**Proceedings of ITP-09**
**2009 Interdisciplinary Transport Phenomena VI**
**Volterra, Italy, October 4-9, 2009**

# ITP-09-49

# SPATIAL STRUCTURE AND VELOCITY FLUCTUATIONS IN TURBULENT BUBBLE JETS IN MICROGRAVITY

**Pau Bitlloch**
Departament d'Estructura i Constituents de la Matèria
Universitat de Barcelona, Spain
Email: bitlloch@ecm.ub.es

**Xavier Ruiz**
Institut d'Estudis Espacials de Catalunya and
Departament de Química Física i Inorgànica
Universtitat Rovira i Virgili, Tarragona, Spain
Email: josepxavier.ruiz@urv.net

**Laureano Ramírez-Piscina**
Institut d'Estudis Espacials de Catalunya and
Departament de Física Aplicada
Universitat Politècnica de Catalunya, Barcelona, Spain
Email: laure@fa.upc.edu

**Jaume Casademunt**
Institut d'Estudis Espacials de Catalunya and
Departament d'Estructura i Constituents de la Matèria
Universitat de Barcelona, Spain
Email: jaume.casademunt@ub.edu

## ABSTRACT

*We study the dynamics of turbulent jets of bubbles created by the injection of a slug flow into a quiescent cavity, both theoretically and experimentally using data from drop tower experiments. The generated bubble jets exhibit a remarkably low degree of coalescence leading to virtually monodisperse suspensions of spherical bubbles. The turbulence of the jet is responsible for the spatial spreading of the bubble distribution in roughly conical jets. We propose a stochastic model for the dispersion of bubbles in the jet, in which the trajectories of individual bubbles can be computed from the stationary solution of a k-epsilon model. Bubble concentration behaves to a good approximation as a passive scalar with a non-homogeneous diffusion. Numerical integration of the model compares well with the experimental data.*

## NOMENCLATURE

$\nu$ kinematic viscosity of the liquid
$\mu$ dynamic viscosity of the liquid
$\mu_t$ eddy viscosity
Re Reynolds number
$k$ turbulent kinetic energy density
$\varepsilon$ turbulent dissipation rate density
$\mathbf{U}$ local mean fluid velocity
$\mathbf{v}_B$ bubble velocity
$\delta v$ fluctuating part of bubble velocity

## INTRODUCTION

Efficient control of bubble formation and management in microgravity environments is an important aspect in multiple applications for space technology. From a fundamental point of view, the statistical physics of bubbly turbulent flows in microgravity is also largely unknown, due precisely to the difficulty to achieve good experimental control of the experimental conditions for bubbles. One of the limiting experimental factors is the control of bubble sizes. This is important both for applications, to control the total gas-liquid contact area, and for fundamental characterization and understanding of the interaction of bubbles and turbulence. Recently a gravity insensitive method has been proposed to generate nearly monodisperse bubble suspensions by injecting into a quiescent cavity, a slug flow previously created in a capillary T-junction [1]. The bubbles have a size of the order of the capillary diameter and only weekly dependent on Weber number. A detailed theory for bubble formation in this set-up has been discussed in detail in Ref. [1]. The experimental characterization of the bubble formation method has been completed in normal gravity in Ref. [2]. The high degree of monodispersivity of the bubbles is also well understood within the theoretical framework of Ref. [1]. When the bubbles are injected into the quiescent cavity, the monodispersivity will be maintained to the extent that bubble coalescence is avoided. In our drop tower experiments, this is achieved to a large degree. In the present contribution we will discuss and model the behavior of such bubble jets in microgravity. In Ref. [1] it was already found that the mean flow associated to such turbulent jets was essentially unaffected by the presence of bubbles, except by a renormalization of the total momentum injected. Therefore, regarding the mean flow, the bubbles could be considered as passive tracers of the flow. It was also pointed out, however, that bubbles may in general affect the degree of turbulence that is, the fluctuating component of the velocity, since they cannot be considered as point-like and sufficiently dilute throughout the whole jet, in particular near the inlet of the cavity.

In this paper we pursue the study of the dispersion of bubbles in the turbulent jet. We construct a model for bubble dynamics in which the instantaneous bubble velocity is calculated as the addition of the local mean flow plus a stochastic term depending on the local degree of turbulence. For this picture the use of effective turbulence models such as the $k$-epsilon model, which yields an approximate closure of the averaged turbulence, appears as specially suitable. The treatment of the spreading of a passive scalar within a $k$-epsilon model has been studied in detail in the literature. In the present case the $k$-epsilon model will provide both the mean flow and local quantities representing the small scale diffusivity associated to the turbulence. The version of the model used here is specifically adapted to jet flows. Our results show that integration of the model, both for individual trajectories and for the concentration field of bubbles, compares well with experiments.

The understanding of the physics of the bubble jets created by this method is an important step towards the aim of producing spatially uniform, dense, monodisperse bubble suspensions in turbulent pipe flows, in microgravity. These could be created by combination of several of such injectors with externally imposed flows, such as in experiments currently in preparation in the ZARM drop tower. The capability of preparation of such suspension opens the door to a large variety of possibilities of interest for practical application but also for a deeper understanding of the interaction of bubbles and turbulence in the absence of buoyancy.

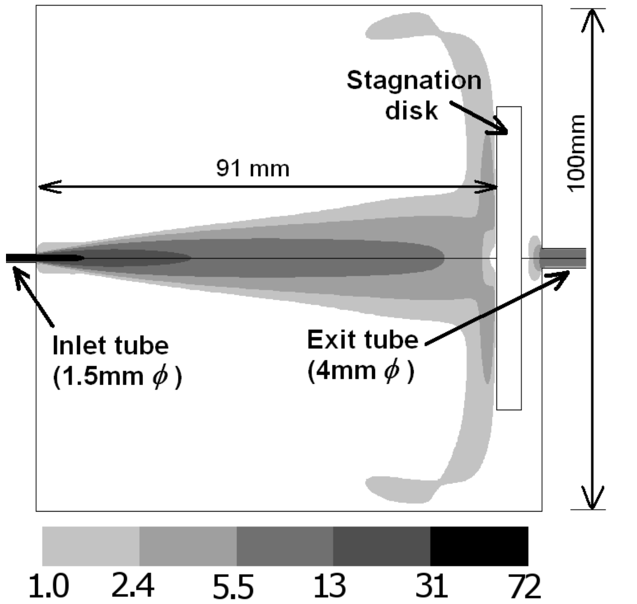


Figure 1. EXPERIMENTAL CELL SCHEMATICS WITH CONTOURS OF CONSTANT MEAN VELOCITY (cm/s)

## BUBBLE SPREADING IN A TURBULENT JET

Drop tower experiments were performed by injecting a pre-generated air-water slug flow in a quiescent cavity as represented in Fig. 1. This injection results in the formation of a turbulent jet which crosses the cavity, and in which the bubbles are spreaded. Details of the used setup can be found in Ref. [1]. A good approximation to this jet can be obtained by ignoring the presence of bubbles but renormalizing the injected momentum to account for the composition of the slug flow [1]. In Fig. 1, we show the structure of the velocity field in this approximation, obtained from a numerical CFD calculation in the case of an injection flow of Re=690, by using a realizable $k$-epsilon model [3]. This variant of the $k$-epsilon model adresses some issues of the original model, which makes it appropriate for the study of jets, and its equations read:

$$\frac{\partial(\rho k)}{\partial t} + \nabla\cdot(\rho k \mathbf{U}) = \nabla\cdot\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\nabla k\right] + 2\mu_t E_{ij}\cdot E_{ij} - \rho\varepsilon \quad (1)$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \nabla\cdot(\rho\varepsilon \mathbf{U}) = \nabla\cdot\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\nabla\varepsilon\right] - \rho C_2 \frac{\varepsilon^2}{k+\sqrt{\nu\varepsilon}} \quad (2)$$

being $\mathbf{U}$ the mean velocity, $\rho$ the density, $E_{ij}$ the rate-of-strain tensor

$$E_{ij} = \frac{1}{2}\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right) , \quad (3)$$

$\mu$ refers to viscosity and $\mu_t$ to eddy viscosity, defined by

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} , \quad (4)$$

$$C_\mu = \frac{1}{A_0 + A_s U^{(*)} \frac{k}{\varepsilon}} , \quad (5)$$

$$U^{(*)} = \sqrt{E_{ij}E_{ij} + \Omega_{ij}\Omega_{ij}} , \quad (6)$$

where

$$\Omega_i = \varepsilon_{ijk} \frac{\partial U_k}{\partial x_j} \quad (7)$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \quad (8)$$

and the other constants, *i.e.*

$$A_0 = 4.04, \; , \quad A_s = \sqrt{6} cos\phi , \quad (9)$$

$$\phi = \frac{1}{3} cos^{-1}\left(\sqrt{6}\frac{E_{ij}E_{jk}E_{ki}}{(\sqrt{E_{ij}E_{ij}})^3}\right) , \quad (10)$$

have been adjusted to the values which offers an optimal performance of the model:

$$C_{1\varepsilon} = 1.44, C_2 = 1.9, \sigma_k = 1.0, \sigma_\varepsilon = 1.2 . \quad (11)$$

In the following we will use the numerical integration of this model as an auxiliary calculation for working out the spreading of bubbles in the jet.

## Stochastic Model

The performed experiments show that the bubble jet has a roughly conical shape, but the characteristic opening angle depends on the degree of turbulence. The bubble cone angle is monotonously increasing with Re and saturates at Re of the order of 800 [1], when Re is defined with the injection capillary diameter. It is customary to decompose the total velocity field in two parts, a mean flow part and a fluctuating component. Since the spatial structure of the mean flow velocity field of a turbulent single-fluid jet is independent of Re [4], and since the local averaged velocities of bubbles coincide to a good approximation with that mean flow [1], the spreading of the spatial distribution of bubbles must be directly related to the fluctuating part of the flow. In Ref. [1] it was argued that if the transversal displacement of bubbles is diffusive, this leads to a conical shape. The natural scale of an effective diffusion coefficient based on the fluctuating part of the flow is $\nu$Re. Since Re is constant along a turbulent jet, then it follows that the opening angle of the cone is well defined, but with a homogeneous diffusivity this angle would not saturate with Re. The observed dependence of the bubble jet opening angle on Re must thus take into account the highly non-homogeneous structure of the flow field. The physical picture behind that analysis can be refined in a more consistent way within the $k$-epsilon model of turbulence. Here we propose a stochastic model of bubble spreading based on this treatment. Within this scheme, we will associate a local diffusivity to bubbles that is inherited from the diffusivity of the kinetic energy of the turbulent component of the flow in the absence of bubbles. The main assumption is thus that bubbles are also passive with respect to the fluctuating component of the flow. This assumption is correct in principle sufficiently far downstream, where the bubble suspension becomes more and more dilute, and the bubble size becomes negligible compared to the scales of the flow, but it may be questionable close to the inlet.

Since bubbles are not point-like and the number of them is relatively small, the aim of the model is to formulate an equation of the probability distribution of finding a bubble at a certain location. The model does not intend to be a good description of the trajectories of bubbles, which are far from diffusive at small scales in the flow, due to strong spatial and temporal correlations. This implies for instance, that the model will be inappropriate to describe properties related to the geometry of the bubble trajectories themselves or the correlations between them, such as the probability of bubble encounters and consequent possible coalescence. Despite this shortcoming of the model, the assumption of a local diffusivity of the probability of finding bubbles may be reasonably justified to describe the spatial distribution of an ensemble of realizations, provided that coalescence events are rare.

To formulate the model we assume the dynamics of bubbles to be that of a biased random walk. Explicitely we write the instantaneous velocity of a bubble as a stochastic differential equation (Langevin equation) of the form:

$$\mathbf{v}_B(t) = \mathbf{U}(\mathbf{r}(t)) + \delta\mathbf{v}(t), \tag{12}$$

where $\mathbf{U}(\mathbf{r}(t))$ is the local mean fluid velocity at the position of the bubble. The term $\delta\mathbf{v}(t)$ is then a fluctuating term of zero mean. Both terms of this decomposition can be obtained from the integration of a $k$-epsilon model. In particular writing the fluctuating term as a gaussian zero-mean white noise with correlation

$$\langle \delta\mathbf{v}(t)\delta\mathbf{v}(t')\rangle = 2D_p\delta(t-t'), \tag{13}$$

the noise intensity $D_p$ is taken as proportional to the diffusivity of the turbulent kinetic energy $k^2/\varepsilon$ in the context of standard $k$-epsilon model:

$$D_p = \frac{\mu_t}{\sigma_p} = \frac{\rho C_\mu}{\sigma_p}\frac{k^2}{\varepsilon} = \alpha\frac{k^2}{\varepsilon}. \tag{14}$$

where $C_\mu = 0.09$ according to the standard model, and $\alpha$ is in principle a fitting parameter. The prediction of this model regarding the spatial structure of the bubble jet does not seem very sensitive to the parameter $\alpha$. From previous studies of diffusion of passive scalars in this framework we take the value $\alpha = 90$ [5].

Associated to the stochastic differential equation Eqn. 12 we can find the Fokker-Planck equation for the probability distribution $P(\mathbf{r},t)$ of finding the bubble in a certain position at any instant of time. This equation has the form of a diffusion equation, and reads:

$$\frac{\partial P(\mathbf{r},t)}{\partial t} + \nabla\cdot(\mathbf{U}P) = \nabla\cdot[D_p\nabla P]. \tag{15}$$

In this framework, the concentration of bubbles, proportional to the probability distribution $P$, diffuses as a passive scalar advected with the mean flow velocity $\mathbf{U}(\mathbf{r},t)$, but with a diffusion coefficient $D_p$ which depends on the local properties of the turbulence through the parameter $k^2/\varepsilon$.

### Numerical Results

Our numerical computations have been carried out with the help of the commercial software FLUENT. In Fig. 1 we show the structure of the velocity field for the turbulent jet as computed within the $k$-epsilon model. For a single-fluid jet, the spatial structure of the jet is rather insensitive to the injection

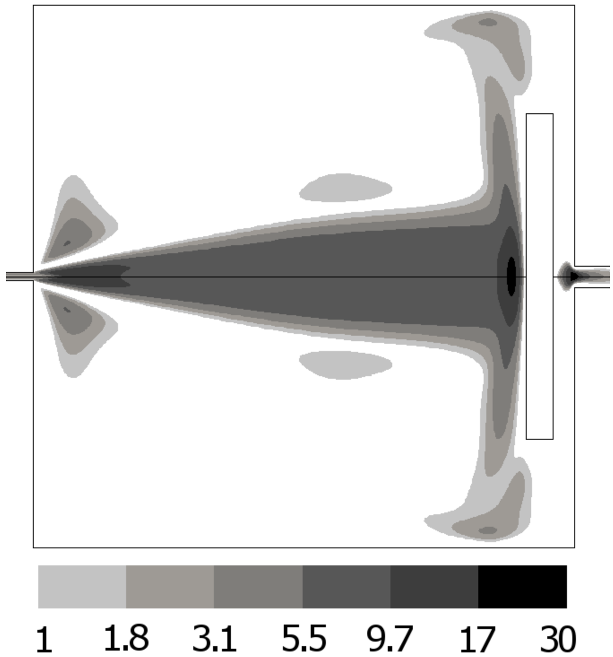


Figure 2. CONTOURS OF CONSTANT $k^2/\varepsilon \times 10^{-5}$

Reynolds number. However, the spatial distribution of a passive scalar advected by this flow is indeed expected do depend on Re. To visualize the degree of inhomogeneity in our model regarding the diffusivity of bubbles we plot in Fig. 2 the quantity $k^2/\varepsilon$, which is in principle proportional to the effective local diffusion coefficient of bubbles. We see that diffusivity values vary in a wide range. The local diffusivity is remarkably homogeneous in a certain central area and abruptly drops on the sides, defining relatively clear-cut jet boundaries. This drop in diffusivity is larger that an order of magnitude in a relatively narrow layer. Therefore, the increase with Re of the opening angle of the bubble jet due to diffusivity has a limit corresponding to such diffusivity drop. We then expect a relatively homogeneous bubble jet with a well defined opening angle, which increases with Reynolds number but which eventually saturate to a maximal value. This is exactly what was observed in experiments [1].

This scenario is supported by the numerical resolution of our model. In Fig. 3 we show the resulting concentration of bubbles by using a inhomogeneous bubble diffusivity locally depending on $k^2/\varepsilon$. Even for large Re bubble spreading is limited by the jet boundaries, and the resulting distribution are similar to experiments. However, this is not the case if an homogeneous diffusivity is used. The use of a single value of diffusivity for the whole system results in a distribution of bubbles that either opens a very small angle, or spreads out of

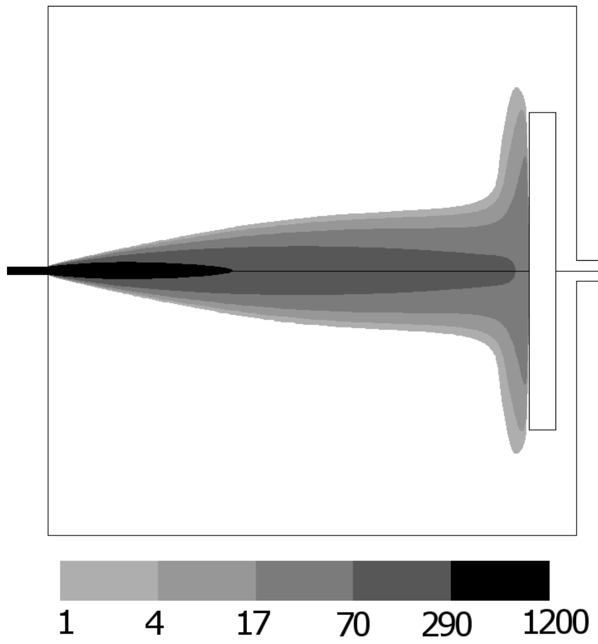


Figure 3. CONTOURS OF CONSTANT BUBBLE CONCENTRATION (OR PROBABILITY DENSITY) WITH AN ARBITRARY NORMALIZATION, CORRESPONDING TO A LOCAL DIFFUSIVITY PROPORTIONAL TO $k^2/\varepsilon$

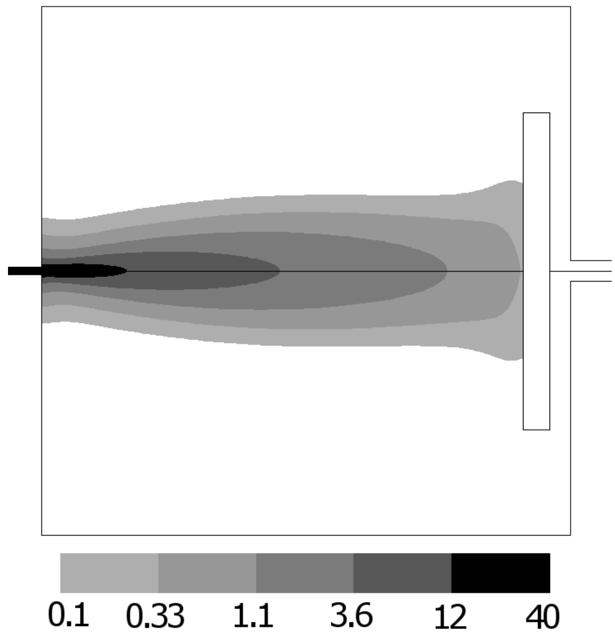


Figure 4. CONTOURS OF CONSTANT BUBBLE CONCENTRATION (OR PROBABILITY DENSITY) WITH AN ARBITRARY NORMALIZATION, CORRESPONDING TO A HOMOGENEOUS DIFFUSIVITY

the limits of the jet, like in the case shown in Fig. 4. We therefore conclude that an inhomogeneous diffusivity is essential to capture the spatial distribution of bubbles.

For a direct comparison of the predicted bubble distribution to the experimental results, in Fig. 5 we have superposed a series of snapshots of an experiment to obtain a sort of ensemble average of the experimental data. We have also drawn concentration profiles given by the theory. These profiles are actually the 2d projections of the cylindrically symmetric jet, in order to allow a direct visual comparison with the experimental snapshots of bubble distributions. The agreement with the numerical profile is reasonable taking into account both the imperfections of the experiment (a certain broken symmetry associated to the preparation process before the microgravity phase) and the approximations involved in the model.

As a final qualitative test of the physical picture we have reconstructed a bubble jet from trajectories consistent with our probabilistic model. That is, we have evolved an ensemble of bubbles undergoing a biased random walk given by the Langevin equation Eqn. 12. Note that the numerical integration of Eqn. 12 permits the simulation of individual trajectories of bubbles, provided they behave as independent. Then the local mean velocity is obtained from the solution of the $k$-epsilon model, and the fluctuating term is calculated with the local diffusion coefficient given by the same model. Although the trajectories may significantly differ from the real ones at short distances, the probability distribution may be much more realistic. In Fig. 6 we plot a representative example of such simulation showing reasonable qualitative agreement with the bubble jets obtained in experiments (see also [1]).

## CONCLUSIONS

We have presented a stochastic model that captures the essential statistics of bubble spatial dispersion in turbulent bubble jets formed by injection of capillary slug flows. The model is based on a simplified description of the turbulent flow within the k-epsilon scheme. The bubbles in the experimental regime of a series of drop tower experiments, although of non-negligible size compared to relevant turbulent scales, have been shown to be passive with respect to the mean flow. Its spatial dispersion is controlled by the fluctuating component of the flow field, so that the angle of dispersion of the jet depends on the Reynolds number and increases up to a saturation value. Even within the simplifications of the model, the treatment of bubbles as passive tracers with a local diffusivity associated to the k-epsilon model seems to capture reasonably well the ensemble dynamics of the bubbles. Numerical results obtained with our model compare very well with experiments.

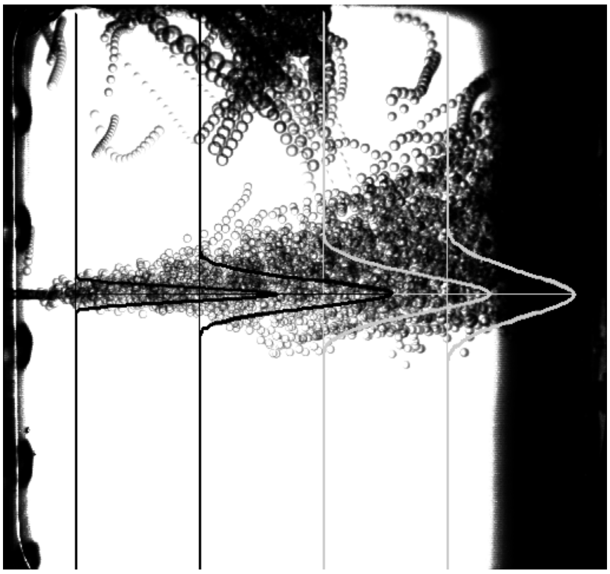

Figure 5. PROJECTED CONCENTRATION PROFILES AND SUPERPOSITION OF EXPERIMENTAL SNAPSHOTS.

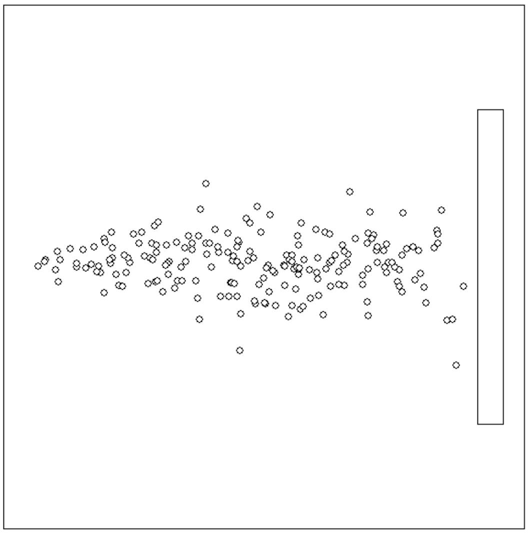

Figure 6. SIMULATION OF LANGEVIN DYNAMICS (SEE TEXT)

On the contrary the use of an homogeneous bubble diffusivity gives results whose qualitative features are distincly different.

A more accurate description of the system should aim at a more realistic modeling of the bubble trajectories. Diffusive trajectories are indeed too erratic on small scales and overestimate significantly the probability of bubble encounters. Introducing a more realistic tracking of the flow trajectories, even if still as passive tracers, should take into account statistical correlations of the flow which would clearly modify the statistics of bubble encounters. A full description of the dynamics of non-dilute suspensions of spherical bubbles, including bubble-bubble interactions and bubble-flow interactions could be approached with large scale Lattice-Boltzmann simulations, in the spirit of Ref. [6]


## ACKNOWLEDGMENT

We gratefully acknowledge the support from the German Space Agency (DLR e.V.) for the funding of the drop tower flights, and the Spanish Ministry of Science and Technology through the Space National Program (Projects ESP2005-25290-E and ESP2007-30637-E). P. B., X.R. and J.C. also acknowledge additional support from Project FIS2006-03525 and L.R.P. from Project FIS2006-11452-C03-02. P.B. ackowledges a grant of the Spanish Ministry of Science and Technology associated to the Project FIS2006-03525.


## REFERENCES


[1] J. Carrera, X. Ruiz, L. Ramírez-Piscina, J. Casademunt and M. Dreyer, "Generation of a Monodisperse Microbubble Jet in Microgravity." *AIAA Journal* **46**:2010-2019 (2008).

[2] S. Arias, X. Ruiz, J. Casademunt, L. Ramírez-Piscina, and R. Gonzàlez-Cinca, "Experimental study of a microchannel bubble injector for microgravity applications." *Microgravity Sci. Technol.* **21(1)**:107-111 (2009)

[3] T.-H. Shih, W. W. Liou, A. Shabbir, Z. Yang, and J. Zhu. "A New *k*- ε Eddy-Viscosity Model for High Reynolds Number Turbulent Flows - Model Development and Validation." *Computers Fluids*, **24(3)**:227-238, (1995).

[4] H. Schlichting, "Boundary-Layer Theory," McGraw-Hill Classic Textbook Reissue. Seventh Edition (1979)

[5] H.K.Versteeg and W.Malalasekera, "An introduction to computational fluid dynamics. The finite volume method" Pearson Prentice Hall (1995)

[6] X. Yin, D. L. Koch and R. Verberg, "Lattice-Boltzmann method for simulating spherical bubbles with no tangential stress boundary conditions", *Phys. Rev. E* **73**:026301 (2006)