Leak Detection, Isolation and Estimation in Pressurized Water Pipe Networks using LPV Models and Zonotopes

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Abstract: In this paper, a leak detection, isolation and estimation methodology in pressurized water pipe networks is proposed. The methodology is based on computing residuals which are obtained comparing measured pressures (heads) in selected points of the network with their estimated values by means of a *Linear Parameter Varying* (LPV) model and zonotopes. The structure of the LPV model is obtained from the non-linear mathematical model of the network. The proposed detection method takes into account modelling uncertainty using zonotopes. The isolation and estimation task employs an algorithm based on the residual fault sensitivity analysis. Finally, a typical water pipe network is employed to validate the proposed methodology. This network is simulated using EPANET software. Parameters of LPV model and their uncertainty bounded by zonotopes are estimated from data coming from this simulator. A leak scenario allows to assess the effectiveness of the proposed approach.

Keywords: Leak Detection, Fault Isolation, Linear Parameter Varying Model, Sensitivity Matrix.

1. INTRODUCTION

Water system networks are used to supply water for industrial and domestic use. Those systems include sources, treatment works, pump stations, reservoirs, pipes, control valves and demand sectors (nodes). Moreover, water systems networks are large scale systems. In those systems, frequently, leaks are present. Therefore, leak detection and isolation methods must be employed to localize leakages. Model-based leak detection and isolation techniques based on pressure measurements and sensitivity analysis have been studied (Perez et al., 2009). These techniques are based on the use of the non-linear model of the network. However, the parameter estimation of non-linear models of water networks is not an easy task (Brdys and Ulanicki, 1994). This is why this paper proposes alternatively to use a non-linear model with LPV structure whose parameters can be more easily estimated using least-squares algorithms as the one proposed by Bamieh and Giarré (2002), among others.

Linear Parameter Varying (LPV) models have recently attracted the attention of the Fault Detection Isolation (FDI) research community. Such models can be used efficiently to represent some non-linear systems (Shamma and Cloutier, 1993). This has motivated some researchers from the FDI community to develop model-based methods using LPV models (see Bokor et al. (2002), among others). But even with the use of LPV models, modeling errors are inevitable in complex engineering systems. So, in order to increase reliability and performance of model-based fault detection, the development of robust fault detection algorithms should be addressed. The robustness of a fault detection system means that it must be only sensitive to faults, even in the presence of model-reality differences according to Chen and Patton (1999). One of the approaches to robustness, known as passive, is based on enhancing the robustness of the fault detection system at the decision-making stage. The aim with the passive approach is to determine, given a set of models, if there is any member in this set that can explain the measurements. A common approach to this problem is to propagate the model uncertainty to the alarm limits of the residuals. When the residuals are outside of the alarm limits, it is argued that model uncertainty alone can not explain the residual and therefore a fault must have occurred. This approach has the drawback that faults that produce a residual deviation smaller than the residual uncertainty due to parameter uncertainty will not be detected.

The typical fault isolation approach proposed in the FDI community uses a set of binary detection tests to compose the observed fault signature. When applying this methodology to leak isolation, since they may exhibit symptoms with different sensitivities, the use of binary codification of the residual produces loss of information (Puig et al., 2005). Alternatively, it is possible to use other additional information associated with the relationship between the residuals and faults, as the residual fault sensitivity, to improve the isolation results (Meseguer et al., 2006).

The innovation of this paper is to present a new leak detection, isolation and estimation method for water distribution systems that can be described by LPV models. The fault detection methodology is based on comparing on-line the real system behaviour of the monitored system obtained by means of sensors with the estimated behaviour using an LPV model. In case of a significant discrepancy (residual) is detected between the LPV model and the measurements obtained by the sensors, the existence of a fault is assumed. To take into account the effect of the uncertain parameters in the detection module, the outputs of LPV models are bounded by a zonotope to avoid false alarms. Analyzing in real-time how the faults affect to the residuals using the residual fault sensitivity, it is possible, to isolate and estimate the leaks.

The structure of this paper is the following: Section 2 presents the modelling principles of water distribution networks and how to obtain LPV models to represent their dynamics. In Section 3, the model identification is presented. Section 4 presents the zonotope-based leak detection methodology while Section 5 presents the leak isolation and estimation methodology using sensitivity analysis. Finally, in Section 6, an application case study based on a hypothetical water distribution network is used to assess the validity of the proposed approach.

2. MODELLING WATER DISTRIBUTION NETWORKS USING LPV MODELS

2.1 Physical modelling principles

The physical components that constitute a water distribution system are given by a set of pipes, pumps and control valves connected by means of nodes that represent junctions with or without demands and also tanks and reservoirs (Brdys and Ulanicki, 1994). Nodes are points in the network where pipes are joined and where water enters or leaves the network. The reservoirs are nodes that represent an infinite external source to the network, for example, rivers, lakes, groundwater aquifers, and also input points to other systems. The tanks are nodes with storage capacity, where the volume of stored water can vary with time. The pipes transport water from one point in the network to another. Flow direction is always from the end at higher hydraulic head (pressure) to at lower head. The hydraulic head lost by water flowing in a pipe due to wall friction can be computed using the Hazen-Williams (H-W) formula (Brdys and Ulanicki, 1994):

$$h_i(k) - h_j(k) = R_{ij} q_{ij}{}^{a}(k) \tag{1}$$

where: h_i is the pressure at the node i, h_j is the pressure at the node j, R_{ij} is the resistance coefficient, q_{ij} is the flow rate through the pipe, a is the flow exponent and k denotes the time time.

In each node, the flow continuity law must be fulfilled indicating that sum of flows in a node must be zero

$$\sum \left(q_{ij}(k) - d_i(k) \right) = 0 \tag{2}$$

where the contribution of d_i is negative because the demand goes out of the node and q_{ij} are the flows that are considered positive if go into the node. Otherwise, they are considered negative. The set of equations that describes the water network dynamics can be represented as nodes head function. Solving Equation (1) with respect q_{ij} the following flow expression is obtained

$$q_{ij}(k) = \sqrt[a]{\left(h_i(k) - h_j(k) \right) / R_{ij}} \tag{3}$$

Then, the set of equations that represent the water network dynamics is obtained by replacing (3) in (2). This set of equations is non-linear since $a \neq 1$ and can not be solved analytically to obtain the node heads, but instead numerical methods should be used. This non-linearity also makes difficult to estimate the parameters of the network (as, f.e. the pipe resistances). For all these reasons, the non-linear model of the network is not very useful for FDI purposes.

2.2 LPV models for water networks

In this paper, is alternatively proposed a LPV model of the water network. LPV models consist of a linear lumped parameters in which the parameters are not constant and depend on system state and/or operating point. There are several ways to obtain an LPV model (Shamma and Cloutier, 1993) (Bamieh and Giarré, 2002). Here, the LPV model structure of the water distribution network is obtained using physical modeling and linearisation around a generic operating point as suggested by Shamma and Cloutier (1993). Parameters are estimated using LPV identification methods (Bamieh and Giarré, 2002). Thus, the water distribution network model can be written using the following LPV MIMO model

$$\boldsymbol{y}(k) = \boldsymbol{\Phi}(k)\boldsymbol{\theta}(\mathbf{p}_k) + \boldsymbol{e}(k) = \hat{\boldsymbol{y}}(k) + \boldsymbol{e}(k) \tag{4}$$

where

- $\boldsymbol{y}(k)$ is the output vector of dimension $n_y \times 1$.
- $\boldsymbol{\Phi}(k)$ is the regressor matrix of dimension $n_y \times n_\theta$ which can contain any function of inputs $\boldsymbol{u}(k)$ and outputs $\boldsymbol{y}(k)$.
- $\mathbf{p}_k \triangleq \mathbf{p}(k)$ is a vector of measurable process variables of dimension $n_p \times 1$ that define the system operating point.
- $\boldsymbol{\theta}(\mathbf{p}_k) \in \boldsymbol{\Theta}_k$ is the LPV parameter vector of dimension $n_\theta \times 1$ whose values can vary according to the system operating point following some known function $\boldsymbol{\theta}(k) = g(\mathbf{p}_k)$.
- $\boldsymbol{\Theta}_k$ is the set that bounds parameter values that can vary according to the system operating point as well.
- $\boldsymbol{e}(k)$ is a vector of dimension $n_y \times 1$ that contains the sensor additive noises whose components are bounded by constants $|e_i(k)| \le \sigma_i$, $i = 1,\dots,n_y$.

In this paper, the uncertain parameter set $\boldsymbol{\Theta}_k$ is described by a zonotope centred in the nominal LPV model :

$$\boldsymbol{\Theta}_k = \boldsymbol{\theta}^0(\mathbf{p}_k) \oplus \mathbf{H}\,\mathbb{B}^n = \left\{ \boldsymbol{\theta}^0(\mathbf{p}_k) + \mathbf{Hz} : \mathbf{z} \in \mathbb{B}^n \right\} \tag{5}$$

where

- $\boldsymbol{\theta}^0(\mathbf{p}_k) \in \mathbb{R}^{n_\theta}$ is the centre of the zonotope and corresponds to the nominal LPV model.
- $\mathbf{H} \in \mathbb{R}^{n_\theta \times n}$ is the shape of the zonoopte (usually $n \ge n_\theta$ and as the bigger n is the more complicated relations between uncertainty component parameters can be taken into account).
- $\mathbb{B}^n \in \mathbb{R}^{n \times 1}$ is a unitary box composed by n unitary ($\mathbb{B} = [-1,1]$) interval vectors.
- $\oplus$ denotes the Minkowski sum.

3. MODEL IDENTIFICATION

3.1 Worst-case approach

Let us consider a sequence of *M* regressor matrix values $\mathbf{\Phi}(k)$ in a fault free scenario and the model of the water network parameterised as in (4). The aim is to estimate nominal model parameters $\boldsymbol{\theta}^0(\mathbf{p}_k)$ and their uncertainty (model set) defined by the matrix $\mathbf{H}$ in such a way that all measured data in a fault free scenario will be covered by the worst-case predicted output $\Upsilon(k)$ ("***worst-case model***"), that is

$$\boldsymbol{y}(k) \in \Upsilon(k) \quad \forall k = 1,...,M \tag{6}$$

where

$$\Upsilon(k) = \hat{\Upsilon}(k) \oplus \mathbf{E}\mathbb{B}^{n_y} \tag{7}$$

with

$$\hat{\Upsilon}(k) = \{\hat{y}(k) = \mathbf{\Phi}(k)\boldsymbol{\theta}(\mathbf{p}_k), \boldsymbol{\theta}(\mathbf{p}_k) \in \mathbf{\Theta}_k\} \tag{8}$$

and $\mathbf{E} = diag\left(\sigma_1, \cdots, \sigma_{n_y}\right)$.

The worst-case approach was first suggested by Ploix (1999) in the context of fault detection. Further works using this approach are Calafiore et *al.* (2002) and Campi et *al.* (2009). In the particular case of representing the uncertain parameter set $\mathbf{\Theta}_k$ using a zonotope, as in (5), (8) can be rewritten as follows

$$\hat{\Upsilon}(k) = \mathbf{\Phi}(k)\boldsymbol{\theta}^0(\mathbf{p}_k) \oplus \mathbf{\Phi}(k)\mathbf{H}\mathbb{B}^n \tag{9}$$

Consequently, $\Upsilon(k)$ can be rewritten as follows:

$$\Upsilon(k) = \hat{\boldsymbol{y}}^0(k) \oplus \hat{\mathbf{\Gamma}}(k) \tag{10}$$

where

$$\hat{\boldsymbol{y}}^0(k) = \mathbf{\Phi}(k)\boldsymbol{\theta}^0(\mathbf{p}_k) \tag{11}$$

and

$$\hat{\mathbf{\Gamma}}(k) = \left(\mathbf{\Phi}(k)\mathbf{H} \quad \mathbf{E}\right) \mathbb{B}^{n+n_y} \tag{12}$$

Notice that $\Upsilon(k)$ is a zonotope centred in the nominal output estimation $\hat{\mathbf{y}}^0(k)$ and with a shape defined by $\hat{\mathbf{\Gamma}}(k)$. Thus, condition (6) can be rewritten as

$$\boldsymbol{y}(k) - \hat{\boldsymbol{y}}^0(k) \in \hat{\mathbf{\Gamma}}(k) \qquad \forall k = 1,...,M \tag{13}$$

3.2 Worst-case parameter estimation

Considering that the parameter set $\mathbf{\Theta}_k$ can be described as the zonotope (5), the optimal zonotope that fulfils the ***"worst-case condition"*** (6) is the one that minimizes the size of the predicted output along the considered set of data as follows

$$J = \sum_{k=1}^{M} (vol(\hat{\Upsilon}(k))) \tag{14}$$

where *vol* is the volume of the output zonotope, considering only uncertainty in parameters, $\hat{\Upsilon}(k)$. This optimization problem with no knowledge assumptions about matrix $\mathbf{H}$ is in general very hard to solve even in the single output case (Campi et al., 2009). In order to reduce the complexity, the zonotope that bounds $\mathbf{\Theta}_k$ can be parameterised such that $\mathbf{H} = \lambda\mathbf{H}_0$ [*], as proposed in Blesa *et al.* (2009), that corresponds to a zonotope with predefined shape (determined by $\mathbf{H}_0$) and a scalar λ. Then (14) can be rewritten as follows

$$J = \sum_{k=1}^{M} vol(\hat{\Upsilon}(k)) = v(|\lambda|) \;{}^{\dagger} \tag{15}$$

and the worst-case optimization problem can be formulated as

Problem 1: "Worst-case Parameter Estimation"

$$\min_{\lambda} \sum_{k=1}^{M} vol(\hat{\Upsilon}(k)) = \min_{\lambda} v(|\lambda|)$$

subject to:

$$\begin{aligned} &y_1(k) - \hat{y}_1^0(k) = \boldsymbol{\varphi}_1(k)\lambda\mathbf{H}_0\mathbf{z}(k) + e_1(k) \\ &\qquad\vdots \qquad\qquad\qquad\qquad \forall k = 1,...,M \\ &y_{n_y}(k) - \hat{y}_{n_y}^0(k) = \boldsymbol{\varphi}_{n_y}(k)\lambda\mathbf{H}_0\mathbf{z}(k) + e_{n_y}(k) \\ &\mathbf{z}(k) \in \mathbb{B}^n, \ \left(e_1, ..., e_{n_y}\right)^t \in \mathbf{E}\mathbb{B}^{n_y} \text{ and } \lambda \geq 0 \end{aligned}$$

where $\boldsymbol{\varphi}_i(k) \in \mathbb{R}^{1\times n_\theta}$ $i = 1,...,n_y$ and $\mathbf{\Phi}(k) = \begin{pmatrix} \boldsymbol{\varphi}_1(k) \\ \vdots \\ \boldsymbol{\varphi}_{n_y}(k) \end{pmatrix}$

Problem 1 is a non-linear polynomial optimization problem that can be solved globally as proposed by Henrion et *al.* (2009).

4. LEAK DETECTION METHODOLOGY

The leakage methodology is based on the evaluation of the residual obtained from the difference between measurements and model prediction using (4)

$$\mathbf{r}(k) = \mathbf{y}(k) - \hat{\mathbf{y}}(k) - \boldsymbol{e}(k) = \mathbf{y}(k) - \mathbf{\Phi}(k)\boldsymbol{\theta}(\mathbf{p}_k) - \boldsymbol{e}(k) \tag{16}$$

Ideally, the residual given by (16), known as parity equation (Iserman, 2006), in case of neglecting modelling errors and noise, it should be different from zero in a faulty scenario and zero, otherwise. However, because of modelling errors and noise, the detection test can be translated to check the following condition

$$\mathbf{0} \in \mathbf{\Gamma}(k) \tag{17}$$

where

$$\mathbf{\Gamma}(k) = \begin{Bmatrix} \mathbf{r}(k) = \mathbf{y}(k) - \mathbf{\Phi}(k)\boldsymbol{\theta}(\mathbf{p}_k) - \boldsymbol{e}(k), \\ \boldsymbol{\theta}(\mathbf{p}_k) \in \mathbf{\Theta}_k, \boldsymbol{e}(k) \in \mathbf{E}\mathbb{B}^{n_y} \end{Bmatrix} \tag{18}$$

and **0** is a vector ($n_y \times 1$) of zeros $\mathbf{0} = (0 \cdots 0)^t$.

Taking into account (4) and (5), $\mathbf{\Gamma}(k)$ can be parameterized as a zonotope

$$\mathbf{\Gamma}(k) = \left(\mathbf{y}(k) - \hat{\mathbf{y}}^0(k)\right) \oplus \hat{\mathbf{\Gamma}}(k) \tag{19}$$

where $\hat{\mathbf{y}}^0(k)$ and $\hat{\mathbf{\Gamma}}(k)$ are defined as in (11) and (12). Then, test (17) is equivalent to test (6) and involves checking if the point **0** belongs to the zonotope $\mathbf{\Gamma}(k)$. Considering matrix **H** is parameterized as was proposed in *Section 3.2* (i.e., $\mathbf{H}=\lambda\mathbf{H}_0$,

[*] $\mathbf{H}_0$ can be obtained using physical knowledge of the system or by the method proposed by Bhattacharyya (1995).

[†] For example if $\mathbf{\Phi}(k)\mathbf{H}_0$ is a square matrix: $v(|\lambda|) = |2\lambda|^n \sum_{k=1}^{M} |\det(\mathbf{\Phi}(k)\mathbf{H}_0)|$

with λ and $\mathbf{H}_0$ known), the detection test can be implemented by determining if the following constraint satisfaction problem is feasible.

Problem 2: "Fault detection test"

$$y_1(k)-\hat{y}_1^0(k)-\boldsymbol{\varphi}_1(k)\lambda\mathbf{H}_0\mathbf{z}(k)-e_1(k)=0$$
$$\vdots$$
$$y_{n_y}(k)-\hat{y}_{n_y}^0(k)-\boldsymbol{\varphi}_{n_y}(k)\lambda\mathbf{H}_0\mathbf{z}(k)-e_{n_y}(k)=0$$
$$\mathbf{z}(k)\in\mathbb{B}^n \text{ and } \left(e_1,\ldots,e_{n_y}\right)^t\in\mathbf{E}\mathbb{B}^{n_y}$$

For every instant k, the feasibility of *Problem 2* can be verified solving a linear programming problem with no objective function.

5. LEAK ISOLATION AND ESTIMATION METHODOLOGY

The proposed leak isolation and estimation method is based on sensitivity analysis of residual (16) to the different leaks. The fault sensitivity transfer function matrix, indicates in what way faults affect into residual and, according to Gertler (1998), can be calculated as

$$\mathbf{S}(q^{-1})=\partial\boldsymbol{r}/\partial\boldsymbol{f}=\left(\partial\boldsymbol{r}/\partial f_1 \quad \cdots \quad \partial\boldsymbol{r}/\partial f_{n_f}\right) \tag{20}$$

where q^{-1} is the shift operator.

Notice that this sensitivity depends on the parameters $\boldsymbol{\theta}$ and faults $\boldsymbol{f}$ when applied to residual (16) coming from the LPV model (4) of the water network. This dependency is the denoted as $\mathbf{S}(q^{-1},\boldsymbol{\theta},\boldsymbol{f})$ .Thus, residual can be expressed as

$$\mathbf{r}(k)=\mathbf{S}(q^{-1},\boldsymbol{\theta},\boldsymbol{f})\boldsymbol{f}(k)+\boldsymbol{e}(k) \tag{21}$$

where

- $\boldsymbol{f}(k)$ is the vector of possible faults of dimension $n_f\times 1$
- $\mathbf{S}(q^{-1},\boldsymbol{\theta},\boldsymbol{f})$ is the matrix sensitivity fault of dimension $n_y\times n_f$.

Considering that leaks are located in the nodes and can be modeled as single abrupt additive faults represented as a step of amplitude f

$$\boldsymbol{f}(k)=\begin{cases}(0 \ \cdots \ 0 \ \cdots \ 0)^t, & k<k_{fault}\\ f\,(0 \ \cdots \ 1 \ \cdots \ 0)^t, & k\geq k_{fault}\end{cases}$$

the problem of fault isolation and estimation implies solving *Problem 3*, for $k\geq k_{fault}$, i.e., once the fault has been detected by solving *Problem 2*.

Problem 3: "Fault isolation and estimation"

$$\underset{i\in\{1,\ldots,n_f\}}{f_i}=\arg\min_{f_1,\ldots,f_{nf}}\left\{J(f_1),\ldots,J(f_{nf})\right\} \quad \text{with}$$

$$J(f_i)=\min_{f_i}\left\{\sum_{l=0}^{\min(k-k_{fault},\ell-1)}\sum_{j=1}^{n_y}\left(r_j(k-l)-S_{ji}(q^{-1},\boldsymbol{\theta},\boldsymbol{f})f_i\right)^2\right\}$$

where $S_{ji}(q^{-1},\boldsymbol{\theta},\boldsymbol{f})=\partial\boldsymbol{r}_j(k)/\partial f_i$ and ℓ is time moving horizon that minimizes the noise effect.

Notice that *Problem 3* implies determining the minimum of n_f values obtained by solving n_f optimization problems.

6. APPLICATION CASE STUDY

6.1 Description

The application case study is based on the water distribution network presented in (Fig. 1). In this particular case, the water network is composed by the following elements: Two reservoirs, three pipes and two nodes with demands d_1 and d_2 expressed in m^3/s. Head sensors (h_{n1} and h_{n2}) (expressed in m) are located in the two nodes. The possible leaks are f_1 and f_2 that are also located in the nodes. The pipe resistance coefficients R_1, R_2 and R_3 in the H-W formula (1) are given by $R=1.2216\cdot 10^{10}L/\left(C^aD^{4.87}\right)$ where L is the pipe length in meters (m), D is the pipe diameter in millimetres (mm) and a is the flow exponent (a = 1.852). The pipes length are $L_1 = L_2$ = 1000m and L_3 = 2000m respectively and the diameters are the same in all pipes $D_1 = D_2 = D_3$ = 200mm.

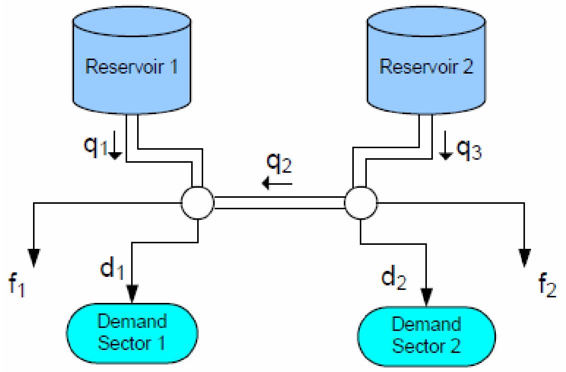

Fig. 1. Water network proposed as case study

Applying the flow Equation (2) to the network under study, the set of equations that describes the flow behaviour is obtained:

$$q_1+q_2-d_1-f_1=0 \tag{22}$$
$$q_3-q_2-d_2-f_2=0 \tag{23}$$

Analogously, the application of the pressure Equation (3) in (22) and (23) leads to

$$\left((h_{d1}-h_{n1})/R_1\right)^{a^{-1}}+\left((h_{n2}-h_{n1})/R_2\right)^{a^{-1}}-d_1-f_1=0 \tag{24}$$
$$\left((h_{d2}-h_{n2})/R_3\right)^{a^{-1}}-\left((h_{n2}-h_{n1})/R_2\right)^{a^{-1}}-d_2-f_2=0 \tag{25}$$

This model is used to develop a high-fidelity simulator using a standard water network simulation software (EPANET).

6.2 LPV Modelling and identification

To obtain the LPV model structure of this network, the non-linear MIMO model defined by (24) and (25), considering no fault is present, is linearised around the operating point characterized by the head measurements (h_{n1}^0 and h_{n2}^0) in nodes. Leading to the following LPV model

$$\begin{pmatrix}\hat{h}_{n1}\\ \hat{h}_{n2}\end{pmatrix}=\begin{pmatrix}a_{11}(d_1,d_2) & a_{12}(d_1,d_2)\\ a_{21}(d_1,d_2) & a_{22}(d_1,d_2)\end{pmatrix}\begin{pmatrix}d_1\\ d_2\end{pmatrix}+\begin{pmatrix}a_{13}(d_1,d_2)\\ a_{23}(d_1,d_2)\end{pmatrix} \tag{26}$$

where $a_{12}(d_1,d_2)=a_{21}(d_1,d_2)$.

Considering bounded additive noise in the pressure sensors

$$h_{ni} = \hat{h}_{ni} + e_i \quad \text{where } |e_i| \le \sigma_i \quad \text{for } i = 1,2 \tag{27}$$

$$\begin{pmatrix} h_{n1} \\ h_{n2} \end{pmatrix} = \begin{pmatrix} d_1 & d_2 & 0 & 1 & 0 \\ 0 & d_1 & d_2 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{11}(d_1,d_2) \\ a_{12}(d_1,d_2) \\ a_{22}(d_1,d_2) \\ a_{13}(d_1,d_2) \\ a_{23}(d_1,d_2) \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \tag{28}$$

and considering parametric modelling uncertainty with no dependences between parameters, then uncertainty can be parameterized as follows

$$a_{ij}(d_1,d_2) \in \left[a_{ij}^0(d_1,d_2) - \lambda_{ij}, a_{ij}^0(d_1,d_2) + \lambda_{ij} \right] \tag{29}$$

Equation (28) can be rewritten in regressor form (4) as follows

$$\boldsymbol{y}(k) = \begin{pmatrix} h_{n1}(k) \\ h_{n2}(k) \end{pmatrix}, \ \boldsymbol{\Phi}(k) = \begin{pmatrix} d_1(k) & d_2(k) & 0 & 1 & 0 \\ 0 & d_1(k) & d_2(k) & 0 & 1 \end{pmatrix},$$

$$\boldsymbol{\theta}(\mathbf{p}_k) = \left(a_{11}(\mathbf{p}_k) \quad a_{12}(\mathbf{p}_k) \quad a_{22}(\mathbf{p}_k) \quad a_{13}(\mathbf{p}_k) \quad a_{23}(\mathbf{p}_k) \right)^t$$

where $\mathbf{p}_k = \begin{pmatrix} d_1(k) \\ d_2(k) \end{pmatrix}$, $\boldsymbol{e}(k) = \begin{pmatrix} e_1(k) \\ e_2(k) \end{pmatrix}$ and

$$\boldsymbol{\Theta}_k = \boldsymbol{\theta}^0(\mathbf{p}_k) \oplus \mathbf{H}\mathbf{B}^5$$

with $\mathbf{H} = diag(\lambda_{11}, \lambda_{12}, \lambda_{22}, \lambda_{13}, \lambda_{23})$ and

$$\boldsymbol{\theta}^0(\mathbf{p}_k) = \left(a_{11}^0(\mathbf{p}_k) \quad a_{12}^0(\mathbf{p}_k) \quad a_{22}^0(\mathbf{p}_k) \quad a_{13}^0(\mathbf{p}_k) \quad a_{23}^0(\mathbf{p}_k) \right)^t \tag{30}$$

The parameter identification of model (4) has been carried out in two steps: first estimation of nominal parameters (30) and second estimation of its uncertainty defined by $\mathbf{H}$.

For the nominal parameter estimation, the LPV parameter estimation algorithm proposed by Bamieh and Giarré (2002) has been applied to a set of head/demand data registered in the network in a non-leak scenario. The parameter dependence of this model with the demand has been parameterized as follows

$$a_{ij} = \alpha_{ij} d_1 + \beta_{ij} d_2 + \gamma_{ij} \tag{31}$$

Once the nominal LPV model has been estimated, parametric modelling uncertainty $\mathbf{H}$ has been obtained solving *Problem 1* considering $\mathbf{H} = \lambda \mathbf{H}_0$ with $\mathbf{H}_0 = \mathbf{I}$ (i.e, the same weight of uncertainty in every parameter), giving $\lambda = 2.05 \cdot 10^{-5}$.

6.3 Leak detection and isolation/estimation implementation

Figure 2 shows the scheme of the leak detection, isolation and estimation procedure described in *Sections 3* and *4*. The faults to be detected, isolated and estimated are the leaks in nodes 1 and 2. These faults can be interpreted as unknown changes in demands d_1 and d_2. The demands considering the leaks are denoted as: $d_{1f} = d_1 + f_1$ and $d_{2f} = d_2 + f_2$, respectively.

Combining (28) with (16), residuals are obtained. Then, the residual leak sensitivity transfer function matrix (20) is

$$\boldsymbol{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \tag{32}$$

with

$$S_{11} = a_{11}(d_{1f}, d_2) + d_{1f}\alpha_{11} + d_2\alpha_{12} + \alpha_{13}$$
$$S_{12} = a_{12}(d_1, d_{2f}) + d_1\alpha_{12} + d_{2f}\alpha_{22} + \alpha_{23}$$
$$S_{21} = a_{12}(d_{1f}, d_2) + d_{1f}\beta_{11} + d_2\beta_{12} + \beta_{13}$$
$$S_{22} = a_{22}(d_1, d_{2f}) + d_1\beta_{12} + d_{2f}\beta_{22} + \beta_{23}$$

Remark: Only single faults have been considered.

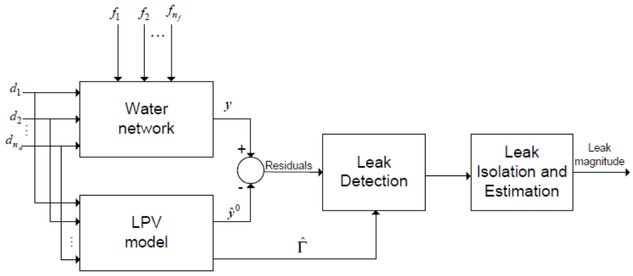

Fig. 2. Scheme of leak detection, isolation and estimation procedure

6.4 Leak scenario

To show the effectiveness of the proposed method a leak scenario corresponding to a leak f_1=0.5 l/s is present in the node 1 at time $1.7 \cdot 10^5$s. Figure 3 shows the residuals (difference between the real and predicted head measurements using the LPV model (26)) in the leak scenario.

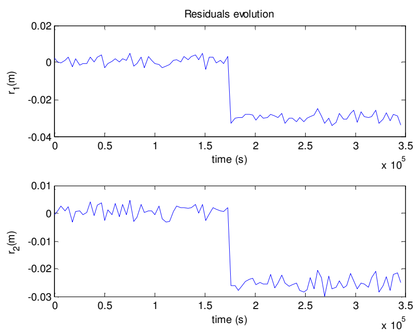

Fig. 3. Residuals evolution

Figure 4 shows the set $\boldsymbol{\Gamma}$, defined by (19), at the fault appearance instant. Notice that since the set $\boldsymbol{\Gamma}$ does not contain the point (0,0). Thus, the leak is detected according to (17).

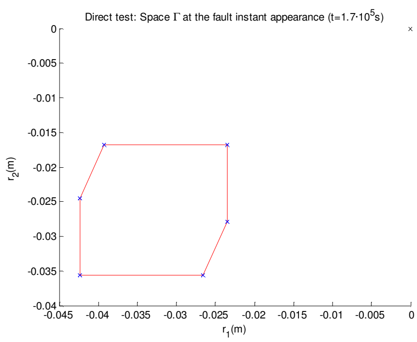

Fig. 4. Leak test in fault scenario

Once the leak has been detected, it should be isolated and estimated. Figure 5 shows the minimum optimization cost functions ($\boldsymbol{J}(f_1)$ and $\boldsymbol{J}(f_2)$) defined in *Problem 3*.

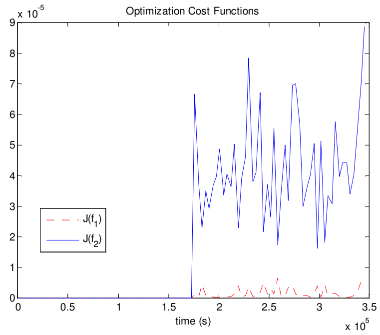

Fig. 5. Optimization cost functions

As $\boldsymbol{J}(f_1)$ is smaller than $\boldsymbol{J}(f_2)$ $\forall k \geq 1.7 \cdot 10^5 s$, then a leak is located in node 1. Thus, the leak magnitude f_1, presented in Figure 6, is obtained as the minimiser of $\boldsymbol{J}(f_1)$ that results from solving *Problem* 3.

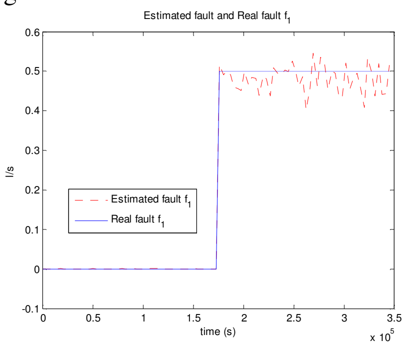

Fig. 6. Fault estimation

7. CONCLUSIONS

In this paper, a leak detection, isolation and estimation method for water pipe network system described by means of LPV model has been proposed. The leak detection methodology is based on checking if head measurements are inside the prediction bounds provided by a zonotope LPV model. The leak isolation and estimation module has been implemented using fault residual sensitivity analysis. This concept has been used to provide additional information to the relationship between residuals and leaks. Moreover, it allows obtaining a leak estimation. Satisfactory results have been obtained using the water pipe network case study. As a further research, the proposed methodology will be applied to a real network.

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