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Interconnection and damping assignment (IDA) passivity-based control (PBC) is currently a well-known viable alternative for solving regulation control problems of a wide class of nonlinear systems. However, a distinctive feature that, in spite of its appearance under several applications, has not been exhaustively exploited, is the flexibility that this technique exhibits for designing output-feedback controllers (OFCs). The purpose of this article is to illustrate this attractive characteristic by approaching the (practically important) case study given by the improvement of the transient stability properties of power systems. The particular system composed by a synchronous generator connected to an infinite bus via a thyristor controlled series capacitor is considered. Two OFCs are presented, one that does not involve the unmeasurable state and another that, although including this state, presents some input-to-state stability properties that allow for establishing a sort of separation principle concerning an observer-based structure for the closed-loop system. The advantages of both controllers are illustrated by numerical simulations when a three-phase short circuit at the generator bus is induced.

**Keywords:** nonlinear control; IDA passivity-based control; output-feedback; SMIB; TCSC



## 1. Introduction

Output-feedback control (OFC) is the branch of control theory that deals with the problem of designing control schemes involving only available for measurement information. Roughly speaking, this task can be carried out following two general approaches, namely: one that synthesises the control law using only the available for measurement states (pure OFC) and another that substitutes the unavailable state for a corresponding estimate obtained from a dynamical observer (observer-based OFC).

Due to the structural richness of nonlinear systems, the solutions that can be found in the literature for solving OFC problems is also varied. However, identification of new ways for approaching this topic still imposes an attractive research challenge. Hence, the aim of this article is to illustrate, by means of a practically important problem, how the interconnection and damping assignment (IDA) passivity-based control (PBC) methodology design (Ortega and Garcia-Canseco 2004) can state a viable alternative in this sense.

The motivation to approach the OFC problem under the IDA-PBC perspective comes from the experiences reported in Ortega and Garcia-Canseco (2004), regarding the control of an electromechanical system, and in Batlle, Doria-Cerezo, Espinosa-Perez, and Ortega (2009), concerning the induction machine control, where it has been shown that exploiting the flexibility offered for solving the *matching equation* (ME), a key step in the controller design, can lead to the proposition of control laws that do not require unmeasurable states.

In this article, this possibility is exploited to propose two controllers that solve the transient stability control problem of a single machine infinite bus (SMIB) system equipped with a thyristor controlled series capacitor (TCSC). The contribution actually consists of one pure and one observer-based OFC that are obtained by tailoring the ME of the IDA-PBC design. For the latter, a novel observer is also introduced and the stability of the closed-loop system is proved by exploiting another feature of the approach that has been also previously identified (Moreno and Espinosa-Pérez 2007), namely, the input-to-state stability (ISS) properties exhibited from the observation to the control errors.

Concerning the case study approached in this article, the importance of the transient stability problem in power networks is evident. The stringent operation conditions imposed to these systems induce oscillations, e.g. by the presence of disturbances,

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that could lead to unstable behaviours (Machowski, Bialek, and Bumby 2008) unless they are damped in a proper way. Usually, this undesirable operation is avoided by means of the power system stabiliser (PSS) and/or the automatic voltage regulator (AVR). However, in many situations, the effect of these devices is not sufficient and the use of power converters has emerged as an efficient complement to achieve the desired behaviour (Hingorani and Gyugyi 2000).

In this work it is assumed that for an SMIB system, the action of the PSS is complemented by the presence of a TCSC. This scheme is currently well known and widely accepted, due to its proved capability for improving transient stability properties besides its primary functions, such as voltage and power flow control, and several controllers for this scheme have been reported dealing with its nonlinear structure and the fact that not all states are available for measurement. Unfortunately, they belong to the class of state-feedback controllers or observer-based schemes, but exhibiting a complicated structure (Messina, Hernandez, Barocio, Ochoa, and Arroyo 2002; Sun, Liu, Song, and Shen 2002; de Leon-Morales, Espinosa-Pérez, and Maya-Ortiz 2004; Manjarekar, Banavar, and Ortega 2008). In this sense, the distinctive feature of the contribution presented in this article lies in the simplicity of the developed controllers.

This article is organised as follows. Section 2 is devoted to formulate the control problem approached in this article, including the considered model for the SMIB–TCSC system and a brief description of the IDA-PBC design. The main contribution is presented in Section 3 while its usefulness is illustrated via a numerical evaluation in Section 4. Section 5 is dedicated to the presentation of some concluding remarks.



## 2. Problem formulation

In this section, the considered model for the SMIB–TCSC system is first presented to later on, after quickly reviewing the controller design methodology, formulate the control problem.

### 2.1 *SMIB–TCSC system*

A widely accepted model for describing the dynamic behaviour of a single synchronous generator connected to an infinite bus, known as an SMIB system, is the so-called *Flux decay model* which is given by the following third-order nonlinear system (Pai 1989)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= P_m - a_1 x_2 - \bar{a}_2 \bar{x}_3 \sin(x_1) \\ \dot{\bar{x}}_3 &= b_3 \cos(x_1) - b_4 \bar{x}_3 + E + \bar{u}, \end{aligned} \tag{1}$$

where $x_1$ is the load angle, $x_2$ is the shaft speed deviation from the synchronous speed and $\bar{x}_3$ is the quadrature axis internal voltage. The *constant* mechanical power delivered to the generator is $P_m$ while the input $E + \bar{u}$ is the field voltage, $E$ being the constant value required to maintain the machine on a stable equilibrium point in the operation region of the system given by $0 \leq x_1 < \frac{\pi}{2}$. Among all the positive coefficients, particularly important in this article is $\bar{a}_2 = \frac{V}{X_\Sigma}$ since it includes the bus voltage $V$ and the total line *reactance* $X_\Sigma$.

If it is considered that the generator is provided with a PSS–AVR control system, one way for improving the transient stability properties of the system, initially introduced in Vithayathil (1986) as a 'rapid adjustment method for network impedance', is to include a switched capacitor, in series connection as shown in Figure 1.

Under this structure, it is possible to consider only the mechanical dynamics of the synchronous generator, the so-called Swing equation, for describing the behaviour of the considered system, while the effect of the included capacitor on the modified line reactance can be modelled as a first-order system (Hingorani and Gyugyi 2000), resulting in a model given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= P_m - a_1 x_2 - a_2 x_3 \sin(x_1) \\ \dot{x}_3 &= b_1\big(-x_3 + x_3^* + u\big), \end{aligned} \tag{2}$$

where $x_3 > 0$ is the total *admittance* of the system, $a_2 = E'V$ stands for the product of the bus voltage and $E'$, the transient voltage of the generator, and $b_1 = 1/T_{dc}$ is a positive constant which depends on the time constant included to model the dynamic response of the TCSC. In this case the control input $u$ is related with the firing angle for the switch while the operation region of the system is still given by $0 \leq x_1 < \frac{\pi}{2}$.

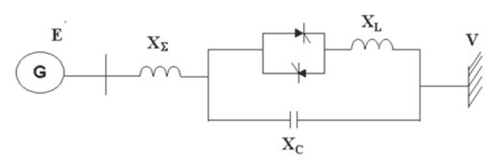


Figure 1. SMIB system with TCSC.

In order to formulate the control problem related with this system, it is necessary to notice that its equilibria is given by two solutions of

$$\begin{aligned} x_2^* &= 0 \\ P_m &= a_2 x_3^* \sin(x_1^*) \\ x_3 &= x_3^*. \end{aligned} \tag{3}$$

However, the practical interest lies on the solution corresponding to $x_1^* = \sin^{-1}(P_m/a_2 x_3^*)$ since the other one is outside the operation region of the system. Then the control problem is posed as the stabilisation of the equilibrium point $x^* = (x_1^*, x_2^*, x_3^*) = (\sin^{-1}(P_m/a_2 x_3^*), 0, x_3^*)$, problem that is further complicated since this objective must be achieved considering that the total admittance of the system $x_3$ is not available for measurement.

**Remark 1:** It is interesting to mention that if instead of using the admittance of the system as third state, the effective line reactance is used, it is still possible to apply IDA-PBC design, as actually has been done in Manjarekar et al. (2008). Unfortunately, the output-feedback case is not approached in this article.

### 2.2 *IDA-PBC design*

The problem of stabilising an equilibrium point of nonlinear systems of the form

$$\dot{x} = f(x, t) + g(x)u, \tag{4}$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^m$ $(m < n)$ is the control action and $g(x)$ is assumed full rank, is approached from the IDA-PBC perspective by finding a control law $u(x)$ that leads to a closed-loop system of the form

$$\dot{x} = F_d(x, t)\nabla_x H_d(x), \tag{5}$$

with[1] $F_d(x, t) + F_d^T(x, t) \leq 0$ and $H_d(x) \geq 0$ is a scalar (energy) function, which satisfies the condition

$$x^* = \arg\min H_d(x),$$

$x^*$ being the equilibrium to be stabilised.

A system with structure as introduced in (5) is known as a Hamiltonian system and the rational behind the selection of this structure is that if $H_d(x)$ is considered as a Lyapunov function of the system, then its time derivative along the trajectories of (5) is given by

$$\dot{H}_d = -\left(\frac{\partial H_d(x)}{\partial x}\right)^T \left(F_d(x) + F_d^T(x)\right)\frac{\partial H_d(x)}{\partial x},$$

proving (e.g. Lemma 3.2.8 of van der Schaft (2000)) that the equilibrium $x^*$ will be asymptotically stable if the system is detectable from $y = g^T(x)\nabla_x H_d(x)$, i.e. if the implication $y(t) \equiv 0 \Rightarrow \lim_{t\to\infty} x(t) = x^*$ is true.

In spite of its clear formulation, the major problem to carry the design out of the controller $u(x)$ comes from the necessity of solving the so-called $ME$ given by

$$g^{\perp}(x) f(x, t) = g^{\perp}(x) F_d(x, t)\nabla H_d(x), \tag{6}$$

where $g^{\perp}(x) \in \mathcal{R}^{(n-m)\times n}$ is a full-rank left-annihilator of $g(x)$, that is, $g^{\perp}(x)g(x) = 0$ and rank $g^{\perp}(x) = n - m$. As can be noticed, Equation (6) involves the under-actuated part of system and the complication for finding a solution comes from the fact that looking for functions $F_d(x, t)$ and $H_d(x)$ that are compatible with the dynamic behaviour of system (4), it is equivalent to solve a nonlinear partial differential equation in the indeterminant $H_d(x)$.

### 2.3 *Output-feedback IDA-PBC problem*

Being the solution of (6), a fundamental element for implementing the IDA-PBC, a lot of research has been devoted to this topic (Ortega et al. 2004). However, the main interest of this article is related with the possibility for solving this equation in an output-feedback way for the system (2). Hence, the problem approached in this article can be formulated as follows:

Consider the system (2) with state vector given by

$$x = [x_1 \quad x_2 \quad x_3]^T,$$

where $x_3$ is unmeasurable. Design a control law $u$ by finding a solution of (6) in such a way that one of the next two conditions is satisfied:

- the control law does not depend on the unmeasurable state $x_3$, i.e. $u = f_u(x_1, x_2)$, or
- the dependency of the proposed controller on the unmeasurable state allows for designing an observer-based control scheme, i.e.

$$\begin{aligned} u &= f_u(x_1, x_2, \zeta) \\ \dot{\zeta} &= f_o(\zeta, x_1, x_2), \end{aligned}$$

whose stability and convergence properties could be stated in a simple as possible way.

## 3. Main result

The main contributions of this article are presented in this section, namely, one pure and one observer-based OFC (equipped with a novel observer) for system (2). This section first introduces the common part of the design for both controllers and later on presents the particular solutions.

### 3.1 *Common step design*

In order to carry out the controller design by following the general procedure presented in Section 2.2, notice that the purpose in this case is to find a solution of the equation given by

$$f(x) + gu = F_d \frac{\partial H_d(x)}{\partial x}, \tag{7}$$

where

$$f(x) = \begin{bmatrix} x_2 \\ P_m - a_1 x_2 - a_2 x_3 \sin(x_1) \\ -b_1(x_3 - x_3^*) \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix}$$

while the matrix $F_d$ is assumed to have the following form:

$$F_d = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix},$$

with each entry $F_{ij}$ of appropriate dimension.

With the above definitions, solving the imposed control problem reduces to find suitable functions that satisfy the following equalities:

$$x_2 = F_{11}\frac{\partial H_d(x)}{\partial x_1} + F_{12}\frac{\partial H_d(x)}{\partial x_2} + F_{13}\frac{\partial H_d(x)}{\partial x_3}, \tag{8}$$

$$\begin{aligned} &P_m - a_1 x_2 - a_2 x_3 \sin(x_1) \\ &\quad = F_{21}\frac{\partial H_d(x)}{\partial x_1} + F_{22}\frac{\partial H_d(x)}{\partial x_2} + F_{23}\frac{\partial H_d(x)}{\partial x_3}, \end{aligned} \tag{9}$$

$$\begin{aligned} &-b_1(x_3 - x_3^*) + b_1 u \\ &\quad = F_{31}\frac{\partial H_d(x)}{\partial x_1} + F_{32}\frac{\partial H_d(x)}{\partial x_2} + F_{33}\frac{\partial H_d(x)}{\partial x_3}, \end{aligned} \tag{10}$$

while guaranteeing, at the same time, that both the equilibrium assignment condition and the stability condition $F_d(x,t) + F_d^T(x,t) \le 0$ are simultaneously satisfied.

Due to the underactuated nature of the system, it is important to notice that Equations (8) and (9) do not depend on the control input $u$ leading to the fact that they must be solved before dealing with the control-dependent equation (10). Accomplishing this step in the controller, the design is carried out in the following proposition.

**Proposition 3.1:** *Consider the input independent equations of the ME* (7) *given by* (8)–(9). *A solution for the corresponding entries of matrix $F_d(x)$ that, at the same time, assigns the equilibrium to be stabilised as a minimum of $H_d(x)$ is given by*

$$\begin{aligned} &F_{11} = 0; \quad F_{12} = \frac{1}{k_1}; \quad F_{13} = 0 \\ &F_{21} = -\frac{1}{k_1}; \quad F_{22} = -\frac{a_1}{k_1}; \quad F_{23} = 0, \end{aligned} \tag{11}$$

*and*

$$\begin{aligned} H_d(x) = &\frac{k_1}{2}x_2^2 + k_1 a_2 x_3\big[\cos(x_1^*) - \cos(x_1)\big] \\ &- k_1 a_2 x_3^* \sin(x_1^*)(x_1 - x_1^*) + H_{d2}(x_3), \end{aligned} \tag{12}$$

*with $k_1$ a positive constant and $H_{d2}(x_3)$ satisfying*

$$\begin{aligned} &\left(\frac{\partial H_{d2}(x_3)}{\partial x_3}\right)_{x_3 = x_3^*} = 0; \\ &\left(\frac{\partial^2 H_{d2}(x_3)}{\partial x_3^2}\right)_{x_3 = x_3^*} > \frac{k_1 a_2 \sin^2(x_1^*)}{x_3^* \cos(x_1^*)}. \end{aligned} \tag{13}$$

**Proof:** From (8) and considering

$$H_d(x) = \frac{k_1}{2}x_2^2 + H_{d1}(x_1, x_3),$$

leads[2] to $F_{11} = F_{13} = 0$ and $F_{12} = \frac{1}{k_1}$. Looking for a possible simple way for assuring the required stability properties, an immediate definition is $F_{21} = -\frac{1}{k_1}$ and $F_{22} = -\frac{a_1}{k_1}$ implying, considering (9) and (14), that

$$-\frac{1}{k_1}\frac{\partial H_d(x)}{\partial x_1} + F_{23}\frac{\partial H_d(x)}{\partial x_3} = -a_2 x_3 \sin(x_1) + P_m.$$

Taking into account, from (3), the equilibrium value of $P_m$, it is possible to define

$$\begin{aligned} H_{d1}(x_1, x_3) = &k_1 a_2 x_3\big[\cos(x_1^*) - \cos(x_1)\big] \\ &- k_1 a_2 x_3^* \sin(x_1^*)(x_1 - x_1^*) + H_{d2}(x_3), \end{aligned}$$

which in its turn requires that

$$F_{23}\left\{a_2\big[\cos(x_1^*) - \cos(x_1)\big] + \frac{\partial H_{d2}(x_3)}{\partial x_3}\right\} = 0,$$

forcing $F_{23}$ to become zero.

The final part of the proof, concerning the equilibrium point assignment, is carried out by noticing that up to this point the structure of the proposed desired energy function is

$$\begin{aligned} H_d(x) = &\frac{k_1}{2}x_2^2 + k_1 a_2 x_3\big[\cos(x_1^*) - \cos(x_1)\big] \\ &- k_1 a_2 x_3^* \sin(x_1^*)(x_1 - x_1^*) + H_{d2}(x_3). \end{aligned} \tag{14}$$

Thus, the first condition in (13) for $H_{d2}(x_3)$ appears from the necessity of satisfying

$$\left(\frac{\partial H_d(x)}{\partial x}\right)_{x=x^*} = \begin{bmatrix} k_1a_2x_3\sin(x_1) - k_1a_2x_3^*\sin(x_1^*) \\ k_1x_2 \\ k_1a_2\cos(x_1^*) - k_1a_2\cos(x_1) + \dfrac{\partial H_{d2}(x_3)}{\partial x_3} \end{bmatrix}_{x=x^*} = 0,$$

while the second condition that appears in (13) comes from the structure of the Hessian matrix associated to $H_d(x)$ which, in order to guarantee that $\operatorname{argmin}\{H_d(x)\} = x^*$, must satisfy that

$$\left(\frac{\partial^2 H_d(x)}{\partial x^2}\right)_{x=x^*} = \begin{bmatrix} k_1a_2x_3^*\cos(x_1^*) & 0 & k_1a_2\sin(x_1^*) \\ 0 & k_1 & 0 \\ k_1a_2\sin(x_1^*) & 0 & \dfrac{\partial^2 H_{d2}(x_3)}{\partial x_3^2} \end{bmatrix}_{x=x^*} > 0,$$

condition that is fulfilled, applying standard Schur's complement arguments, if and only if

$$\left(\frac{\partial^2 H_{d2}(x_3)}{\partial x_3^2}\right)_{x_3=x_3^*} > \frac{k_1a_2\sin^2(x_1^*)}{x_3^*\cos(x_1^*)}.$$

□

The following remarks are in order about the result presented above.

**Remark 2:** It is interesting to notice how the construction of matrix $F_d$ was carried out by defining in an advantageous way each of their entries with the aim of facilitating the stability proof. In addition, it is also important to mention that the designer arrives to the controller design step equipped with several degrees freedom, given by the entries of matrix $F_d$ that have not yet been defined and the function $H_{d2}(x_3)$.

Once the fixed part of matrix $F_d$ has been defined, the rest of the design is related with choosing its free entries. In this sense, notice that under the definition of the desired energy function (14), Equation (10) takes the form

$$\begin{aligned} &-b_1(x_3 - x_3^*) + b_1u \\ &\quad = F_{31}\left[-k_1a_2x_3^*\sin(x_1^*) + k_1a_2x_3\sin(x_1)\right] \\ &\qquad + F_{32}k_1x_2 + F_{33}\left\{k_1a_2\left[\cos(x_1^*) - \cos(x_1)\right] + \frac{\partial H_{d2}(x_3)}{\partial x_3}\right\}. \end{aligned} \tag{15}$$

As can be seen, there exist several possibilities for solving this equation. Among them, the designer must look for those that while satisfying condition $F_d(x,t) + F_d^T(x,t) \le 0$ at the same time hold with the constraint $\operatorname{argmin}\{H_d(x)\} = x^*$. In the rest of this section two different solutions are presented, the first gives a control law that does not require the unmeasurable state $x_3$ as a result while the second, although depending on this variable, exhibits some properties that simplify the design of an observer-based control.

### 3.2 *Pure output feedback control*

The first OFC proposed in this article is presented in the next proposition. As will be clear in the proof of the result, the fact that its structure does not depend on the unmeasurable state $x_3$ is due to the favourable steps on which the available degrees of freedom in (15) were chosen.

**Proposition 3.2:** *Consider the dynamic behaviour of a SMIB system equipped with a TCSC described by* (2). *Assume that*

**A.1** *The only available for measurement states are* $x_1$ *and* $x_2$
**A.2** *All the model parameters are known.*

*Under these conditions, a pure OFC that locally asymptotically stabilises the equilibrium point* $x^* = (x_1^*, x_2^*, x_3^*) = (\sin^{-1}(P_m/a_2x_3^*), 0, x_3^*)$ *is given by*

$$u = \frac{kk_1}{b_1}x_2 - \frac{k_1a_2}{\gamma}\left[\cos(x_1^*) - \cos(x_1)\right], \tag{16}$$

*with* $k$, $k_1$ *and* $\gamma$ *positive constants that satisfy*

$$\gamma > \frac{k_1a_2\sin^2(x_1^*)}{x_3^*\cos(x_1^*)}; \quad \sqrt{\frac{4a_1b_1}{k_1\gamma}} > k. \tag{17}$$

**Proof:** Considering the structure of constraint (15), one way for simultaneously dealing with both the required elimination of $x_3$ in the control law and the equilibrium point assignment is to define

$$H_{d2}(x_3) = \frac{\gamma}{2}(x_3 - x_3^*)^2, \quad \gamma > 0,$$

and $F_{33} = -\frac{b_1}{\gamma}$, since under these definitions the conditions imposed in (13) are satisfied under the first inequality listed in (17), while it is obtained that

$$\begin{aligned} b_1u = {} & F_{31}\left[-k_1a_2x_3^*\sin(x_1^*) + k_1a_2x_3\sin(x_1)\right] + F_{32}k_1x_2 \\ & - \frac{b_1k_1a_2}{\gamma}\left[\cos(x_1^*) - \cos(x_1)\right]. \end{aligned} \tag{18}$$

Then, if $F_{31}=0$ and $F_{32}=k$, the pure output feedback control law (16) is obtained with

$$F_d = \begin{bmatrix} 0 & \frac{1}{k_1} & 0 \\ -\frac{1}{k_1} & -\frac{a_1}{k_1} & 0 \\ 0 & k & -\frac{b_1}{\gamma} \end{bmatrix},$$

which satisfies the condition $F_d + F_d^T \leq 0$ if and only if the second condition listed in (17) is satisfied.

In order to prove that the assigned equilibrium point is asymptotically stable, notice that the time derivative of $H_d(x)$ along the trajectories of the closed-loop system can be written as

$$\dot{H}_d(x) = -\frac{2a_1}{k_1}z_2 - \frac{2b_1}{\gamma}z_3 + 2kz_2z_3,$$

where $z_2 = k_1x_2$ and $z_3 = k_1a_2[\cos(x_1^*) - \cos(x_1)] + \gamma(x_3 - x_3^*)$. From this expression it is easy to see that if $x_2=0$, the only condition that leads to $H_d(x)=0$ is $z_3=0$, but this last requirement holds only when $x=x^*$, then the proof is completed by invoking standard *La Salle* arguments. □

**Remark 3:** An interesting feature of the proposed controller is related with the structure of the desired energy function which in its complete form reads as

$$H_d(x) = \frac{k_1}{2}x_2^2 + k_1a_2x_3\left[\cos(x_1^*) - \cos(x_1)\right] - k_1a_2x_3^*\sin(x_1^*)(x_1 - x_1^*) + \frac{\gamma}{2}(x_3 - x_3^*)^2.$$

This function has the structure similar to the Lyapunov function proposed in Pai (1989) for studying the open-loop stability of the flux decay model (1), in this case the advantage is given by the design parameters $k_1$ and $\gamma$ that can be used to change its shape. In Pai (1989), it is recognised that the use of this function leads to conservative stability conditions since system (1) remains asymptotically stable even for operation conditions that are not captured in the analysis. This situation also appears with the proposed controller since it is possible to numerically illustrate, as will be done in Section 4, that the closed-loop stability properties are preserved under more relaxed conditions than that stated in the previous proposition. Evidently, this uncertainty would be eliminated if the actual region of attraction of the equilibrium point is computed. Unfortunately, it is well known that carrying out this task is quite difficult and all the efforts on this topic have been practically abandoned.

**Remark 4:** From a tuning perspective, the main constraint on the controller gains is imposed by the value of $\gamma$. Due to the fact that it defines the damping coefficient for $x_3$, remember that $F_{33} = -\frac{b_1}{\gamma}$, its value must be small. This in turn also forces the value of $k_1$ to be small, since it is required that

$$\frac{4a_1b_1}{k_1k^2} > \gamma > \frac{k_1a_2\sin^2(x_1^*)}{x_3^*\cos(x_1^*)},$$

leading to the following tuning procedure: propose a small value of $k_1$ and, depending on the chosen value for $\gamma$, select the value of $k$ that achieves a better performance while the inequality is also satisfied.

**Remark 5:** Another characteristic of the proposed controller (16) is that it exhibits some robustness properties since it can be implemented independently of the parameters $a_1$ and $b_1$. Indeed, defining $C_1 = \frac{k_1a_2}{\gamma}$ and $C_2 = \frac{kk_1}{b_1}$, the control law can be written as

$$u = C_2x_2 - C_1[\cos(x_1^*) - \cos(x_1)],$$

leading to the stability conditions given by

$$C_1 < \frac{x_3^*\cos(x_1^*)}{\sin^2(x_1^*)}; \quad C_2 < \frac{4C_1}{kk_1}.$$

Under these conditions the controller gains depend only on the parameters $P_m$ and $a_2$, since $x_1^*$ (and in its turn $C_1$) depends on them, see (3), and the free design parameters $k$, $k_1$. However, due to the purpose of this article, this additional advantage will not be further developed leaving its study to be reported somewhere else.

### 3.3 *Observer-based OFC*

In contrast to the pure OFC design, the observer-based controller design requires to cover several steps in order to achieve the posed stabilisation objective. Specifically, in addition to the (in this case state-feedback) controller design, a dynamic observer must be proposed and the stability of the whole system must be guaranteed. In this section these three topics are approached.

#### 3.3.1 *State-feedback design*

Concerning the state-feedback design, the key point that must be taken into account is the possibility to decide how the unmeasurable state appears in the control law defined by (15). If $F_{31}$ is different from zero, $x_3$ will appear in the control law in a nonlinear fashion while the definition of $F_{33}$ will determine whether this state appears in a linear way (due to the definition of $H_{d2}$) in the scheme.

In Proposition 3.3, a state-feedback control law that locally asymptotically stabilises the desired equilibrium point is presented. It is developed considering that a linear dependency on the unmeasurable state imposes a more treatable structure, hence the nonlinear dependency is to be avoided.

**Proposition 3.3:** *Consider the dynamic behaviour of a SMIB system equipped with a TCSC described by* (2). *Assume* A.2 *and the following*:

**A.3** *The complete model state is available for measurement.*

*Under these conditions, a state-feedback controller that locally asymptotically stabilises the equilibrium point* $x^* = (x_1^*, x_2^*, x_3^*) = (\sin^{-1}(P_m/a_2 x_3^*), 0, x_3^*)$ *is given by*

$$u = \frac{kk_1}{b_1}x_2 - \frac{(b_1+k_2)k_1a_2}{b_1\gamma}\left[\cos(x_1^*) - \cos(x_1)\right] - \frac{k_2}{b_1}(x_3 - x_3^*), \tag{19}$$

*with* $k$, $k_1$, $k_2$ *and* $\gamma$ *positive constants that satisfy*

$$\gamma > \frac{k_1 a_2 \sin^2(x_1^*)}{x_3^* \cos(x_1^*)}; \quad \sqrt{\frac{4a_1(b_1+k_2)}{k_1\gamma}} > k. \tag{20}$$

**Proof:** Consider the constraint (15) and select $F_{31} = 0$, $F_{32} = k$ and $F_{33} = -\frac{(b_1+k_2)}{\gamma}$. Under these conditions the resulting control law takes the form given by (19) with

$$F_d = \begin{bmatrix} 0 & \frac{1}{k_1} & 0 \\ -\frac{1}{k_1} & -\frac{a_1}{k_1} & 0 \\ 0 & k & -\frac{b_1+k_2}{\gamma} \end{bmatrix}.$$

The fact that $\operatorname{argmin}\{H_d(x)\} = x^*$ is proved exactly in the same way as in the pure OFC design case. This justifies the first condition in (20). Concerning the stability properties, the proof is similar to the previous case, with the difference that in this case

$$F_d + F_d^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{2a_1}{k_1} & k \\ 0 & k & -\frac{2(b_1+k_2)}{\gamma} \end{bmatrix},$$

leading to the second condition listed in (20) and allowing to apply standard La Salle arguments in order to conclude asymptotic stability. □

**Remark 6:** Since the structure of the state-feedback control is similar to the structure of the pure OFC introduced in Proposition 3.2, then the tuning procedure previously proposed can be followed in this case. The advantage is given by the inclusion of $k_2$, which offers the possibility of improving the damping of $x_3$ even for restricted values of $k$. However, this flexibility must be carefully considered since the scheme will include a dynamic observer that could compromise the closed-loop operation.

### 3.3.2 *Observer design*

Regarding the dynamic observer design, it is useful to recognise that in model (2), $x_3$ appears in a linear way with respect to the measurable states $x_1$, $x_2$. Due to this characteristic, this model can be equivalently written as

$$\dot{\sigma} = \psi_1(\sigma)x_3 + \psi_0(\sigma) \tag{21}$$

$$\dot{x}_3 = -b_1x_3 + B(u), \tag{22}$$

where $B(u) = b_1(x_3^* + u)$, $\sigma = [x_1, x_2]^T$ and

$$\psi_1(\sigma) = \begin{bmatrix} 0 \\ -a_2\sin(x_1) \end{bmatrix}; \quad \psi_0(\sigma) = \begin{bmatrix} x_2 \\ -a_1x_2 + P_m \end{bmatrix}.$$

In the next proposition a dynamic observer is proposed to estimate the state $x_3$ which exploits this linear structure at a fundamental level.

**Proposition 3.4:** *Consider the dynamic behaviour of an SMIB system equipped with a TCSC described by* (2). *Assume* A.1 *and* A.2. *Under these conditions a globally convergent observer for the unmeasurable state* $x_3$ *is given by*

$$\begin{aligned} \dot{s} &= -\left[b_1 + a_2k_3\sin^2(x_1)\right](s + \beta(\sigma)) + B(u) - K(\sigma)\psi_0(\sigma) \\ \hat{x}_3 &= s + \beta(\sigma), \end{aligned} \tag{23}$$

*with*

$$\beta(\sigma) = -k_3x_2\sin(x_1), \quad k_3 > 0, \tag{24}$$

*and*

$$K(\sigma) = \frac{\partial\beta(\sigma)}{\partial\sigma}. \tag{25}$$

**Proof:** Consider the alternative representation of model (2) given by (21)–(22) and define the variable $y = \dot{\sigma} - \psi_0(\sigma)$. Under this definition the system reads as

$$\begin{aligned} \dot{x}_3 &= -b_1x_3 + B(u) \\ y &= \psi_1(\sigma)x_3, \end{aligned}$$

exhibiting a structure that allows for proposing, using classical arguments, an observer of the form

$$\dot{\hat{x}}_3 = -b_1\hat{x}_3 + B(u) + K(\sigma)(y - \hat{y}) \tag{26}$$

$$\hat{y} = \psi_1(\sigma)\hat{x}_3, \tag{27}$$

where $\hat{x}_3$ is the estimate of $x_3$ and $K(\sigma)$ is a time-varying gain that depends on the measurable state, to be determined below.

Under these conditions the dynamic of the estimation error $\tilde{x}_3 = x_3 - \hat{x}_3$ is given by

$$\dot{\tilde{x}}_3 = -[b_1 + K(\sigma)\psi_1(\sigma)]\tilde{x}_3,$$

where, to guarantee convergence, the gain $K(\sigma)$ must be chosen such that

$$b_1 + K(\sigma)\psi_1(\sigma) > 0, \tag{28}$$

for all time.

Since observer (26)–(27) is not implementable, due to the fact that the variable $y = \dot{\sigma} - \phi(\sigma, u)$ depends on the time derivative of $\sigma$ which in turn depends on the unmeasurable state $x_3$, consider the alternative representation of the observer given by

$$\begin{aligned} \dot{\hat{x}}_3 - K(\sigma)\dot{\sigma} &= -[b_1 + K(\sigma)\psi_1(\sigma)]\hat{x}_3 + B(u) - K(\sigma)\psi_0(\sigma) \\ \hat{y} &= \psi_1(\sigma)\hat{x}_3 \end{aligned}.$$

Defining the availability for measurement variable as

$$s = \hat{x}_3 - \beta(\sigma),$$

with $\beta(\sigma)$ a function that holds with (25), the observer takes the implementable form

$$\begin{aligned} &\dot{s} = -[b_1 + K(\sigma)\psi_1(\sigma)](s + \beta(\sigma)) + B(u) - K(\sigma)\psi_0(\sigma) \\ &\hat{x}_3 = s + \beta(\sigma) \end{aligned},$$

that coincides with (23) if it is considered the function (24). The convergence properties of the scheme are proved noting that condition (28) is satisfied since

$$b_1 + K(\sigma)\psi_1(\sigma) = b_1 + a_2 k_3 \sin^2(x_1) > 0.$$

□

**Remark 7:** It is interesting to point out that the proposed observer resembles the obtained by the application of immersion and invariance (I&I) techniques (Astolfi, Karagiannis, and Ortega 2007). Current research is under development with the aim to explain and may further exploit this similarity.

### 3.3.3 *Output-feedback stability analysis*

The final step in the observer-based control design is related with the stability proof of the system composed by the plant, the state-feedback control and the observer. In this sense, the advantage of developing the controller design, as above, lies in the fact that guaranteeing the stability properties of the closed-loop system can be achieved in a (relatively) simple way.

For instance, as reported in Moreno et al. (2007), it is possible to attain this objective by proving that the map from the observation error $\tilde{x}_3 = x_3 - \hat{x}_3$ to the control error $e = x - x^*$ exhibits some ISS properties (Angeli, Ingalls, Sontag, and Wang 2004). The motivation for guaranteeing this kind of property comes from the fact that under ISS, for a bounded (zero) observation error, the control error will be bounded (resp., zero), establishing a sort of separation principle since these convergence properties are guaranteed without considering any particular structure for the estimation scheme, which can be designed in an independent way. In the proposition below, the desired ISS properties of the system under study are established.

**Proposition 3.5:** *Consider the dynamic behaviour of a SMIB system equipped with a TCSC described by* (2) *in closed-loop with the output-feedback version of controller* (19) *given by*

$$\begin{aligned} u_o = {}& \frac{kk_1}{b_1}x_2 - \frac{(b_1 + k_2)k_1 a_2}{b_1\gamma}\left[\cos(x_1^*) - \cos(x_1)\right] \\ & - k_2(\hat{x}_3 - x_3^*). \end{aligned} \tag{29}$$

*Under these conditions the map*

$$\Sigma: \ \tilde{x}_3 \ \rightarrow \ \|x - x^*\|$$

*is locally input-to-state stable.*

**Proof:** The first point to be noticed is that the output feedback controller can be written as $u_o = u + k_2\tilde{x}_3$ with $u$ the original state feedback controller (19). Under these conditions, the closed-loop system takes the form

$$\dot{x} = F_{\rm d}\frac{\partial H_d(x)}{\partial x} + gk_2\tilde{x}_3,$$

with $F_d$ and $H_d(x)$ as in the state-feedback design.

The procedure to prove the claimed ISS properties closely follows as presented in Khalil (2002). In this sense, notice that if $\tilde{x}_3 = 0$ then $x^*$ is locally asymptotically stable, as proved in Section 3.3.1, while if $\tilde{x}_3 \neq 0$, the time derivative of $H_d(x)$ along the trajectories of the closed-loop system reads as

$$\dot{H}_d(x) = -\left(\frac{\partial H_d(x)}{\partial x_{23}}\right)^T \bar{R}_d \frac{\partial H_d(x)}{\partial x_{23}} + \frac{\partial H_d(x)}{\partial x_3}k_2\tilde{x}_3,$$

where, under the conditions found in the state-feedback design, $\bar{R}_d = \bar{R}_d^T > 0$ is given by

$$\bar{R}_d = \begin{bmatrix} \dfrac{2a_1}{k_1} & -k \\ -k & \dfrac{2(b_1 + k_2)}{\gamma} \end{bmatrix}$$

and

$$\frac{\partial H_d(x)}{\partial x_{23}} = \begin{bmatrix} \dfrac{\partial H_d(x)}{\partial x_2} \\ \dfrac{\partial H_d(x)}{\partial x_3} \end{bmatrix}.$$

From this last expression it is possible to show that

$$\dot{H}_d(x) \leq -(1-\theta)\left(\frac{\partial H_d(x)}{\partial x_{23}}\right)^T \bar{R}_d \frac{\partial H_d(x)}{\partial x_{23}}, \quad 0 < \theta < 1,$$

provided the following constraint holds:

$$\frac{\|k_2\tilde{x}_3\|}{\theta\lambda_{\min}\{\bar{R}_d\}} \leq \frac{\|\frac{\partial H_d(x)}{\partial x_{23}}\|^2}{\|\frac{\partial H_d(x)}{\partial x_3}\|} \leq \gamma_1 \|\frac{\partial H_d(x)}{\partial x}\|; \quad \gamma_1 > 1. \quad (30)$$

On the other hand, notice that $\frac{\partial H_d(x)}{\partial x}$ can be equivalently written as

$$\frac{\partial H_d(x)}{\partial x} = \begin{bmatrix} k_1 a_2 x_3^*\{\sin(x_1) - \sin(x_1^*)\} + k_1 a_2 \sin(x_1)(x_3 - x_3^*) \\ k_1 x_2 \\ -k_1 a_2\{\cos(x_1) - \cos(x_1^*)\} + \gamma(x_3 - x_3^*) \end{bmatrix},$$

leading to the fact that $\|\frac{\partial H_d(x)}{\partial x}\| \leq \sqrt{\gamma_2}\|x - x^*\|$ with

$$\gamma_2 \geq \max\{k_1^2 a_2^2(1 + x_3^*)^2, (k_1^2 a_2^2 + \gamma^2)\},$$

allowing for guaranteeing that

$$0 < c_1\|\tilde{x}_3\| \leq \|x - x^*\|; \quad c_1 = \frac{|k_2|}{\sqrt{\gamma_2}\theta\lambda_{\min}\{\bar{R}_d\}}.$$

The proof is completed by a direct application of the Theorem 4.19 reported in Khalil (2002, p. 176). For this, notice that since $H_d(x)$ is composed by quadratic and locally bounded terms, this function can be locally upper and lower bounded by class-$\mathcal{K}_\infty$ functions. Then, the only point is to find a class-$\mathcal{K}$ function $\rho$ that satisfies $\|x - x^*\| \geq \rho(\|\tilde{x}_3\|) > 0$. However, this function can be readily identified, from the inequalities presented above, as $\rho(r) = c_1 r$, proving that the map $\Sigma : \tilde{x}_3 \rightarrow e$ is ISS. □

## 4. Numerical evaluation

The usefulness of the proposed OFCs is illustrated in this section via some numerical simulations. The purpose is to show that, in addition to the achievement of the stabilisation objective, the proposed schemes offer some performance advantage over the open-loop behaviour. To carry out this evaluation, the parameters of the SMIB model (given in pu) were taken from de Leon-Morales et al. (2004) as $P_m = 16$, $a_1 = 1$, $a_2 = 21.3358$ and $b_1 = 20$. Under these conditions the equilibrium point that must be stabilised is given by $x^* = (x_1^*, x_2^*, x_3^*) = (0.984936, 0, 0.9)$ while the longest fault duration allowable for open-loop stability, i.e. the critical clearing time, is $t_{cl} = 180$ ms.

To evaluate the controllers, it was considered that at the beginning of the experiment the system was operating in the desired equilibrium point, i.e. the initial conditions of the states were defined by $x^*$, while the initial condition of the estimated state (in the case of the observer-based scheme) was given by $\hat{x}_3(0) = 0$, with the aim to consider the worst operating case. Under this scenario, a three-phase short circuit at the generator bus was induced by letting the value of the parameter $a_2$ to take the zero value (this condition is equivalent to drop the generated power to be zero). The length of the fault was equal to $t_{cl}$ starting at $t = 0.5$ s.

Regarding the controller gains, for both the pure and the observer-based OFC the considered values were $k_1 = 0.01$, $\gamma = 100$ and $k = 8.5$, which satisfy the stated stability conditions. In addition, for the second one it was considered that $k_2 = 10$ while the observer gain was set at $k_3 = 0.1$. The reason that justifies these values was the intention to evaluate both controllers under similar conditions allowing, at the same time, to illustrate the performances that they can achieve. In this sense, as usual, special attention was given to the first overshot in the time response of the load angle, since this value is fundamental in determining whether if the variables will remain or not in the region of attraction of the equilibrium point.

Figure 2 shows, in comparison with the open-loop behaviour (in continuous–line), the load angle behaviour under both the pure (in dashed-line) and the observer-based (in dotted-line) schemes. Besides the fact that with the two controllers the stabilisation objective is achieved improving the open-loop transient response, the superiority of the latter can be noticed,

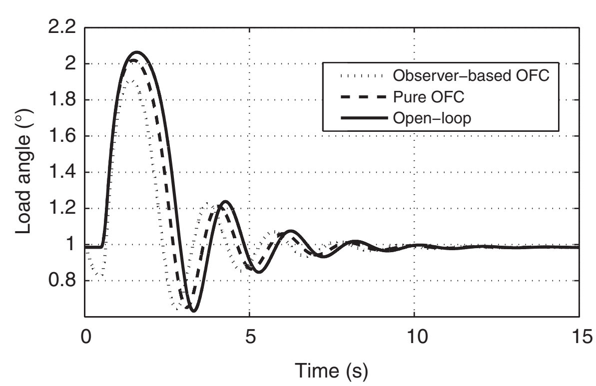


Figure 2. Load angle behaviour under a three-phase short circuit at the generator bus of duration $t_{cl} = 180$ ms and starting at $t = 0.5$ s.

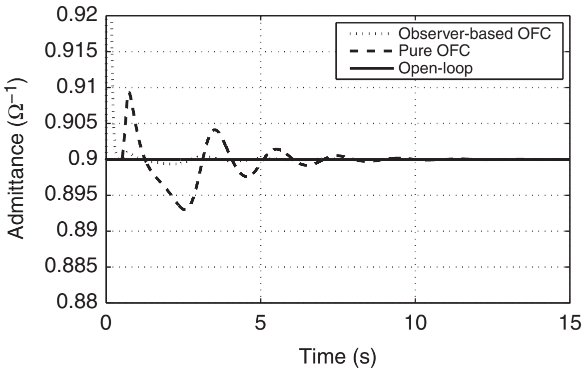


Figure 3. Angular speed behaviour under a three-phase short circuit at the generator bus of duration $t_{cl} = 180$ ms and starting at $t = 0.5$ s.

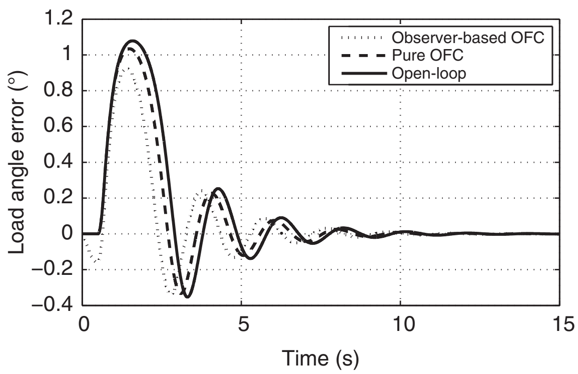


Figure 4. Total admittance behaviour under a three-phase short circuit at the generator bus of duration $t_{cl} = 180$ ms and starting at $t = 0.5$ s.

since it reduces the overshot in about 10% in contrast to the 5% reduction exhibited by the pure OFC. This advantage is less notorious concerning the speed behaviour, which is presented in Figure 3, but is drastically different regarding the total admittance, included in Figure 4, where it can be observed that after a peak value of about $1.5\,\Omega^{-1}$, produced by the uncertainty introduced in the initial condition of the estimated state, the observed-based scheme reaches the corresponding value of the equilibrium point faster than the pure OFC. In Figures 5 and 6 the error signals for $x_1$ and $x_3$ are included (the corresponding picture for $x_2$ is the same than Figure 3) while, with the aim to illustrate the internal stability properties of the observer-based algorithm, in Figure 7 the behaviour of the estimated state is presented.

Although at this point of the evaluation procedure the observer-based OFC has exhibited a better performance, it is quite interesting to illustrate some stabilisation properties of the pure OFCs that are not

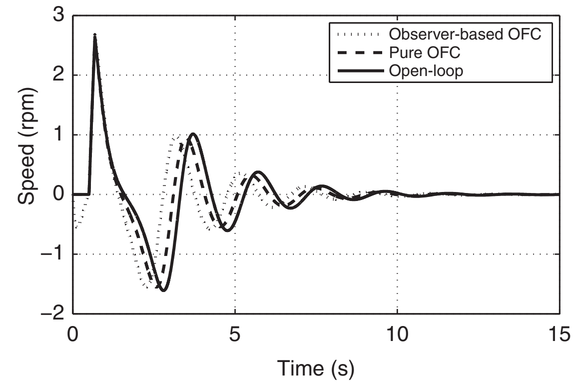


Figure 5. Load angle error behaviour under a three-phase short circuit at the generator bus of duration $t_{cl} = 180$ ms and starting at $t = 0.5$ s.

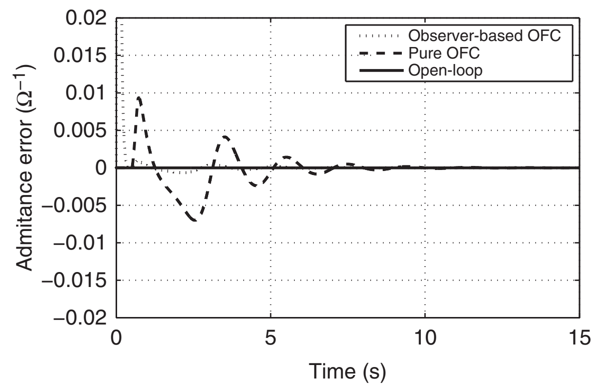


Figure 6. Total admittance error behaviour under a three-phase short circuit at the generator bus of duration $t_{cl} = 180$ ms and starting at $t = 0.5$ s.

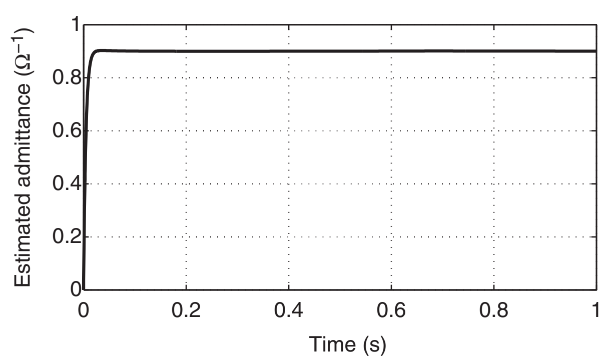


Figure 7. Estimated admittance behaviour when $\hat{x}_3(0) = 0$.

captured in the stability analysis presented in Section 3.2. In Figures 8–10 the closed-loop behaviour of the three states of the system are presented when $k = 8.5$, in continuous line and when $k = 170$, in dashed-line. In these pictures it is evident that the

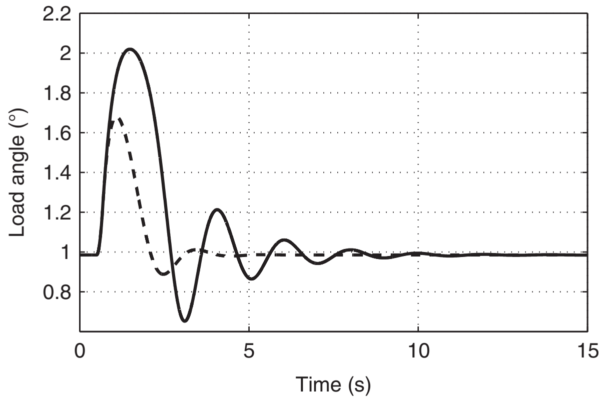


Figure 8. Load angle behaviour under operation conditions ($k = 170$) outside the gain range determined by the stability analysis.

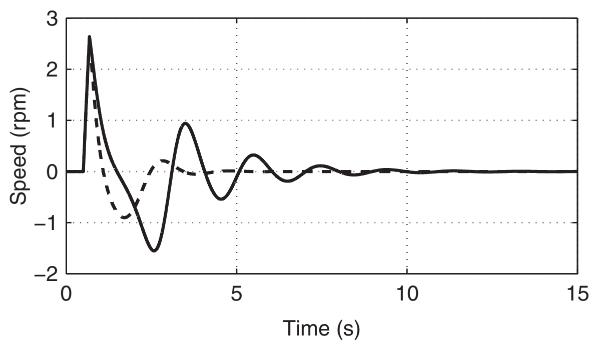


Figure 9. Angular speed behaviour under operation conditions ($k = 170$) outside the gain range determined by the stability analysis.

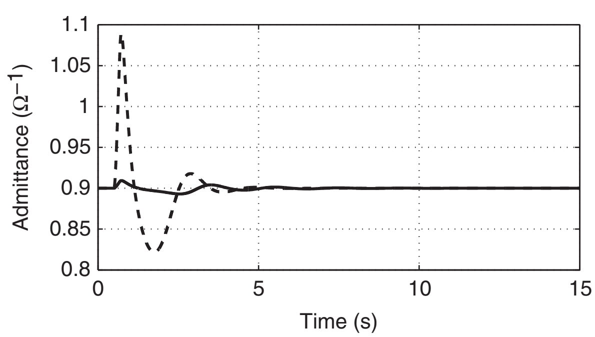


Figure 10. Total admittance behaviour under operation conditions ($k = 170$) outside the gain range determined by the stability analysis.

superior performance is achieved under the second condition, since the overshot for the load angle is reduced in ~15% with respect to the first condition (and 19% with respect to the open-loop behaviour). However, as can be verified, the value $k = 170$ does not hold with the stability condition found in Section 3.2. As mentioned before, this kind of behaviour is due to the conservative structure of the desired energy function viewed as a Lyapunov function and deserves a deeper study (which is currently developed), but authors believe that it could be exploited even if its formal justification is still under study.

**Remark 8:** Even though it is difficult to carry out a fair comparison, it is interesting to point out that the presented result exhibits some advantages with respect to previously reported results. To illustrate this point, the scheme presented by the authors in de Leon-Morales et al. (2004) can be considered where exactly the same model with the same parameters were used. In this case, the overshot reduction achieved by the reported scheme is around 15%, however the structure of the proposed observer is remarkably complex. Thus, if it is considered that similar performances are achieved with a much simpler controller, then the advantage of the contribution of this article is clear.

## 5. Concluding remarks

In this article it has been illustrated how the flexibility offered for solving the ME in the application of the IDA-PBC design methodology can be used to generate OFCs. This illustration was carried out by considering the practically important problem of improving the transient stability properties of a power system composed by a synchronous generator connected to an infinite bus via a TCSC. Two controllers were proposed, a pure and an observer-based OFC, and both of them, in addition to achieve the stabilisation objective, have shown a better transient response with respect to the open-loop behaviour. Although the observer-based scheme showed a superior performance, it was illustrated that the pure OFC can achieve remarkable responses considering operating conditions that are not captured in the stability analysis. Current research is carried out with the aim to explain this advantageous behaviour. In addition, it was also illustrated how the aforementioned flexibility in tailoring the ME can be further exploited with the aim of simplifying the design of the observer-based control. Specifically, it was shown that deciding the structure of the state-feedback control could allow for the use of some tools, ISS, for establishing a sort of separation principle which in turn gives some freedom to the designer for approaching the observer design problem in an independent way with respect to the controller proposition.

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## Notes

1. All vectors in this article are *column* vectors, even the gradient of a scalar function denoted $\nabla_{(\cdot)} = \frac{\partial}{\partial(\cdot)}$. When clear from the context, the subindex will be omitted.
2. Notice that $x_2 - x_2^* = x_2$ is due to the fact that $x_2^* = 0$.

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