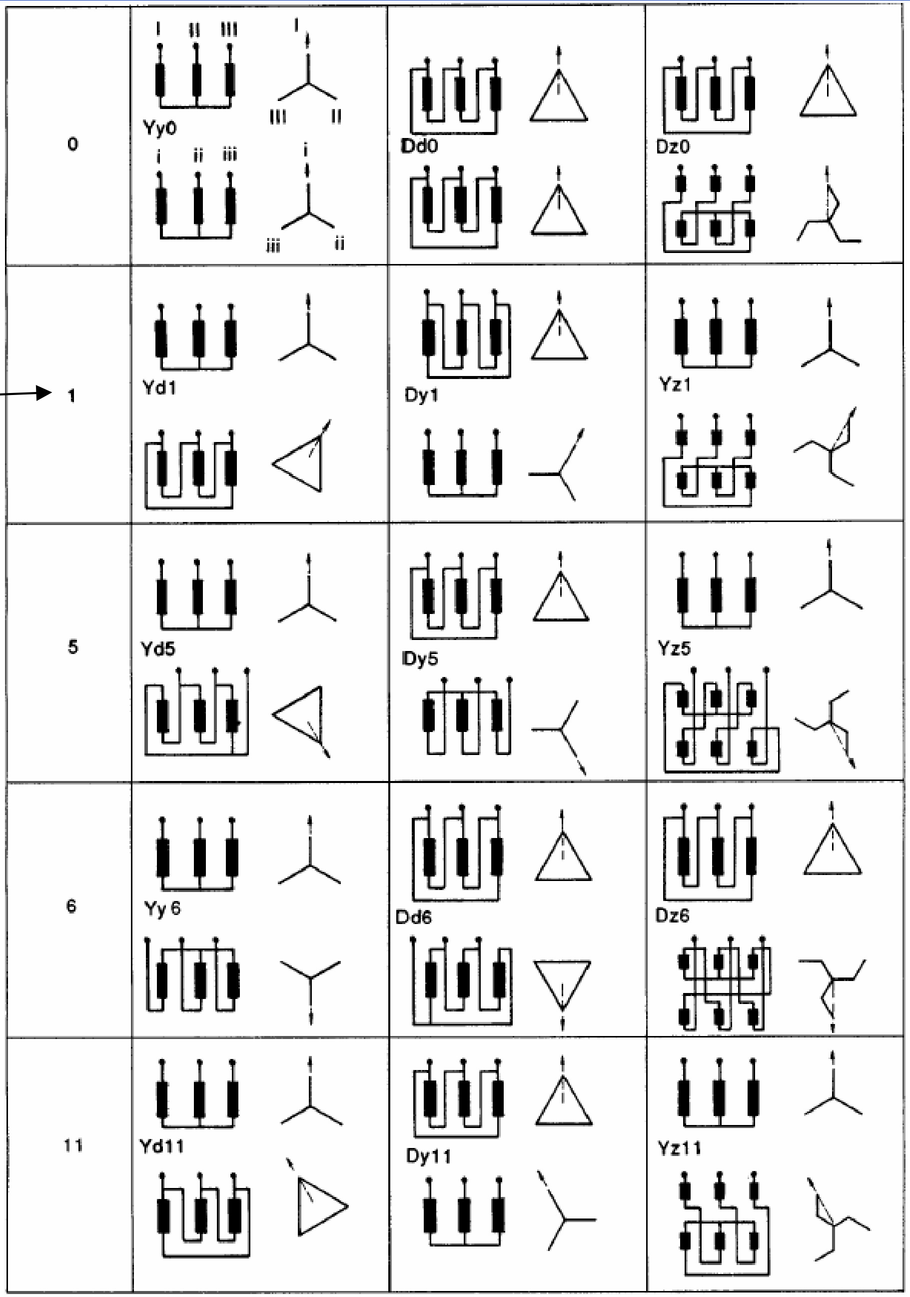


Dz 0

1 1 1

THREE-PHASE TRANSFORMERS. COMMON CONNECTIONS

Clock number
or
vector number



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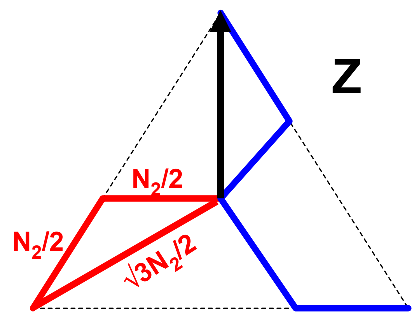

	Y	D	Z
V_{PN}	n	n/√3	√3n/2
V_{PP}	√3n 1.73	n	3n/2 1.5

Y winding requires less turns

Voltages ratio (V_{1n}/V_{2n})

Yy: n_1/n_2

Dd: n_1/n_2

Yd: $\sqrt{3}n_1/n_2$

Dy: $n_1/(\sqrt{3}n_2)$

Yz: $2n_1/(\sqrt{3}n_2)$

Dz: $2n_1/(3n_2)$

PREFERRED CONNECTIONS

- One way to keep 3rd harmonic currents from spreading over an entire system is to use Δ-connected windings in transformers. (motors generate 3rd harmonics)

- **Line-to-neutral voltages are distorted due to third harmonics. Delta loops minimize third harmonic content because line-to-line voltages are obtained as the subtraction of two line-to neutral voltages (third-order harmonic components in each phase of a 3-phase system are in phase).**

- It is also desirable that a three-phase system have a delta to provide a path for third-harmonic currents in order to eliminate or reduce third-harmonic voltages in the waveform.

- Step-down transmission ending transformers usually have the HV winding delta connected and the LV star connected with the neutral earthed.

- Step-up transformers in power stations usually have the LV winding star connected and the HV delta connected.

PREFERRED CONNECTIONS

Yy

- $V_{pp\text{-}Y}$=173%$V_{pp\text{-}\Delta}$ → $n_Y<n_\Delta$ → $S_{Y,Cu}>S_{\Delta,Cu}$
- Unbalanced load: poor performance
- Problems with 3rd harmonics
- Solution: Double secondary in Δ
- No phase shifting between 1y and 2y

Dd

- No phase shifting between 1y and 2y
- Needs > isolation space than Yy
- Used only in LV applications
- No problems with 3rd harmonics

Yd and Dy

- Unbalanced load: good performance
- **No** problems with 3rd harmonics
- Phase shifting between 1y and 2y
- **Yd**: Step-down transformer in HV lines
- **Dy**: Step-down transf. in distribution lines because allows a grounded neutral in LV side

Yz

- No problems with 3rd harmonics
- Unbalanced load: good performance
- Same phase shifting as Yd
- Allows grounded Neutral in 2y side
- $V_{pp\text{-}Y}$=115%$V_{pp\text{-}Z}$ → $n_Y<n_Z$ → $S_{y,Cu}>S_{Z,Cu}$

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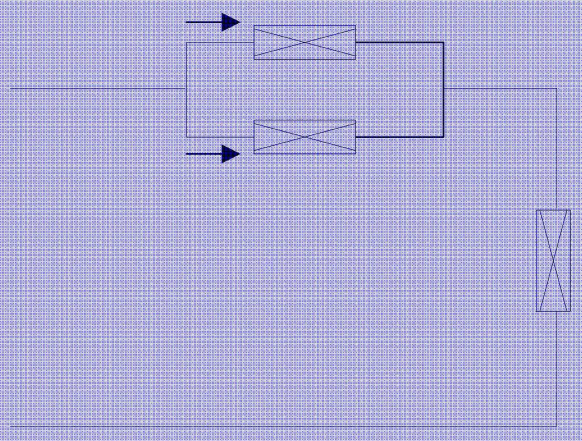
2. Similar ε_{sc}: $\varepsilon_{scA} \approx \varepsilon_{scB}$ ($c_A\varepsilon_{scA} = c_B\varepsilon_{scB}$)

3. Same angular displacement/clock number

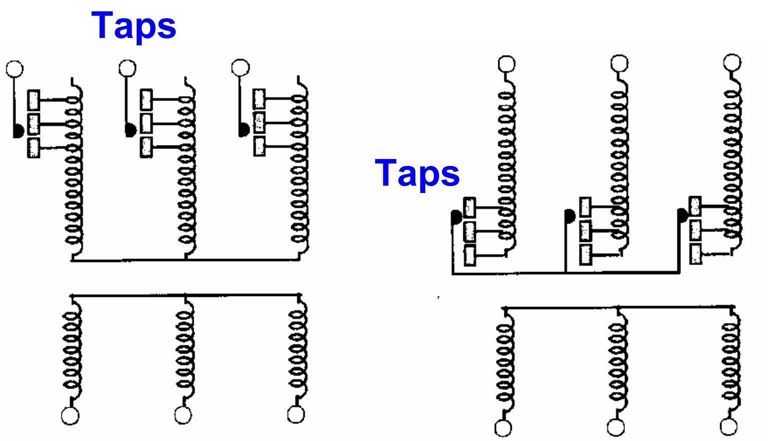


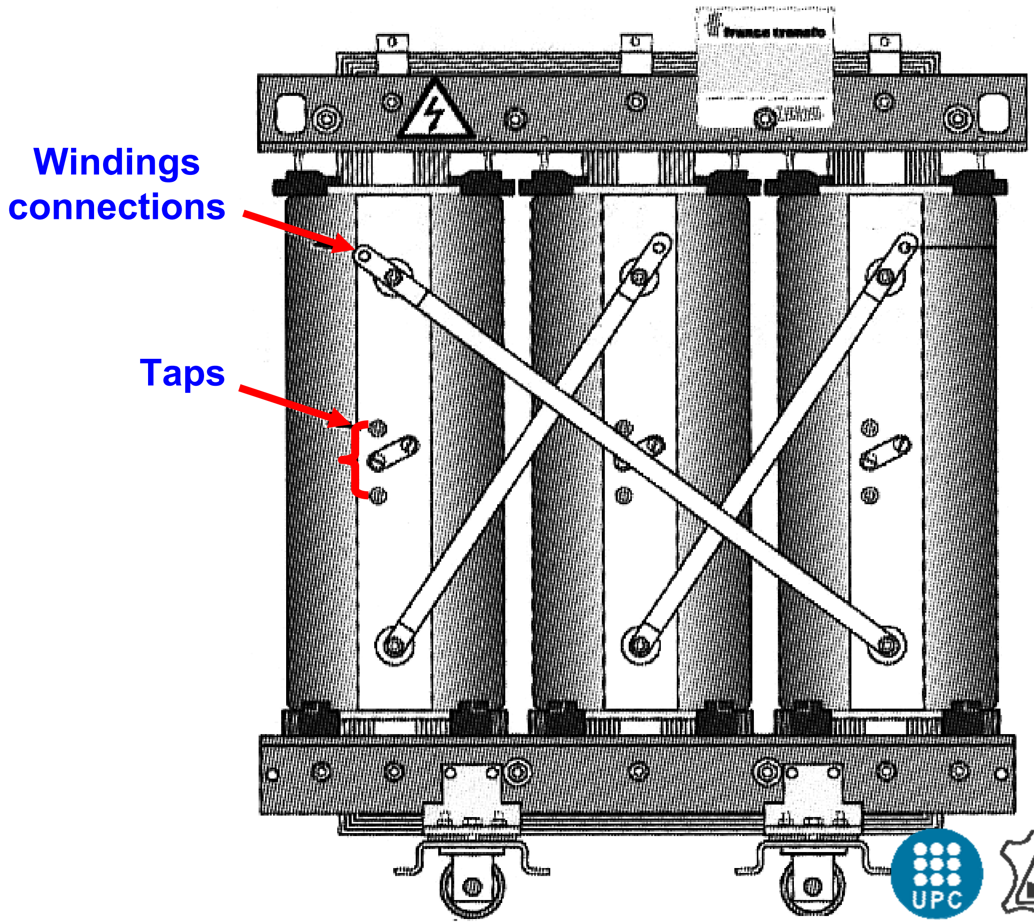
$$I_{2A} Z_{2scA} = I_{2B} Z_{2scB} \rightarrow c_A I_{2An} Z_{2scA} = c_B I_{2Bn} Z_{2scB}$$

$$c_A \frac{I_{2An} Z_{2scA}}{V_{2n}} = c_B \frac{I_{2Bn} Z_{2scB}}{V_{2n}} \rightarrow c_A \varepsilon_{scA} = c_B \varepsilon_{scB}$$

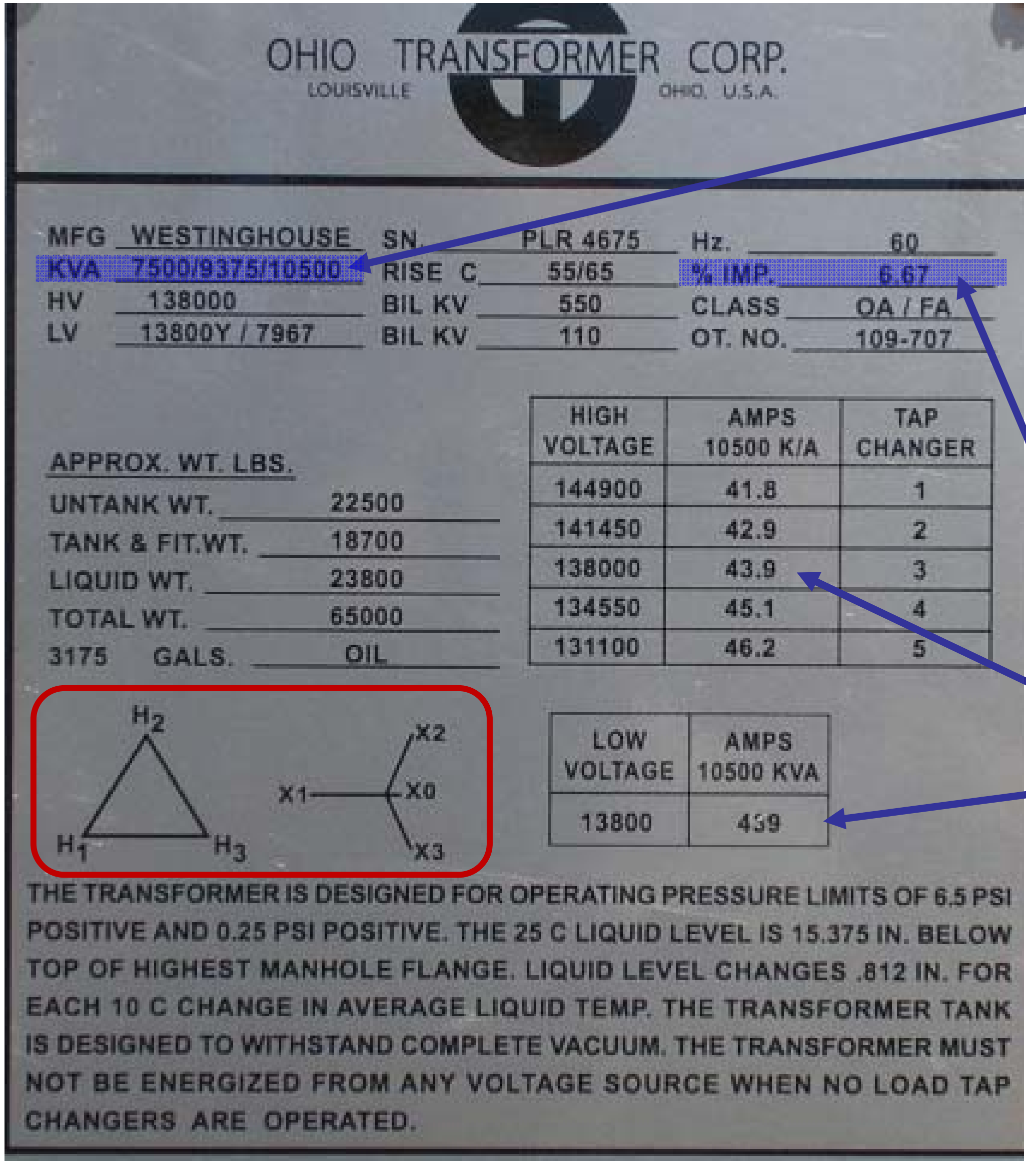
TAPPED TRANSFORMERS

- Transformer tap: connection point along a transformer winding that allows a certain number of turns to be selected.
- It produces variable turns ratio and enables voltage regulation of the output.
- The tap selection is made via a tap changer mechanism
- In step-down distribution transformers, taps are usually found in HV windings





TRANSFORMER NAMEPLATE



kVA: 7500/9375/10500

ONAN(55ºC)/ONAF(55ºC)/ONAF(65ºC)

ONAN = Oil Natural, Air Natural.
ONAF = Oil Natural, Air Forced.
ONAF 65ºC the expected life is halved

Dy 138kV/13.8kV

S_n =10500 kVA

ε_{sc} = 6.67%

r_t =10

$I_{1n,line}$= 43.92 A

$I_{2n,line}$ = 439.2 A

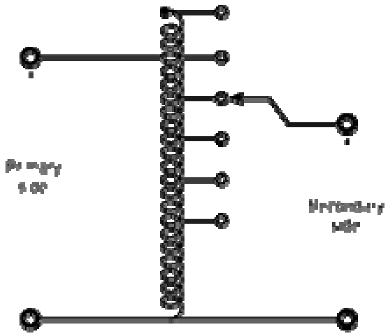
$Z_{1sc,\Delta} = \varepsilon_{sc} \cdot V_{1n,\Delta}/I_{1n,\Delta} = 363\ \Omega$

$Z_{2sc,Y} = \varepsilon_{sc} \cdot V_{2n,Y}/I_{2n,Y} = 1.21\ \Omega$

$I_{1sc,line} = I_{1n,line}/\varepsilon_{sc}$ =658.47 A

$I_{2sc,line} = I_{2n,line}/\varepsilon_{sc}$ =6584.7 A

AUTOTRANSFORMERS AND VARIACs



ADVANTAGES

- **Conductor length savings (less N_2 turns are required)**
- **Smaller magnetic circuit**
- **Reduced leakage flux and no-load current**
- **Reduced ε_{cc}**
- **Reduction of electrical and magnetic loss**

LIMITATIONS

- **Primary and secondary not insulated!**
- An insulation failure can result in full input voltage applied to the output.
- Higher short circuit current (lower ε_{cc})

VARIAC: adjustable autotransformer

INSTRUMENT TRANSFORMERS

- Instrument transformers are used for measuring voltage, current, power and energy in electrical systems, and for protection and control.
- Where a voltage or current is too large to be conveniently measured by an instrument, it can be scaled down to a standardized low value.
- Instrument transformers isolate measurement and control circuitry from the high currents or voltages present on the circuits being measured or controlled.

VOLTAGE TRANSFORMERS

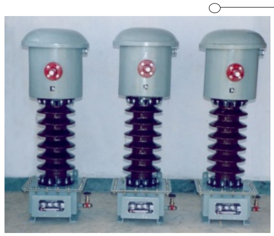
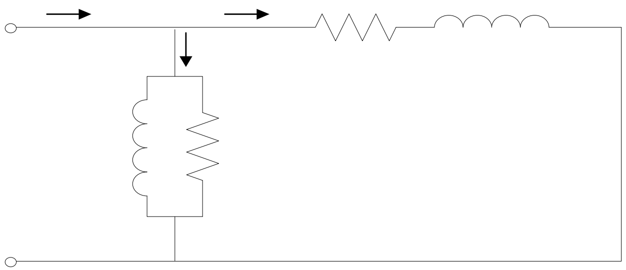
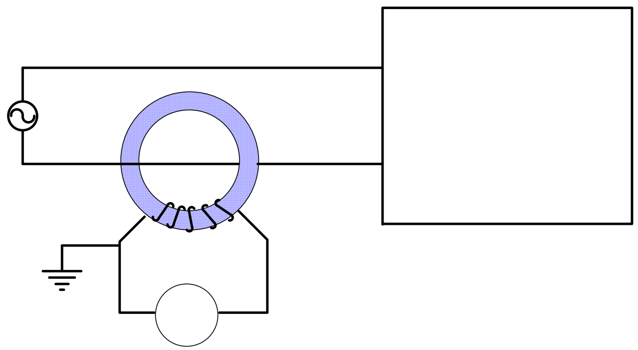
- Voltage transformers (VTs) are used in high-voltage circuits
- VTs are designed to present a negligible load to the supply being measured
- VTs allow protective relay equipment to be operated at a lower voltages
- VTs have a precise winding ratio for accurate metering
- Secondary voltage standardized to **110 V**

INSTRUMENT TRANSFORMERS

CURRENT TRANSFORMERS

- A current transformer is a transformer designed to provide a current in its secondary coil proportional to the current flowing in its primary coil.
- They work with low magnetic flux (linear zone).
- Standardized secondary currents: **5 A** and 1 A.
- Current transformers are connected in series with the line, measuring I_{line}.
- The secondary has connected an ammeter in series (nearly in short circuit)
- Normal operation: $I_{line} = I_1 = I_0 + I_2' \rightarrow I_1 \approx I_2'$
- Secondary open: $I_{line} = I_1 \approx I_0 \rightarrow I_0 \uparrow\uparrow \rightarrow \Phi \uparrow\uparrow \rightarrow V_2 \uparrow\uparrow \rightarrow P_{iron} \uparrow\uparrow$ **DANGER!**



$N_1 = 1$

ISOLATION TRANSFORMERS

ISOLATION TRANSFORMERS (IT)

- An IT is a 1:1 power transformer which is used as a safety precaution.
- It is used to decouple two circuits. They are also used for the power supply of devices not on ground potential.
- The IT has a secondary which is non-grounded. Thus one can avoid being shocked by touching just one wire and earth ground.
- Since the neutral wire of an outlet is directly connected to ground, grounded objects near the device under test (lamp, concrete floor, oscilloscope ground lead, etc.) may be at a hazardous potential difference with respect to that device.
- By using an isolation transformer, the bonding is eliminated, and the shock hazard is entirely contained within the device.
- Industrial power applications have also used isolated power outputs to power devices that must not be stopped in case of single fault to ground. An isolated output can withstand one fault to either of wires to ground without disturbing the equipment operation (just makes safe floating to normal grounded power supply).
- The "floating" output ITs are used in medical applications, electronics testing,…

THREE-PHASE DISTRIBUTION TRANSFORMERS FEATURES

	TECHNICAL FEATURES OF 24 kV THREE-PHASE TRANSFORMERS												
Power (kVA)	50	75	100	125	160	200	250	315	400	500	630	800	1000
Group	Yyn 0	Yyn 0	Yyn 0	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11	Dyn 11
P_0 (kW)	0.24	0.33	0.40	0.48	0.58	0.69	0.82	0.98	1.17	1.38	1.64	1.96	2.15
P_{sc} (kW)	1.39	1.87	2.20	2.53	2.97	3.49	4.10	4.86	5.80	6.89	8.22	10.24	13.30
ε_{sc} (%)	4	4	4	4	4	4	4	4	4	4	4	6	6
I_0 (%I_n)	4.7	4.1	3.3	3.0	2.7	2.4	2.2	2.1	2.0	2.0	1.9	1.8	1.6
Weight (kg)	385	481	570	655	731	834	976	1100	1422	1640	1930	2267	2645

IEC76, UNE 20101

ELECTRICAL CHARACTERISTICS

Rated Power kVA			250	400	630	800	1000	1250	1600	2000	2500
Rated Voltage		Primary	Maximum Voltage 24 kV								
		Secondary no load*	420 V between phases - no load								
Tapping Range*			± 2,5 ± 5% ó +2,5 + 5 + 7,5 + 10% (other regulation voltages on demand)								
Vector Group*			Dyn 11								
No Load Losses (W)*			650	930	1300	1550	1700	2130	2600	3100	3800
Load Losses (W)*			3250	4600	6500	8100	10500	13500	17000	20200	26500
Short Circuit Impedance % at 75° C *			4	4	4	6	6	6	6	6	6
No Load current at 100% of Vn*			2	1,8	1,6	1,4	1,3	1,2	1,1	1	0,9
Max. Sound Power Level (dB)*			62	65	67	68	68	70	71	73	76
Voltage drop at full load %		cos φ = 1	1,4	1,2	1,1	1,2	1,2	1,3	1,2	1,2	1,2
		cos φ = 0,8	3,3	3,2	3,1	4,4	4,4	4,4	4,4	4,4	4,4
Efficiency (%)	Load 100%	cos φ = 1	98,5	98,6	98,8	98,8	98,8	98,8	98,8	98,9	98,8
		cos φ = 0,8	98,1	98,3	98,5	98,5	98,5	98,5	98,5	98,6	98,5
	Load 75%	cos φ = 1	98,7	98,8	99,0	99,0	99,0	99,0	99,0	99,1	99,0
		cos φ = 0,8	98,4	98,6	98,7	98,7	98,8	98,7	98,8	98,8	98,8

Ormazabal Datasheet

EXAMPLE 1

A 900 kVA, 15000/3000-V, ε_{sc} = 8% three-phase transformer, has the primary and the secondary windings connected as in the table. Determine the parameters of the table.

l: line p: phase	Yy	Yd	Dd	Dy
Voltages ratio: r_V	15000/3000 = 5	5	5	5
Turns ratio: r_t	(15000/√3)/ 3000/√3 = 5	5/√3	5	5√3
V_{l1}	15000 V	15000 V	15000 V	15000 V
V_{p1}	15000/√3 V	15000/√3 V	15000 V	15000 V
I_{l1}	900000/(√3·15000) = 34.64 A	34.64 A	34.64 A	34.64 A
I_{p1}	34.64 A	34.64 A	34.64/√3 = 20 A	20 A
$V_{p,sc1} = \varepsilon_{sc} \cdot V_{p1}$	0.08·15000/√3 = 692.8 V	692.8 V	0.08·15000 = 1200 V	1200 V
$Z_{sc1} = V_{p,sc1}/I_{p1}$	692.8/34.64 = 20 Ω	20 Ω	1200/20 = 60 Ω	60 Ω
V_{l2}	3000 V	3000 V	3000 V	3000 V
V_{p2}	3000/√3 V	3000 V	3000 V	3000/√3 V
I_{l2}	900000/(√3·3000) = 173.21 A	173.21 A	173.21 A	173.21 A
I_{p2}	173.21 A	173.21/√3 = 100 A	100 A	173.21 A
$V_{p,sc2} = \varepsilon_{sc} \cdot V_{p2}$	0.08·3000/√3 = 138.56 V	0.08·3000 = 240 V	240 V	138.56 V
$Z_{sc2} = V_{p,sc2}/I_{p2}$	138.56/173.21 = 0.8 Ω	240/100 = 2.4 Ω	2.4 Ω	0.8 Ω

EXAMPLE 2

A Dy, 320 kVA, ε_{sc}=4%, P_{sc}=3584 W, 10800/400-V three-phase transformer feeds a balanced load which absorbs 300 A with PF = 0.8(i). In this situation V_1 =11100 V. Calculate the voltage in the load

From the statement it results: $I_{1n} = 17.11\ \text{A}$ $\quad I_{2n} = 461.88\ \text{A}$ $\quad r_t = 27$

$$P_{sc} = 3 \cdot R_{1sc\,Y} \cdot I_{1n\,Y}^2 \longrightarrow R_{1sc\,Y} = \frac{P_{sc}}{3 \cdot I_{1nY}^2} = \frac{3584}{3 \cdot 17.11^2} = 4.08\ \Omega$$

$$V_{1sc} = \varepsilon_{sc}.V_{1n} = 0.04 \cdot 10800 = 432\ \text{V} \longrightarrow Z_{1sc\,Y} = \frac{V_{1scY}}{I_{1nY}} = \frac{432/\sqrt{3}}{17.11} = 14.58\ \Omega$$

$$X_{1sc\,Y} = \sqrt{Z_{1scY}^2 - R_{1scY}^2} = \sqrt{14.58^2 - 4.08^2} = 13.997\ \Omega$$

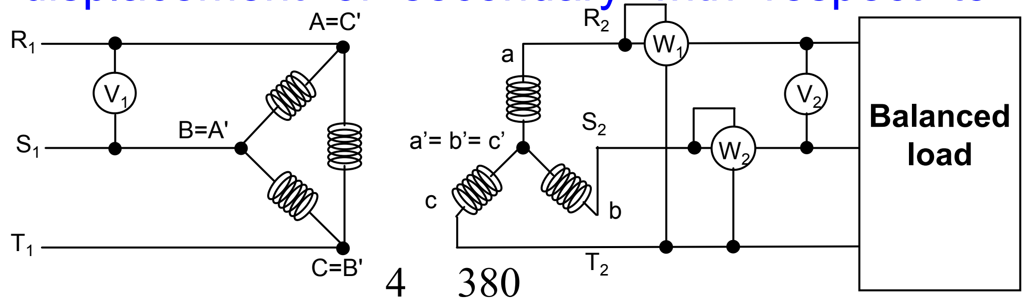
$$\varphi_{sC} = \text{arctg}\frac{X_{1scY}}{R_{1scY}} = 73.74^{\circ}$$

$$\overline{V}_{1Y} = \overline{V}'_{2Y} + \overline{I}'_{2Y} \cdot \overline{Z}_{1scY} \longrightarrow \frac{11100}{\sqrt{3}}^{\varphi_{V1}{}^{\circ}} = \overline{V}'_2{}^{0^{\circ}} + \frac{300}{27}^{-36.87^{\circ}} \cdot 14.58^{73.74^{\circ}}$$

$$V'_{2Y} = 6278.28\ \text{V} \longrightarrow V_2' = \sqrt{3} \cdot 6278.28 = 10874.30\ \text{V} \longrightarrow \mathbf{V_2 = \frac{10874.30}{27} = 402.75\ V}$$

EXAMPLE 3

A Dy, 125 kVA ε_{sc} = 4%, P_o = 480 W, P_{sc} = 2530 W, 24000/380-V, three-phase transformer has a secondary winding with R_2= 0.01 Ω. The wattmeter readings are W_1=60186.56 W, W_2=23813.44 W, while V_2=368.6 V. Compute: a) R_1 b) V_1 c) The transformer efficiency d) The angular displacement of secondary with respect to primary voltages.



From the statement it results:

$I_{1n} = 3.007\text{ A} \quad I_{2n} = 189.92\text{ A} \quad r_t = 63.158$

a) $P_{sc} = 3 \cdot R_{2sc\,Y} \cdot I_{2n\,Y}^2$

$$R_{2sc\,Y} = \frac{P_{sc}}{3 \cdot I_{2n_Y}^2} = \frac{2530}{3 \cdot 189.9178^2} = 0.02338\,\Omega \qquad Z_{2sc\,Y} = \frac{V_{2sc\,Y}}{I_{2n\,Y}} = \frac{\frac{4}{100} \cdot \frac{380}{\sqrt{3}}}{189.9178} = 0.0462\,\Omega$$

$$X_{2sc\,Y} = \sqrt{Z_{2sc\,Y}^2 - R_{2sc\,Y}^2} = \sqrt{0.0462^2 - 0.02338^2} = 0.03985\,\Omega \qquad \overline{Z}_{2sc\,Y} = (0.02338 + j0.03985)\,\Omega$$

$$0.02338 = R_{2Y} + \frac{R_{1Y}}{r_t^2} \qquad R_{1Y} = 53.37\,\Omega \qquad \mathbf{R_{1\Delta} = 3 \cdot R_{1Y} = 160.11\,\Omega}$$

b) $P_T = W_1 + W_2 = 84000$

$Q_T = \sqrt{3} \cdot (W_1 - W_2) = 63000$

$$\varphi_T = \text{arctg}(Q_T/P_T) = 36.87^\circ \qquad I_{2Y} = \frac{P_T = 84000}{\sqrt{3} \cdot 368.6 \cdot 0.8} = 164.465\text{ A}$$

$$\overline{V}_{1Y}' = \overline{V}_{2Y} + \overline{I}_{2Y} \cdot \overline{Z}_{2,sc\,Y} \qquad \overline{V}_{1Y}' = \frac{368.6}{\sqrt{3}}\,{}^{0^\circ} + 164.465\,{}^{-36.87^\circ} \cdot (0.02338 + j0.03985) = 219.839\,{}^{0.72^\circ}\text{ V}$$

$$\mathbf{V_1 = 24048.8\text{ V}}$$

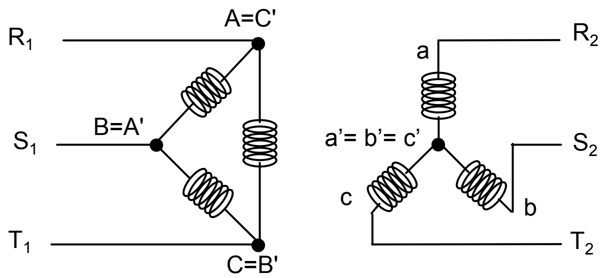
EXAMPLE 3

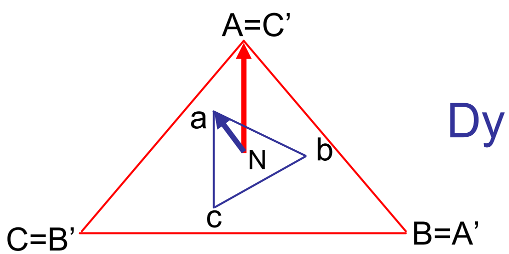
c) $$\eta = \frac{P_2}{P_2 + P_{Fe} + P_{Cu}} = \frac{P_2}{P_2 + P_o \cdot (V_1/V_{1n})^2 + c^2 \cdot P_{sc}}$$

$$c = I_2/I_{2n} = 164.465/189.92 = 0.866$$

$$\eta = \frac{84000}{84000 + 480 \cdot \left(\frac{24048.8}{24000}\right)^2 + 0.866^2 \cdot 2530} = \frac{84000}{86379.34} = 0.972$$

d) Clock number = angular displacement between secondary and primary voltages

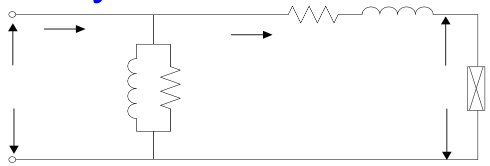




Dy11

EXAMPLE 4

A 50 kVA Dy1 15000/380-V and three-phase transformer has relative voltage drops ε_{sc} = 10% and ε_{Xsc} = 8%. The transformer feds a balanced wye-connected load which phase impedances are 5^{0° Ω. Determine: a) R_{1sc}, X_{1sc}, Z_{1sc}. b) The voltage in the secondary, being the primary fed at 15000-V and the secondary



1/3 Eq. Circuit Yy

Solved from the point of view of the electrical system: Yy

$$V_{1n,Y} = 15000/\sqrt{3}\text{ - V} \quad V_{2n,Y} = 380/\sqrt{3}\text{ - V} \quad r_t = V_{1n,Y}/V_{2n,Y} = 39.47$$

$$S_n = 50000\text{VA} = 3\cdot V_{1n,Y}\cdot I_{1n,Y} = 3\cdot\frac{15000}{\sqrt{3}}\cdot I_{1n,Y} \rightarrow I_{1n,Y} = 1.925\text{ A}$$

$$\varepsilon_{sc} = 0.1 = \frac{Z_{1scY}\cdot 1.925}{15000/\sqrt{3}} \rightarrow Z_{1scY} = 450\,\Omega \qquad \varepsilon_{Xsc} = 0.08 = \frac{X_{1scY}\cdot 1.925}{15000/\sqrt{3}} \rightarrow X_{1scY} = 360\,\Omega$$

$$\overline{Z}_{1scY} = 270 + j\cdot 360 = 450^{53.13^\circ}\ \Omega$$

$$\overline{Z}_{LoadY}' = r_t^{\,2}\cdot\overline{Z}_{LoadY} = 39.47^2\cdot 5^{0^\circ} = 7789.4^{0^\circ}\ \Omega$$

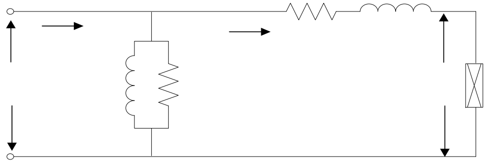
$$\overline{V}_{1Y} = \overline{I}_{2Y}'\cdot(\overline{Z}_{1scY} + \overline{Z}_{LoadY}') \rightarrow 15000/\sqrt{3}^{\,0^\circ} = \overline{I}_{2Y}'\cdot(450^{53.13^\circ} + 7789.4^{0^\circ}) \rightarrow \overline{I}_{2Y}' = 1.0735^{-2.558^\circ}\text{ A}$$

$$\overline{V}_{2Y}' = \overline{I}_{2Y}'\cdot\overline{Z}_{LoadY}' \rightarrow \overline{V}_{2Y}' = 1.0735^{-2.558^\circ}\cdot 7789.4^{0^\circ} = 8361.92^{-2.558^\circ}\text{ V}$$

$$V_{2Y} = V_{2Y}'/r_t = 8361.92/39.47 = 211.835\text{ V} \rightarrow V_2 = \sqrt{3}\cdot V_{2Y} = 366.91\text{ V}$$

EXAMPLE 5

A 50 kVA Dy1 15000/380-V and three-phase transformer has relative voltage drops ε_{sc} = 10% and ε_{Xsc} = 8%. The transformer feds a balanced wye-connected load which phase impedances are 5^{0^o} Ω. Determine: a) R_{1sc}, X_{1sc}, Z_{1sc}. b) The voltage in the secondary, being the primary fed at 15000-V and the secondary



1/3 Eq. Circuit Δy

Solved from the point of view of the electrical machine: Dy

$$V_{1n,\Delta} = 15000\text{ - V} \quad V_{2n,Y} = 380/\sqrt{3}\text{ - V} \quad r_t = V_{1n,\Delta}/V_{2n,Y} = 68.37$$

$$S_n = 50000\text{VA} = 3\cdot V_{1n,\Delta}\cdot I_{1n,\Delta} = 3\cdot 15000\cdot I_{1n,\Delta} \rightarrow I_{1n,\Delta} = 1.11\text{ A}$$

$$\varepsilon_{sc} = 0.1 = \frac{Z_{1sc\Delta}\cdot 1.11}{15000} \rightarrow Z_{1sc\Delta} = 1350\,\Omega \qquad \varepsilon_{Xsc} = 0.08 = \frac{X_{1sc\Delta}\cdot 1.11}{15000} \rightarrow X_{1sc\Delta} = 1080\,\Omega$$

$$\overline{Z}_{1sc\Delta} = 810 + j\cdot 1080 = 1350^{53.13^o}\ \Omega$$

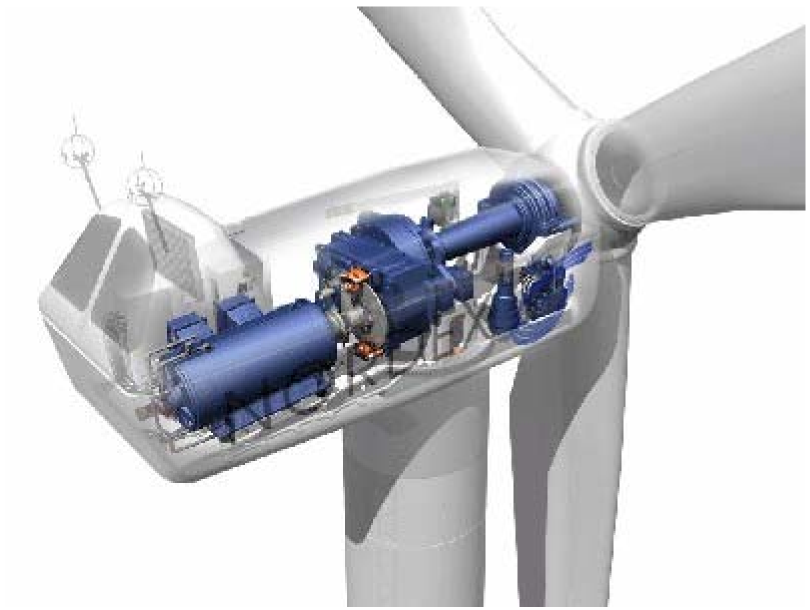
$$\overline{Z}_{Load\Delta}' = r_t^{\,2}\cdot\overline{Z}_{LoadY} = 68.37^2\cdot 5^{0^o} = 23372.6^{0^o}\ \Omega$$

$$\overline{V}_{1\Delta} = \overline{I}_{2\Delta}'\cdot(\overline{Z}_{1sc\Delta} + \overline{Z}_{Load\Delta}') \rightarrow \quad 15000^{0^o} = \overline{I}_{2\Delta}'\cdot(1350^{53.13^o} + 23372.6^{0^o}) \rightarrow \overline{I}_{2\Delta}' = 0.6197^{-2.557^o}\text{ A}$$

$$\overline{V}_{2\Delta}' = \overline{I}_{2\Delta}'\cdot\overline{Z}_{Load\Delta}' \rightarrow \overline{V}_{2\Delta}' = 0.6197^{-2.557^o}\cdot 23372.6^{0^o} = 14484.00^{-2.557^o}\text{ V}$$

$$V_{2Y} = V_{2\Delta}'/r_t = 14484/68.37 = 211.85\text{ V} \rightarrow V_2 = \sqrt{3}\cdot V_{2Y} = 366.93\text{ V}$$

THE INDUCTION MACHINE

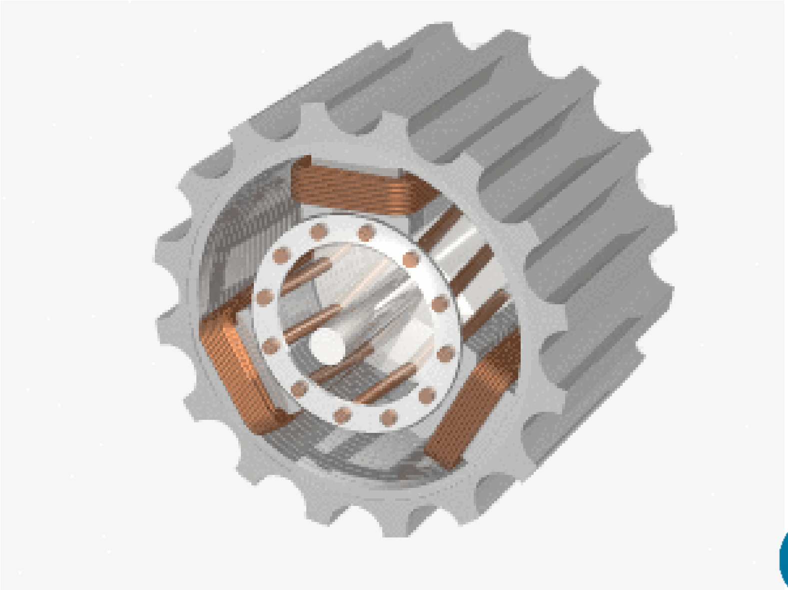


Unit 5. THE INDUCTION MACHINE

CONTENTS LIST:

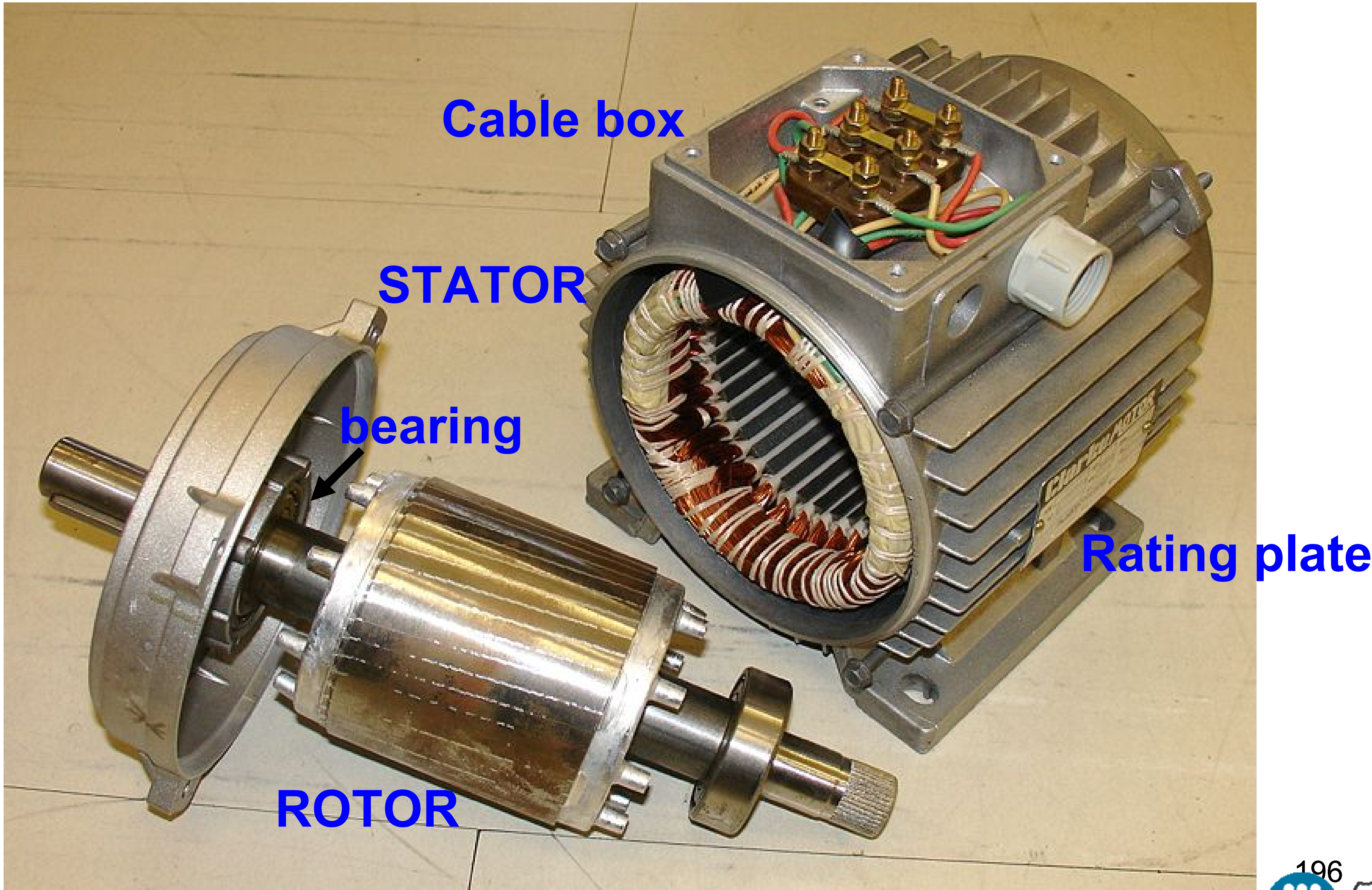
INTRODUCTION

- It is estimated that as much as 60 % of total power generation capacity is consumed by electric motors
- Of this total consumption, more than 80 percent is utilized by motors greater than 20 HP (horse power)
- The induction motor is the most commonly motor used in industrial applications
- It is very simple, rugged and presents relatively low manufacturing costs.
- Efficiency can reach > 95%

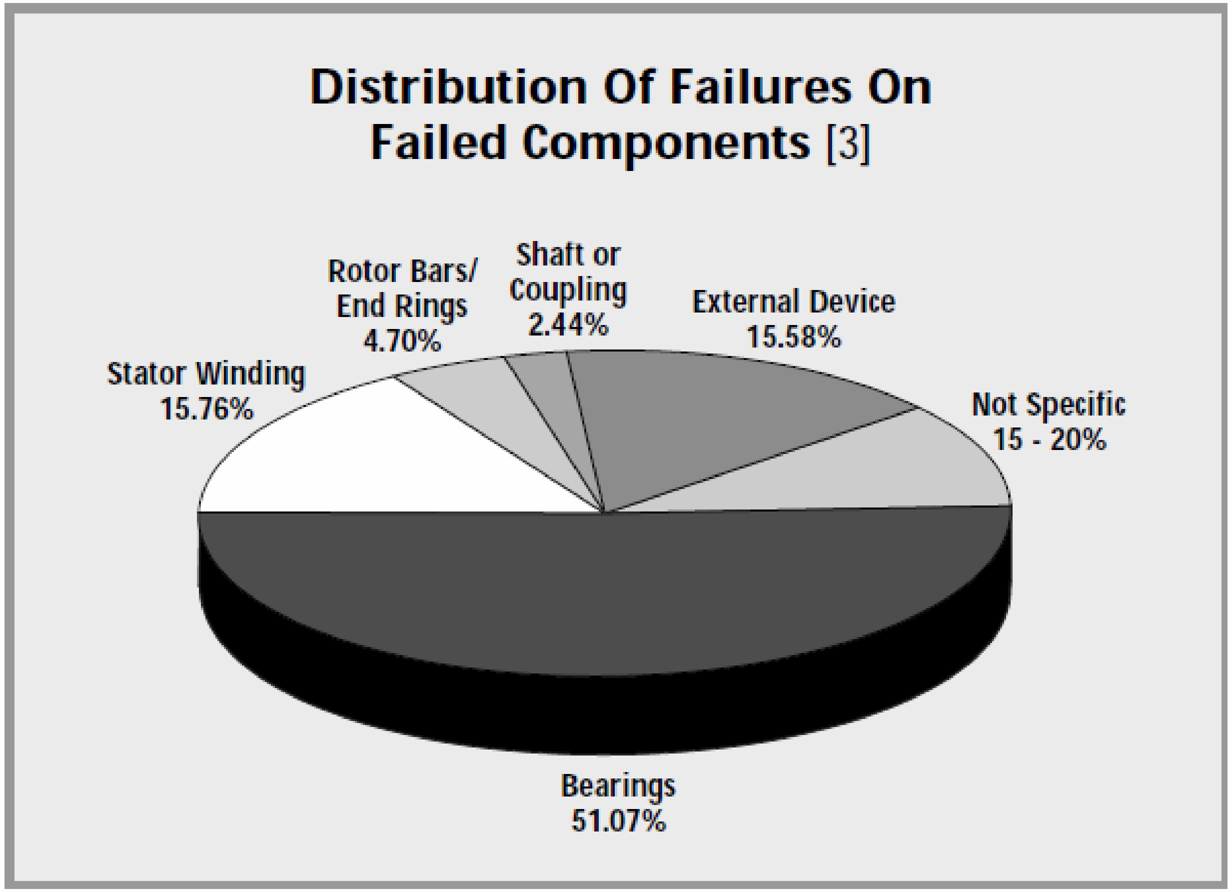


INTRODUCTION

SIMPLE AND RUGGED !!



INTRODUCTION

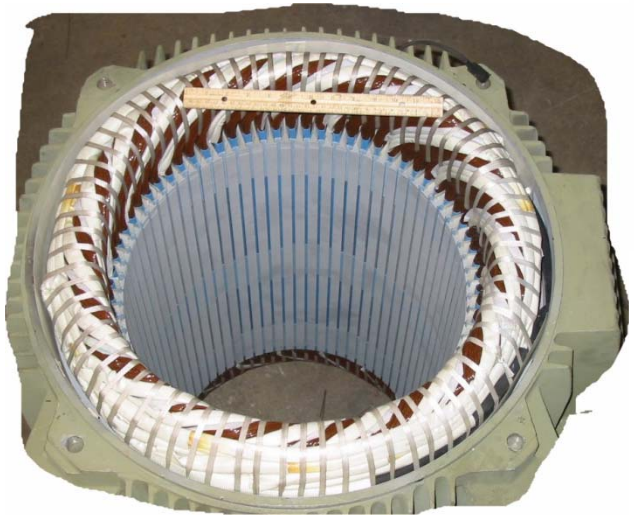


Olav Vaag Thorsen and Magnus Dalva, "A Survey Of Faults On Induction Motors In Offshore Oil Industry, Petrochemical Industry, Gas Terminals, And Oil Refineries," *IEEE Transactions On Industry Applications*, Vol. 31, No. 5, (September/October 1995).

INTRODUCTION

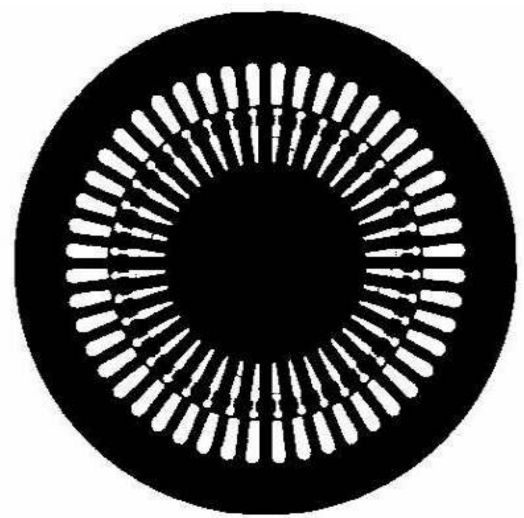
- It can acts as motor, generator and brake.
- The IM has two main parts: the stator and the rotor

STATOR

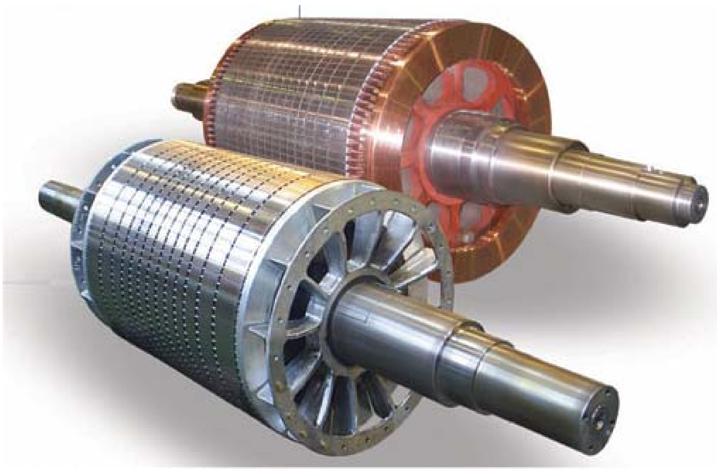


LAMINATIONS

Stator and rotor



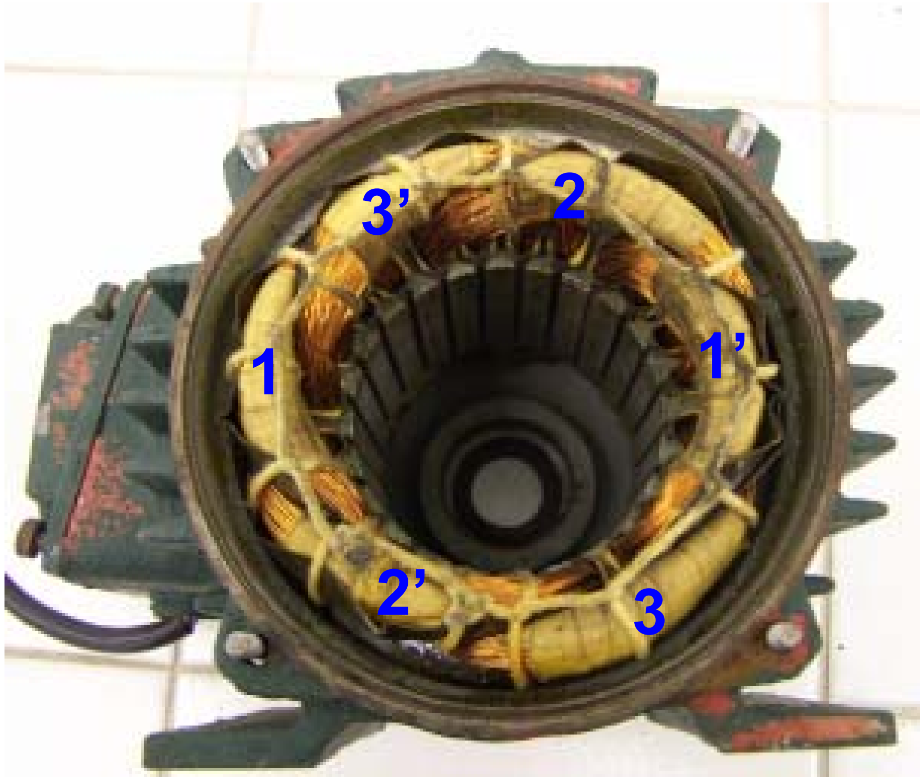
ROTOR



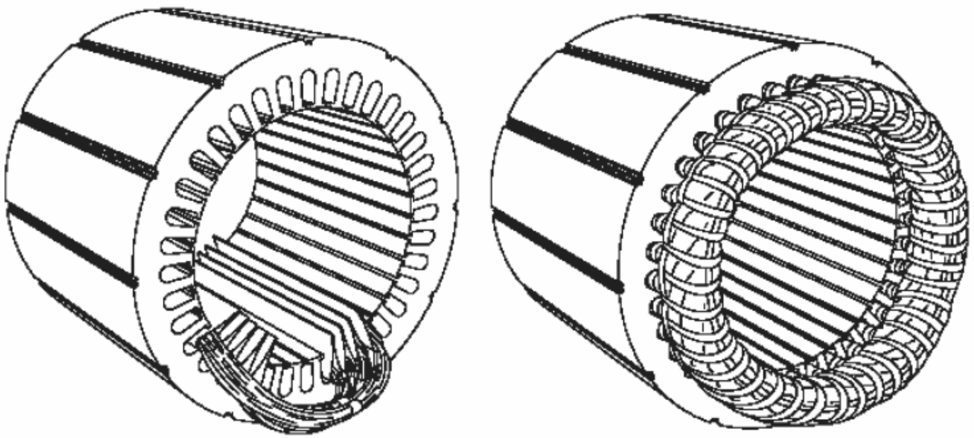
Squirrel cage

IM STATOR WINDINGS

- The stator core of a motor is made up of several hundred thin laminations
- Stator laminations are stacked together forming a hollow cylinder.
- Coils of insulated wire are inserted into slots of the stator core

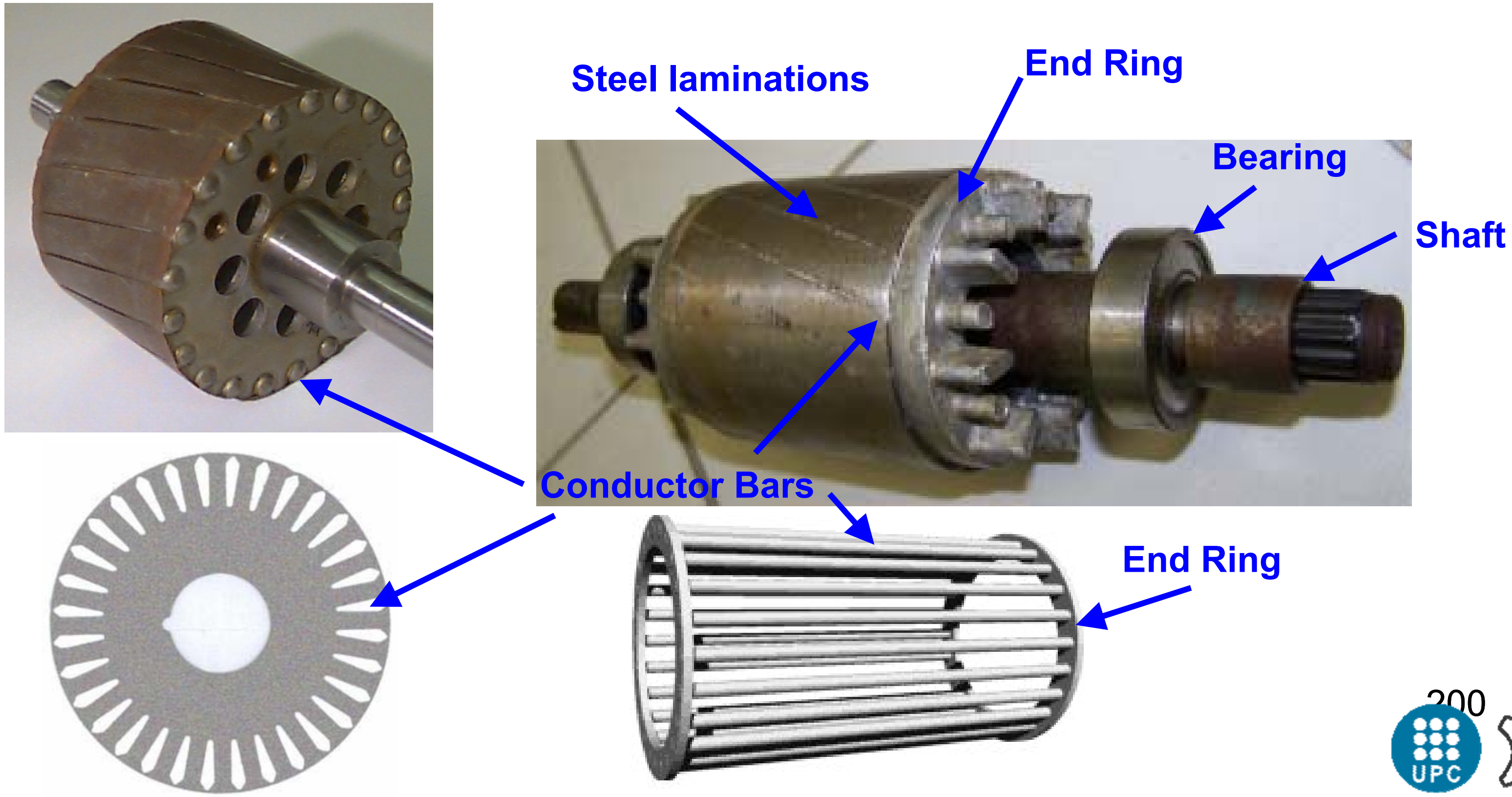


2 poles pairs



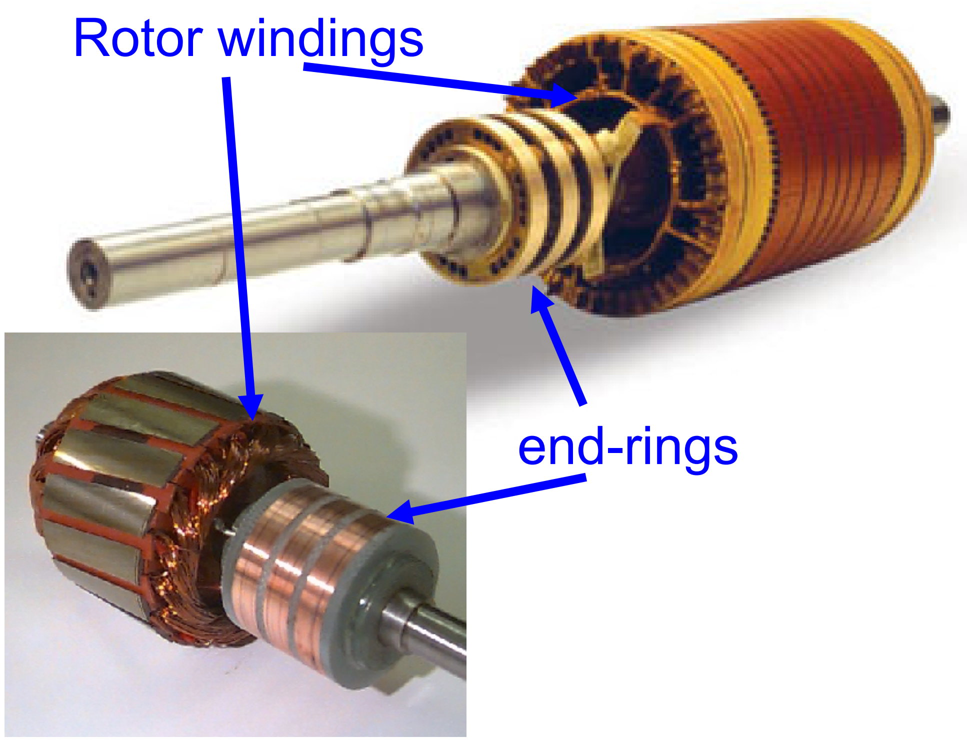
IM SQUIRREL CAGE ROTOR

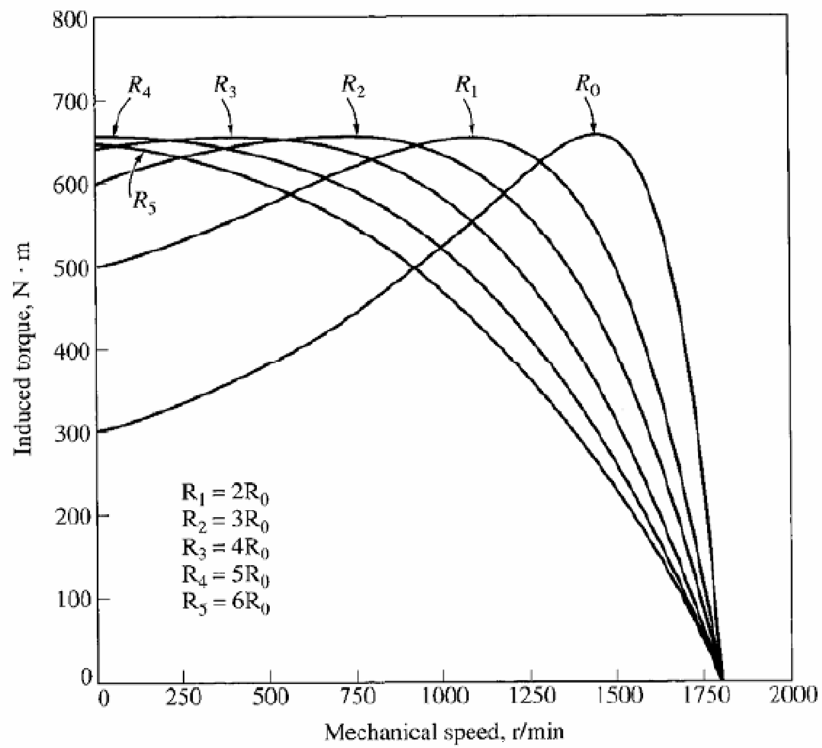
- The laminations are stacked together to form a rotor core.
- Aluminum is die-cast in the slots of the rotor core to form a series of conductors around the perimeter of the rotor.
- The conductor bars are mechanically and electrically connected with end rings. The rotor core mounts on a steel shaft to form a rotor assembly.



IM WOUND-ROTOR

- More expensive than the squirrel cage motor
- Wound-rotor motors suitable for starting heavy loads and accelerating them gradually and smoothly.
- **Typical applications include conveyer belts, hoists and elevators, wind-driven generators, paper mills, cement factory ovens, …**

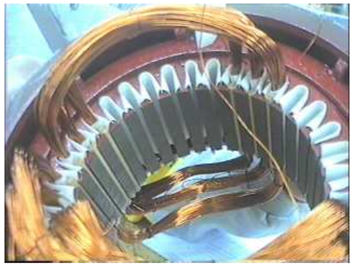




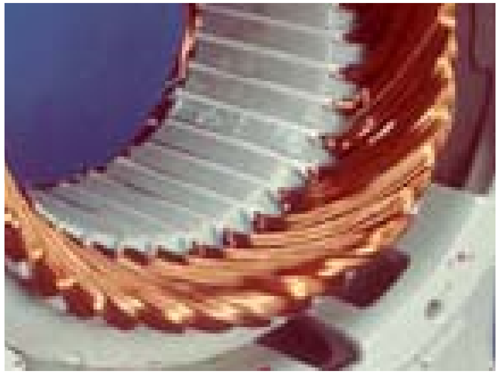
IM WOUND-ROTOR

- The rotor windings have the coils connected together to a set of rings that make contact with carbon-composite brushes.
- The brushes are connected to a set of impedances such as a resistor bank. This design allows for a varying resistance from almost short-circuit condition to an open-circuit condition
- By modifying the resistance, the speed-torque characteristics can be altered.
- This allows for the torque to remain high, the inrush low, and the speed varied.
- Wound-rotor motors suitable for starting heavy loads and accelerating them gradually and smoothly.

ASSEMBLING AN IM. The stator



Concentric winding



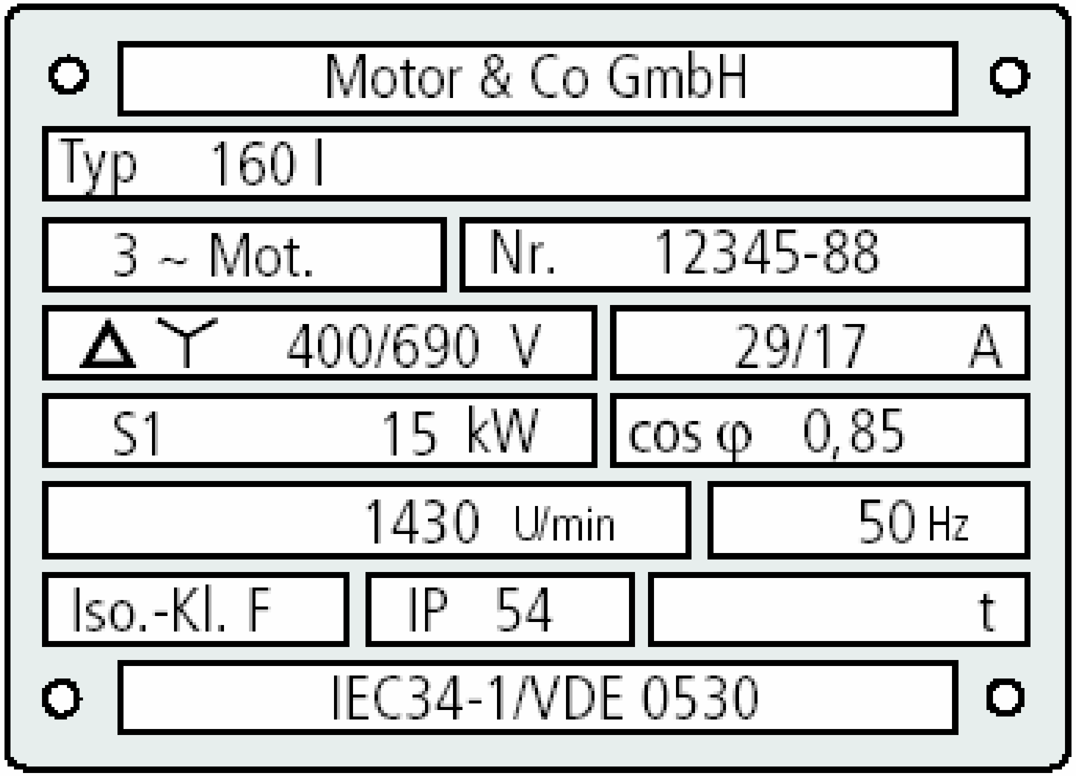
Eccentric winding (wave coil) the most applied because only one type of coil is required (easy of manufacture)

Eccentric (wave coil: imbricat)

Concentric

RATING PLATE or NAMEPLATE

Rated voltages, rated full-load current, rated power, power factor at rated load, frequency and speed at rated-load are specified on the **rating plate** or **nameplate**.

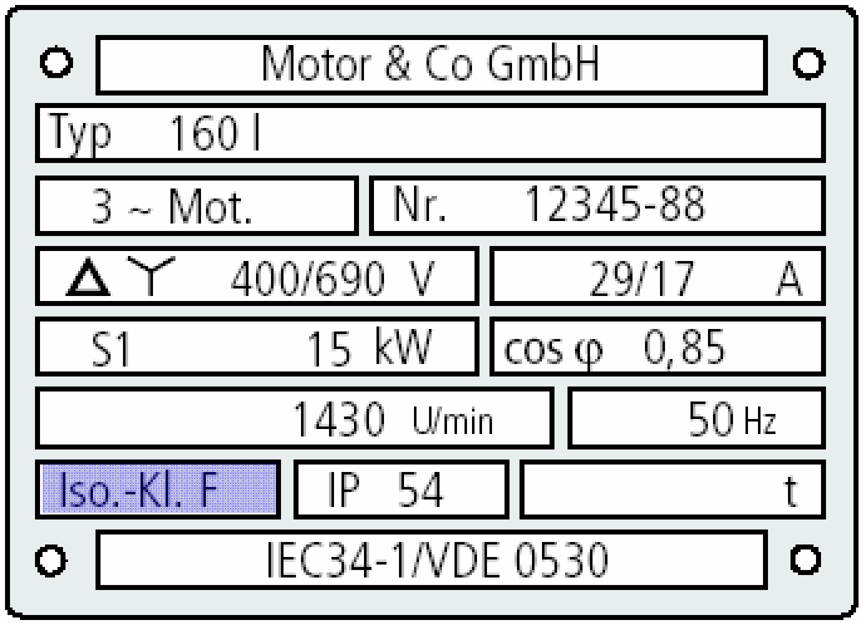


IMs Isolation Class

Isolation class (IEC 600 34-1)

Isolation class	Maximum Temperature ºC
Y	90
A	105
E	120
B	130
F	155
H	180
200	200
220	220
250	250

- The isolation class determines the maximum temperature for which the windings of the machine can safely operate without loss of their properties.



IMs DEGREE OF PROTECTION: IP

IP Code (IEC 600 34-5 – UNE EN 50102)

- Protection of persons against getting in contact with (or approaching) live parts and against contact with moving parts inside the enclosure.
- Also protection of the machine against ingress of solid foreign objects and against the harmful effects due to water ingress.

IP 5 5

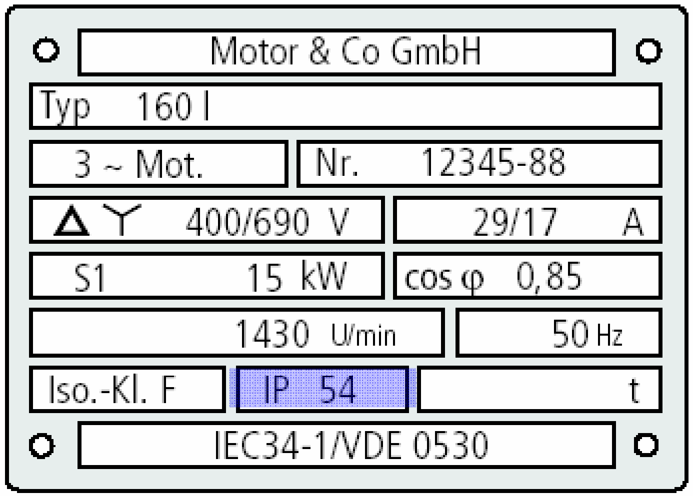
Characteristic letter

Degree of protection to persons and to parts of the motors inside the enclosure

2: Motors protected against solid objects greater than 12 mm
4: Motors protected against solid objects greater than 1 mm
5: Dust-protected motors
6: Dust-tight motors

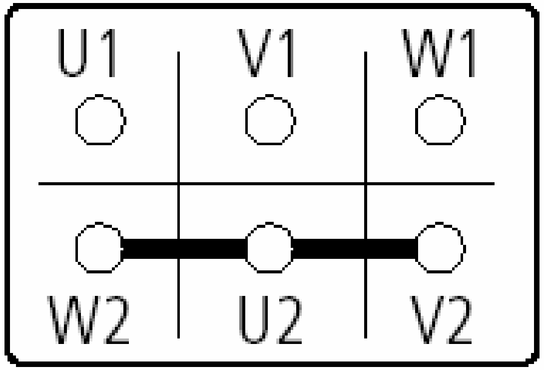
Degree of protection provided by the enclosure with respect to harmful effects due to ingress of water

3: Motors protected against spraying water
4: Motors protected against splashing water
5: Motors protected against water jets
6: Motors protected against heavy seas

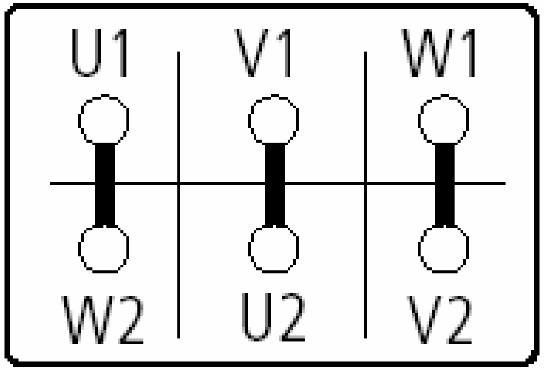


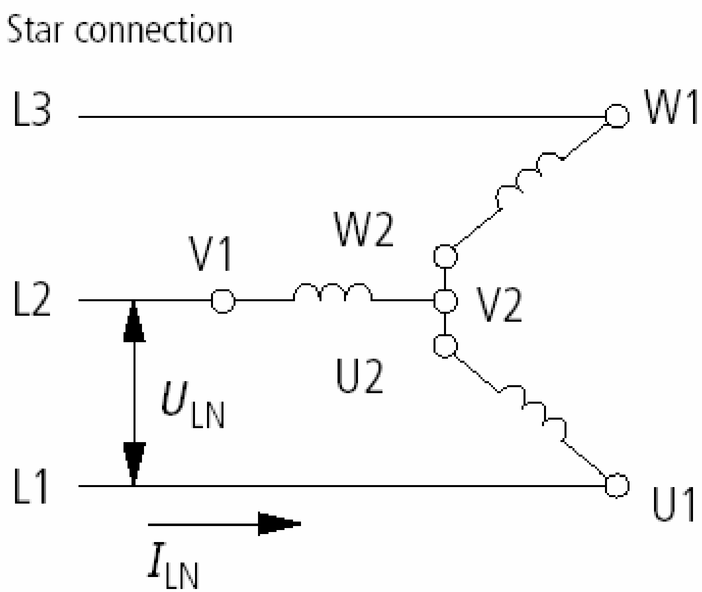
STAR AND DELTA CONNECTIONS

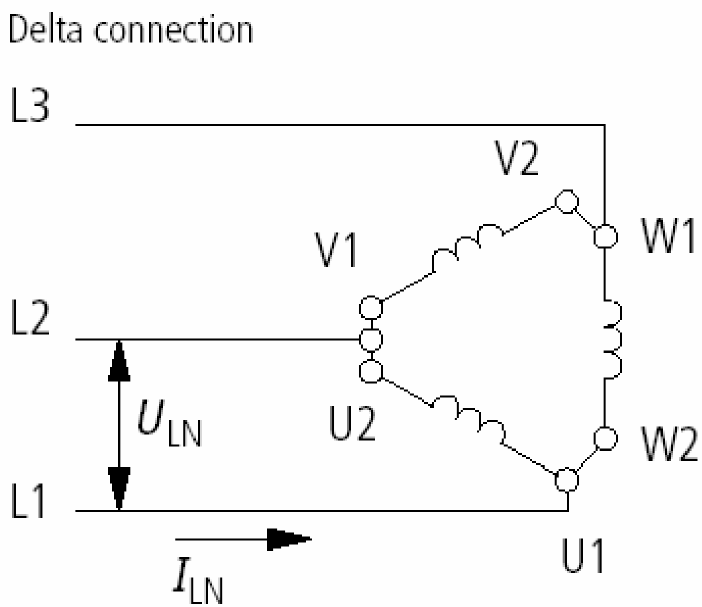
STAR



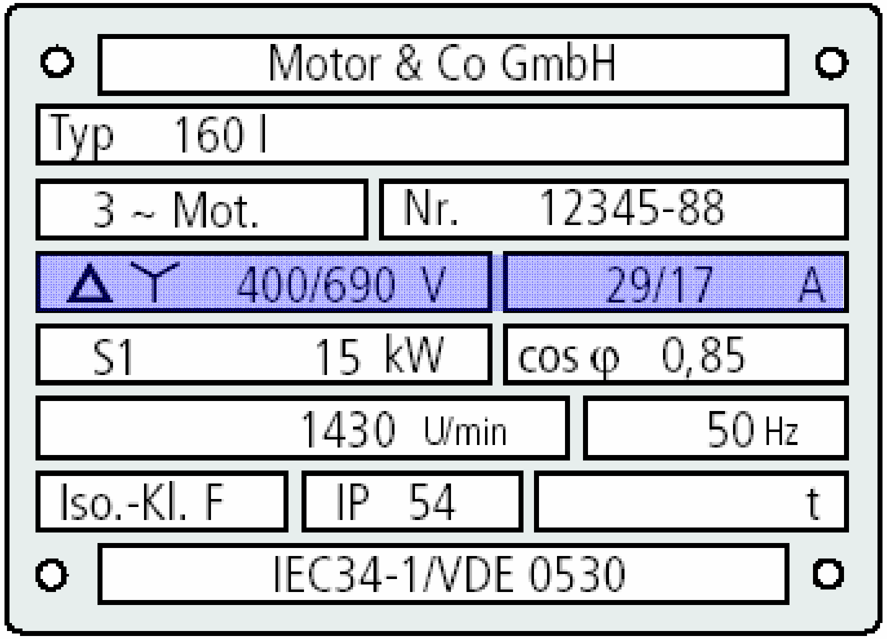
DELTA







STAR AND DELTA CONNECTIONS



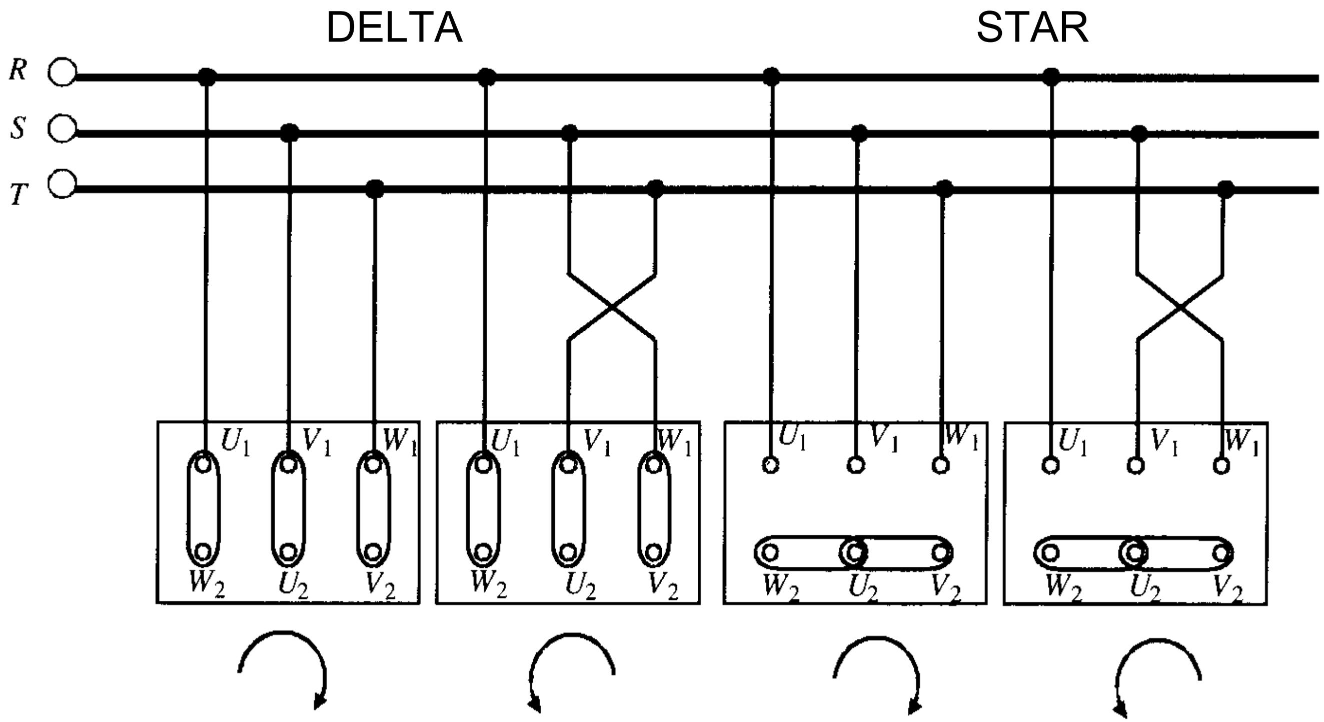
- In case of a 400 V line-voltage, this motor must be connected in Δ
- In case of a 690 V line-voltage, this motor must be connected in Y
- In both cases, each winding is feed at 400 V.

$$P_{input,n} = \sqrt{3}\cdot 400\cdot 29\cdot 0.85 = \sqrt{3}\cdot 690\cdot 17\cdot 0.85 = 17.08\,\mathrm{kW}$$

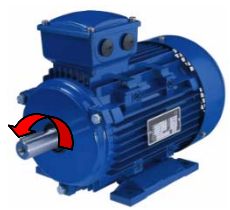
$$\eta_{\%} = 100\cdot\frac{P_{output}}{P_{input}} = 100\cdot\frac{15\,\mathrm{kW}}{17.08\,\mathrm{kW}} = 87.8\%$$

DIRECTION OF ROTATION

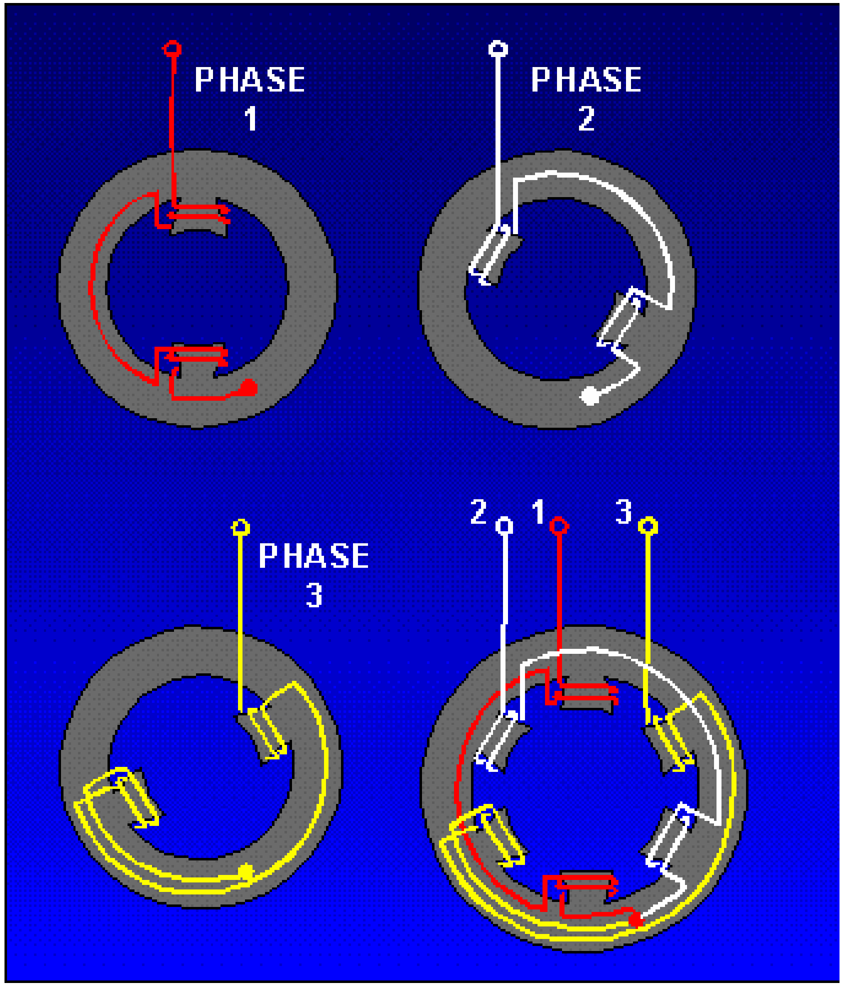
When the mains supply is connected to the stator terminals marked U_1,V_1 and W_1, of a three phase motor, and the mains phase sequence is L_1(R), L_2(S), L_3(T), the motor will rotate clockwise, as viewed from the D-end.



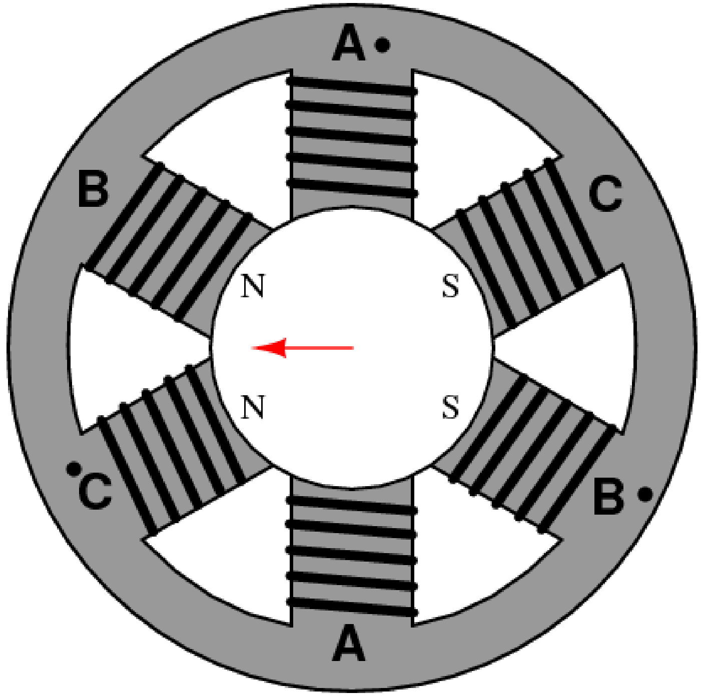
Direction of rotation



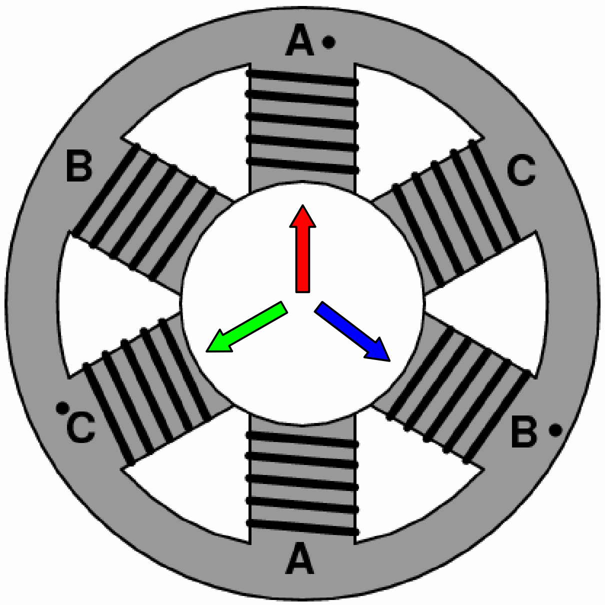
ROTATING MAGNETIC FIELD



Three-phase Y-connected stator



ROTATING MAGNETIC FIELD



$$\vec{B}_A = B\cos\omega t(0\hat{x} + 1\hat{y})$$

$$\vec{B}_B = B\cos(\omega t - 120^o)(\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{y})$$

$$\vec{B}_C = B\cos(\omega t + 120^o)(-\frac{\sqrt{3}}{2}\hat{x} - \frac{1}{2}\hat{y})$$

The resultant magnetic field is:

$$\vec{B}_{res} = \vec{B}_A + \vec{B}_B + \vec{B}_C = \frac{3}{2}B(\sin\omega t \cdot \hat{x} + \cos\omega t \cdot \hat{y})$$

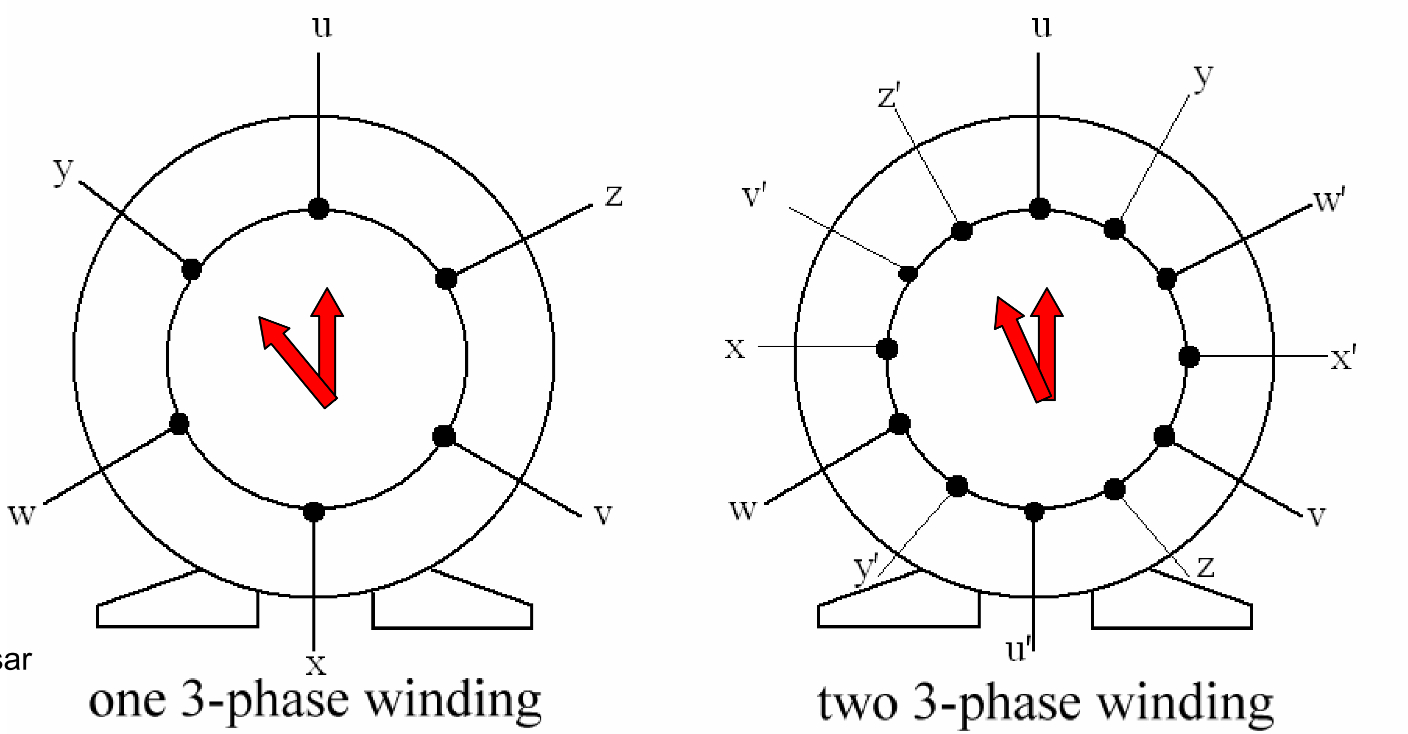
The former expression is a rotating magnetic with angular speed $\omega = 2\pi \mathbf{f_1}$ and a constant modulus given by: $B_{res} = 3B/2$

If $p \neq 1$, the angular displacement between stator windings is divided by p, and the synchronous speed is also divided by p.

NUMBER OF POLES PAIRS: SYNCHRONOUS SPEED

- Number of poles pairs = number of 3-phase windings
- n_0: synchronous speed or B-field speed

$$w_0 = 2\cdot\pi\cdot\frac{f}{p} \quad \text{(rad/s)} \qquad n_0 = 60\cdot\frac{f}{p} \quad \text{(r.p.m.)}$$



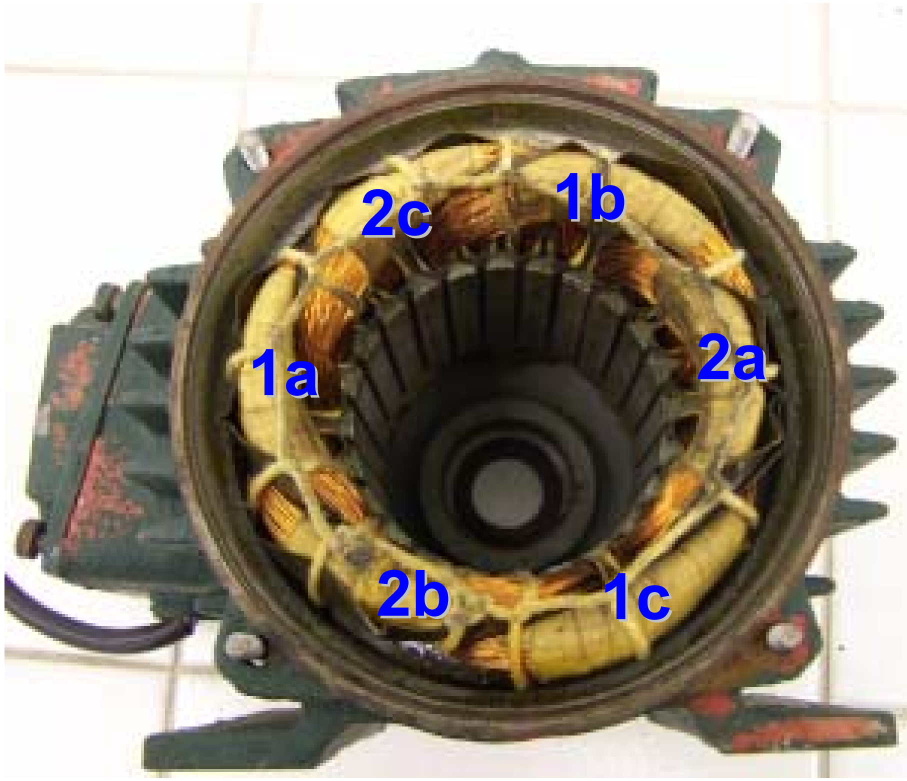
Boldea&Nasar

$p = 1$ | $p = 2$

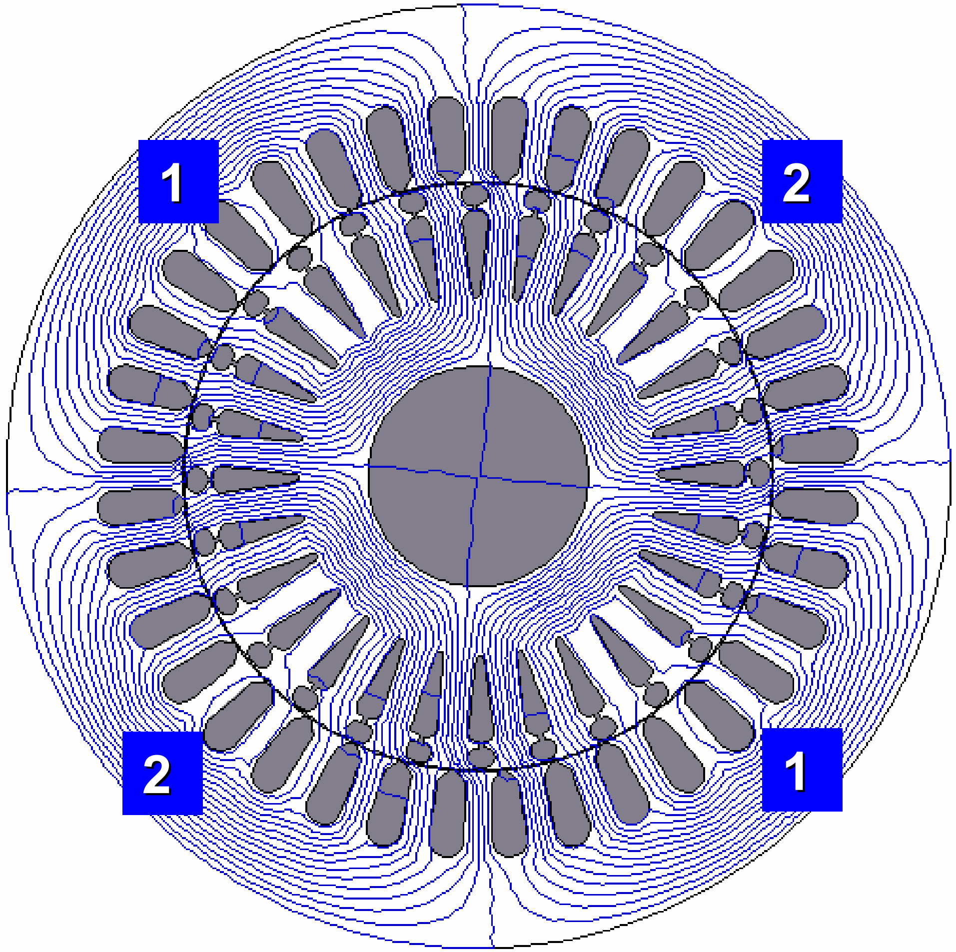
120°/1 mechanical degrees between phase axes | 120°/2 mechanical degrees between phase axes

SYNCHRONOUS SPEED

TWO POLES PAIRS: SYNCHRONOUS SPEED



Two pairs of poles



HOW THE IM WORKS

Three-phase winding shifted 120º

↓

Rotating field at $60 \cdot f_1/p$

↓

Induces voltage in the rotor bars

↓

A current flows through the rotor bars

↓

Force applied on the rotor bars $\vec{F} = I\vec{l} \wedge \vec{B}$

↓

The rotor produces mechanical torque

↓

The IM rotates

SLIP: ROTOR SPEED

- Rotor speed always lags B-field
- Slip (s): speed difference between rotor and B-field
- n_0: synchronous speed or B-field speed

Slip:

$$s = \frac{w_0 - w}{w_0} = \frac{n_0 - n}{n_0}$$

Rotor angular speed:

$$w = w_0 .(1 - s) \quad (\text{rad/s})$$

Rotor speed:

$$n = 60 \cdot \frac{f_1}{p} (1 - s) \quad (\text{r.p.m.})$$

ROTOR CURRENTS FREQUENCY AND EMF

- Frequency of rotor currents: f_2

$$\boxed{f_2 = s \cdot f_1}$$

EMF one phase of stator:

$$E_1 = 2.\pi.N_1.f_1.\Phi_o.\xi_1$$

EMF one phase of rotor when rotor at standstill (s = 1):

$$E_2 = 2.\pi.N_2 f_1.\Phi_o.\xi_2$$

EMF one phase of rotor when rotor rotates:

$$\mathbf{E_{2s}} = 2.\pi.N_{2}.f_2.\Phi_o.\xi_2 = s.2.\pi.N_{2}.f_1.\Phi_o.\xi_2 = \mathbf{sE_2}$$

ξ_i: factor which groups: form, distribution and pass factors

EXAMPLE 1a

Example. A 3-phase, 200-hp, 50-Hz, 4-pole, induction motor runs with a slip s = 0.02. Determine: a) The angular speed of the magnetic field (synchronous speed) b) The frequency of the rotor currents c) The speed of the motor

a) Synchronous speed:

$$4 \text{ poles} \rightarrow p = 2 \rightarrow n_0 = 60 \cdot f_1/p = 60 \cdot 50/2 = 1500 \text{ rpm}$$

b) Frequency of the rotor currents:

$$f_2 = s \cdot f_1 = 0.02 \cdot 50 = 1 \text{ Hz}$$

c) Motor speed = rotor speed:

$$n = n_0 \cdot (1 - s) = 1500 \cdot (1 - 0.02) = 1470 \text{ rpm}$$

EXAMPLE 1b

A 3-phase induction motor runs at 465 rpm at rated load. The same motor rotates at a speed of 490 rpm under no load conditions. Supposing a line-frequency of 50 Hz, determine: a) The angular speed of the magnetic field (synchronous speed) b) The slip under no-load conditions c) The slip at rated load. d) The frequency of the rotor currents under no-load, under rated-load conditions and at starting

a) Synchronous speed:

$n_0 = 60 \cdot f_1/p = 60 \cdot 50/p = 3000/p$ rpm $\rightarrow$ **$p = 6$** poles-pairs $\rightarrow n_0 = 500$ rpm

b) Slip under no-load conditions:

$$s = (n_0 - n)/n_0 = (500 - 490)/500 = 0.02\ (2\%)$$

c) Slip at rated load:

$$s = (n_0 - n)/n_0 = (500 - 465)/500 = 0.07\ (7\%)$$

d) Frequency of the rotor currents:

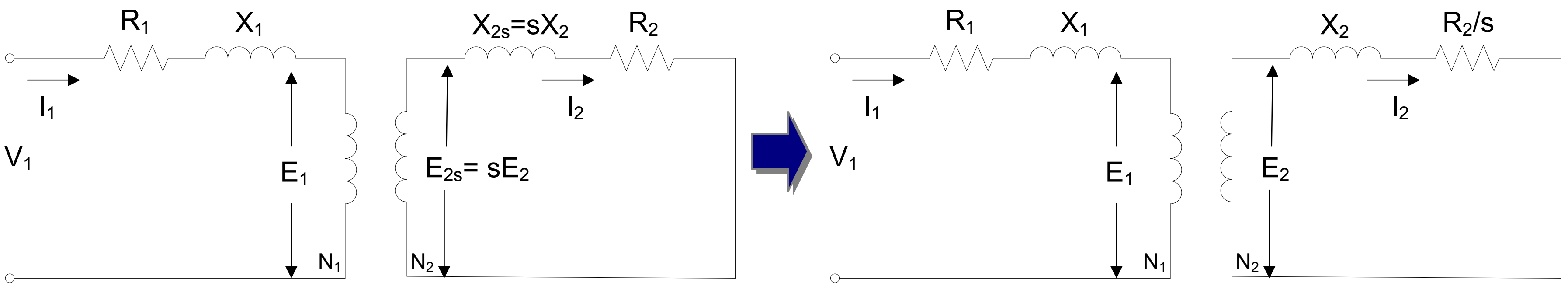
no-load: $f_2 = s \cdot f_1 = 0.02 \cdot 50 = 1$ Hz

rated load: $f_2 = s \cdot f_1 = 0.07 \cdot 50 = 3.5$ Hz

at starting: $f_2 = s \cdot f_1 = 1 \cdot 50 = 50$ Hz

Exact equivalent circuit (1/3)

- Electrical resistance winding losses: R_1, R'_2, X_1, X'_2
- Iron losses: R_m, X_m
- Mechanical load and torque: $R'_2(1/s-1)$

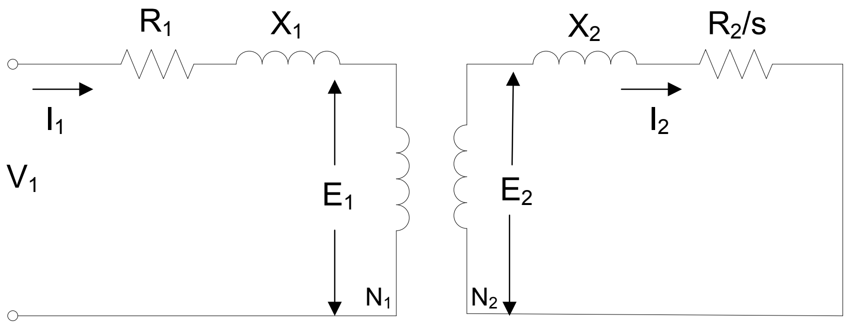


Rotating rotor $f_2 = sf_1$, $N_1 \neq N_2$

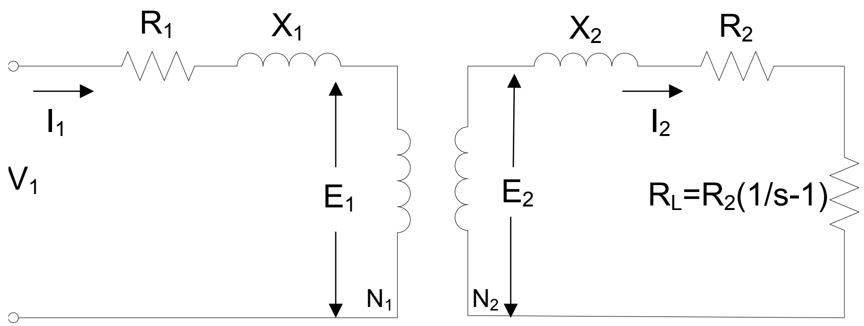
Static rotor (s=1) $f_2 = f_1$, $N_1 \neq N_2$

$$\bar{I}_2 = \frac{s\bar{E}_2}{R_2 + jsX_2} \quad \Rightarrow \quad \bar{I}_2 = \frac{\bar{E}_2}{R_2/s + jX_2}$$

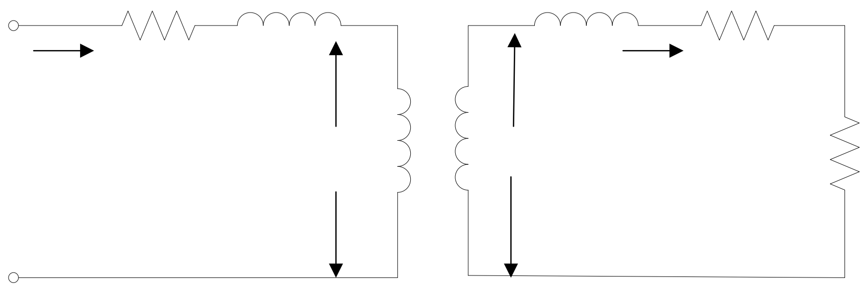
Exact equivalent circuit (1/3)



Static rotor $f_2 = f_1$, $N_1 \neq N_2$



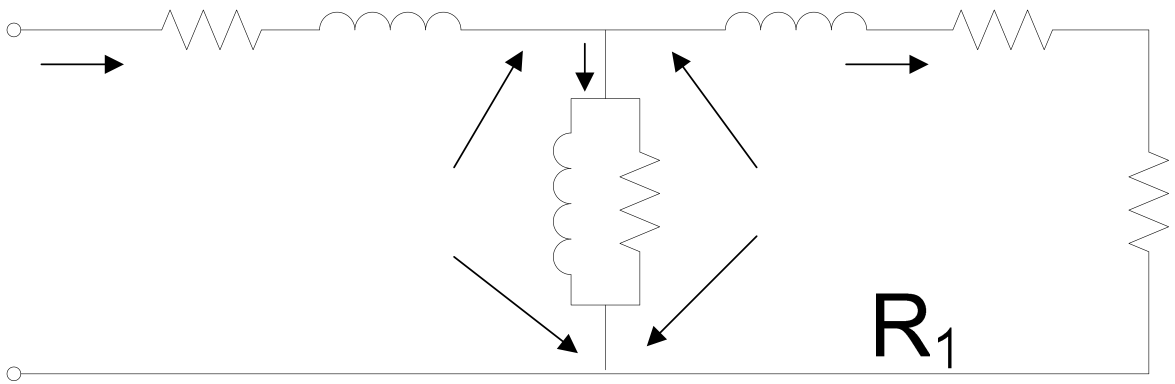
Static rotor $f_2 = f_1$, $N_1 \neq N_2$



Static rotor $f_2 = f_1$, $N_1 = N_2$

THE PRACTICAL IM. EQUIVALENT CIRCUIT

Exact equivalent circuit (1/3)



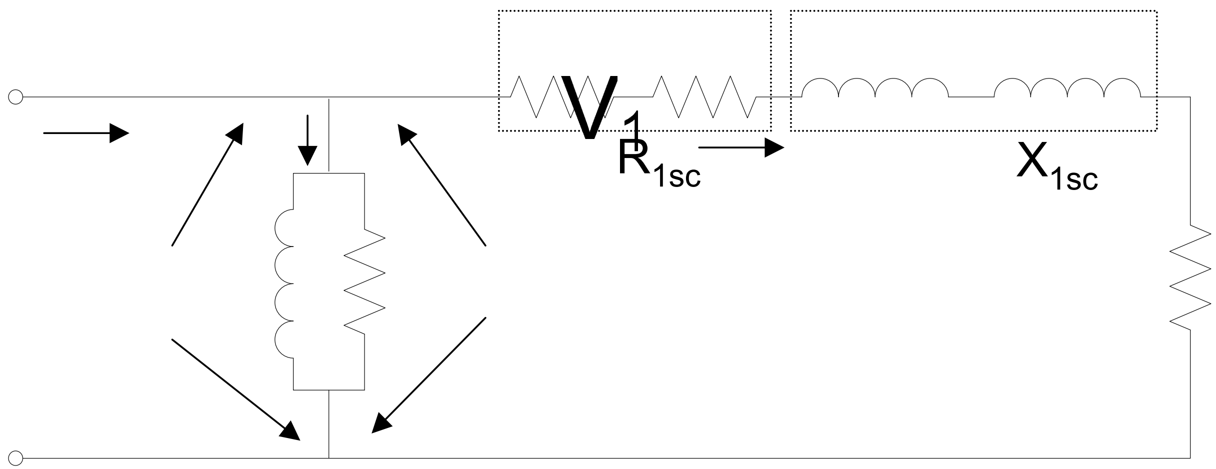
R_1 X_1

1/3 Simplified equivalent circuit (R_m >> R_1,R_2' X_m >> X_1,X_2')

P_{motor} > 10 kW

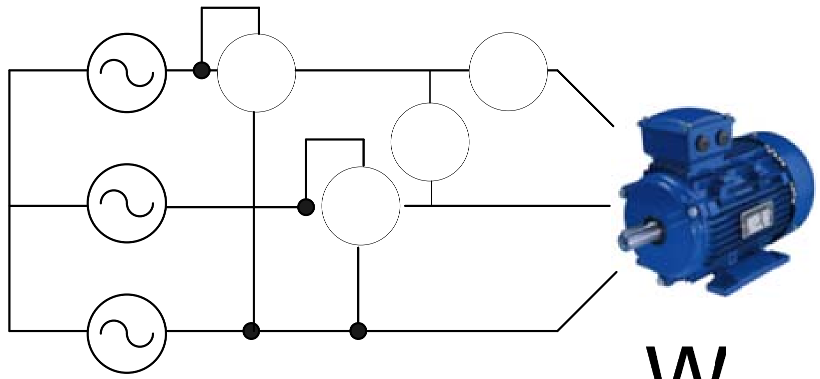
I_o

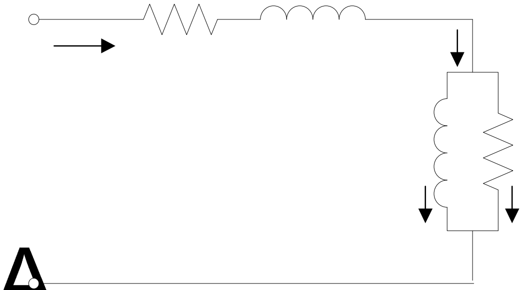
I_1



V_1 E_1 X_m

NO LOAD TEST (Low current, rated voltage)





No load $s \approx 0 \Rightarrow R_L' = R_2'(1/s-1) \rightarrow \infty$

1.- Measure R_1

2.- Feed the IM at different V_1 (V_{1n} - $0.4V_{1n}$)

3.- Linear regression $\rightarrow$ P_{mec}= ct and $P_{iron} \sim V_1^2$

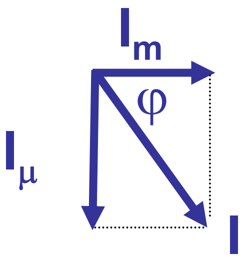
4.- $\cos\varphi_0 = \dfrac{P_{iron,V1n}}{3V_{1p,n} I_{0p}}$

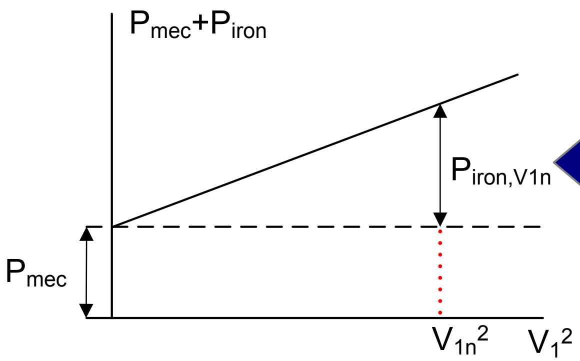
5.- $I_m = I_{0,p}\cos\varphi_0 \qquad I_\mu = I_{0,p}\mathrm{sen}\varphi_0$

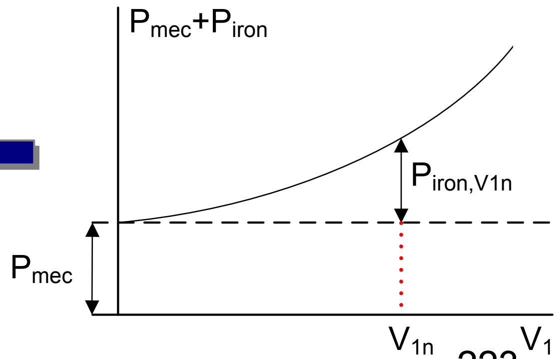
6.- $R_m = \dfrac{V_{1p,n}}{I_m} \qquad X_m = \dfrac{V_{1p,n}}{I_\mu}$

$$W_1 + W_2 = P_{iron} + P_{Cu1} + P_{mec}$$

$$\underbrace{P_{iron}}_{\sim V_1^2} + \underbrace{P_{mec}}_{\sim ct} = W_1 + W_2 - P_{Cu1} = W_1 + W_2 - 3R_1 I_{0p}^2$$



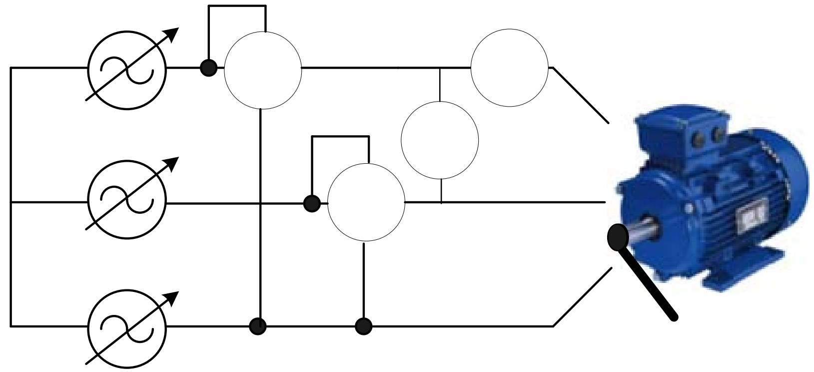


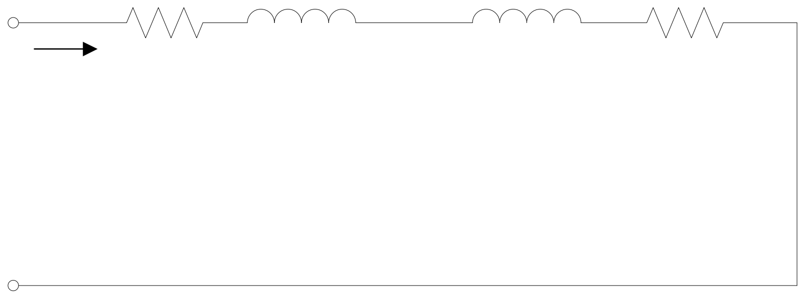


Linear regression

BLOCKED ROTOR TEST (Low voltage, rated current)

Test realized at very low voltage starting from $V_1 = 0$





W1 A V W2

1.- Measure R_1

2.- $P_{sc} = W_1 + W_2 = 3I_{1n}^2(R_1 + R_2') \rightarrow R_2' = \dfrac{W_1 + W_2}{3I_{1n}^2} - R_1$

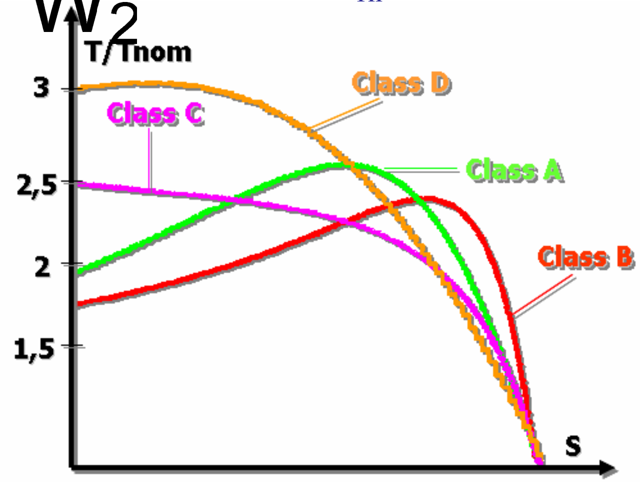
3.- $Q_{sc} = \sqrt{3}(W_1 - W_2) = 3I_{1n}^2(X_1 + X_2') \rightarrow X_1 + X_2' = \dfrac{\sqrt{3}(W_1 - W_2)}{3I_{1n}^2}$

NEMA A: $X_1 = X_2'$

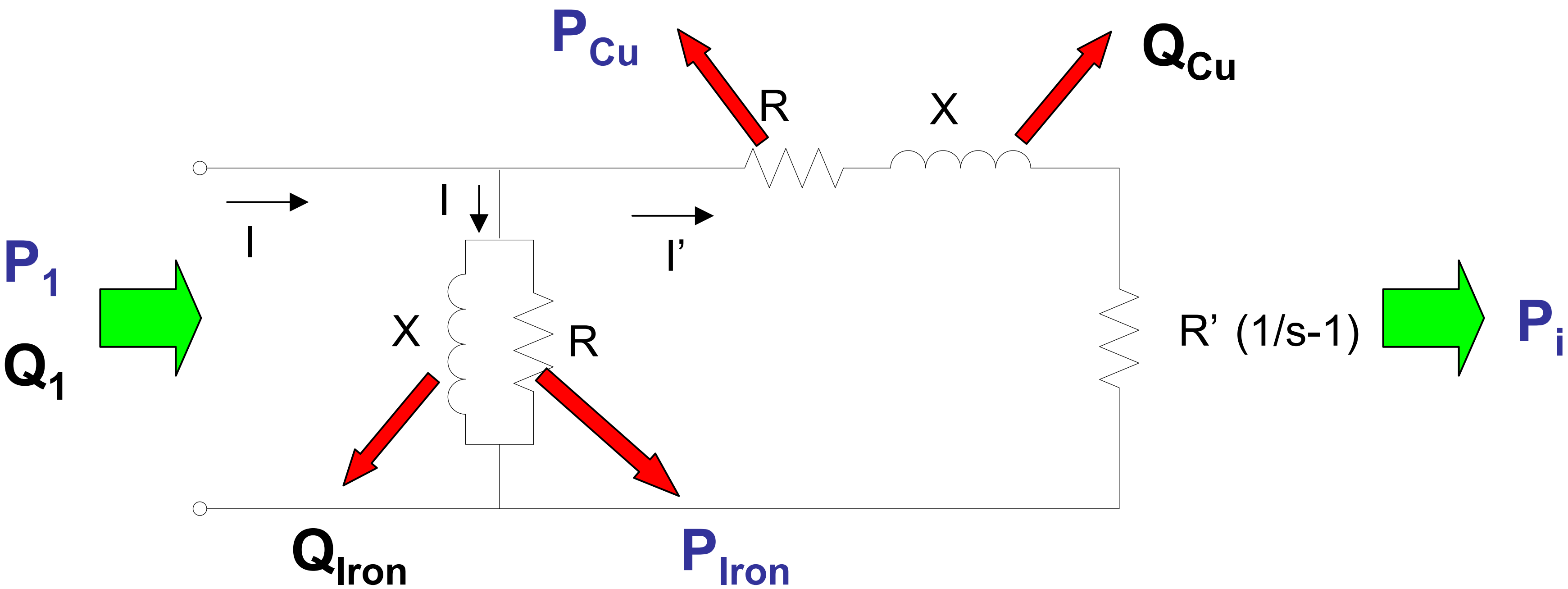
NEMA B: $X_1 = 0.4(X_1 + X_2')$

NEMA C: $X_1 = 0.3(X_1 + X_2')$

NEMA D: $X_1 = X_2'$



POWER LOSSES AND EFFICIENCY



$$\text{Input electrical power}: P_1 = 3V_{1,\text{phase}} I_{1,\text{phase}} \cos\varphi$$

$$\text{Iron losses}: P_{\text{iron}} = 3V_{1,\text{phase}}^2 / R_m$$

$$\text{Copper losses}: P_{Cu} = 3I'^2_{2,\text{phase}} \, R_{1sc}$$

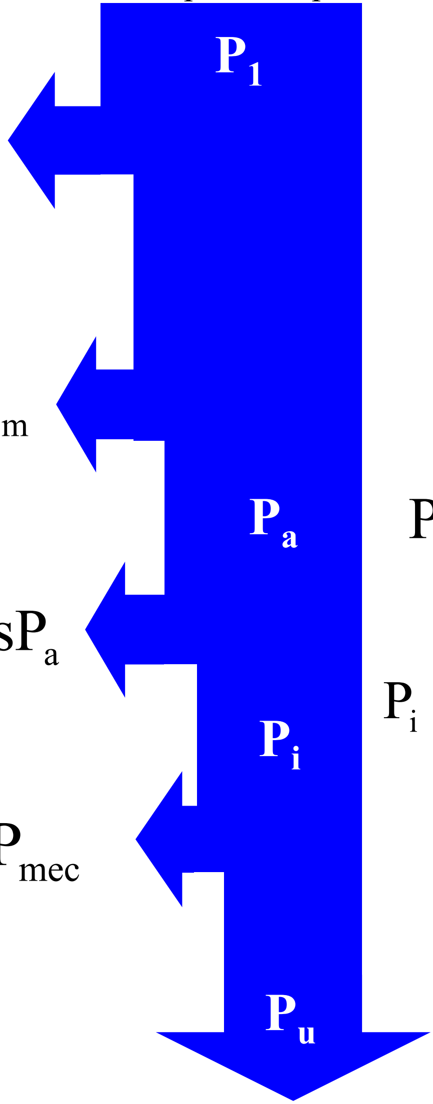
$$\text{Internal power}: P_i = T_i \cdot \omega = 3I'^2_2 \, R'_2 \left(\frac{1}{s} - 1\right)$$

$$\text{Mechanical losses (friction and forced ventilation)}: P_{\text{mec}}$$

$$\eta = \frac{P_{\text{useful}}}{P_1} = \frac{P_1 - P_{\text{iron}} - P_{Cu} - P_{\text{mec}}}{P_1}$$

POWER LOSSES AND EFFICIENCY

$$P_1 = 3V_{1,phase} I_{1,phase} \cos\varphi$$



$$\eta = \frac{P_{useful}}{P_1}$$

$$P_{Cu,1} = 3I^2_{1,phase} R_1$$

$$P_{iron} = 3V^2_{1,phase} / R_m$$

$$P_a = P_1 - P_{Cu,1} - P_{iron}$$

$$P_{Cu,2} = 3I'^2_{2,phase} R'_2 = sP_a$$

$$P_i = T_i.\omega = 3I'^2_{2,phase} R'_2 \left(\frac{1}{s} - 1\right) = \underbrace{3I'^2_{2,phase} R'_2}_{P_{Cu,2}} \frac{1-s}{s}$$

$$= (1-s)P_a$$

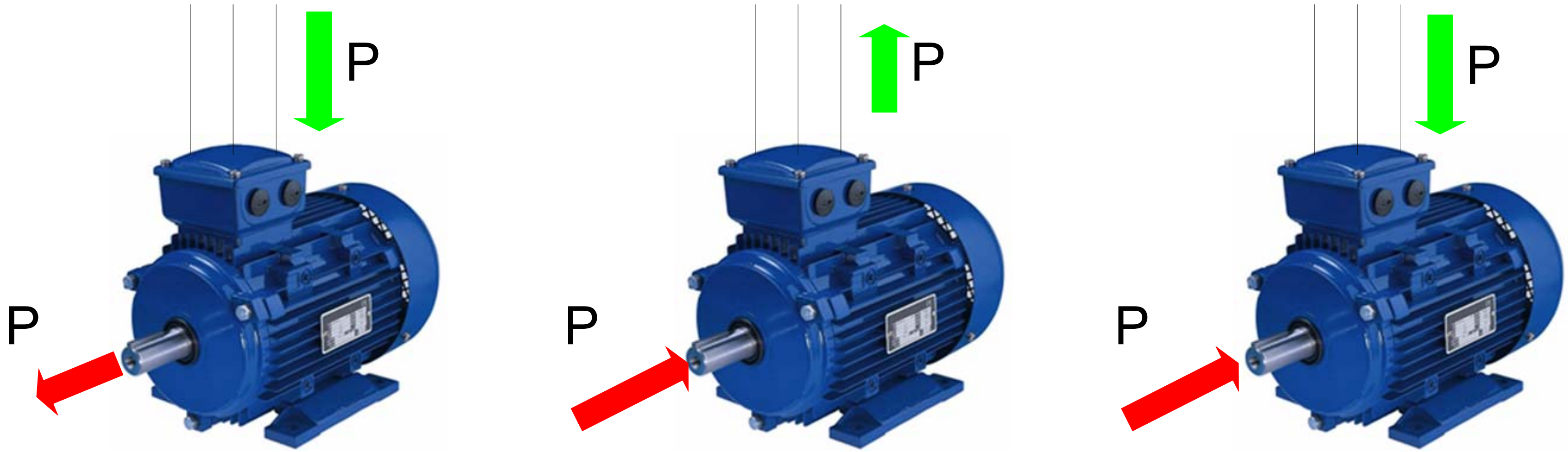
$$P_{mec}$$

$$P_u = T_u.\omega$$

OPERATIONAL MODES OF AN IM

The IM can operate as a:

1) Motor
2) Generator
3) Brake



MOTOR	GENERATOR	BRAKE
$0 \leq n < n_0$	$n > n_0$	n and n_0 opposite sign
$s \in (0,1)$	$s < 0$	$s > 1$

R S T

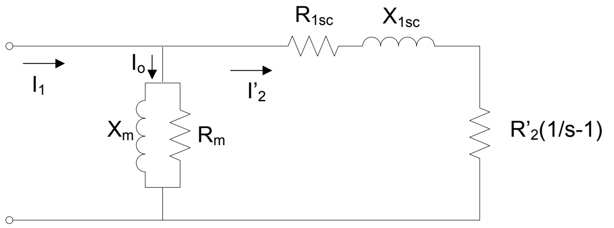
R S

elec

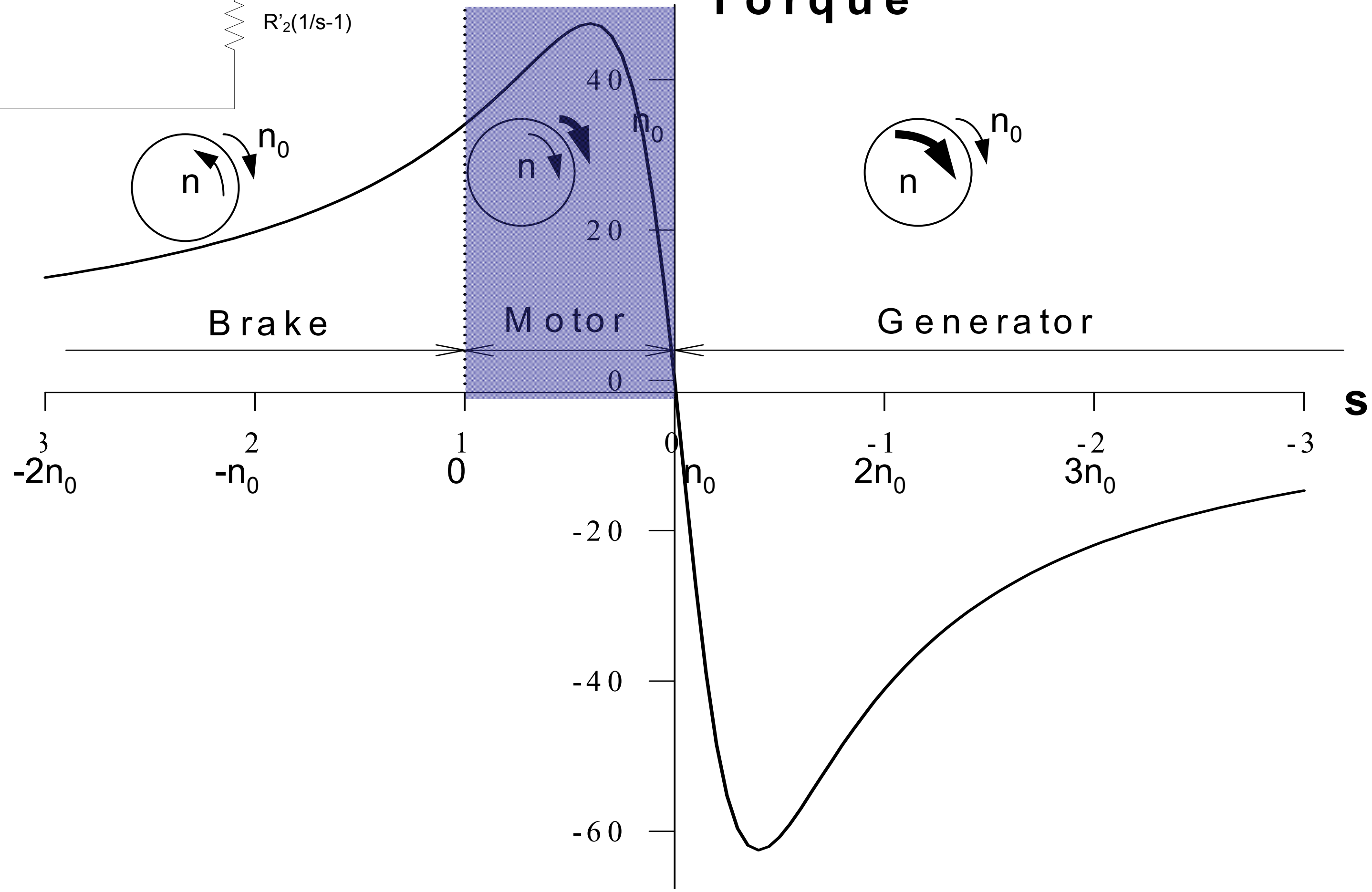
TORQUE

$$P_i = T_i . \omega = 3 I'^2_2 R'_2 \left(\frac{1}{s} - 1\right)$$

$$I_2' = \frac{V_{1,phase}}{\sqrt{(R_1 + R_2'/s)^2 + X_{1sc}^2}}$$

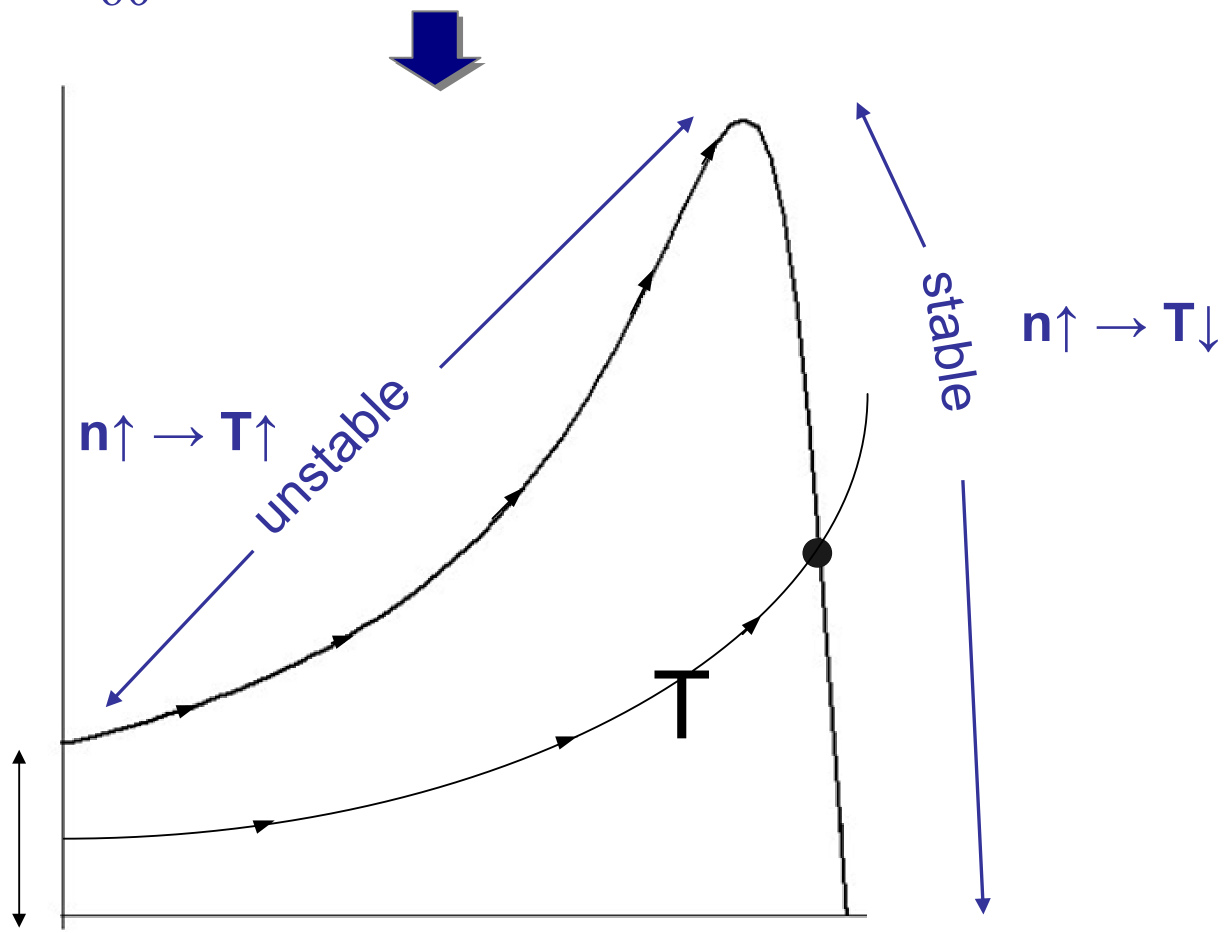


$$T_i = \frac{3 \cdot R_2' \cdot \frac{V_{1,phase}^2}{s}}{\frac{2\pi}{60} n_0 [(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$



TORQUE: OPERATING AS A MOTOR

$$T_i = \frac{3 \cdot R_2' \cdot \frac{V_{1,phase}^2}{s}}{\frac{2\pi}{60} n_0 [(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$

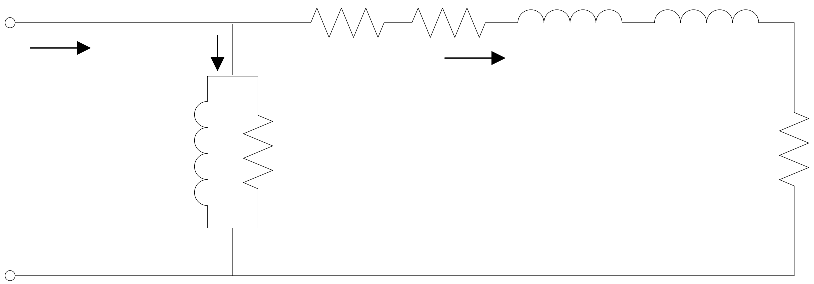


START UP TORQUE

Starting ➡ **s = 1** ➡

$$T_{i,start} = \frac{3 \cdot R_2' \cdot \frac{V_{1,phase}^2}{1}}{\frac{2\pi}{60} n_0 [(R_1 + R_2'/1)^2 + (X_1 + X_2')^2]}$$

$$T_{i,start} = \frac{3 \cdot R_2' \cdot V_{1,phase}^2}{\frac{2\pi}{60} n_0 [(R_1 + R_2')^2 + (X_1 + X_2')^2]}$$



$$I'_{2,start} = \frac{V_{1,phase}}{\sqrt{[(R_1 + R_2')^2 + (X_1 + X_2')^2]}}$$

$$T_{i,start} = \frac{3}{\frac{2\pi}{60} n_0} \cdot R_2' I'^2_{2,start}$$

$$\mathbf{R_2' \uparrow \rightarrow T_{i,start} \uparrow}$$

TORQUE: MAXIMUM TORQUE

$$T_i = \frac{3 \cdot R_2' \cdot \dfrac{V_{1,phase}^2}{s}}{\dfrac{2\pi}{60} n_0 [(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$

$$s_{Ti,max} = \pm \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}}$$

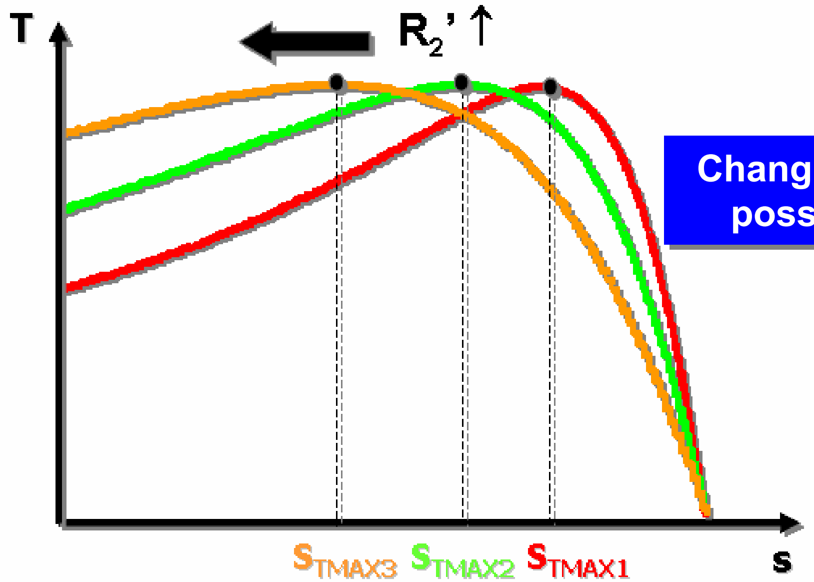
$$T_{i,max} = \pm \frac{3 \cdot V_{1,phase}^2}{2 \cdot \dfrac{2\pi}{60} n_0 [\pm R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}]}$$

+: motor

-: generator

s_{Tmax} depends of R_2'

The maximum torque is independent of R_2'



Changing R_2' (wound rotor) it is possible to shift T- s curves

R_2'↑
- Starting torque ↑
- Efficiency ↓

IMs EFFICIENCY

Efficiency limit values according to IEC 60034-30 (2008)

Output kw	IE1 Standard efficiency			IE2 High efficiency			IE3 Premium efficiency		
	2 pole	4 pole	6 pole	2 pole	4 pole	6 pole	2 pole	4 pole	6 pole
0.75	72.1	72.1	70.0	77.4	79.6	75.9	80.7	82.5	78.9
1.1	75.0	75.0	72.9	79.6	81.4	78.1	82.7	84.1	81.0
1.5	77.2	77.2	75.2	81.3	82.8	79.8	84.2	85.3	82.5
2.2	79.7	79.7	77.7	83.2	84.3	81.8	85.9	86.7	84.3
3	81.5	81.5	79.7	84.6	85.5	83.3	87.1	87.7	85.6
4	83.1	83.1	81.4	85.8	86.6	84.6	88.1	88.6	86.8
5.5	84.7	84.7	83.1	87.0	87.7	86.0	89.2	89.6	88.0
7.5	86.0	86.0	84.7	88.1	88.7	87.2	90.1	90.4	89.1
11	87.6	87.6	86.4	89.4	89.8	88.7	91.2	91.4	90.3
15	88.7	88.7	87.7	90.3	90.6	89.7	91.9	92.1	91.2
18.5	89.3	89.3	88.6	90.9	91.2	90.4	92.4	92.6	91.7
22	89.9	89.9	89.2	91.3	91.6	90.9	92.7	93.0	92.2
30	90.7	90.7	90.2	92.0	92.3	91.7	93.3	93.6	92.9
37	91.2	91.2	90.8	92.5	92.7	92.2	93.7	93.9	93.3
45	91.7	91.7	91.4	92.9	93.1	92.7	94.0	94.2	93.7
55	92.1	92.1	91.9	93.2	93.5	93.1	94.3	94.6	94.1
75	92.7	92.7	92.6	93.8	94.0	93.7	94.7	95.0	94.6
90	93.0	93.0	92.9	94.1	94.2	94.0	95.0	95.2	94.9
110	93.3	93.3	93.3	94.3	94.5	94.3	95.2	95.4	95.1
132	93.5	93.5	93.5	94.6	94.7	94.6	95.4	95.6	95.4
160	93.7	93.8	93.8	94.8	94.9	94.8	95.6	95.8	95.6
200	94.0	94.0	94.0	95.0	95.1	95.0	95.8	96.0	95.8
250	94.0	94.0	94.0	95.0	95.1	95.0	95.8	96.0	95.8
315	94.0	94.0	94.0	95.0	95.1	95.0	95.8	96.0	95.8
355	94.0	94.0	94.0	95.0	95.1	95.0	95.8	96.0	95.8
375	94.0	94.0	94.0	95.0	95.1	95.0	95.8	96.0	95.8

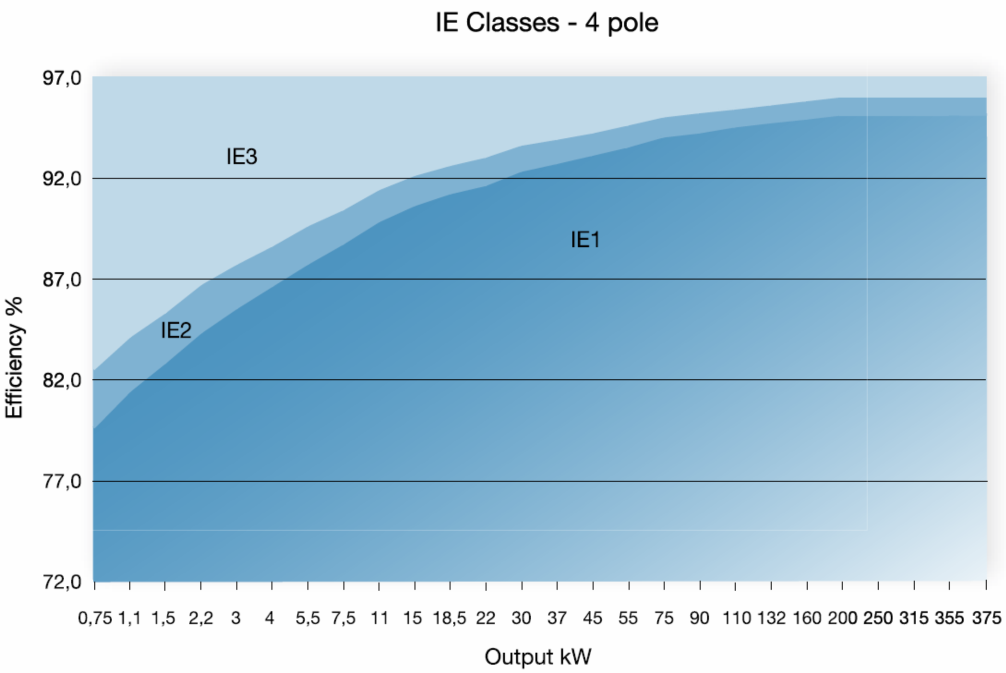


ABB motor guide

IMs CHARACTERISTICS

IP 55 – IC 411 – Insulation class F, temperature rise class B
IE2 efficiency class according to IEC 60034-30, 2008

Measured at rated current

Output kW	Motor type	Speed r/min	Efficiency IEC 60034-2-1; 2007 Full load 100%	Efficiency IEC 60034-2; 1996 Full load 100%	Power factor cos φ 100%	Current I_N A	Speed r/min	Efficiency IEC 60034-2-1; 2007 Full load 100%	Efficiency IEC 60034-2; 1996 Full load 100%	Power factor cos φ 100%	Current I_N A	Moment of inertia J = 1/4 GD^2 kgm^2	Weight kg	Sound pressure level L_P dB(A)
3000 r/min = 2-poles		**380 V 50 Hz**					**415 V 50 Hz**				**CENELEC design**			
0.18	**M3AA 63 A**	2815	74.6	75.1	0.69	0.53	2830	72.5	72.9	0.60	0.58	0.00013	3.9	54
0.25	**M3AA 63 B**	2800	78.5	79.0	0.75	0.64	2830	76.2	76.6	0.67	0.69	0.00016	4.4	54
0.37	**M3AA 71 A**	2740	74.1	74.4	0.84	0.9	2815	73.9	74.2	0.74	0.95	0.00035	4.9	58
0.55	**M3AA 71 B**	2765	75.5	75.9	0.85	1.3	2820	75.7	76.1	0.75	1.35	0.00045	5.9	58
0.75	**M3AA 80 A**	2795	76.5	76.8	0.85	1.85	2835	76.5	76.8	0.77	1.75	0.00069	8.5	60
1.1	**M3AA 80 C**	2875	82.0	83.1	0.83	2.4	2905	82.3	83.2	0.76	2.4	0.0011	11	60
1.5	**M3AA 90 L**	2885	84.0	84.4	0.90	3.05	2905	84.2	85.2	0.87	2.8	0.0024	16	63
2.2	**M3AA 90 LB**	2865	84.0	85.6	0.90	4.65	2890	84.8	86.2	0.85	4.25	0.0027	18	63
3	**M3AA 100 LB**	2920	87.0	87.3	0.89	6.2	2935	87.0	87.5	0.85	5.9	0.005	25	62
4	**M3AA 112 MB**	2860	86.4	87.5	0.92	7.6	2895	87.3	88.1	0.92	6.95	0.0062	30	68
5.5	**M3AA 132 SB**	2905	87.6	88.3	0.89	11.2	2925	88.0	89.0	0.86	10.4	0.016	42	73
7.5	**M3AA 132 SC**	2890	88.5	88.5	0.92	14.4	2920	88.8	90.2	0.93	12.14	0.022	56	73
11	**M3AA 160 MLA**	2920	89.3	90.4	0.91	20.5	2935	90.1	91.2	0.90	18.6	0.045	91	69
15	**M3AA 160 MLB**	2925	90.3	91.3	0.92	27.5	2938	90.9	91.9	0.90	25.5	0.049	98	69
18.5	**M3AA 160 MLC**	2928	91.0	92.1	0.91	33.5	2941	91.4	92.5	0.88	32	0.054	106	69
22	**M3AA 180 MLA**	2944	91.3	92.3	0.90	40.5	2953	91.7	92.7	0.86	38.5	0.078	132	69
30	**M3AA 200 MLA**	2946	92.0	93.1	0.90	55	2956	92.5	93.6	0.89	50	0.163	198	72
37	**M3AA 200 MLB**	2942	92.4	93.5	0.90	67	2953	92.9	94.0	0.89	62	0.181	211	72
45	**M3AA 225 SMA**	2960	93.4	94.2	0.88	83	2968	93.7	94.5	0.87	77	0.25	264	74
55	**M3AA 250 SMA**	2964	93.7	94.4	0.88	101	2971	94.0	94.7	0.87	93	0.517	305	75
75	**M3AA 280 SMA**	2965	94.3	95.0	0.89	136	2972	94.6	95.3	0.88	125	0.593	390	75
90	**M3AA 280 SMB**	2967	94.4	95.3	0.89	162	2973	94.7	95.6	0.88	150	0.654	4[illegible]	75

IMs CHARACTERISTICS

IP 55 – IC 411 – Insulation class F, temperature rise class B
IE2 efficiency class according to IEC 60034-30, 2008

Output kW	Motor type	Speed r/min	Efficiency IEC 60034-2-1; 2007 Full load 100%	Efficiency IEC 60034-2; 1996 Full load 100%	Power factor cos φ 100%	Current I_N A	Speed r/min	Efficiency IEC 60034-2-1; 2007 Full load 100%	Efficiency IEC 60034-2; 1996 Full load 100%	Power factor cos φ 100%	Current I_N A	Moment of inertia J = 1/4 GD² kgm²	Weight kg	Sound pressure level L_P dB(A)
3000 r/min = 2-poles			**380 V 50 Hz**				**415 V 50 Hz**			**CENELEC design**				
75	**M2CA 280 SA**	2974	94.1	94.8	0.89	137	2980	94.2	94.8	0.87	127	0.8	480	77
90	**M2CA 280 SMA**	2970	94.4	95.1	0.90	159	2978	94.5	95.1	0.89	147	0.9	545	77
110	**M2CA 315 SA**	2980	94.6	95.1	0.87	202	2983	94.6	95.1	0.85	190	1.2	695	80
132	**M2CA 315 SMA**	2980	95.0	95.4	0.89	238	2983	95.0	95.7	0.87	222	1.4	770	80
160	**M2CA 315 MB**	2979	95.3	96.1	0.90	282	2982	95.3	96.1	0.89	262	1.7	840	80
200 [2)]	**M2CA 315 LA**	2977	95.5	96.3	0.90	350	2981	95.6	96.3	0.90	321	2.1	975	80
200	**M2CA 355 SA**	2975	95.4	95.4	0.90	353	2980	95.6	95.5	0.89	327	2.5	1200	83
250	**M2CA 355 MA**	2981	96.0	95.9	0.90	439	2984	96.2	96.0	0.88	410	2.7	1260	83
280 [2)]	**M2CA 355 MB**	2978	96.0	96.0	0.90	492	2982	96.1	96.1	0.88	460	2.7	1260	83
315 [2)]	**M2CA 355 LA**	2978	96.3	96.3	0.89	560	2982	96.4	96.4	0.89	515	3.4	1480	83
355 [2)]	**M2CA 355 LB**	2981	96.5	96.5	0.89	627	2985	96.5	96.5	0.87	585	3.5	1520	83
400 [2)]	**M2CA 400 MLA**	2983	96.7	96.7	0.89	705	2986	96.8	96.8	0.88	652	6.3	2050	85
450 [2)]	**M2CA 400 MLB**	2986	97.0	96.9	0.90	782	2988	97.0	96.9	0.89	724	6.9	2150	85
500 [2)]	**M2CA 400 LKA**	2986	97.1	97.0	0.91	858	2988	97.2	97.0	0.89	802	7.8	2450	85
560 [2)]	**M2CA 400 LKB**	2986	97.2	97.2	0.90	980	2989	97.2	97.2	0.88	910	7.9	2500	85

START UP CURRENT

- The instruction ITC-BT-47 of the REBT (Low-voltage Spanish Regulation) limits the AC motor's maximum start up current as:

Motor rated power	$I_{màx,start\ up}/I_{rated}$
0.75 kW – 1.5 kW	4.5
(1.5 kW, 5.0 kW]	3.0
(5.0 kW,15.0 kW]	2.0
> 15.0 kW	1.5

It is necessary to limit the start up current

HOW TO OBTAIN VARIABLE SPEED

$$n = 60 \cdot \frac{f}{p}(1 - s) \quad (\text{r.p.m.})$$

- **n** depends on three variables: **f**, **p** and **s**
- **s** is no practical to change
- **p** (number of poles pairs) has an integer value: 1,2,3,4,…
  For example: if f = 50 Hz n = 3000, 1500, 1000, 750, 600 rpm
- The most effective way to change motor speed is changing frequency **f** (POWER ELECTRONICS, 80s decade)

STARTING METHODS

Direct starting	Low power motors
Wye-delta starting	The cheaper and widely used reduced voltage starting system
Autotransformer starting	Voltage reduction during starting
Static soft-starting	An electronic equipment governs the motor during starting
Variable R in the rotor circuit	Only for wound-rotor induction motors

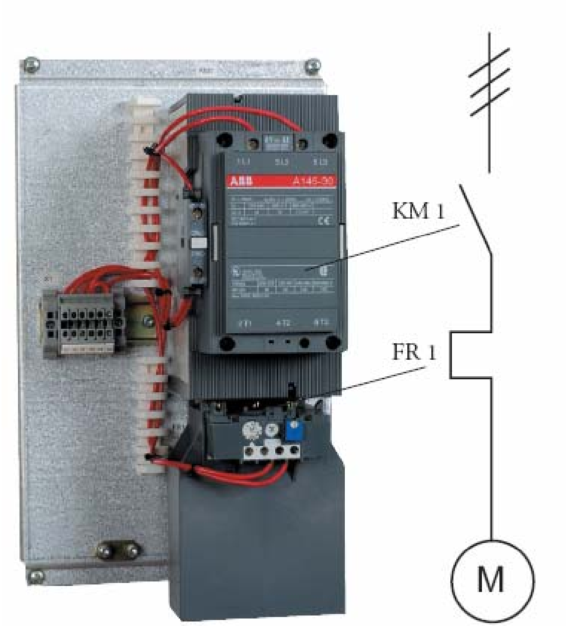
Reduced voltage starting methods

- Wye-delta and solid-state method are the simplest methods to apply.
- Over the years the dominant voltage reduction method world wide has been the wye-delta starting technique.

DIRECT-ON-LINE STARTING (DOL)

- **Short-circuit protection**
- **Overload protection**

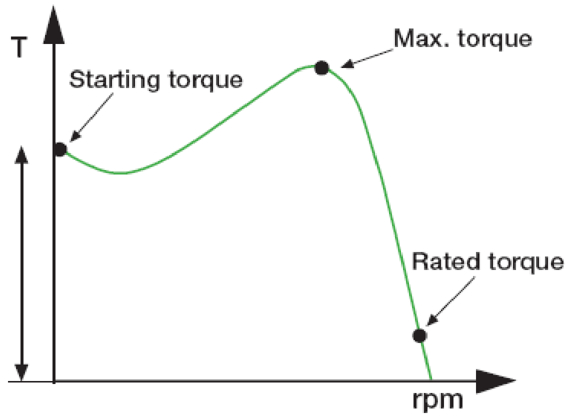
The disadvantage with this method is that it gives the highest possible starting current.

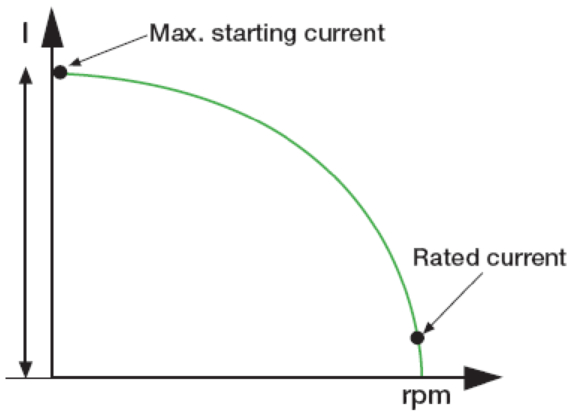


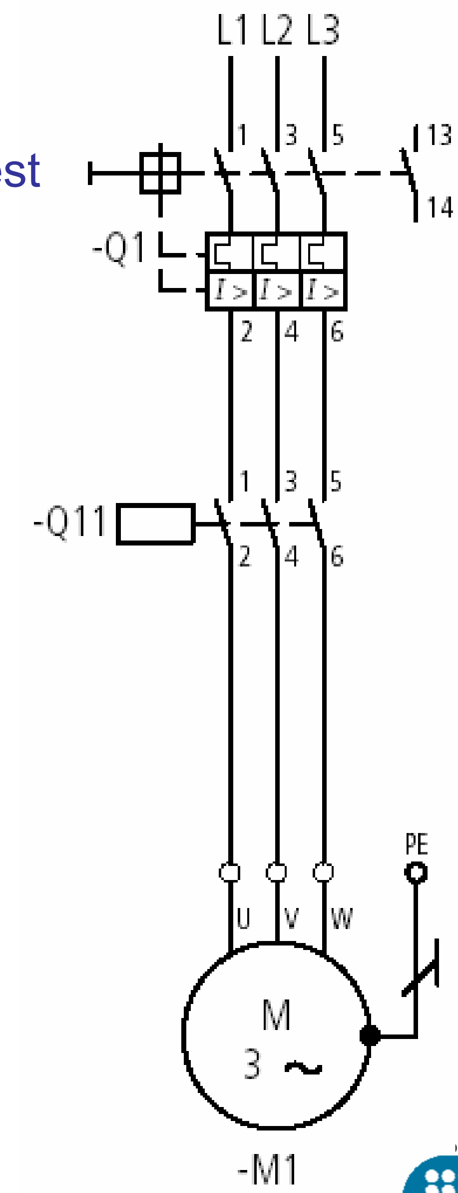
D.O.L. starter with contactor and O/L relay

Single line diagram for a D.O.L.

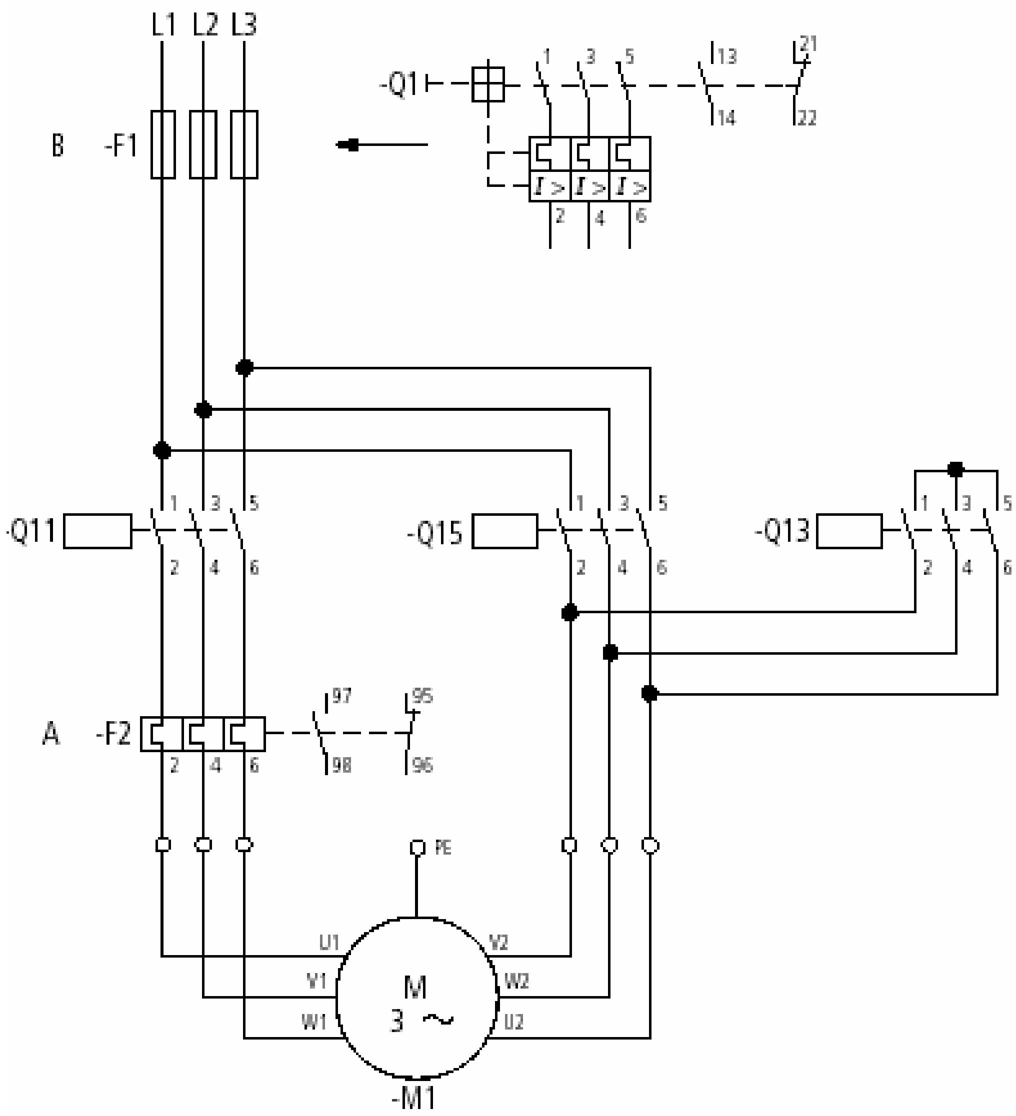
KM 1 Main contactor
FR 1 Overload relay





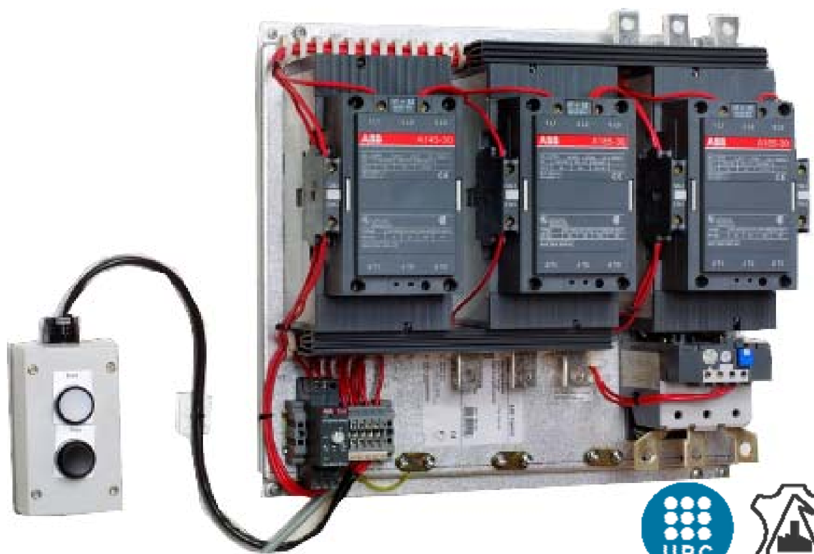


WYE-DELTA STARTING



- **Short-circuit protection**
- **Overload protection**

- Starting method that reduces the starting current and starting torque.
- Consists of three contactors, an overload relay and a timer for setting the time in the star-position (starting position).
- The motor must be delta connected during a normal run.



WYE-DELTA STARTING

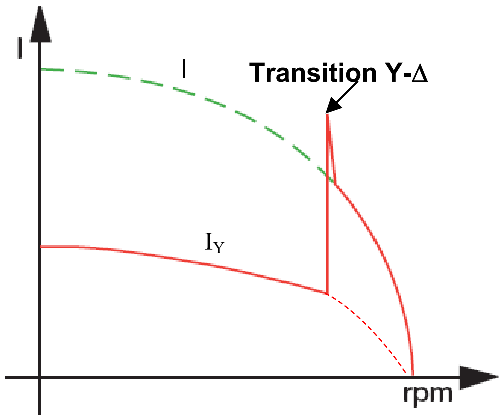
- Lowest cost electromechanical reduced voltage starter that can be applied
- Disadvantage: it requires more panel space and, more components (3 contactors and a timer relay)
- Only works when the application is light loaded during the start. If the motor is too heavily loaded, there will not be enough torque to accelerate the motor up to speed before switching over to the delta position.
- Useful for starting up pumps and fans (low load torque during starting, T ~ n^2).

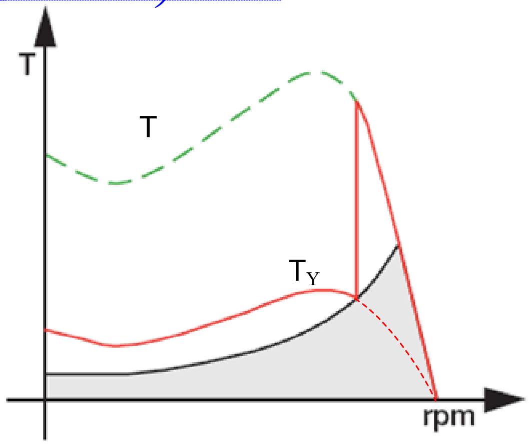
Starting Current Reduction

$$V_{phase,Y} = \frac{V_{phase,\Delta}}{\sqrt{3}} \rightarrow I_{phase,Y} = \frac{I_{phase,\Delta}}{\sqrt{3}} \rightarrow I_{line,Y} = \frac{I_{phase,\Delta}}{\sqrt{3}} = \frac{I_{line,\Delta}/\sqrt{3}}{\sqrt{3}} = \frac{I_{line,\Delta}}{3}$$

$$V_{phase,Y} = \frac{V_{phase,\Delta}}{\sqrt{3}} \rightarrow \frac{T_Y}{T_\Delta} = \left(\frac{V_{phase,Y}}{V_{phase,\Delta}}\right)^2 = \left(\frac{V_{phase,\Delta}/\sqrt{3}}{V_{phase,\Delta}}\right)^2 = \frac{1}{3}$$

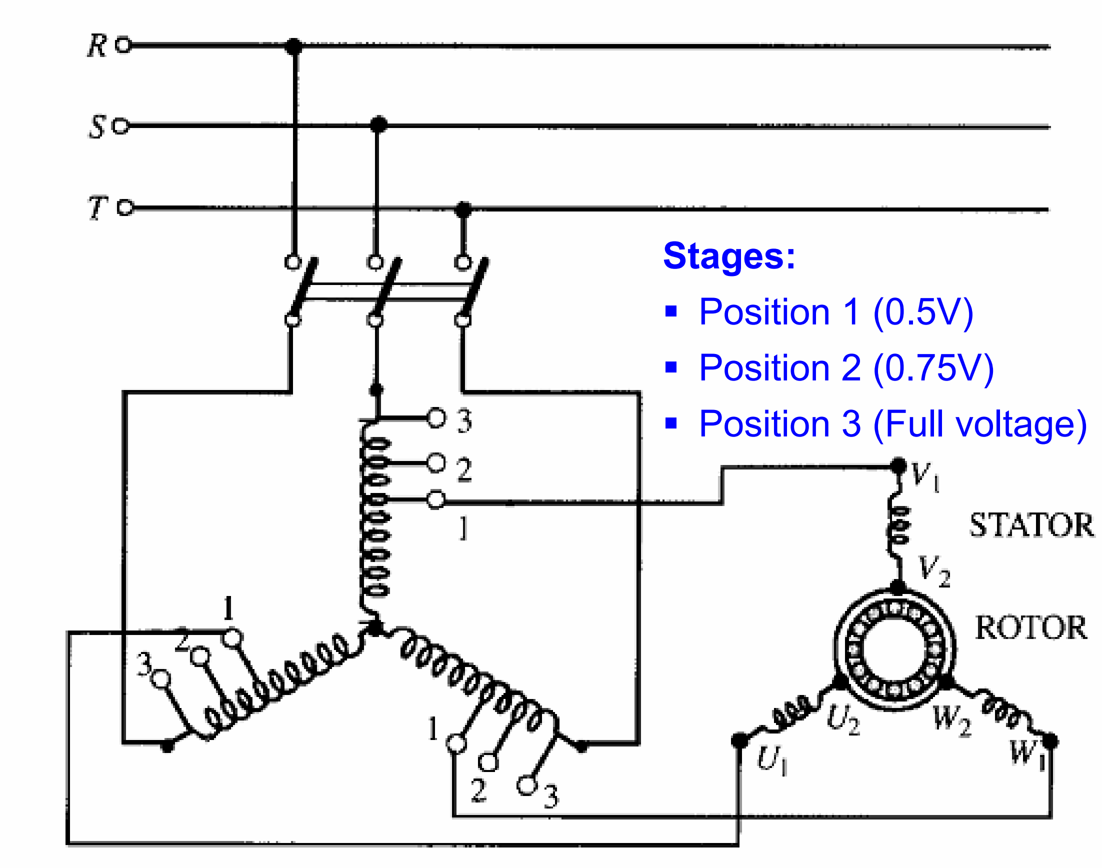
Starting Torque Reduction





AUTOTRANSFORMER STARTING

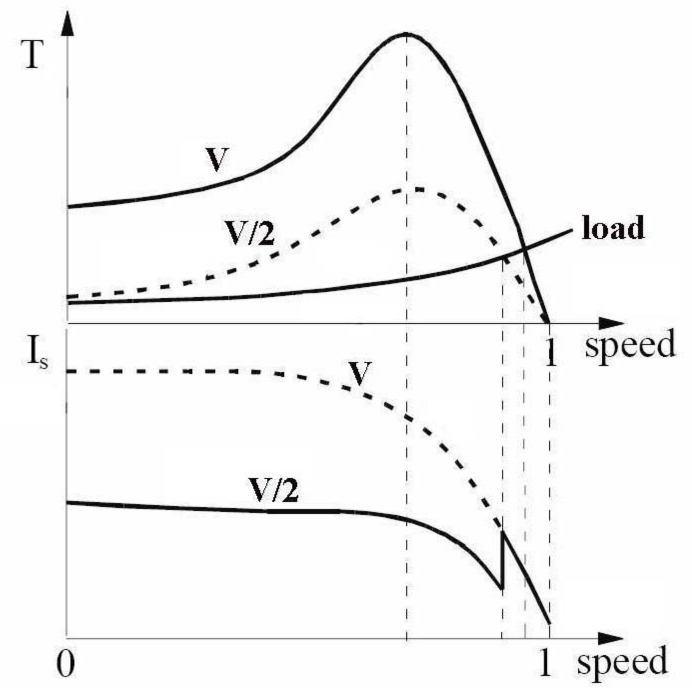
- Autotransformer voltage reduction is adequate only with light starting loads (fan, ventilator, pumps).
- Process can be performed in 2 or 3 steps: the motor is fed through the autotransformer, at the voltage: 0.5, 0.65, 0.8 V_{line}.
- Autotransformers are preferred due to their smaller size



Stages:

- Position 1 (0.5V)
- Position 2 (0.75V)
- Position 3 (Full voltage)

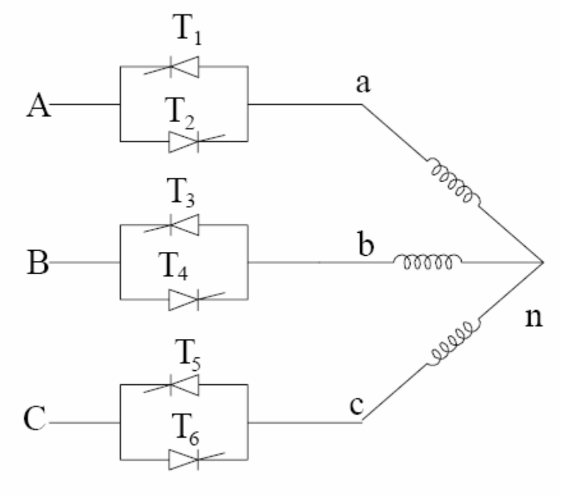
Two-stages autotransformer starting

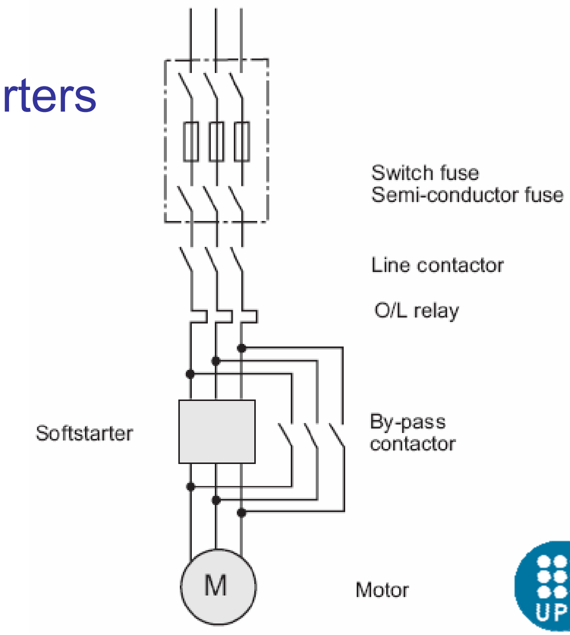


One-stage autotransformer starting

SOFT-STARTING

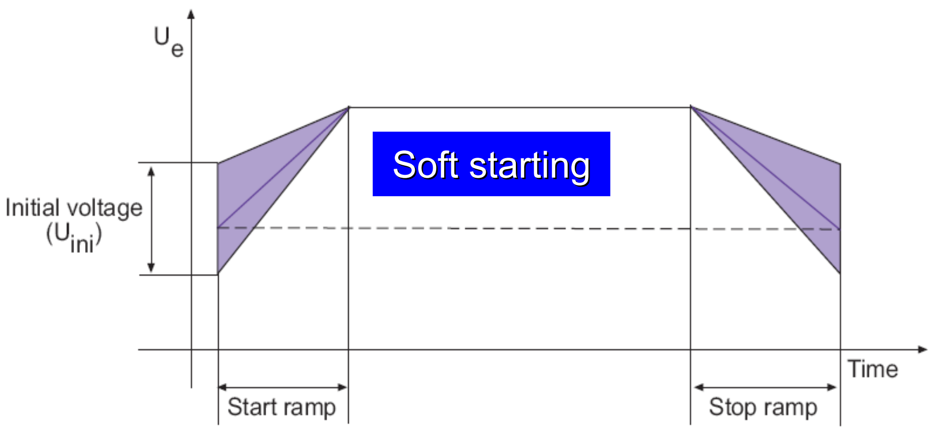
- Solid-state starters offer another technique of reduced voltage starting
- Soft-starters use thyristors and enjoy natural commutation (from the power grid).
- In numerous applications such as fans, pumps, or conveyors, soft-starters are now common practice when speed control is not required.
- Their cost is reasonably low to be competitive.
- During starting, either the stator current or the torque may be controlled.
- The soft starter incorporates a bypass. After the start ends, the bypass is switched on and the soft-starter is isolated.
- Dynamic braking can also be performed by soft-starters

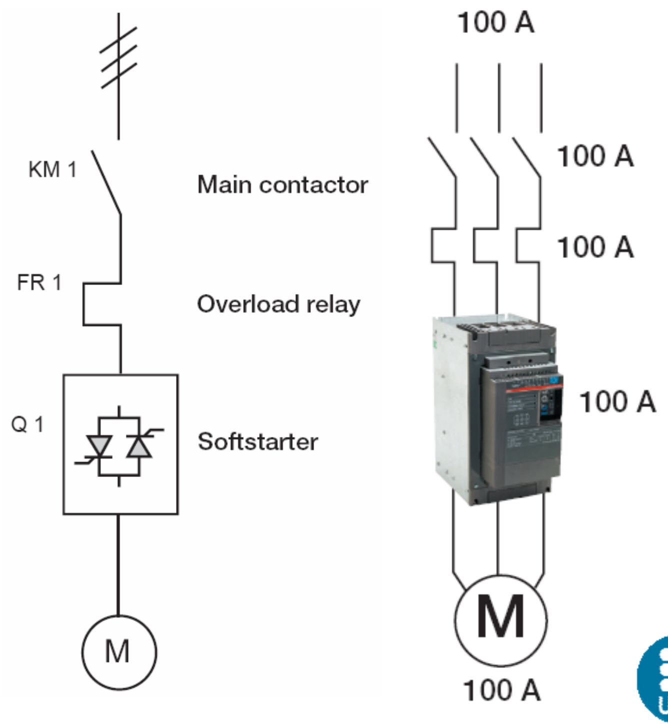




SOFTSTARTING

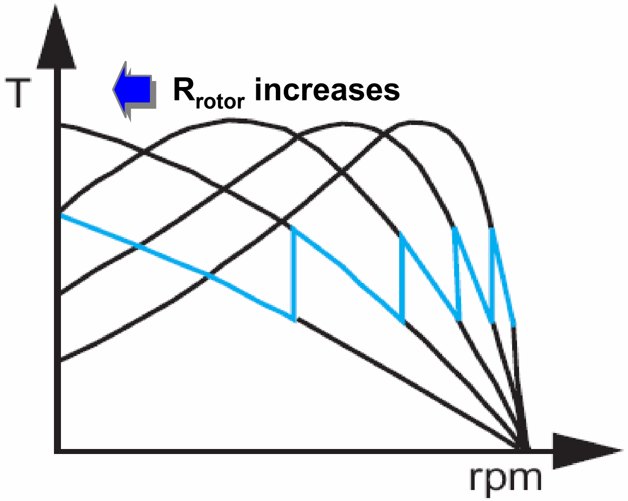
- Soft-starting eliminates unnecessary jerks during the start.
- Gradually, the voltage and the torque increase so that the machinery starts to accelerate.
- Possibility to adjust the torque to the exact need, whether the application is loaded or not.
- It allows soft-stop, which is very useful when stopping pumps where the problem is water hammering in the pipe system at direct stop as for star-delta starter and direct-on-line starter.

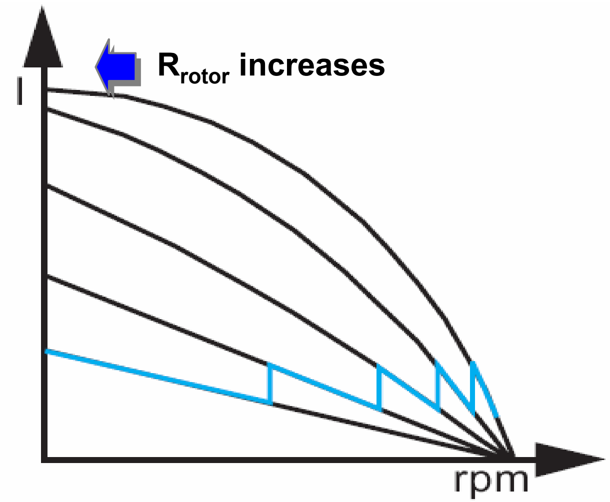




VARIABLE R IN THE ROTOR

- Method applied only in wound rotor with slip-ring motors.
- It is possible to adjust the starting torque up to the maximum torque (Y-Δ starting presents reduced starting torque).
- The motor is started by changing the rotor resistance.
- When speeding up, the resistance is gradually removed until the rated speed is achieved.
- The advantage of a slip-ring motor is that the starting current will be lower and it is possible to adjust the starting torque up to the maximum torque.





VARIABLE SPEED DRIVES (Drives)

- In developed countries, 10% of all induction motor power is converted in variable speed drives applications. The forecast is that, in the next decade, up to 50% of all electric motors will be fed through power electronics
- By controlling the frequency, the rated motor torque is available at a low speed and the starting current is low, between 0.5 and 1.0 times the rated motor current, maximum 1.5 In.
- Drives allow soft-stop.
- It is very common to install a filter together with the drive in order to reduce the levels of emission and harmonics generated.

R
S
T
f = 50 Hz

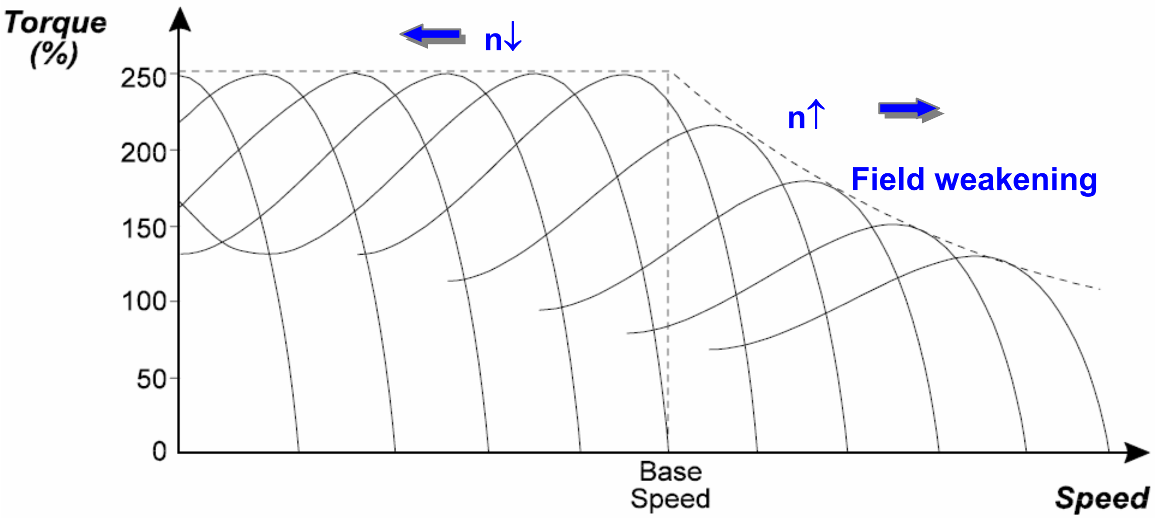
R'
S'
T'
V',f ' variable

Variable speed drive

3-phase induction motor

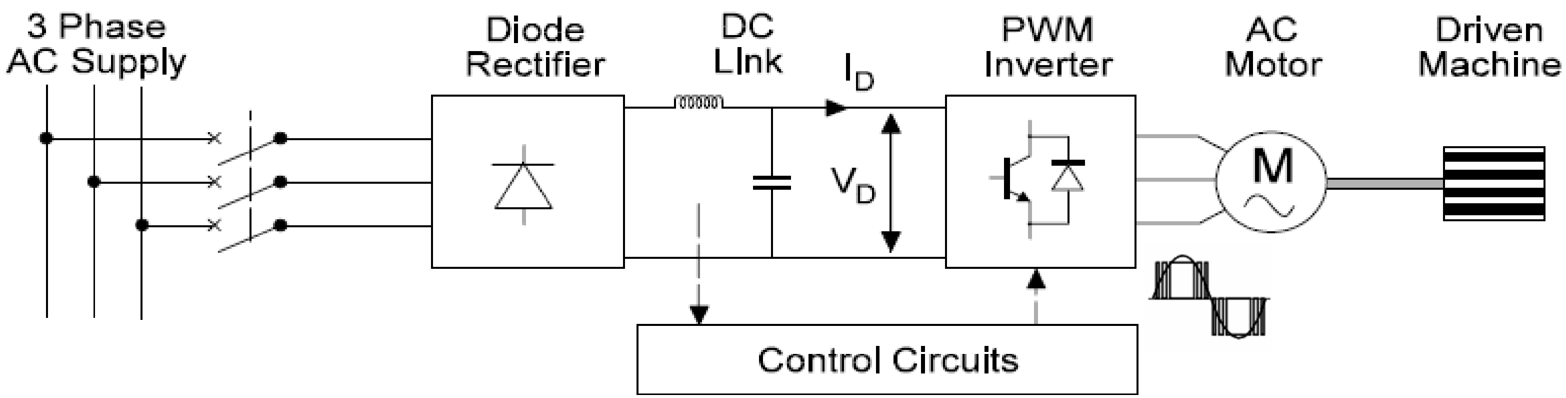
VARIABLE SPEED APPLICATIONS

- The speed of the IM can be controlled by adjusting the frequency and magnitude of the stator voltage.
- Motor speed is proportional to frequency, but the voltage must be simultaneously adjusted to avoid over-fluxing the motor.
- The AC motor is able to develop its full torque over the normal speed range, provided that the flux is held constant, (*V/f* ratio kept constant).
- The speed of an AC induction motor can be increased above its nominal 50 Hz rating, but the *V/f* ratio will fall because the stator voltage cannot be increased any further. This results in a fall of the air-gap flux and a reduction in output torque. As with the DC motor, this is known as the *field weakening* range. The performance of the AC motor in the field weakening range is characterized by constant power, thus reduced torque.



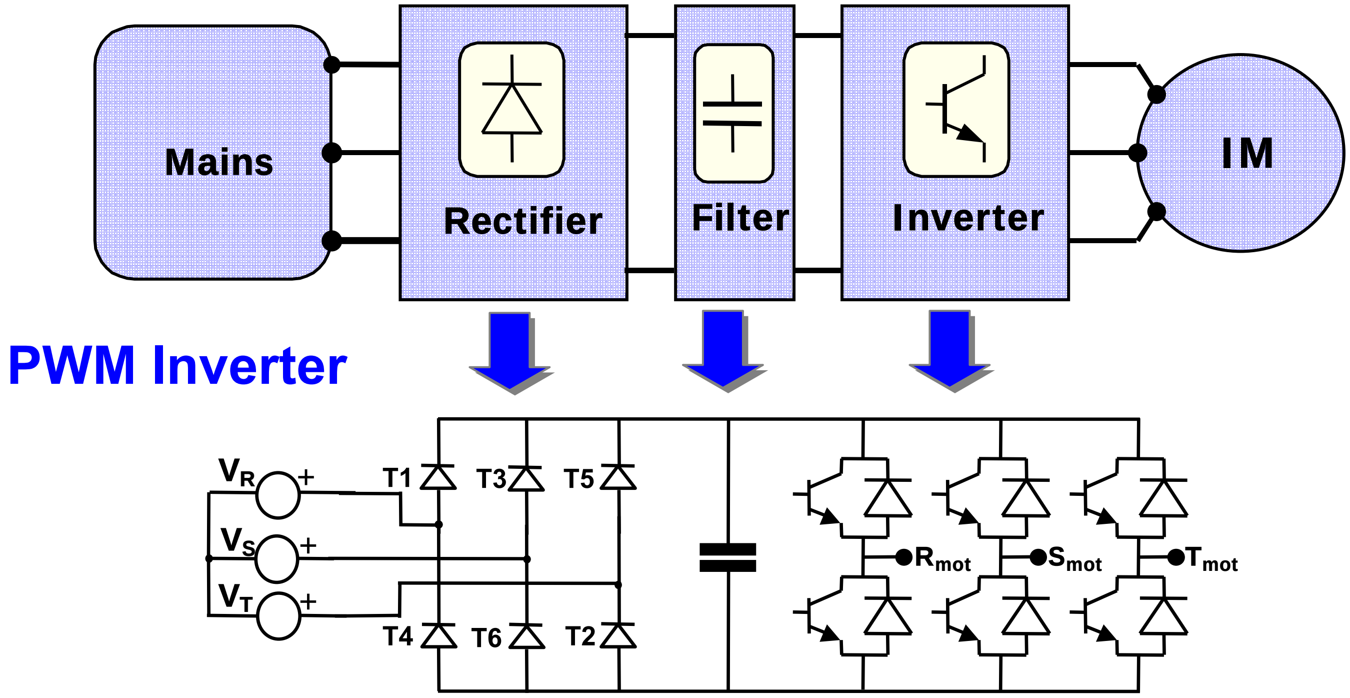
FREQUENCY INVERTER

- Frequency inverters convert constant sinusoidal mains voltage and frequency into a DC voltage.
- From the DC voltage they generate a new three-phase supply with variable voltage and frequency for the three-phase motor.
- The frequency inverter draws almost only active power (p.f. ~ 1) from the supplying mains.
- The reactive power needed for motor operation is supplied by the DC link. This eliminates the need for p.f. correction on the mains side.



FREQUENCY INVERTER

- IGBT's switching is made by means of PWM (Pulse Width Modulation) technique.
- PWM consists of comparing a triangular carrier signal with a sin-wave modulator signal.
- A three-phase stepped sinusoidal-like signal is obtained.
- By changing module and frequency of carrier and modulator signals, waves of variable voltage and frequency are obtained at the inverter's output.



STARTING METHODS COMPARATIVE

	Direct-on-line	Star-delta start	Drives	Softstarter
Slipping belts and heavy wear on bearings	Yes	Medium	No	No
High inrush current	Yes	No	No	No
Heavy wear and tear on gear boxes	Yes	Yes (loaded start)	No	No
Damaged goods / products during stop	Yes	Yes	No	No
Water hammering in pipe system when stopping	Yes	Yes	Best solution	Reduced
Transmission peaks	Yes	Yes	No	No

Drives = variable speed drive

Auto transformer start has similar problems to the star-delta start

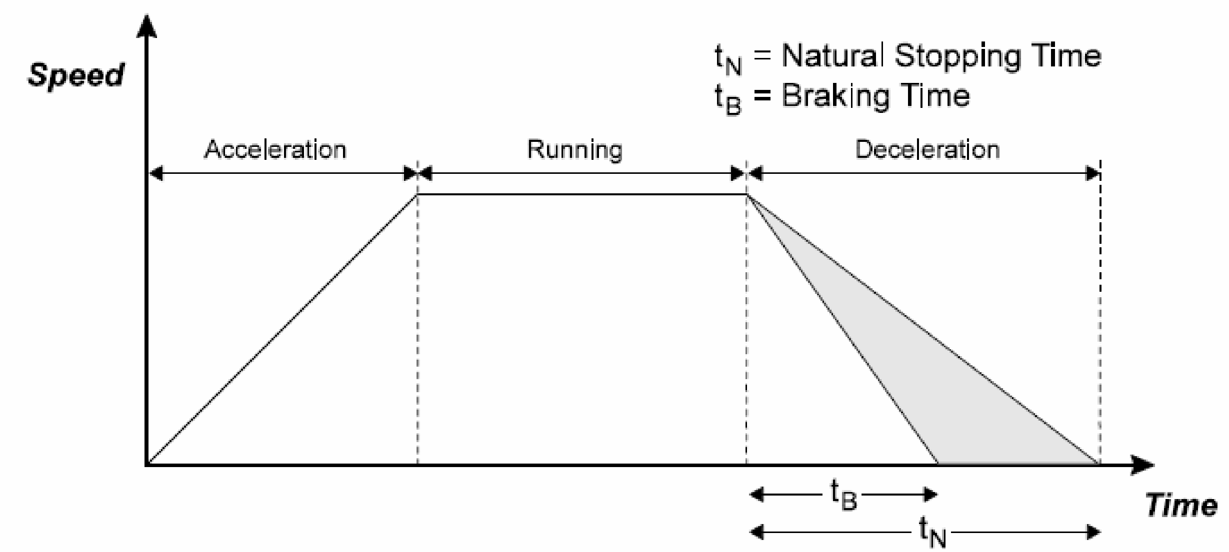
BRAKING OF IM

- In some applications motors must be braked in order to stop them fast.
- The properties of the machine can be used to stop it.
- Applications examples: elevators, conveyor belts, electrical traction, etc.
- Three types of braking:

Regenerative braking (dynamic braking)

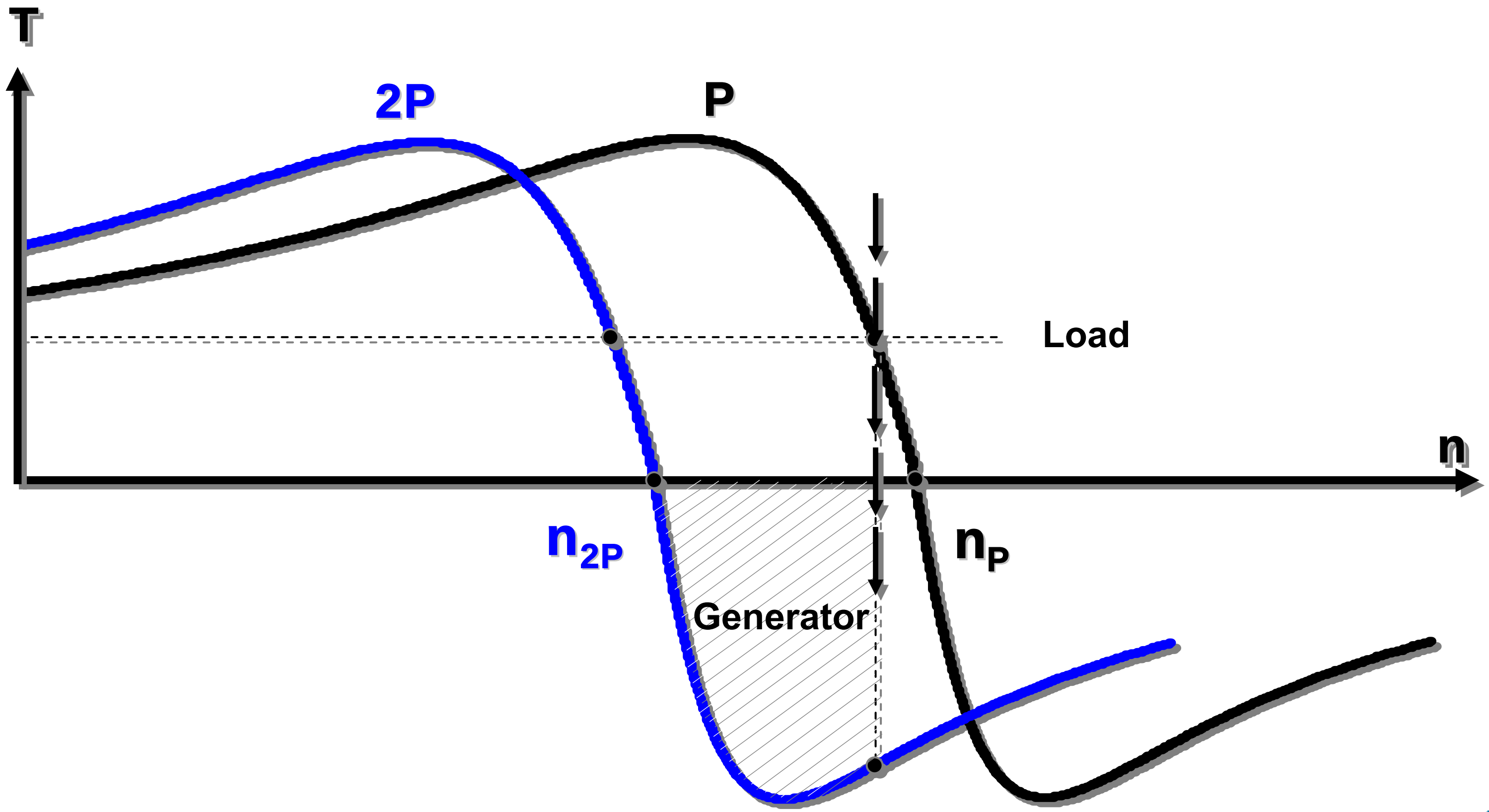
Reverse-current braking

DC injection braking



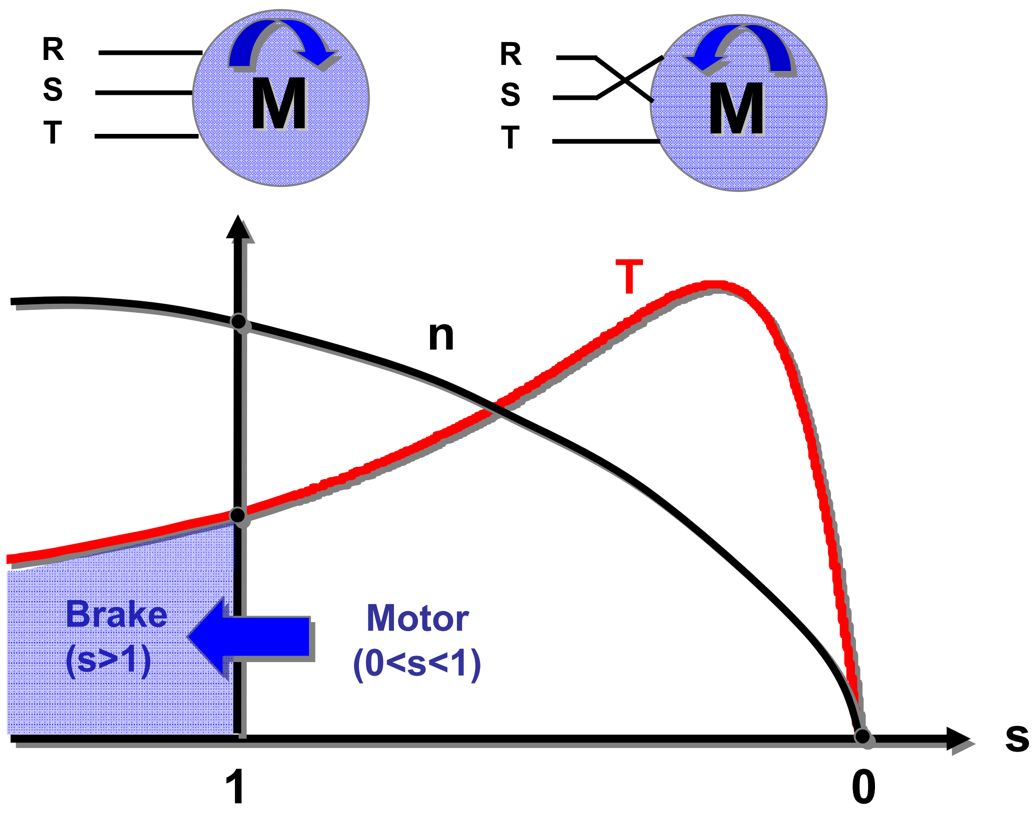
BRAKING OF IM: REGENERATIVE OR DYNAMIC BRAKING

- It requires a motor with P > 1 (pairs of poles)
- Breaking is performed by transforming P (n) to 2P (n/2).
- The motor passes to behave as a generator.
- The energy generated can be dissipated in resistances or returned to the net. In this last case a four-quadrant inverter is required.



BRAKING OF IM: REVERSE-CURRENT BRAKING

- The motor acts in the brake region by permuting two phases
- Then, the magnetic field rotates in opposite to the rotation direction
- The braking torque is low
- High braking current and high braking power losses in stator windings and rotor bars
- Special motors are required



BRAKING OF IM: DC INJECTION BRAKING

- The motor is disconnected from the voltage source and a DC current is injected into the stator windings.
- This can be achieved by connecting two phases of the induction motor to a DC supply.
- The injected current should be roughly equal to the excitation current or no-load current of the motor.
- The stator generates a static magnetic field.
- The rotor bars cut through this field. A current will be developed in the rotor with a magnitude and frequency proportional to speed.
- This results in a braking torque that is proportional to speed.
- The braking energy is dissipated as losses in the rotor windings, which in turn, generate heat.
- The braking energy is limited by the temperature rise permitted in the motor.

EXAMPLE 2

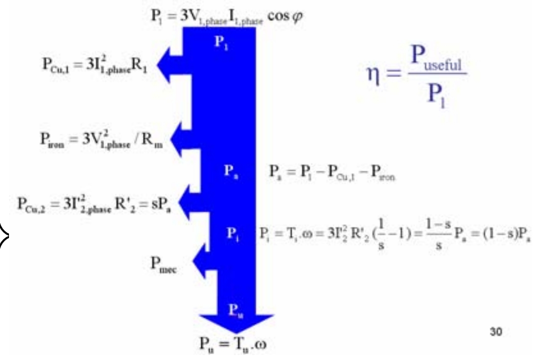
A 3-phase, 200-hp, 3300-V, 50-Hz, 4-pole, star-connected induction motor has the following equivalent circuit parameters (per phase):$R_1=R'_2=0.8\ \Omega$, $X_1=X'_2=3.5\ \Omega$. a) Calculate the slip and the torque at full load if the friction loss is 3 kW. b) Calculate the current absorbed by the motor during starting.

a) $P_{useful} = 200 \cdot 746 = 149200$ watt

$$P_i = P_{useful} + P_{mec} = 152200 \text{ watt} = 3 \cdot I'^2_2 \cdot \overset{=0.8}{R'_2} \cdot (1/s - 1)$$

Being

$$I'_2 = \frac{V_{1star}}{\sqrt{(R_1 + R'_2/s)^2 + X_{1sc}^2}} = \frac{3300/\sqrt{3}}{\sqrt{(0.8 + 0.8/s)^2 + 7^2}}$$



It results a quadratic equation in s: $s^2 - 0.52359s + 0.005988 = 0$

The results are: $s_1 = 0.5119$ (not possible) $s_2 = 0.0118$

Thus, from $s = 0.0118$ it results: $I'_2 = 27.63$ A

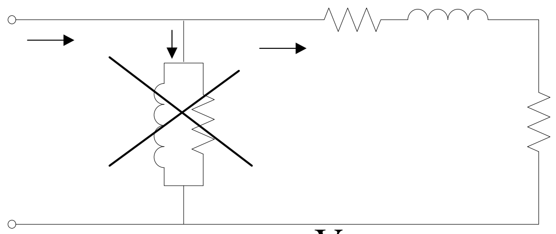
$$P_i = \frac{2\pi}{60} n T_i = 3 I_2'^2 R_2' (1/s - 1) \rightarrow \frac{2\pi}{60} \cdot 1500 \cdot (1 - 0.0118) \cdot T_i = 3 \cdot 27.63^2 \cdot 0.8 \cdot (1/0.0118 - 1)$$

Resulting in, $T_i = 988.5$ Nm

b) $$I_{1starting} \approx I'_{2starting} = \frac{V_{1Y}}{\sqrt{(R_1 + R'_2/1)^2 + X_{1sc}^2}} = \frac{3300/\sqrt{3}}{\sqrt{(0.8 + 0.8/1)^2 + 7^2}} = 265.34 \text{ A}$$

EXAMPLE 3

A 3-phase, 6-pole, 50 Hz induction motor has the following equivalent circuit parameters per phase: $R_1 \approx 0\ \Omega$, $R'_2 = R$, $X_1 = X'_2 = 5R\ \Omega$, $I_{starting}/I_n = 4.10264$. Calculate a) the nominal speed, b) the PF during starting and under rated load as well as c) $T_{starting}/T_n$. Neglect the parallel branch of the equivalent circuit



$$\bar{Z}_{eq} = (R_1 + R_2'/s) + j(X_1 + X_2') = R/s + j10R$$

$$I_{1,phase} = \frac{V_{1,phase}}{\sqrt{(R/s)^2 + (10R)^2}}$$

a) $$\frac{I_{1,starting}}{I_{1,n}} = \frac{\dfrac{V_{1,phase}}{\sqrt{(R/1)^2 + (10R)^2}}}{\dfrac{V_{1,phase}}{\sqrt{(R/s_n)^2 + (10R)^2}}} = 4.10264 \rightarrow s_n = 0.025 \rightarrow n_n = 60 \cdot \frac{f_1}{p} \cdot (1 - s_n) = 975 \text{ rpm}$$

R_{1sc} X_{1sc}

b) $$\bar{Z}_{eq,starting} = R/1 + j \cdot 10 \cdot R = 10.0499 \cdot R^{84.29^\circ}\,\Omega \rightarrow PF_{starting} = \cos(84.29^\circ) = 0.1(i)$$

I_o

$$\bar{Z}_{eq,n} = R/0.025 + j10R = 41.231R^{14.04^\circ}\,\Omega \rightarrow PF_n = \cos(14.04^\circ) = 0.97(i)$$

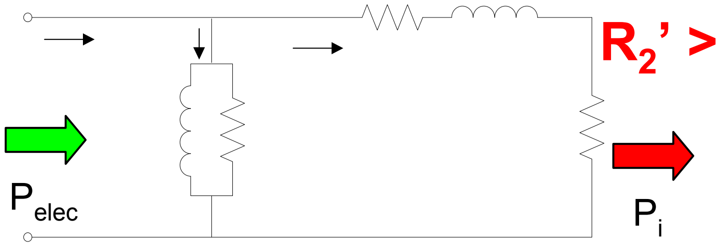
I_2

$X_{\mu l}$ R_m $R'_2(1/s-1)$

c) $$\frac{T_{i,starting}}{T_{i,n}} = \frac{\dfrac{3 \cdot R_2' \cdot \dfrac{V_{1,phase}^2}{1}}{\dfrac{2\pi}{60} n_0 [(R_1 + R_2'/1)^2 + (X_1 + X_2')^2]}}{\dfrac{3 \cdot R_2' \cdot \dfrac{V_{1,phase}^2}{s_n}}{\dfrac{2\pi}{60} n_0 [(R_1 + R_2'/s_n)^2 + (X_1 + X_2')^2]}} = s_n \frac{(R_1 + R_2'/s_n)^2 + (X_1 + X_2')^2}{(R_1 + R_2')^2 + (X_1 + X_2')^2} = 0.025 \frac{R^2/0.025^2 + 100R^2}{R^2 + 100R^2} = 0.4208$$

EXAMPLE 4a

A 3-phase, 440-V 50-Hz, Δ-connected, 4-pole induction motor runs at a speed of 1447 rpm when operating at its rated load. Equivalent circuit per-phase parameters: R_1=0.2 Ω, R'_2=0.4 Ω, X_1=X'_2= 2 Ω, R_m=200 Ω, X_m=40 Ω. Determine, for rated load, the values of a) line current, b) power factor, c) torque, d) output power and e) efficiency. The mechanical loss are estimated in 1000 watts.



s = (1500-1447)/1500 = 0.0353 $\quad V_{1\Delta} = 440^{0^\circ}$ V

$$\bar{I}_0 = \frac{440}{200} + \frac{440}{j40} = 2.2 - j11 = 11.218^{-78.69^\circ}\ \mathrm{A}$$

$$\bar{I}_2' = \frac{440^{0^\circ}}{0.6 + 0.4(1/0.0353 - 1) + j4} = 34.08 - j11.8 = 36.08^{-19.147^\circ}\ \mathrm{A}$$

$$\bar{I}_{1,\Delta} = \bar{I}_0 + \bar{I}_2' = 36.28 - j22.8 = 42.85^{-32.147^\circ}\ \mathrm{A}$$

$$\text{a) } \Delta\text{-connection}: I_{1,line} = \sqrt{3} I_{1,\Delta} = \sqrt{3}\ 42.85 = 74.2\mathrm{A}$$

$$\text{b) } PF = \cos(32.147^\circ) = 0.847\ (i)$$

$$\text{c) } P_i = \frac{2\pi}{60} n T_i = 3 I_2'^2 R_2'(1/s - 1) \ \rightarrow\ T_i = 281.5\ \mathrm{Nm}$$

$$T_i = \frac{3.R_2'.\frac{V_{1,\Delta}^2}{s}}{\frac{2\pi}{60} n_0[(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$

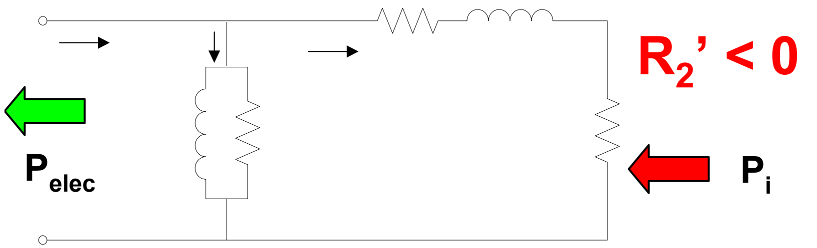
$$\text{d) } P_{useful} = P_i - P_{mec} = 3 \cdot 36.08^2 \cdot 0.4 \cdot (1/0.0353 - 1) - 1000 = 41.65\ \mathrm{kW}$$

$$\text{e) } P_{input} = \sqrt{3} V_{1,line} I_{1,line} \cos\varphi = \sqrt{3} \cdot 440 \cdot 74.2 \cdot 0.847 = 47.90\ \mathrm{kW} \rightarrow \eta_{\%} = 100 \frac{P_{useful}}{P_{input}} = 86.95\%$$

EXAMPLE 4b

Determine the same quantities as in example 4a, if the same machine runs as a generator (for example a wind-driven generator) with the same numerical value of slip, when connected at the same three-phase line.

Generator: s < 0 → s = - 0.0353 → n = 1553 rpm $V_{1\Delta} = 440^{0^o}$V



$$\bar{I}_0 = 2.2 - j11 = 11.218^{-78.69^o}\text{ A (same as Example 4a)}$$

$$\bar{I}_2' = \frac{440^{0^o}}{0.6 + 0.4(1/-0.0353 - 1) + j4} = -35 - j12.6 = 37.23^{-160.22^o}\text{ A}$$

$$\bar{I}_{1,\Delta} = \bar{I}_0 + \bar{I}_2' = -32.8 - j23.6 = 40.41^{-144.26^o}\text{ A}$$

a) $\Delta\text{-connection}: I_{1,\text{line}} = \sqrt{3}\, I_{1,\Delta} = \sqrt{3}\; 40.41 = 70\text{A}$ R_{1sc} X_{1sc}

b) $\text{PF} = |\cos(144.26^o)| = 0.81\,(\text{i})$

c) $P_i = \frac{2\pi}{60} n T_i = 3I_2'^2 R_2'(1/s - 1) \rightarrow T_i = -299.7\text{ Nm (it is a braking torque)}$

I_o I_1 I'_2

$$T_i = \frac{3.R_2'.\frac{V_{1,\Delta}^2}{s}}{\frac{2\pi}{60} n_0[(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$

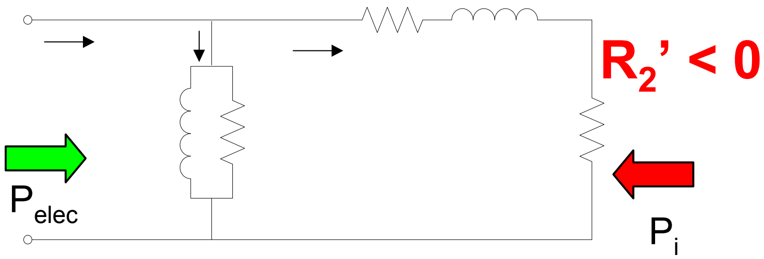
X_μ R_m $R'_2(1/s-1)$

d) $P_{\text{mechanical, input}} = |P_i| + P_{\text{mec}} = |3 \cdot 37.23^2 \cdot 0.4(1/-0.0353 - 1)| + 1000 = 49.7\text{ kW}$

e) $P_{\text{electrical, output}} = \sqrt{3} V_{1,\text{line}} I_{1,\text{line}} \cos\varphi = \sqrt{3}.440 \cdot 70 \cdot 0.81 = 43.2\text{ kW} \rightarrow \eta_\% = 100 \frac{P_{\text{electrical, output}}}{P_{\text{mechanical, input}}} = 87\%$

EXAMPLE 4c

The same machine as in example 4a, which is connected at the same generator, runs at 1447 rpm when, suddenly and instantaneously, phases R and S are permuted (the machine runs as a brake). In this instant determine the values of a) line current b) power factor c) the braking torque d) the electrical input power e) Joule losses



$$n = 1447 \text{ rpm} \quad \text{and} \quad n_o = -1500 \text{ rpm}$$

$$s = \frac{n_0 - n}{n_0} = \frac{-1500 - 1447}{-1500} = 1.965 \qquad \overline{V}_{1\Delta} = 440^{0^\circ} \text{ V}$$

$$\overline{I}_0 = 2.2 - j11 = 11.218^{-78.69^\circ} \text{ A} \quad (\text{same as Example 4a})$$

$$\overline{I}_2' = \frac{440^{0^\circ}}{0.6 + 0.4(1/1.965 - 1) + j4} = 10.99 - j108.89 = 109.44^{-84.23^\circ} \text{ A}$$

$$\overline{I}_{1,\Delta} = \overline{I}_0 + \overline{I}_2' = 13.19 - j119.89 = 120.61^{-83.722^\circ} \text{ A}$$

a) Δ-connection: $I_{1,line} = \sqrt{3} I_{1,\Delta} = \sqrt{3}\, 120.61 = 208.9 \text{ A}$

b) $PF = \cos(+83.72^\circ) = 0.109 \text{ (i)}$

R_{1sc} X_{1sc}

c) $P_i = \frac{2\pi}{60} n T_i = 3 I_2'^2 R_2'(1/s - 1) = \text{-}7.06 \text{ kW (input power)} \rightarrow T_i = -46.57 \text{ Nm (braking torque)}$

I_o I_1 I'_2

$$T_i = \frac{3.R_2'.\frac{V_{1,\Delta}^2}{s}}{\frac{2\pi}{60} n_0[(R_1 + R_2'/s)^2 + (X_1 + X_2')^2]}$$

e) $P_{electrical,\, input} = \sqrt{3} V_{1,line} I_{1,line} \cos\varphi = \sqrt{3}.440 \cdot 208.9 \cdot 0.109 = 17.35 \text{ kW}$

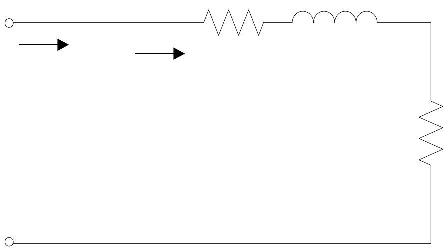
X_μ R_m $R'_2(1/s-1)$

f) $P_{Cu} = 3 I_2'^2 R_{1sc} = 3 \cdot 109.44^2 \cdot 0.6 = 21.56 \text{ kW} \qquad P_{iron} = 3V_1^2/R_m = 3 \cdot 440^2/200 = 2.90 \text{ kW}$

Note that $P_{Cu} + P_{iron} = 24.4 \text{ kW} = P_{i,input} + P_{electical,input}$

EXAMPLE 5

A 3-phase, 400V-50 Hz wound-rotor IM (stator and rotor windings are wye-connected) has a full-load speed of 1440 rpm. The following parameters are known: R_1=1Ω, R_2'=0.8Ω, X_{1sc}=5 Ω. Neglecting the mechanical losses and the parallel branch of the equivalent circuit, determine: a) The full-load current and the internal torque. b) The current and the internal torque during starting being the rings in short-circuit. c) The current and the torque during starting when a resistance R_{add}'=4.3 Ω has been added in series with the rotor through the rings.



$$\text{a) Full load: } s=\frac{n_o-n}{n_o}=\frac{1500-1440}{1500}=0{,}04 \qquad (4\%)$$

$$I_{1,\text{Full-Load}}=I_{1,Y}=\frac{V_{1,Y}}{Z_{1,Y}}=\frac{V_{1,Y}}{\sqrt{(R_1+R_2'/s)^2+X_{1cc}^2}}=\frac{400/\sqrt{3}}{\sqrt{(1+0.8/0.04)^2+5^2}}=10{,}70\text{ A}$$

$$M_{i,\text{Full-Load}}=\frac{3\cdot V_{1,Y}{}^2\cdot\frac{R_2'}{s}}{\frac{2\pi}{60}\cdot n_o\left[\left(R_1+\frac{R_2'}{s}\right)^2+\left(X_{1,sc}\right)^2\right]}=43.73\text{ Nm}$$

$$\text{b) } I_{1,\text{Start}}=\frac{V_{1,Y}}{Z_{1,Y}}=\frac{V_{1,Y}}{\sqrt{(R_1+R_2'/s)^2+X_{1cc}^2}}=\frac{400/\sqrt{3}}{\sqrt{(1+0.8/1)^2+5^2}}=43.46\text{ A}$$

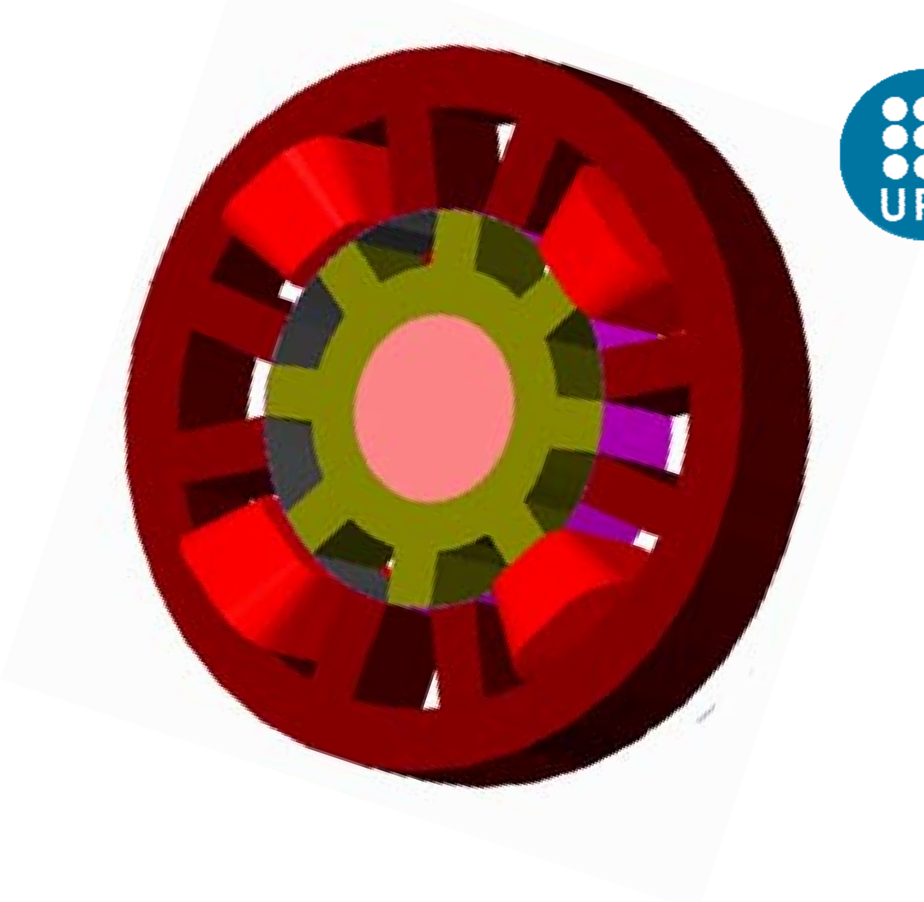
$$M_{i,\text{start}}=\frac{3\cdot V_{1,Y}{}^2\cdot\frac{R_2'}{1}}{\frac{2\pi}{60}\cdot n_o\left[\left(R_1+\frac{R_2'}{1}\right)^2+\left(X_{1,sc}\right)^2\right]}=28.86\text{ Nm}$$

$$\text{c) } R_{2new}'=R_2'+R_{add}'=5.1\,\Omega$$

$$I_{1,\text{Start}}=\frac{V_{1,Y}}{Z_{1,Y}}=\frac{V_{1,Y}}{\sqrt{(R_1+R_{2,new}'/s)^2+X_{1cc}^2}}=\frac{400/\sqrt{3}}{\sqrt{(1+5.1/1)^2+5^2}}=29.82\text{ A}$$

$$M_{i,\text{start}}=\frac{3\cdot V_{1,Y}{}^2\cdot\frac{R_{2,new}'}{1}}{\frac{2\pi}{60}\cdot n_o\left[\left(R_1+\frac{R_{2,new}'}{1}\right)^2+\left(X_{1,sc}\right)^2\right]}=83.50\text{ Nm}$$

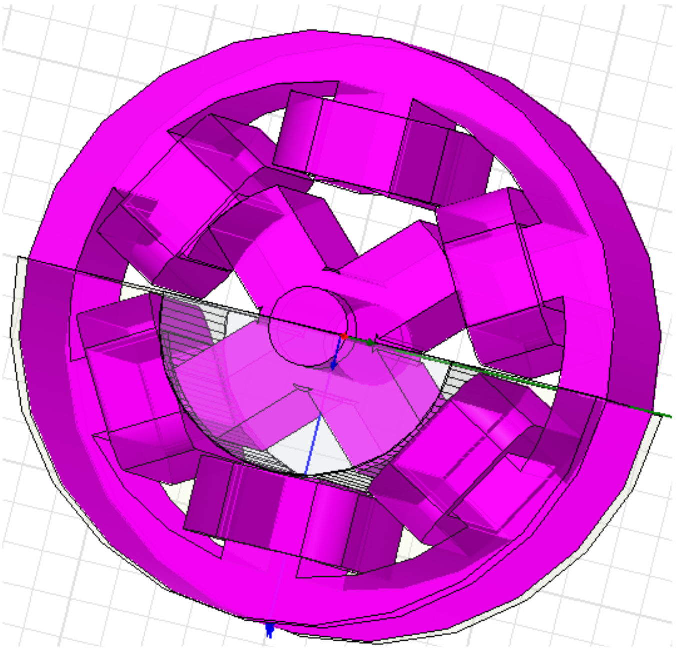
Other Types of Electrical Machines



Unit 6. OTHER TYPES OF ELECTRICAL MACHINES

CONTENTS LIST:

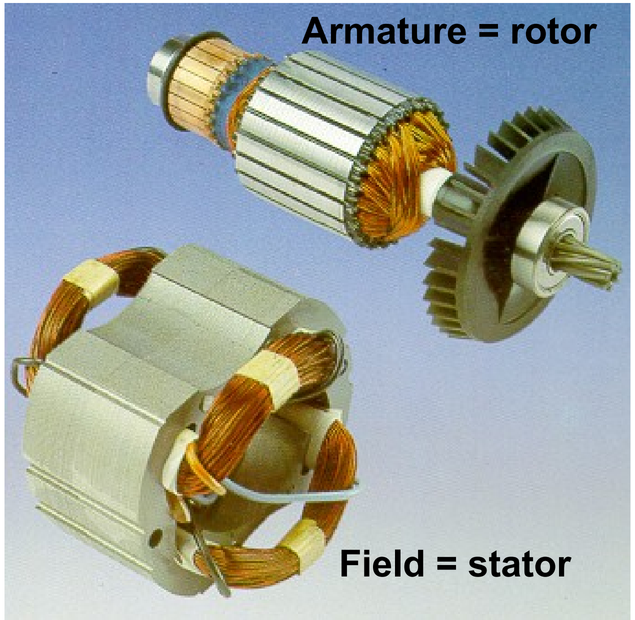
- Brushed DC machines (DC)
- Brushless DC machines (BLDC)
- Permanent Magnet Synchronous machines (PMSM)
- Switched reluctance machines (SRM)

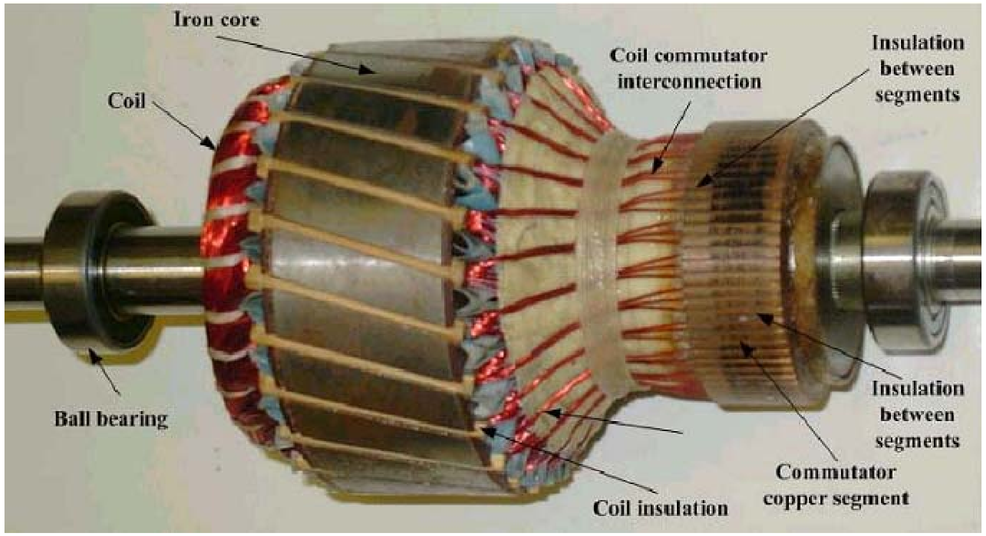


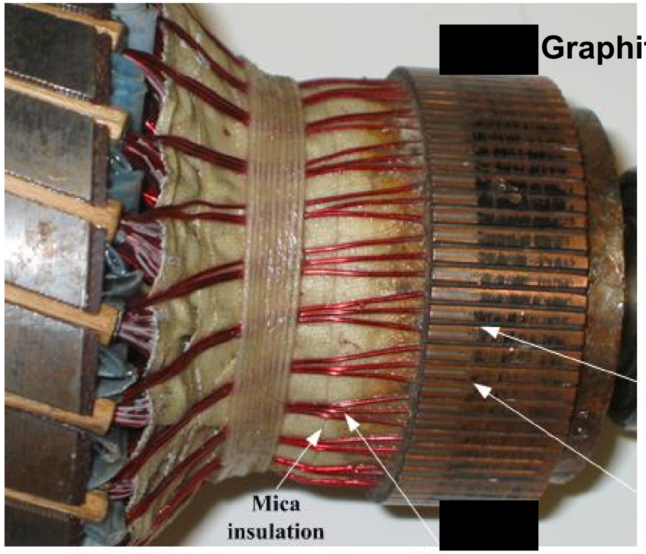
BRUSHED DC MACHINES

Brushed DC machines consists of :

- An electromagnetic or permanent magnetic structure called field or stator which is static
- An armature or rotor which rotates
- The Field produces a magnetic field
- The Armature produces voltage and torque under the action of the magnetic field







BRUSHED DC MACHINES

The stator or field

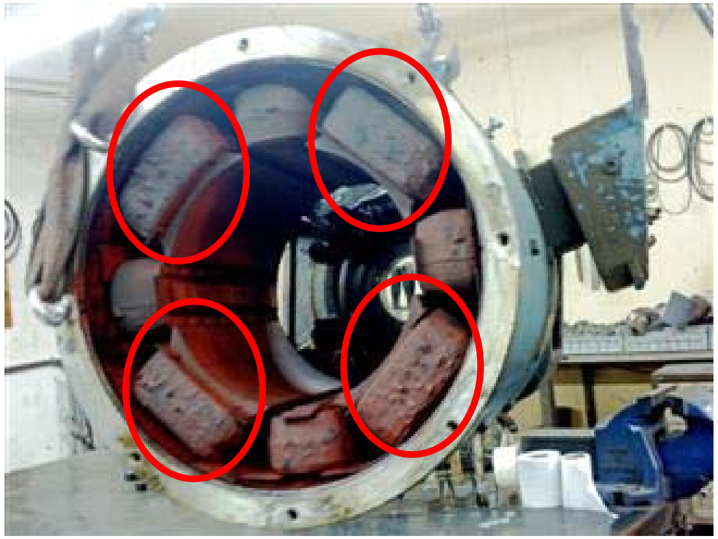
Two types of stators:

1) **Permanent magnet stator**

2) **Wound stator**

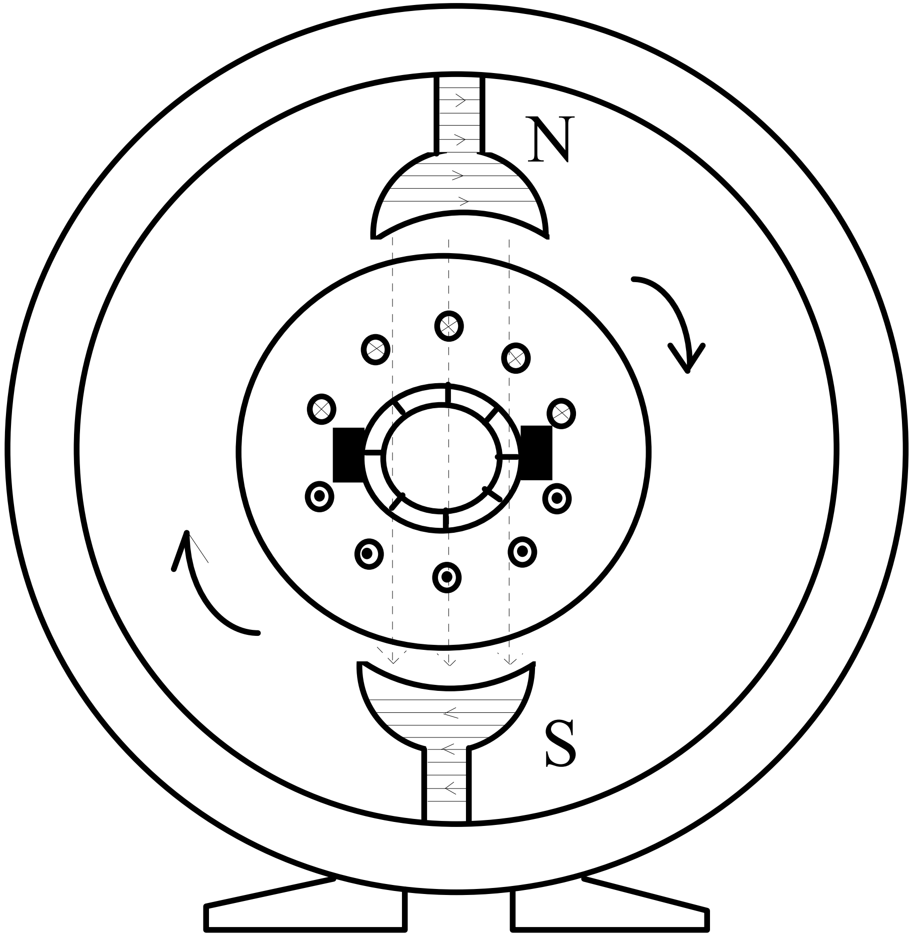
- The field winding takes the form of a concentric coil wound around the main poles.
- Field windings carry the excitation current and produce the main field in the machine.
- Poles are created electromagnetically.

2 and 6-poles permanent magnet stators (fields)

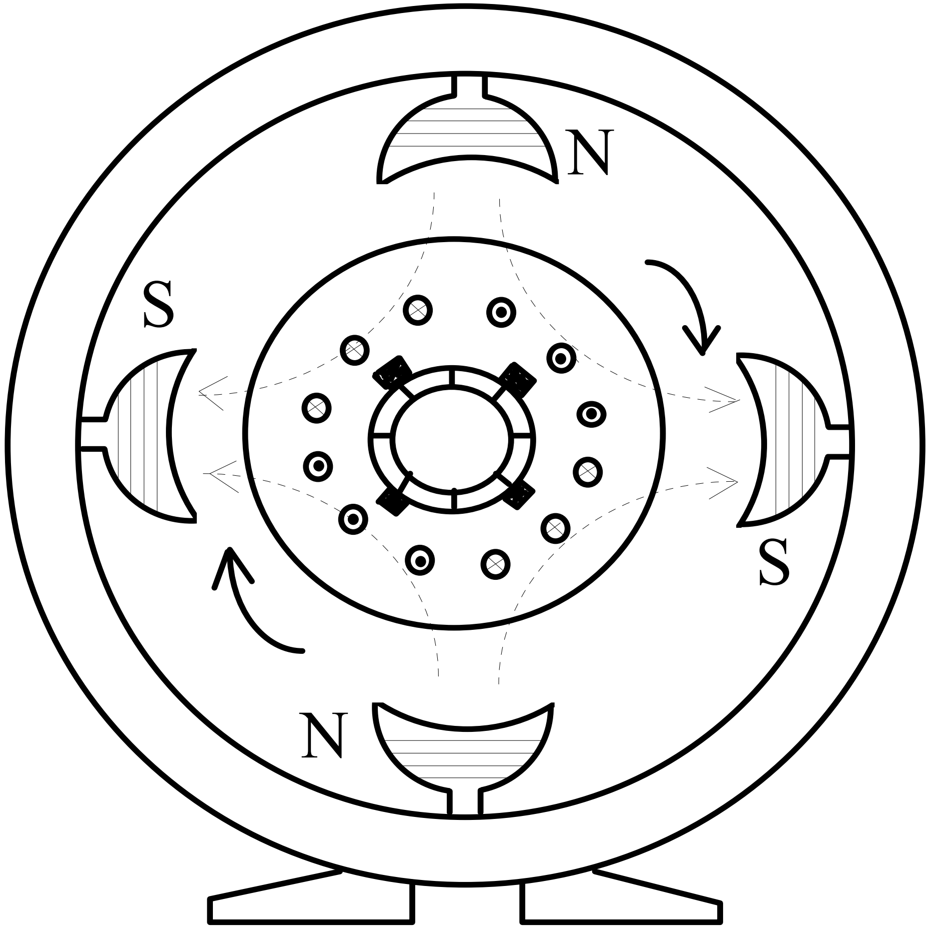


4-poles wound stator (field)

The stator or field



2-poles wound field



4-poles wound field

BRUSHED DC MACHINES

Armature reaction

- The current flowing in the armature conductors also creates a magnetic field that distorts and weakens the flux coming from the stator poles.
- Under no-load conditions, the small current flowing in the armature does not appreciably affect the flux coming from the stator poles (Fig. 1).
- When the armature carries its normal current, it produces a strong magnetic field which, if it acted alone, would create a flux as in Fig. 2.
- Fig. 3 shows the resulting flux.

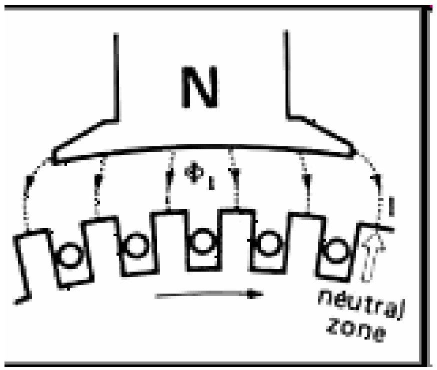


Fig. 1 Motor running at no-load.

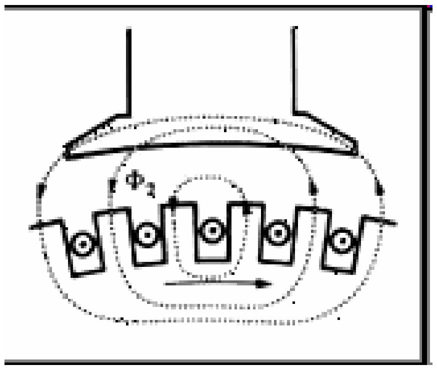


Fig. 2 Flux due to the armature current when running at full load

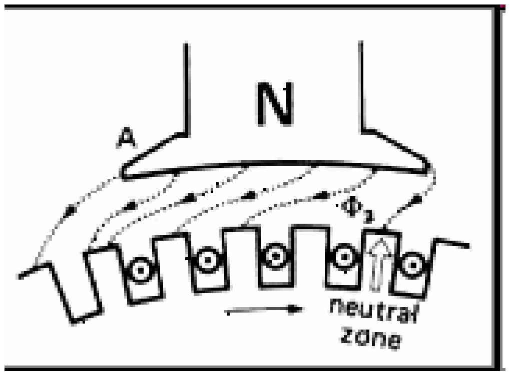
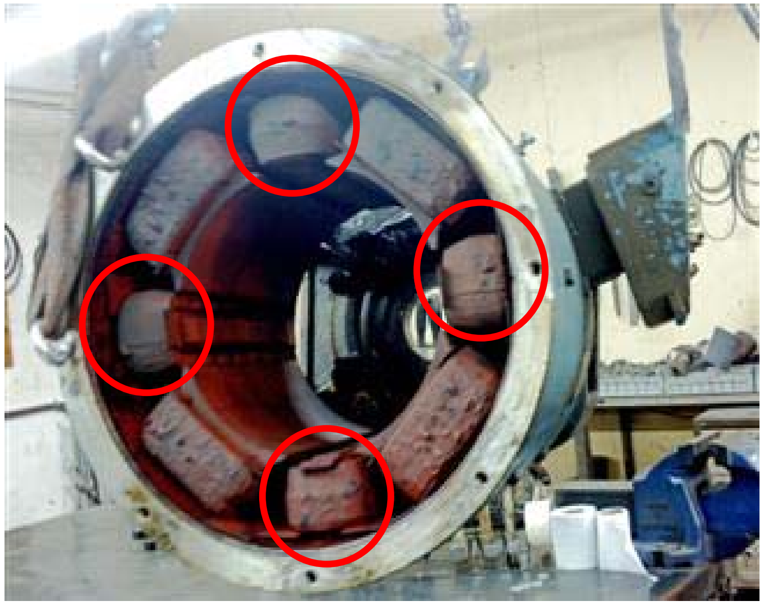
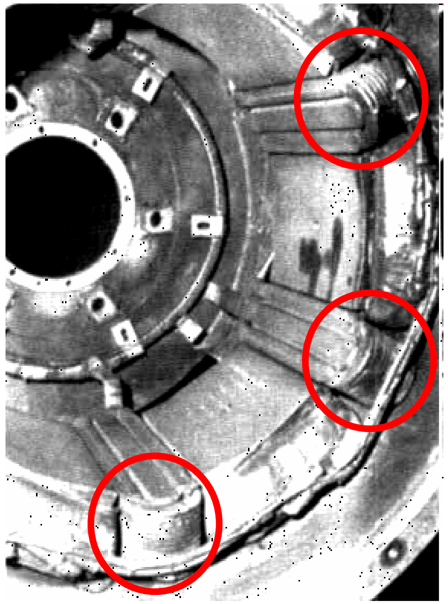


Fig. 3 Resulting flux when running at full load

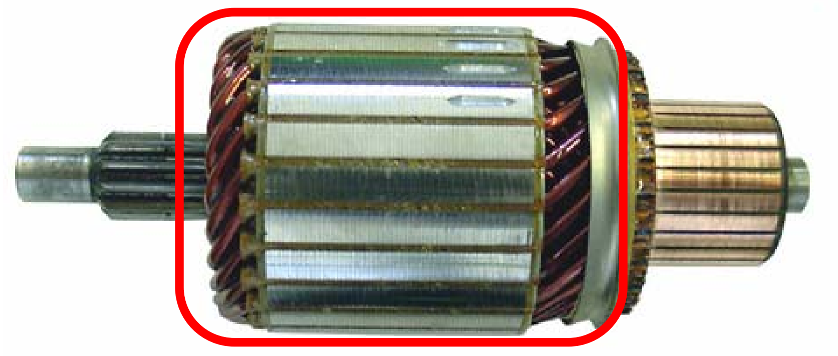
Commuting stators poles

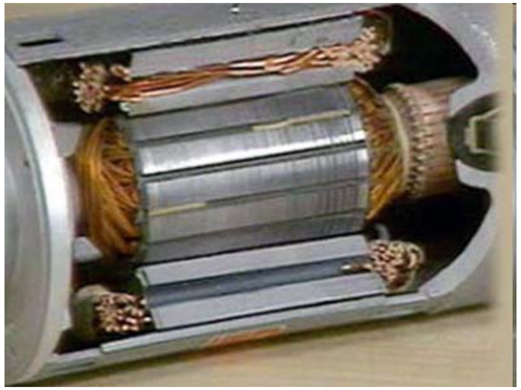
- Narrow poles placed between the main poles
- Counter the effect of armature reaction by developing a magnetic field opposite to the B field of the armature.
- Applied in medium- and large-power dc motors
- Commutation is greatly improved





The armature winding

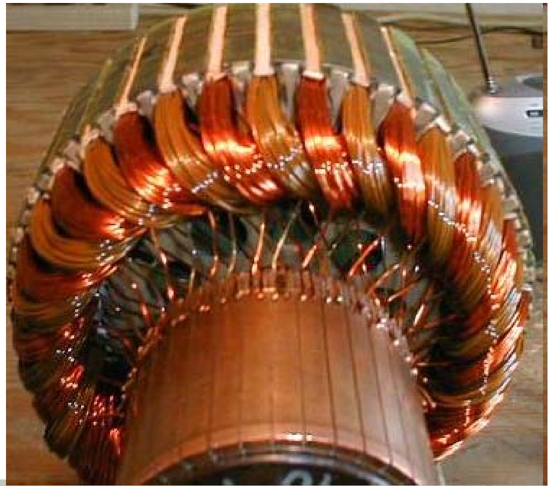

Rotor or armature winding

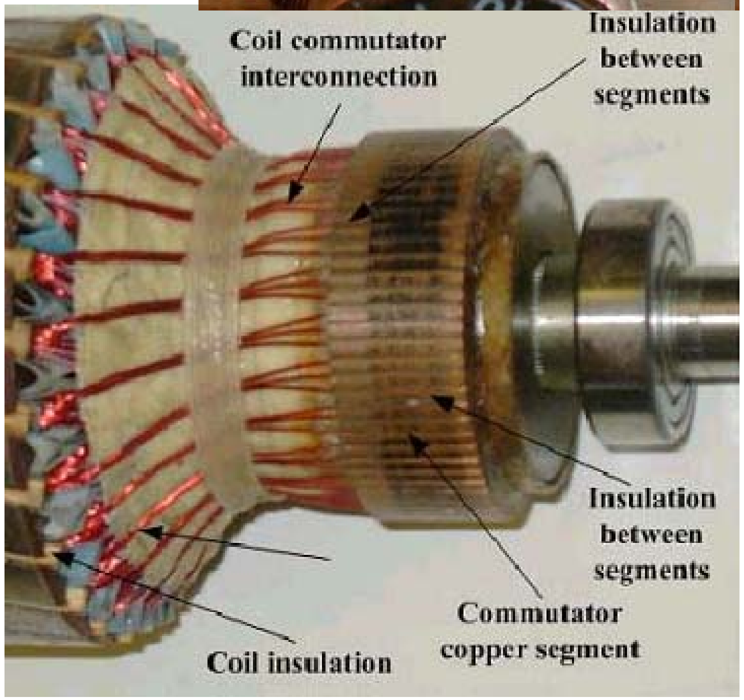

Section of the motor

- Armature winding is made of coils which are wound into open slots lowered on the armature.
- They generate a magnetic field that interacts with the B field of the stator, generating a pull.
- The coils are prevented from flying out due to the centrifugal forces by means of bands of steel wire on the surface of the rotor in small groves cut into it.
- The end portion of the windings are taped at the free end and bound to the winding carrier ring of the armature at the commutator end.

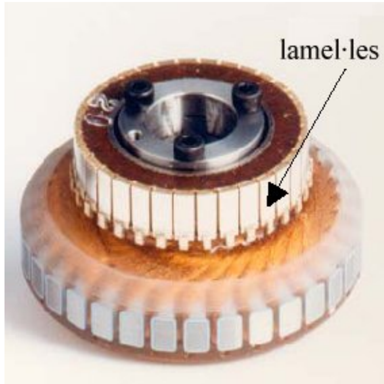
The commutator

- It is a key element of the DC machines
- Consists of copper segments tightly fastened together with mica insulating separators on an insulated base.
- Each commutator segment is provided with a 'riser' where the ends of the armature coils get connected.
- The surface of the commutator is machined and surface is made concentric with the shaft and the current collecting brushes rest on the same.





The commutator



BRUSHED DC MACHINES

Brushes

- Brushes rest on the surface of the commutator.
- Electro-graphite is used as brush material.
- The hardness of the graphite brush is selected to be lower than that of the commutator.
- More number of relatively smaller width brushes are preferred in place of large broad brushes.
- The brush holders provide slots for the brushes to be placed.
- **Brushes cause wear, thus reducing the expected life of the machine!!**

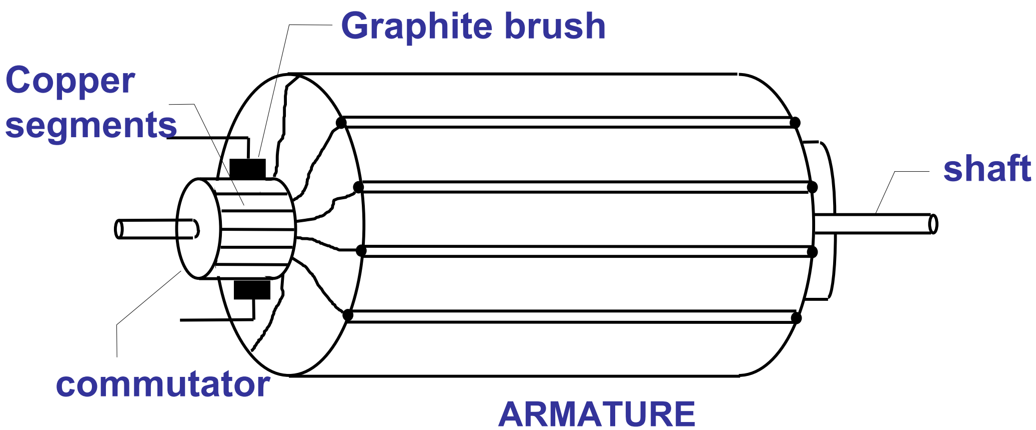
Brushes and brush holders

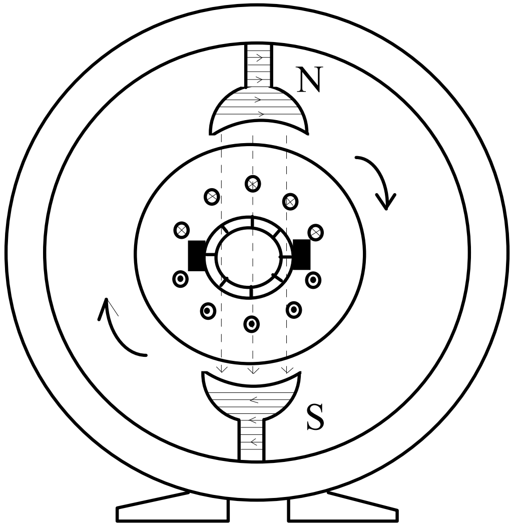
Brush holders

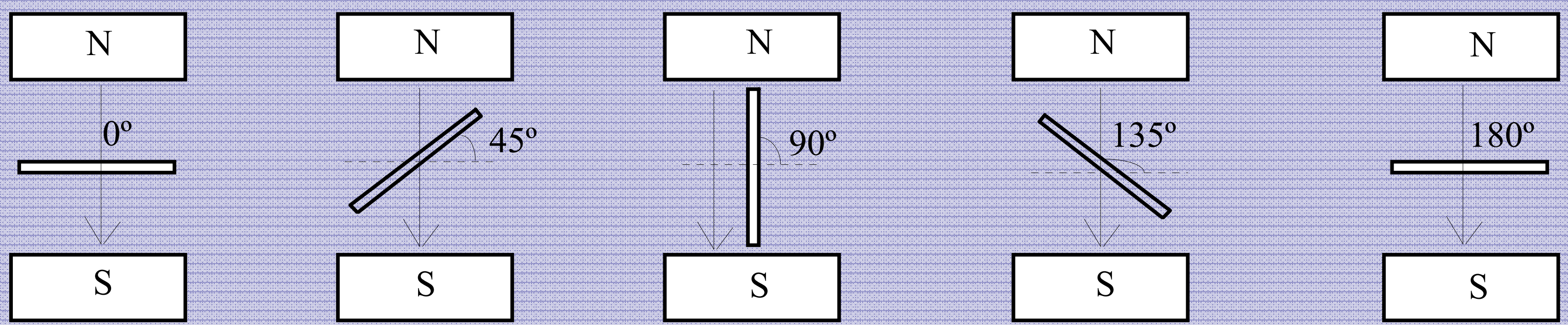
- Brush holders provide slots for the brushes to be placed.
- Brush holders ensure that the carbon brush is retained in a defined position to establish the electrical contact with the rotating part of the machine.
- It is decisively important to ensure optimum, uniform brush pressure over the entire wear path of the carbon brush.

BRUSHED DC MACHINES

Magnetic flux Φ







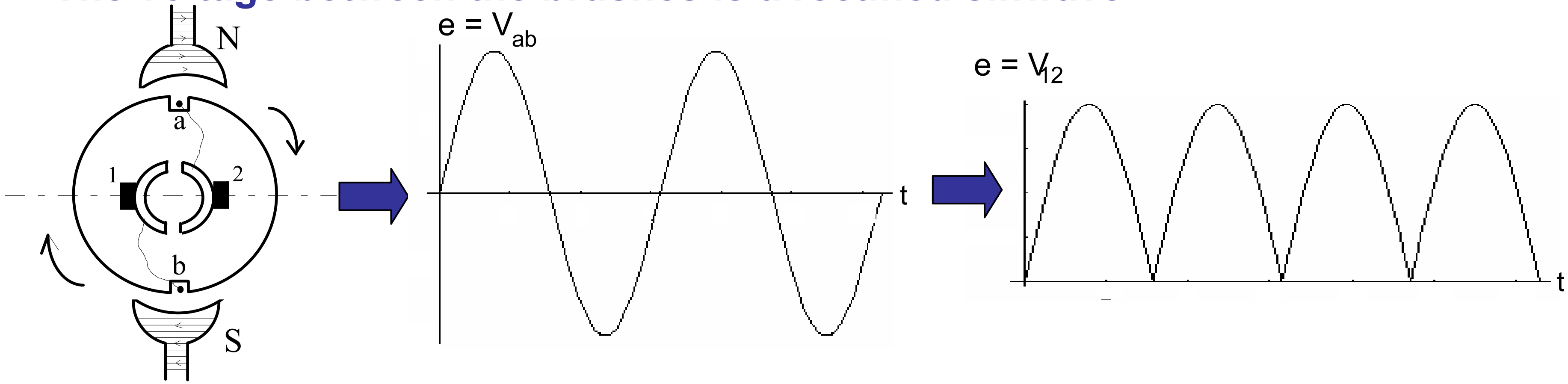
1) $\Phi = \Phi_o$ 2) $\Phi = \Phi_o.\sin 45°$ 3) $\Phi = 0$ 4) $\Phi = -\Phi_o.\sin 45°$ 5) $\Phi = -\Phi_o$

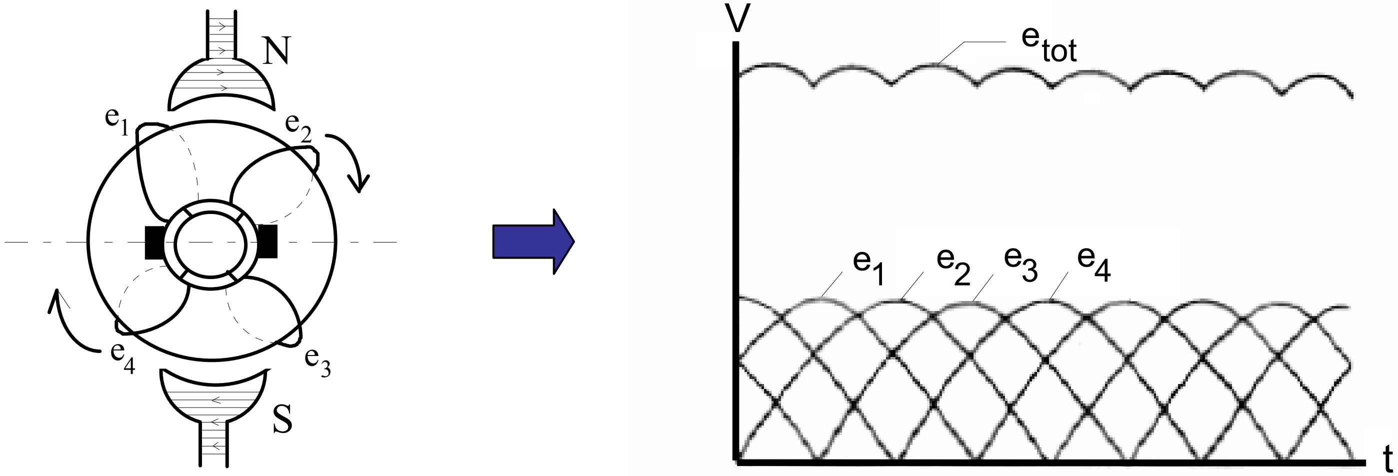
The magnetic flux Φ is a sinwave

BRUSHED DC MACHINES

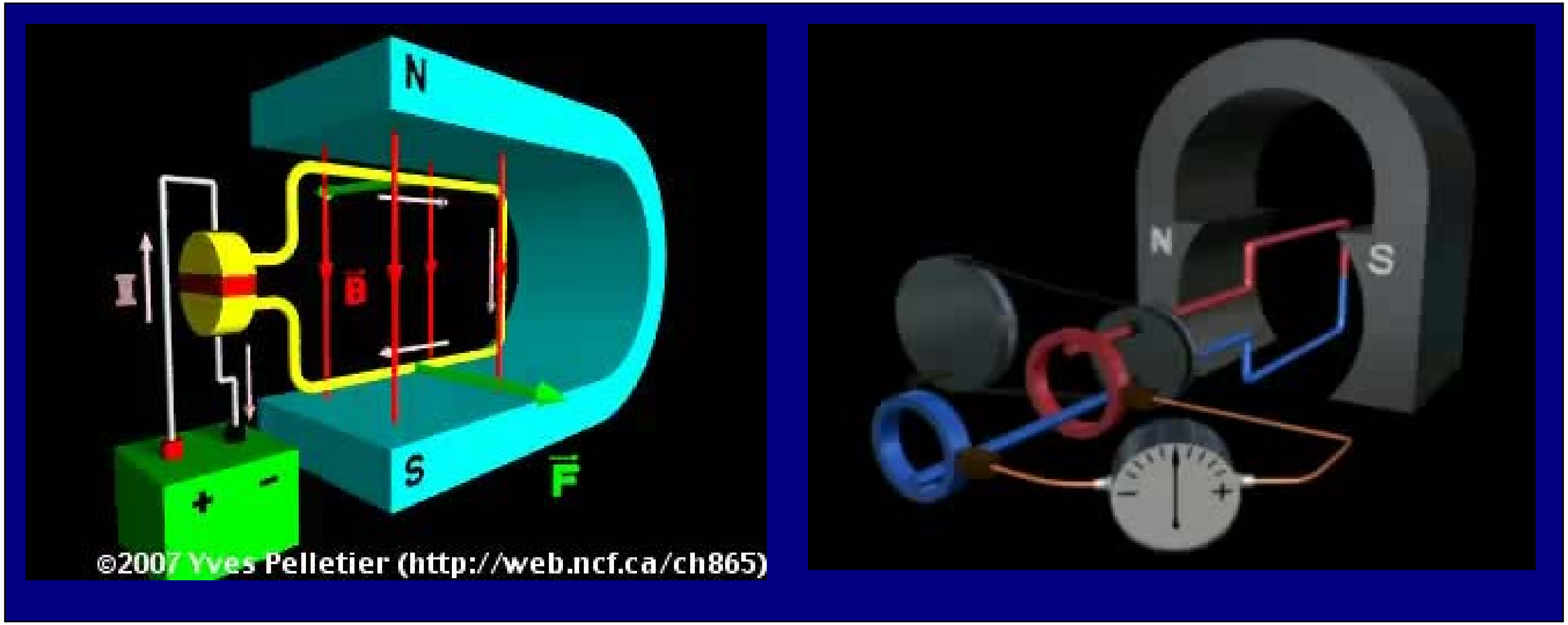
Operation of the commutator

- The commutator acts as a mechanical rectifier.
- The e.m.f. induced in each coil is a sinwave.
- **The voltage between the brushes is a rectified sinwave.**

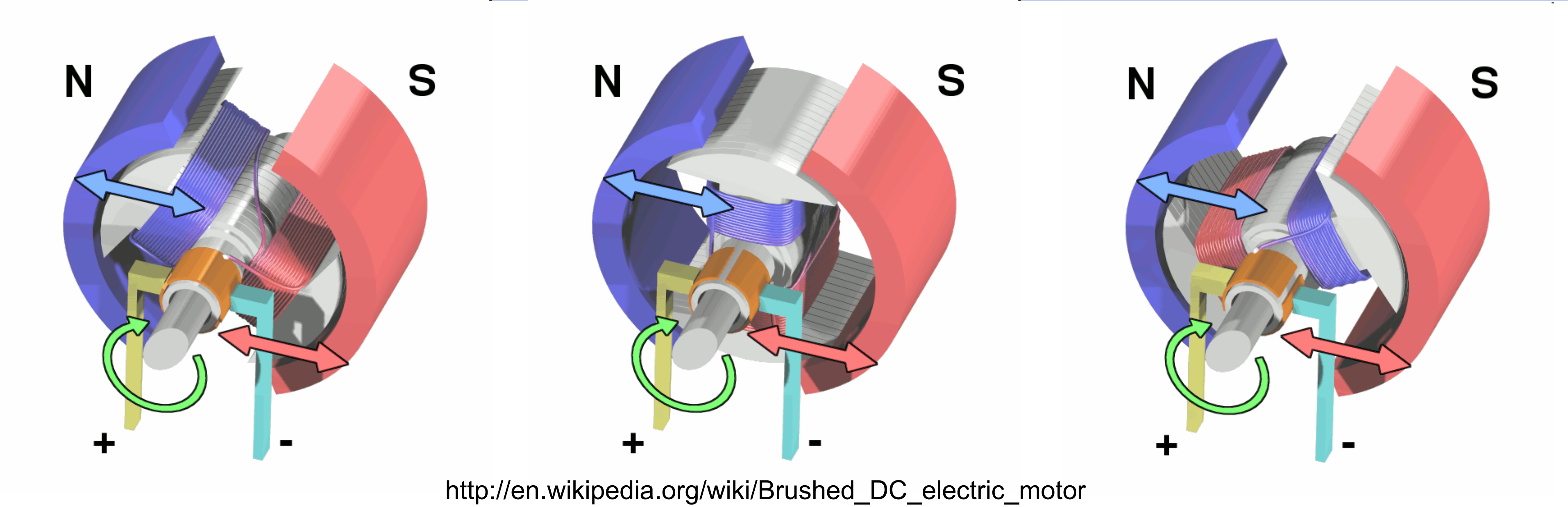




Operation as motor and generator



BRUSHED DC MACHINES



http://en.wikipedia.org/wiki/Brushed_DC_electric_motor

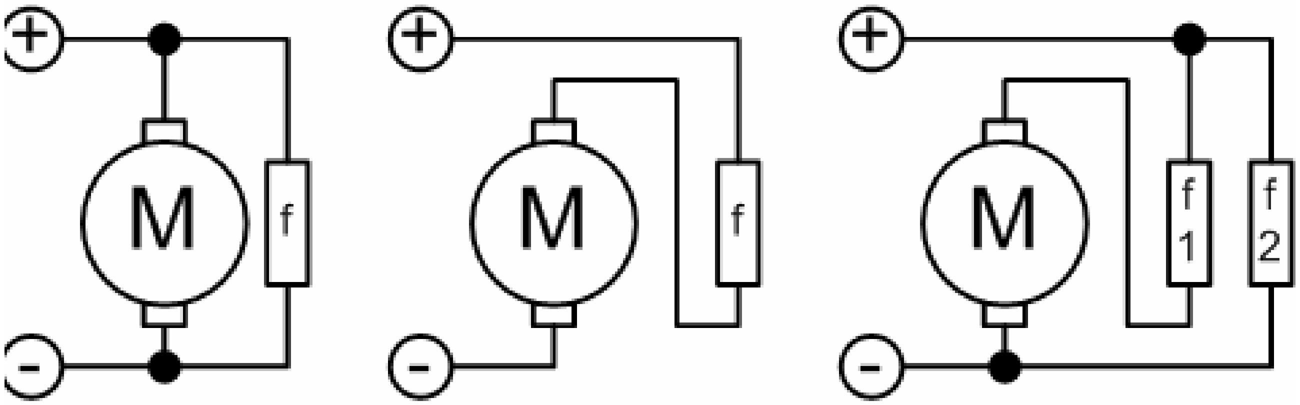
When the coil is powered, a magnetic field is generated around the armature. The left side of the armature is pushed away from the left magnet and drawn toward the right, causing rotation.

The armature continues to rotate.

When the armature becomes horizontally aligned, the commutator reverses the direction of current through the coil, reversing the magnetic field. The process then repeats.

BRUSHED DC MACHINES

Armature and field connection



Shunt connection **Series** connection **Compound** connection

f = field coil M: armature

Basic equations

$$\begin{cases} E = K \cdot \omega \cdot \Phi \\ T_i = K \cdot I \cdot \Phi \\ P_{i(e)} = P_{i(m)} \Rightarrow E \cdot I_i = T_i \cdot \omega \end{cases}$$

K: constructive machine constant

ω: angular speed in rad/s

Φ: magnetic flux

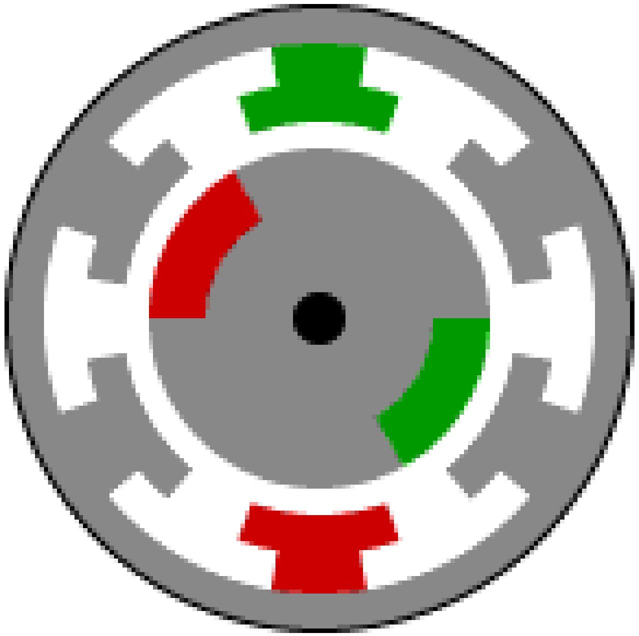
I: armature current

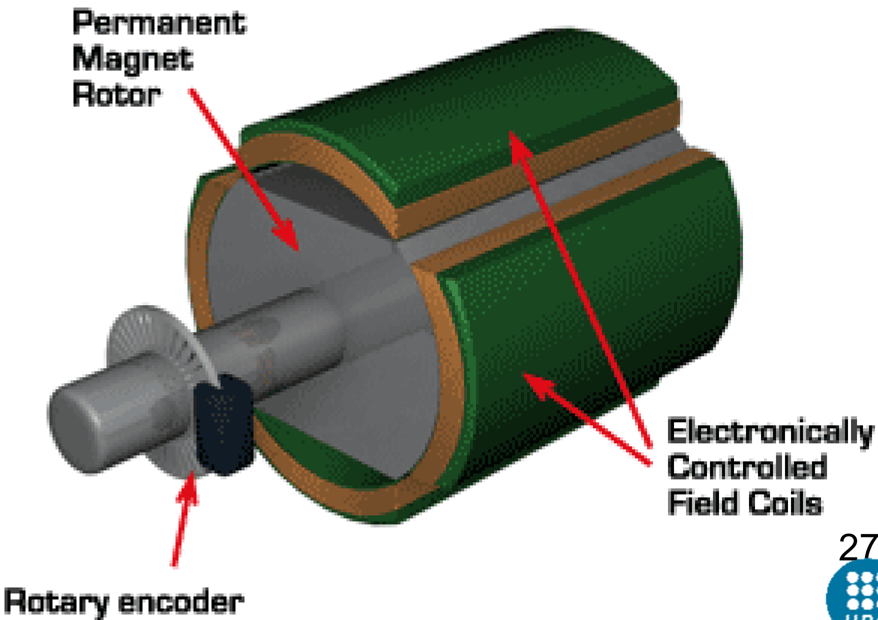
T_i: torque

E: e.m.f.

BRUSHLESS DC MACHINES (BLDC motors)

- Brushless DC motors (BLDC motors) are synchronous electric motors powered by direct-current (DC).
- BLDC motors do not use brushes for commutation; instead, they are electronically commutated.
- The current-to-torque and voltage-to-speed relationships of BLDC motors are linear.
- BLDC motors have fixed permanent magnets on the rotor and possibly more poles on the stator than the rotor.
- Without the use of an electronic converter they can not rotate.

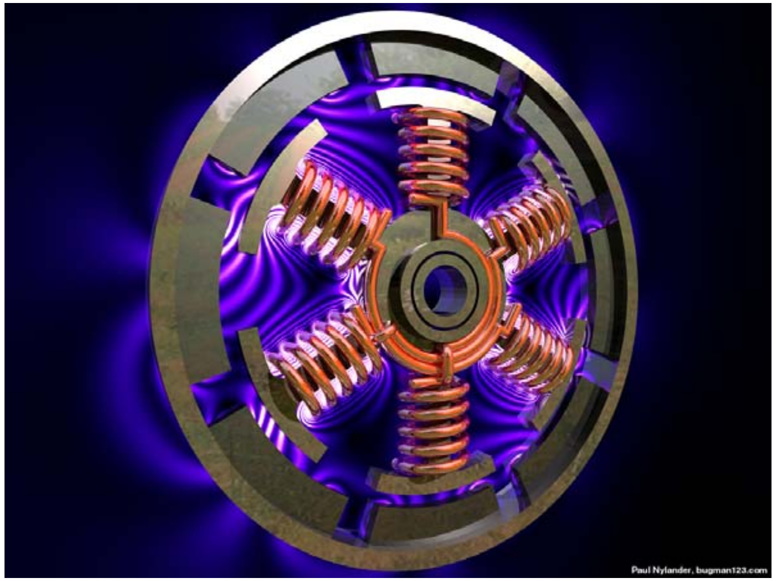


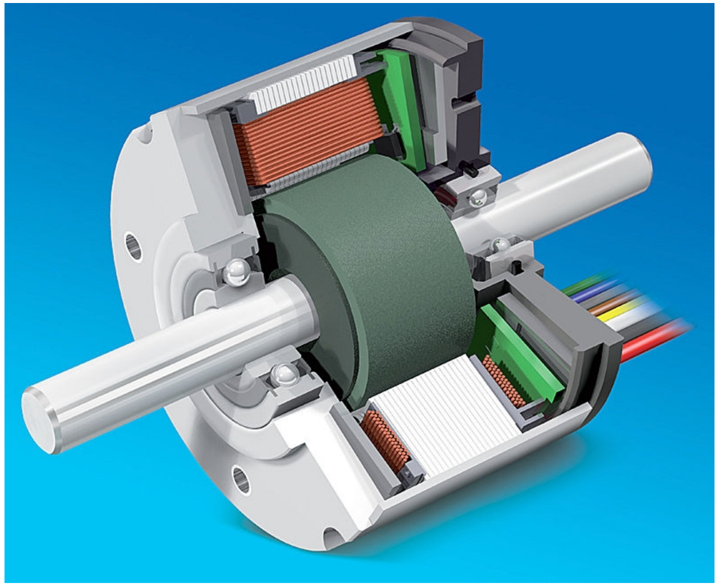


BRUSHLESS DC MACHINES (BLDC motors)

BLDC motors have many advantages over brushed DC motors and IM:

- Better speed versus torque characteristics
- High dynamic response
- High efficiency
- Long operating life (brushes and commutator limit life)
- Noiseless operation
- Higher speed ranges (brushes limit speed)
- High torque to size ratio and power to size ratios, making it useful in applications where space and weight are critical factors.





BRUSHLESS DC MACHINES (BLDC motors)

Additional advantages compared to brushed DC motors and IM:

- BLDC motors require less maintenance, so they have a longer life
- BLDC motors produce more output power per frame size
- Because the rotor is made of permanent magnets, the rotor inertia is less, compared with other types of motors. This improves acceleration and deceleration characteristics, shortening operating cycles.
- Speed regulation is easier because of their linear speed/torque characteristics.
- With brushless motors, brush inspection is eliminated, making them ideal for limited access areas and applications where servicing is difficult.
- BLDC motors operate much more quietly than brushed DC motors, reducing Electromagnetic Interference (EMI).
- Low-voltage models are ideal for battery operation, portable equipment or medical applications.

BRUSHLESS DC MACHINES (BLDC motors)

BLDC motors are used in industries such as

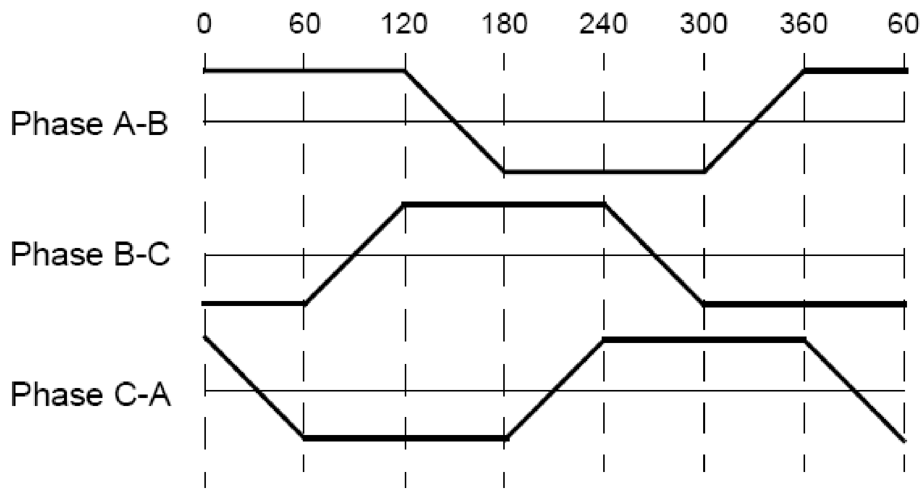
- Appliances
- Automotive
- Aerospace
- Consumer
- Medical
- Industrial Automation
- Equipment and Instrumentation
- Forty-eight volts, or less voltage rated motors are used in automotive, robotics, small arm movements and so on. Motors with 100 volts, or higher ratings, are used in appliances, automation and in industrial applications.

Stator of a BLDC

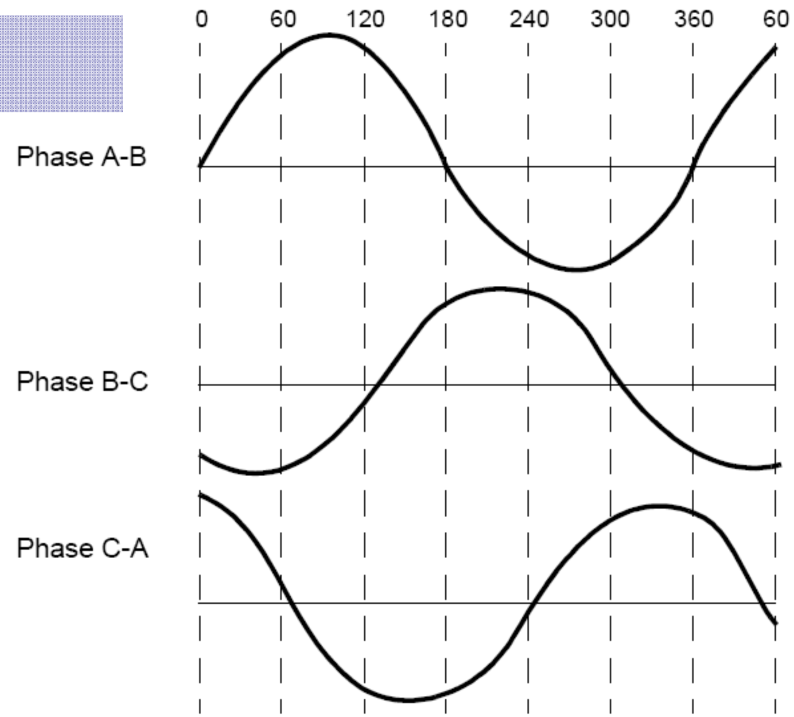
- Consists of stacked steel laminations with windings placed in the slots that are axially cut along the inner periphery
- Traditionally, the stator resembles that of an IM; however, the windings are distributed in a different manner.
- Most BLDC motors have three stator windings connected in star fashion. Each of these windings are distributed over the stator periphery to form an even numbers of poles.
- There are two types of stator windings variants: trapezoidal and sinusoidal motors.
- Stator windings are single-phase, 2-phase and 3-phase. Out of these, 3-phase motors are the most popular and widely used.

BRUSHLESS DC MACHINES (BLDC motors)

Stator of a BLDC



TRAPEZOIDAL BACK EMF



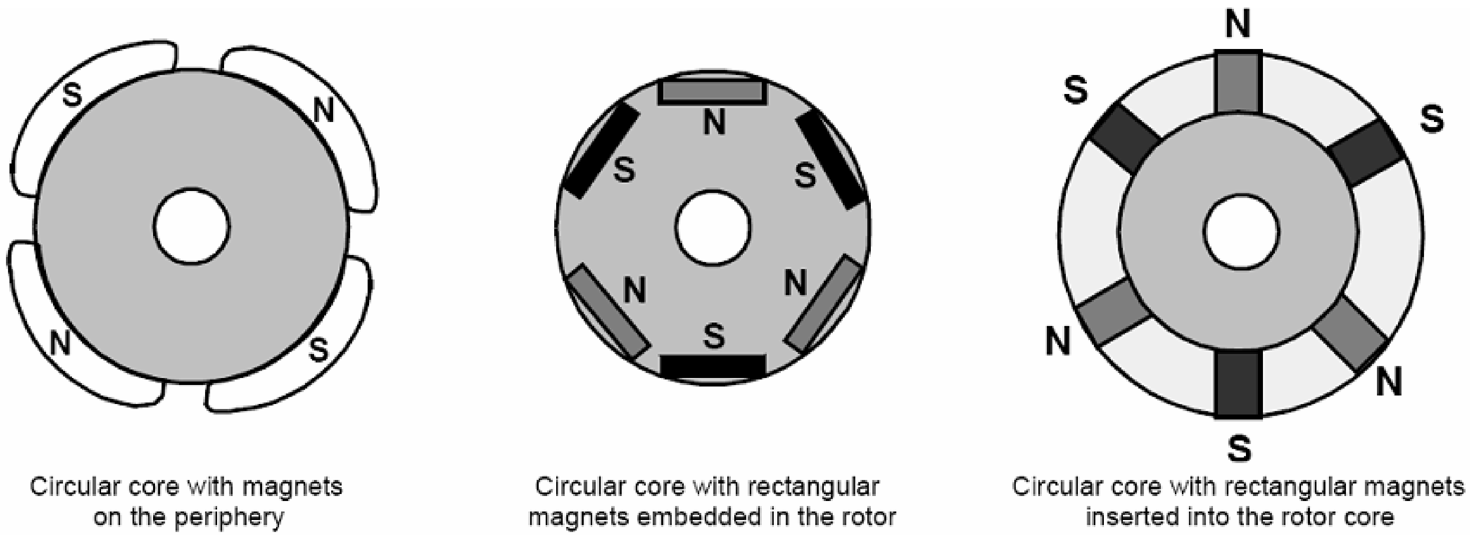
SINUSOIDAL BACK EMF

- The trapezoidal motor gives a back EMF an phase currents in trapezoidal fashion and the sinusoidal motor’s back EMF and currents are sinusoidal.
- The torque output of a sinusoidal motor is smoother than that of a trapezoidal motor.
- However, sinusoidal motors have an extra cost, as they take extra winding interconnections because of the coils distribution on the stator periphery, thereby increasing the copper intake by the stator windings.

BRUSHLESS DC MACHINES (BLDC motors)

Rotor of a BLDC

- The rotor is made of permanent magnet and can vary from 2 to 8 pole pairs with alternate North and South poles.
- Traditional magnets are made of ferrita (low cost and low flux density for a given volume)
- Rare earth alloy magnets (Nd, SmCo, NdFeB) are gaining popularity because of their high magnetic density per volume and enables the rotor to compress further for the same torque. Also, these alloy magnets improve the size-to-weight ratio and give higher torque for the same size motor using ferrite magnets.

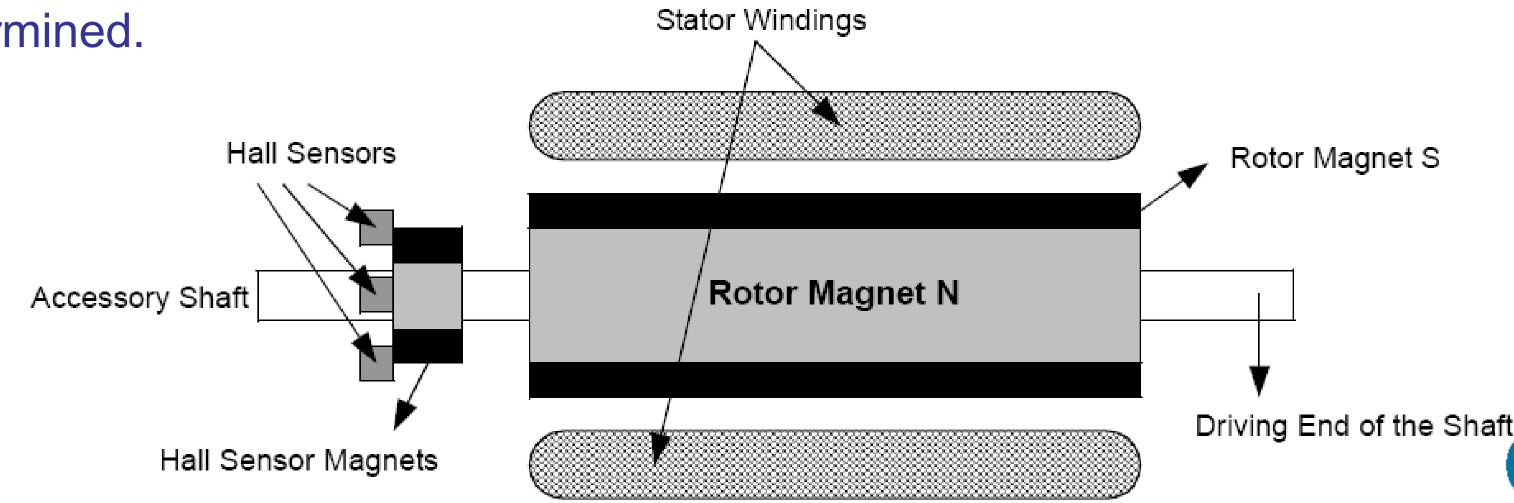


Cross sections of different arrangements of magnets in a rotor

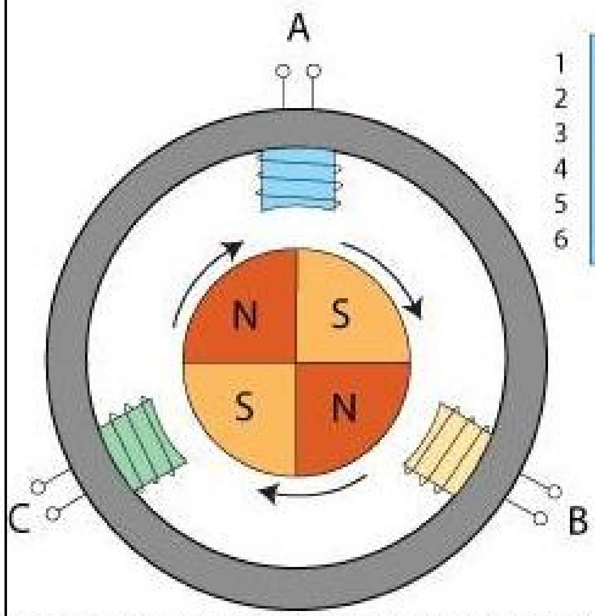
BRUSHLESS DC MACHINES (BLDC motors)

Hall sensors

- Unlike a brushed DC motor, the commutation of a BLDC motor is controlled electronically. To rotate the BLDC motor, the stator windings should be energized in a sequence.
- It is important to know the rotor position in order to decide which winding will be energized following the energizing sequence.
- Rotor position is sensed using three Hall effect sensors embedded into the stator on the non-driving end of the motor.
- Whenever the rotor magnetic poles pass near the Hall sensors, they give a high or low signal, indicating the N or S pole is passing near the sensors. Based on the combination of these signals, the exact sequence of commutation can be determined.



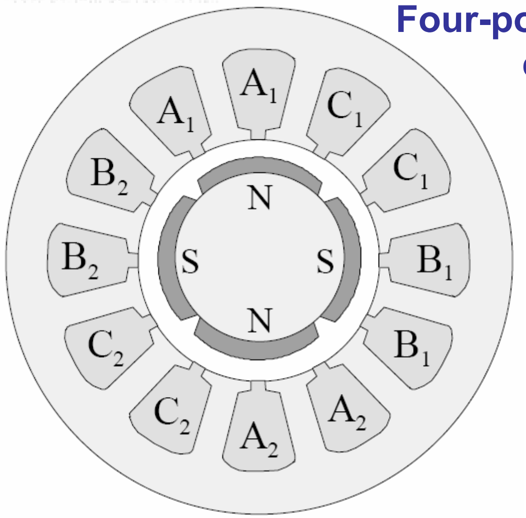
BRUSHLESS DC MACHINES (BLDC motors)



	Coil A	Coil B	Coil C
1	+	-	off
2	+	off	-
3	off	+	-
4	-	+	off
5	-	+	off
6	off	-	+

Six commutation phases "move" the electromagnetic field, which causes the permanent magnets on the rotor to move the motor shaft.

Eight-pole trapezoidal BLDC motor with inner rotor



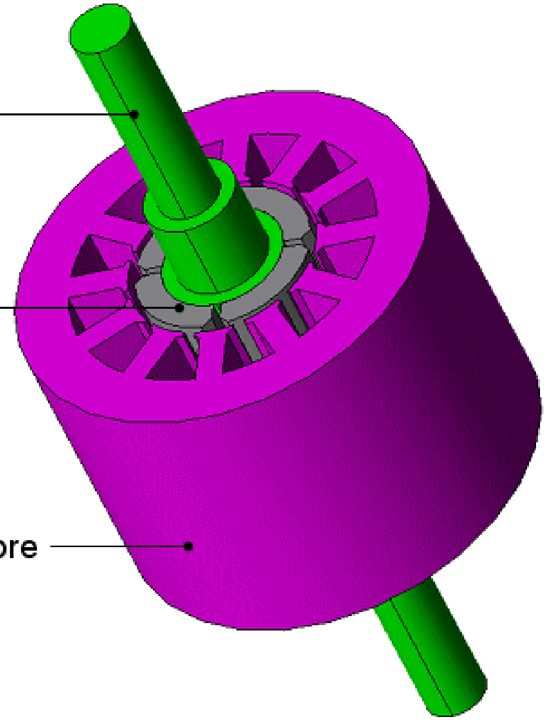
Four-pole BLDC motor with 3-phase concentrated winding

Electronic board to control a BLDC motor

PMSM main features

PMSMs are very accepted in high-performance applications due to:

- High power density
- Precise torque control
- High torque to current ratio
- Maximum power and torque density: **COMPACTNESS**
- High efficiency
- Low noise emissions
- Robustness
- High reliability



Comparison between a PMSM and an IM with the same power

PMSM main features

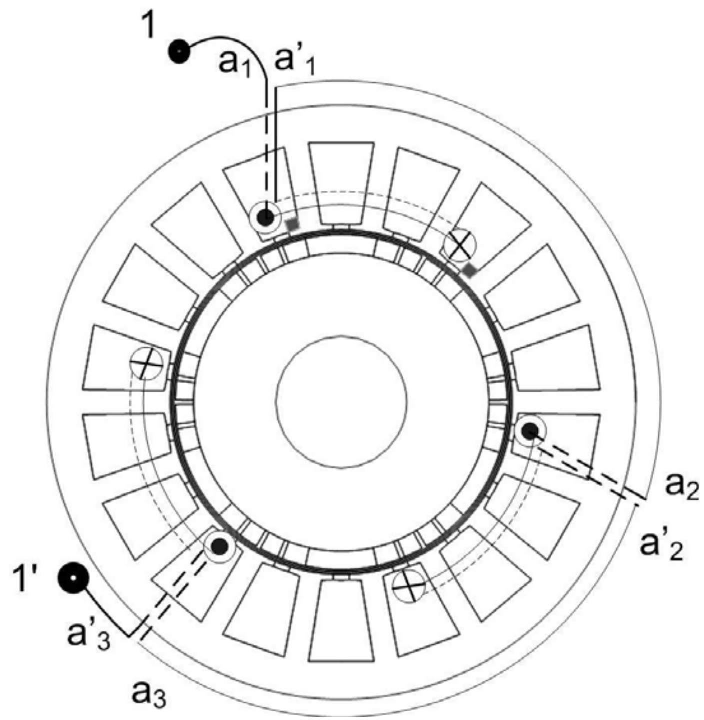
- PMSMs are synchronous machines. It means that both, stator and rotor magnetic fields rotates exactly at the same speed (slip =0).
- While the excitation current waveform was rectangular with a BLDC, **sinusoidal excitation** is used with PMSMs, which eliminates the torque ripple caused by the commutation.
- The construction of a PMSM does not differ from that of the BLDC, although **distributed windings** are more often used.
- PMSMs are typically fed by **voltage source inverters**, which allow controlling both speed and output torque.
- PMSM can be realized with either **embedded or surface magnets** on the rotor. The location of the magnets has a significant effect on the motor's characteristics.
- As in the case of BLDC motors, a position sensor is also required.

PMSM

The stator

- The stator is made usually of distributed three-phase windings. Small motors have concentrated windings.
- Three-phase sinusoidal excitation is applied by means of a voltage source inverter.

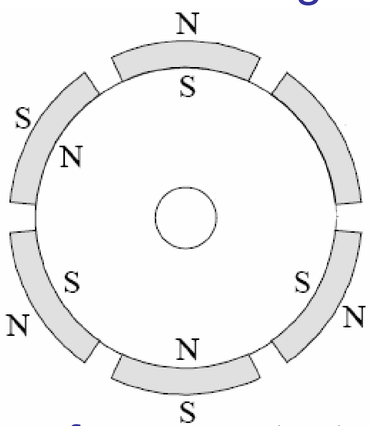
Stator of a PMSM



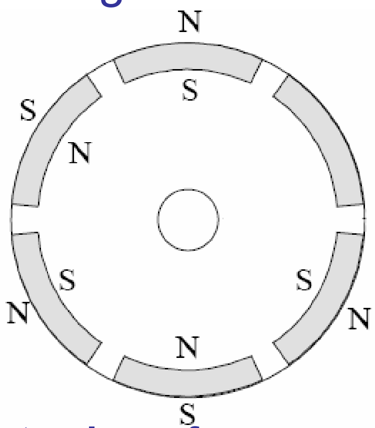
Stator of a SPMSM with 3-poles pairs

The rotor

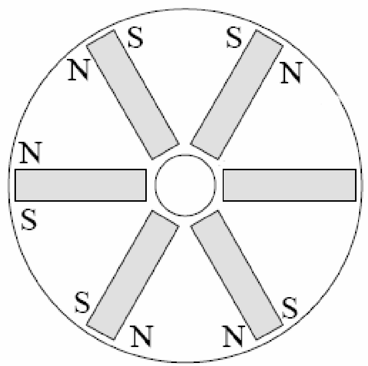
- As in the BLDC motors, the rotor is made of electrical steel laminations and permanent magnets (Nd, SmCo, NdFeB).
- PMSM can be realized with either **embedded or surface magnets** on the rotor. The location of the magnets has a significant effect on the motor's characteristics.



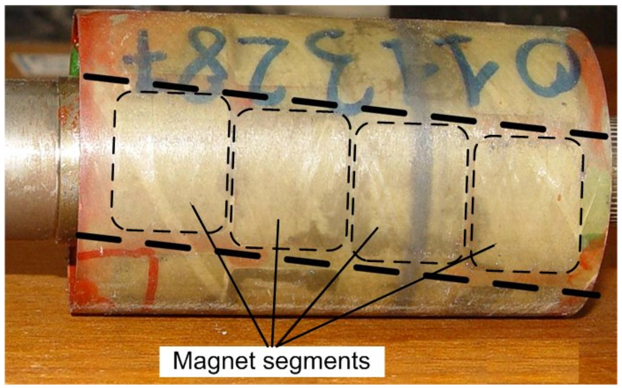
Non-salient surface magnet rotor

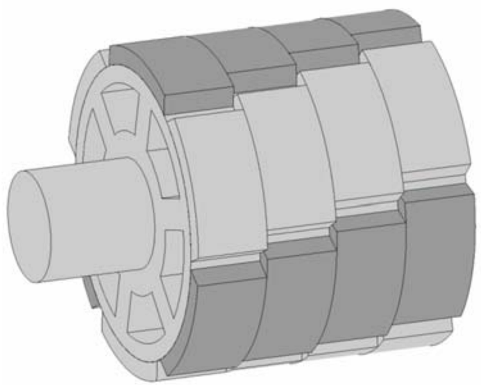


Salient pole surface magnet rotor



Embedded magnets in the rotor

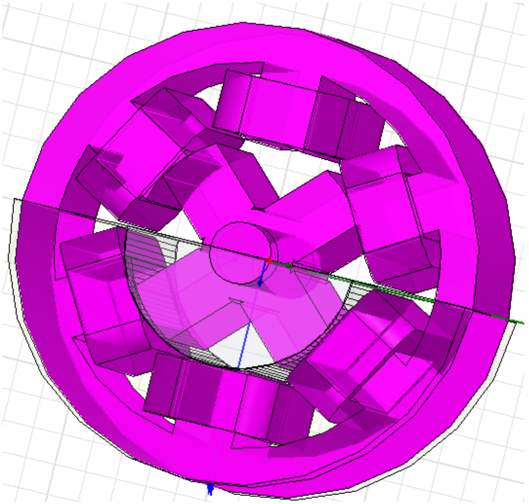


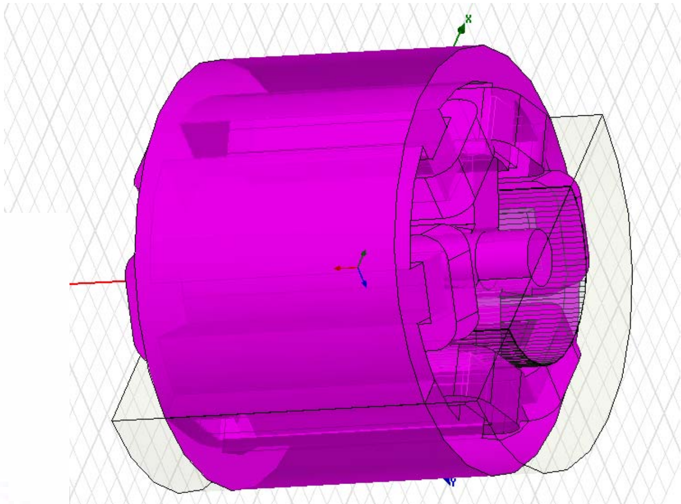


Surface magnet rotors with skewed magnets to reduce torque ripple and vibrations

SWITCHED RELUCTANCE MACHINES (SRM)

- **Magnetic reluctance** is analogous to resistance in an electrical circuit.
- An electric field causes an electric current to follow the path of least resistance.
- A magnetic field causes magnetic flux to follow the path of least magnetic reluctance.



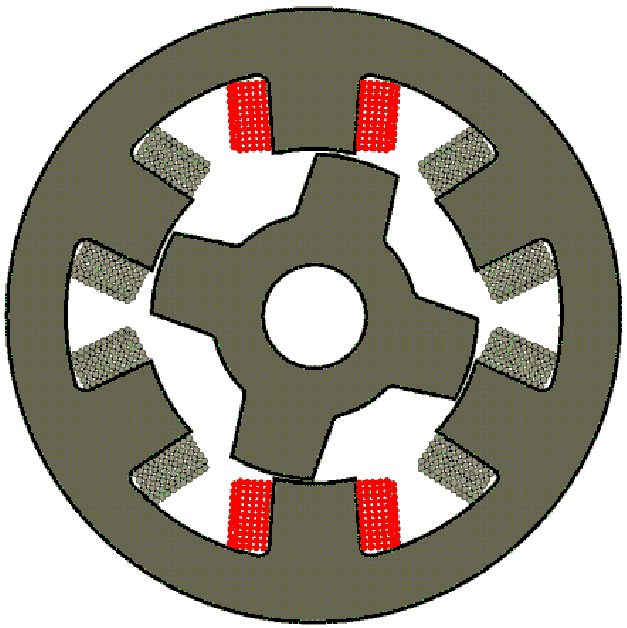


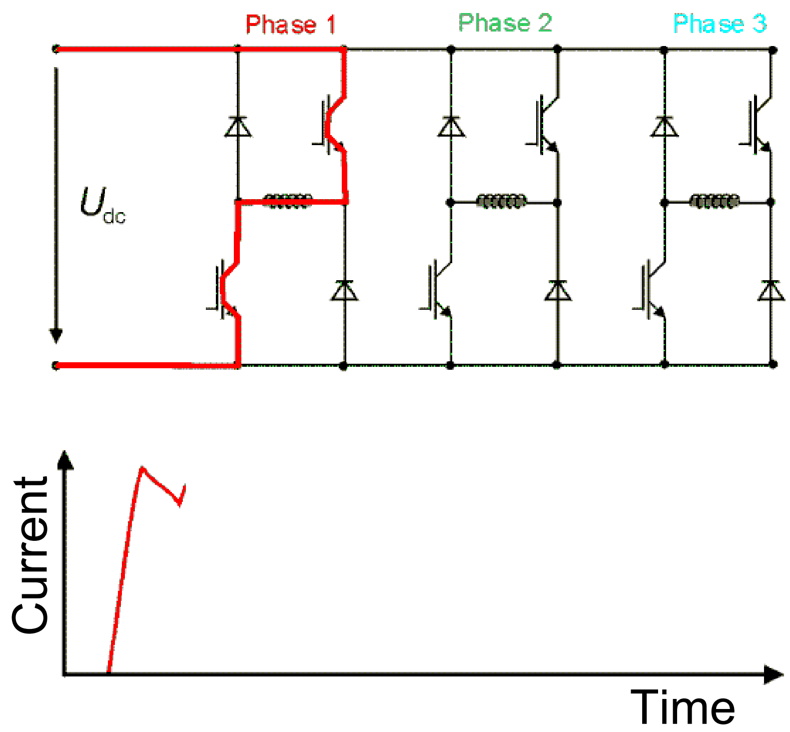
3-phase SRM with topology 12/8

w.fleadh.co.uk/srm.htm

SWITCHED RELUCTANCE MACHINES (SRM)

- **Magnetic reluctance** is analogous to resistance in an electrical circuit.
- An electric field causes an electric current to follow the path of least resistance.
- A magnetic field causes magnetic flux to follow the path of least magnetic reluctance.
- The force that attracts iron or steel to permanent magnets follows the minimum reluctance path. Reluctance is minimized when the magnet and metal come into physical contact.





http://www.isea.rwth-aachen.de/images/research/electricaldrives/researchFocus_reluctance1_med.gif

SWITCHED RELUCTANCE MACHINES (SRM)

Electrical machines can be classified into two categories on the basis of how they produce torque:

Electromagnetic torque

- Motion is produced by the interaction of two magnetic fields, one generated by the stator and the other by the rotor.
- Two magnetic fields, mutually coupled, produce an electromagnetic torque tending to bring the fields into alignment.
- Some of the familiar ways of generating these fields are through energized windings, with permanent magnets, and through induced electrical currents.

Variable reluctance torque

- Motion is produced as a result of the variable reluctance in the air gap between the rotor and the stator.
- When a stator winding is energized, producing a single magnetic field, reluctance torque is produced by the tendency of the rotor to move to its minimum reluctance position.

SWITCHED RELUCTANCE MACHINES (SRM)

The stator

- The stator is made usually of DC windings conforming three-phase or four-phase arrangements.
- DC excitation is applied and an electronic bridge is applied to properly feed the adequate phase.

The rotor

- The rotor contains no conductors or permanent magnets.
- It consists simply of steel laminations stacked onto a shaft.

http://www2.eti.uni-karlsruhe.de/wolfju/summary.htm

SRM rotor with 12 poles

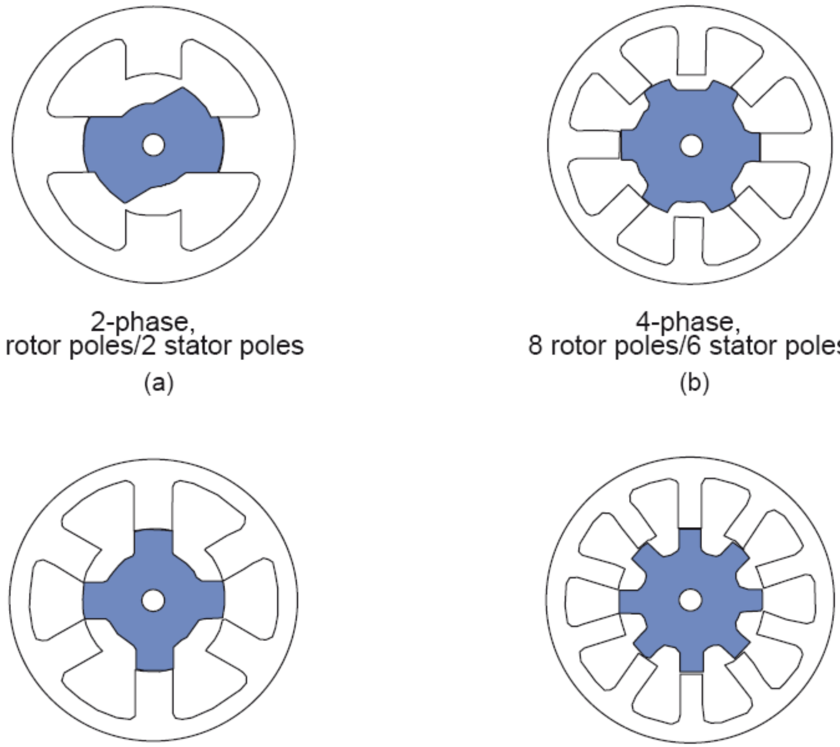
3-phase SRM with topology 6/4

PhD Thesis: Contribución al control de motores de reluctancia autoconmutados; José I. Perat Benavides

SWITCHED RELUCTANCE MACHINES (SRM)

Principle of operation

- As current is passed through one of the stator windings, torque is generated by the tendency of the rotor to align with the excited stator pole.
- The direction of torque generated is a function of the rotor position with respect to the energized phase, and is independent of the direction of current flow through the phase winding.
- Continuous torque can be produced by intelligently synchronizing each phase's excitation with the rotor position.
- Many different SRM geometries can be realized by varying the number of phases, the number of stator poles, and the number of rotor poles,.



2-phase, 4 rotor poles/2 stator poles (a)

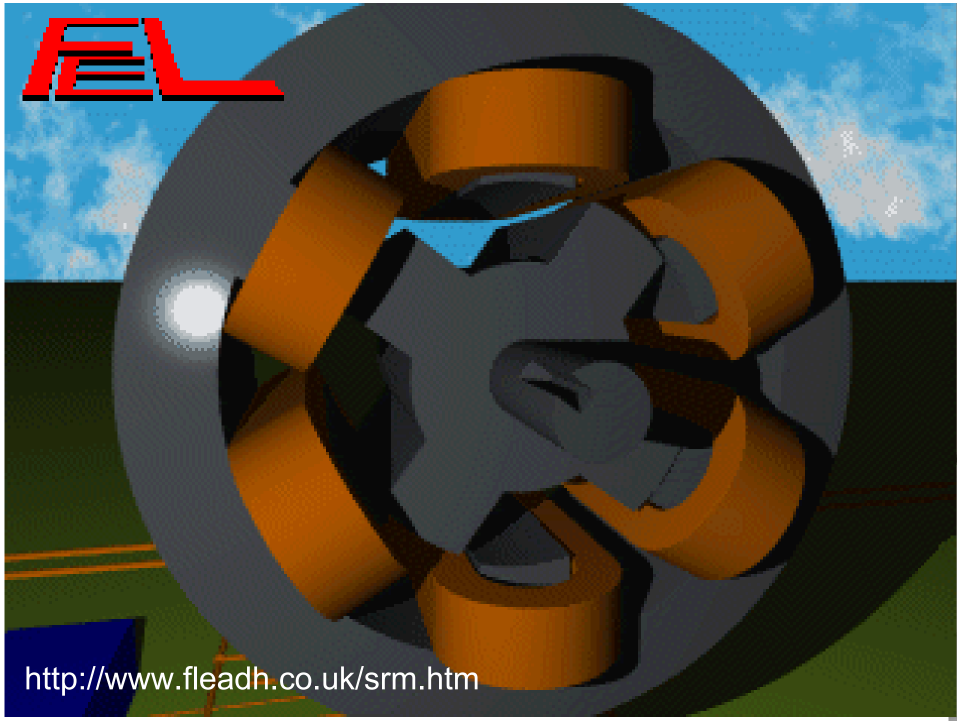
4-phase, 8 rotor poles/6 stator poles (b)

3-phase, 6 rotor poles/4 stator poles

5-phase, 10 rotor poles/8 stator poles

Switched Reluctance Motor Control – Basic Operation and Example Using the TMS320F240; Michael T. DiRenzo; Texas Instruments

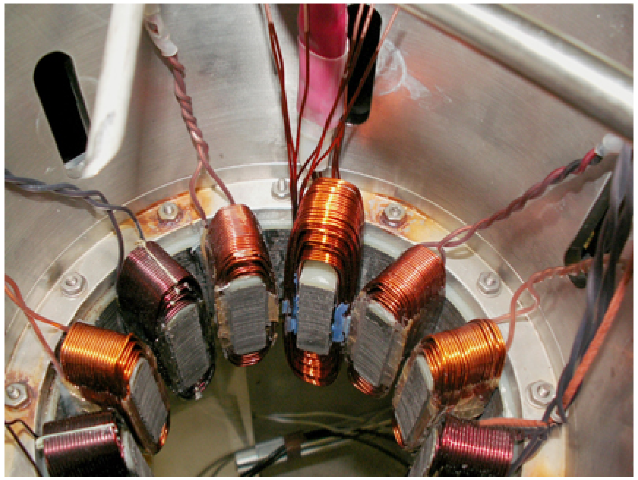
SWITCHED RELUCTANCE MACHINES (SRM)


http://www.fleadh.co.uk/srm.htm

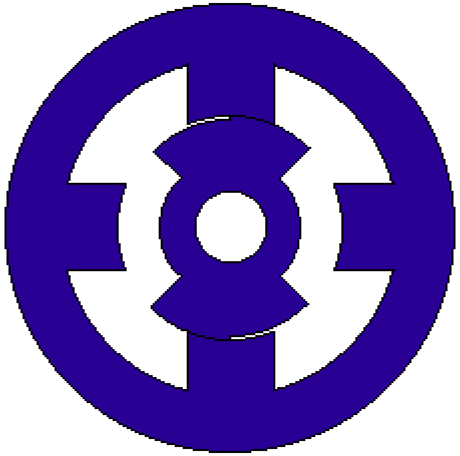

PhD Thesis: Contribución al control de motores de reluctancia autoconmutados; José I. Perat Benavides

3-phase SRM with topology 6/4

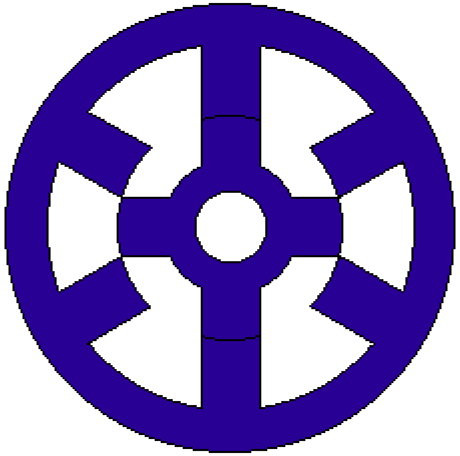
3-phase SRM with topology 12/8


http://www.grc.nasa.gov/WWW/RT/2006/images/RX51S-brown-fig.jpg

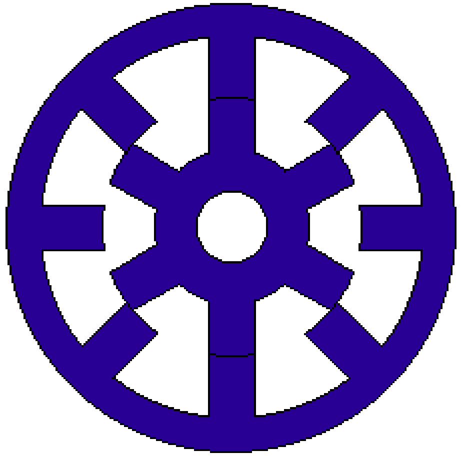
SWITCHED RELUCTANCE MACHINES (SRM)



2-phase 4/2

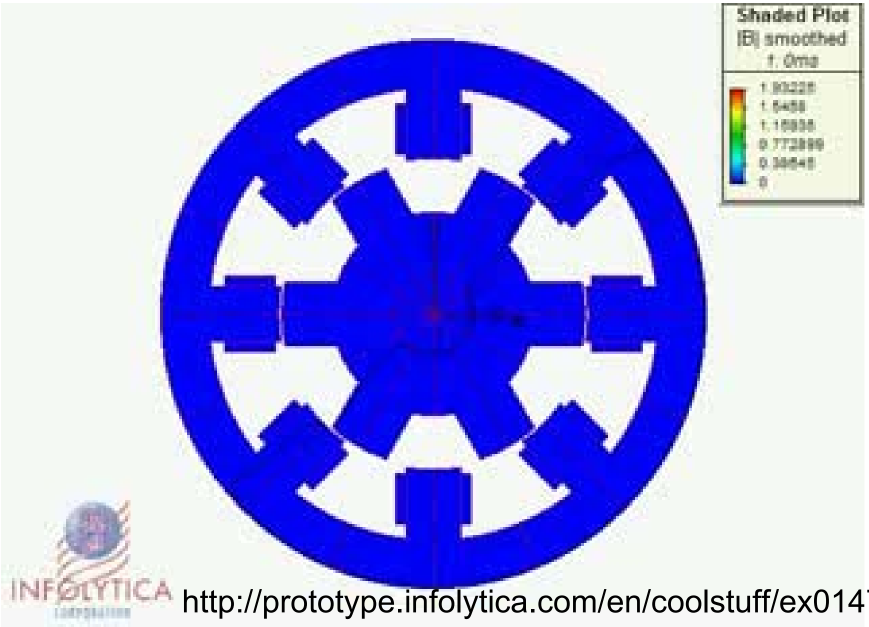


3-phase 6/4



4-phase 8/6

http://www.fleadh.co.uk/srm.htm



http://prototype.infolytica.com/en/coolstuff/ex0147/

SWITCHED RELUCTANCE MACHINES (SRM)

Advantages

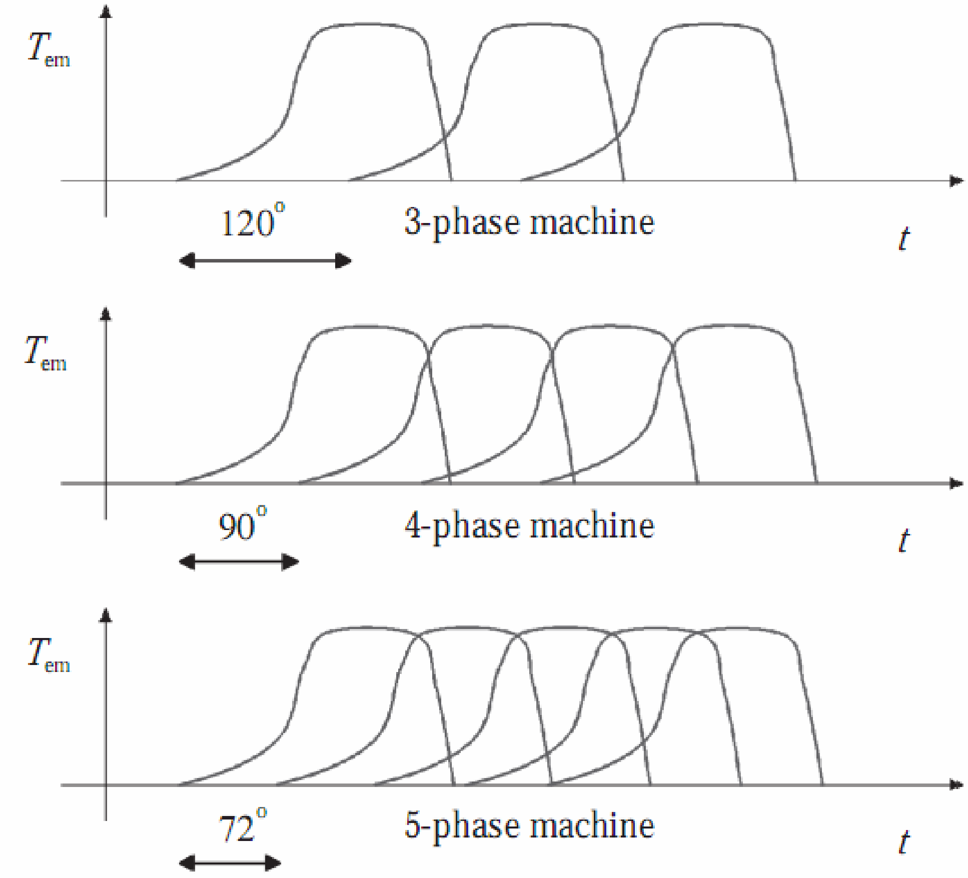
- In construction, the SRM is the simplest of all electrical machines.
- Only the stator has windings.
- Because of the lack of conductors or magnets on the rotor, very high speeds can be achieved.
- Low cost
- **Mechanical simplicity!! -> Rugged**

Limitations

- Like the BLDC and PMSM motors, SRMs can not run directly from a DC bus or an AC line. **It must always be electronically commutated.**
- The saliency of the stator and rotor, necessary for the machine to produce reluctance torque, causes strong non-linear magnetic characteristics, complicating the analysis and control of the SRM.
- SRMs **are difficult to control** and require a shaft position sensor to operate.
- They tend to be **noisy**.
- SRMs have more **torque ripple** than other types of motors.

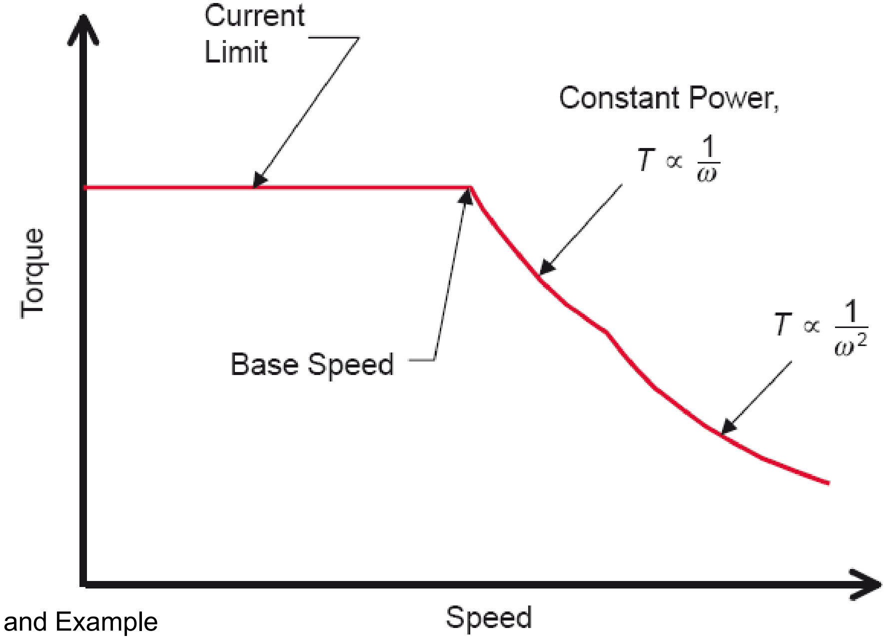
Torque ripple

- Generally, increasing the number of SRM phases reduces the torque ripple
- More phases require more electronics. Cost increase.
- At least two phases are required to guarantee starting.
- At least three phases are required to ensure the starting direction.



Torque-Speed Characteristics

- The torque-speed operating point of an SRM is determined almost entirely by the control.
- This is one of the features that makes the SRM an attractive solution.
- The envelope of operating possibilities, is limited by physical constraints such as the supply voltage and the allowable temperature rise of the motor under increasing load.
- Like other motors, torque is limited by maximum allowed current, and speed by the available bus voltage.

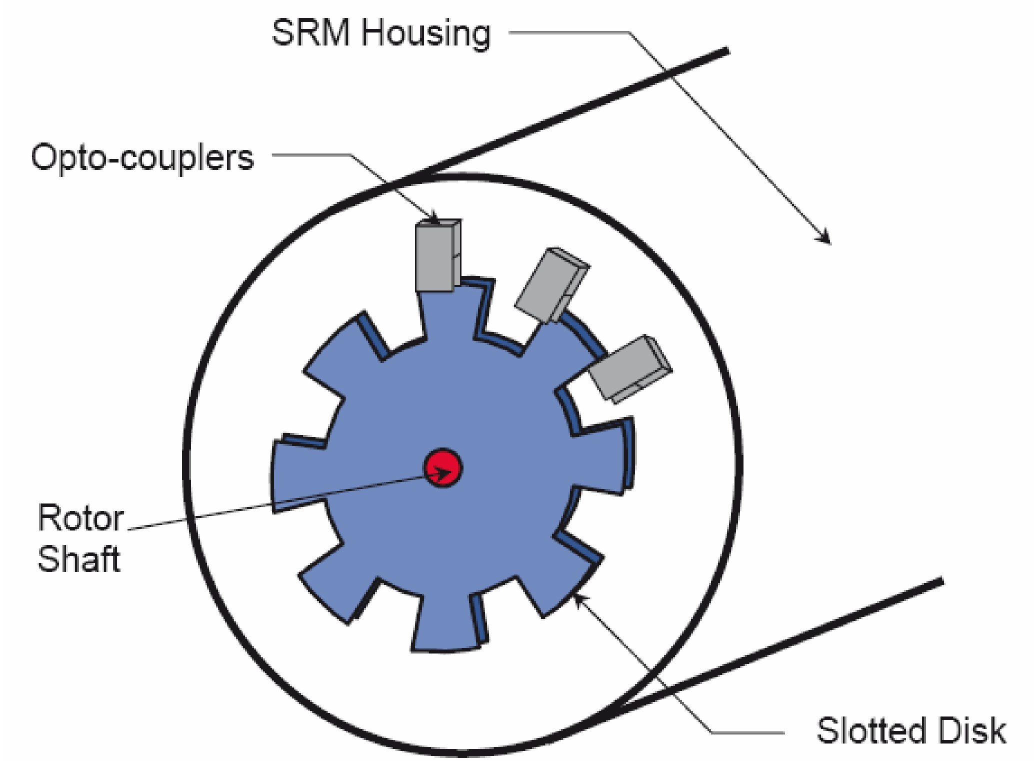


SRM Torque-Speed envelope

Switched Reluctance Motor Control – Basic Operation and Example Using the TMS320F240; Michael T. DiRenzo; Texas Instruments

Position sensor

Shaft position information is provided using a slotted disk connected to the rotor shaft and three opto-couplers mounted to the stator housing.

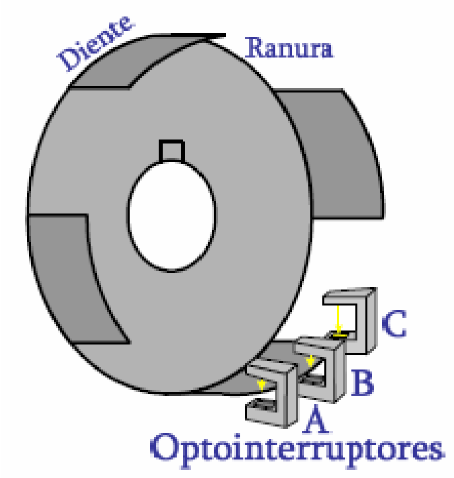


SRM Shaft Position Sensor

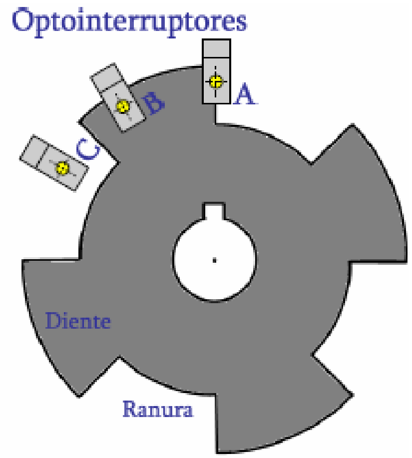
Switched Reluctance Motor Control – Basic Operation and Example Using the TMS320F240; Michael T. DiRenzo; Texas Instruments

SWITCHED RELUCTANCE MACHINES (SRM)

Position sensor

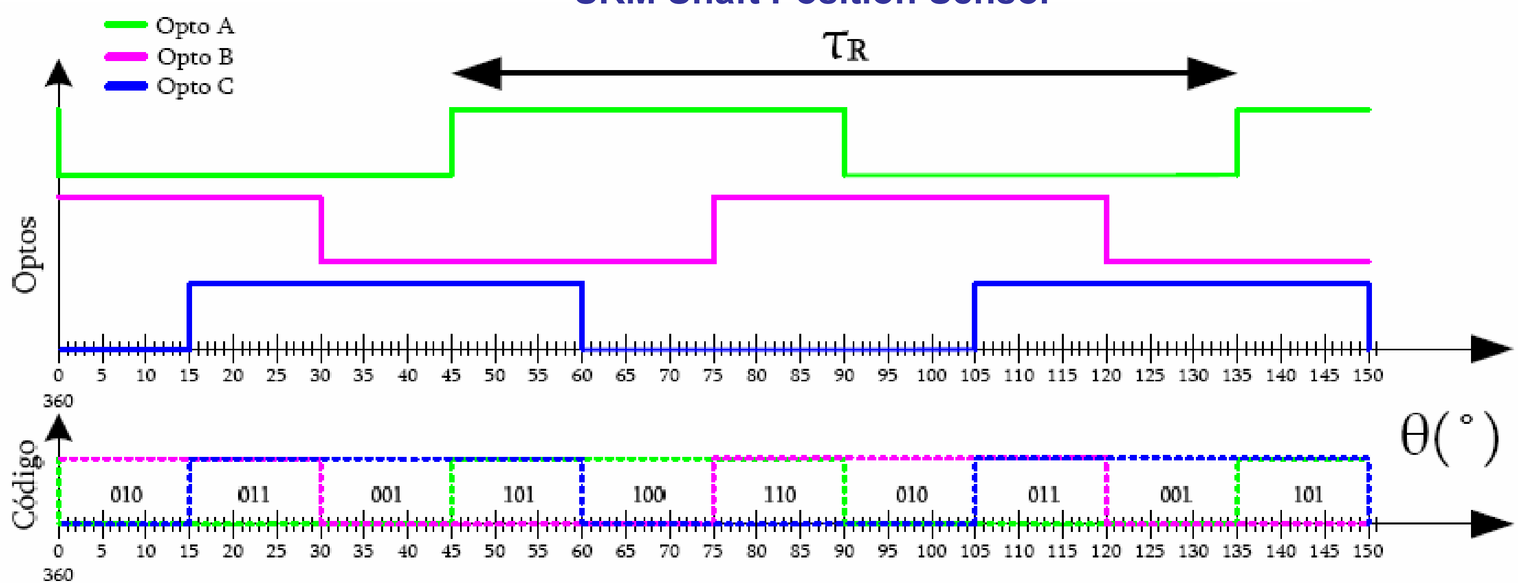


Disco ranurado cilíndrico



Disco ranurado plano

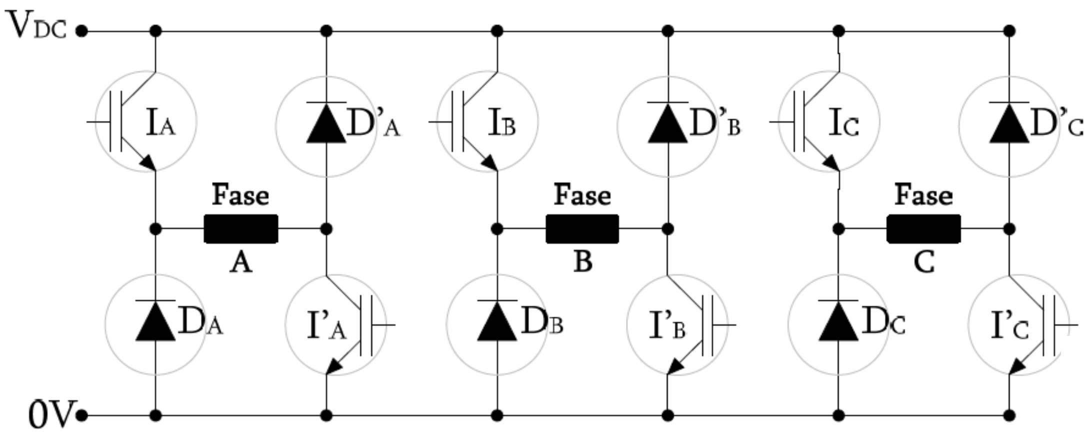
SRM Shaft Position Sensor

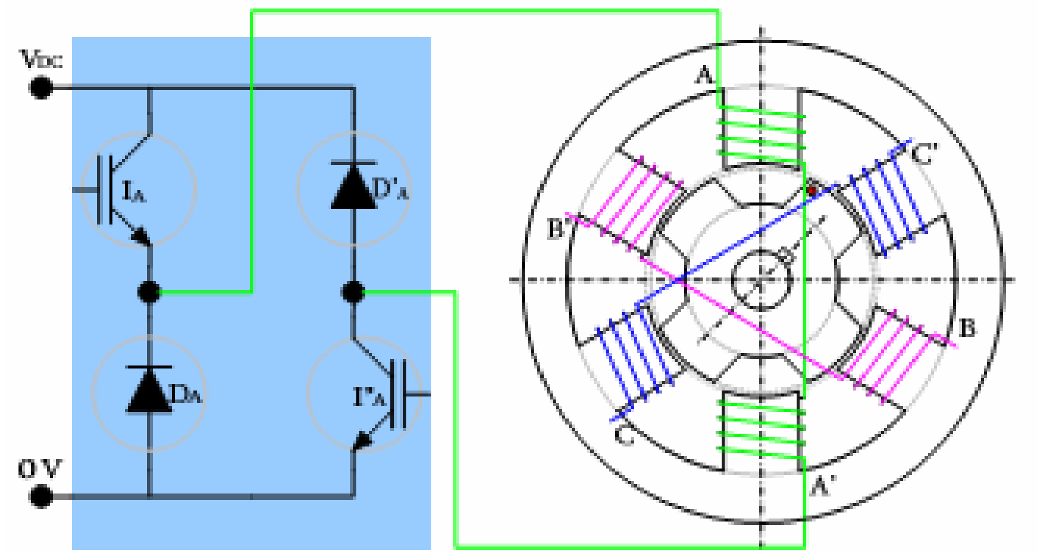


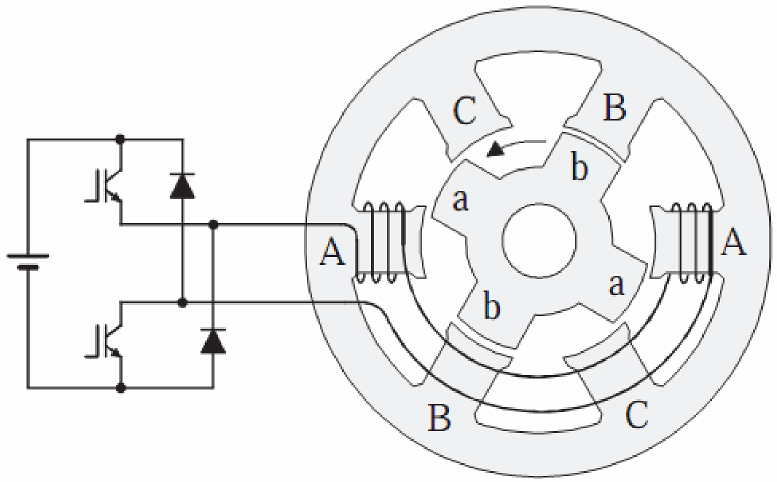
PhD thesis: Simulación interactiva de motores de reluctancia Autoconmutados; Balduino Blanqué Molina

SWITCHED RELUCTANCE MACHINES (SRM)

Electronic converter







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