A Recommender System Based on Group Consensus

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Abstract

This paper presents the foundation for a new methodology for a collaborative recommender system (RS). This methodology is based on the degree of consensus of a group of users stating their preferences via qualitative orders-of-magnitude. The structure of distributive lattice is considered in defining the distance between users and the RSs new users. This proposed methodology incorporates incomplete or partial knowledge into the recommendation process using qualitative reasoning techniques to obtain consensus of its users for recommendations.

Introduction

The RS proposed is a collaborative memory based system where the user is recommended items based on users with similar profiles and preferences. Several different approaches have been discussed in the literature to address the problem of searching users' similarities such as: correlation-based [26, 21], cosine-based [3, 25], and graph theoretic [2]. This RS differs from others because it uses a heuristic that allows different levels of precision to be considered simultaneously. We present recommendations that search user's similarities in terms of consensus to other users. Rather than using classical methods, we put forth an approach to recommending by searching the most similar neighbors using a level of degree of consensus directly or through a dive function that permit consensus based on underlying common values.

Before users interact with the RS they must answer a questionnaire about features concerning the product to be recommended using labels of an absolute order-of-magnitude space. This is used to calibrate a user's precision of opinion to a feature. This is relevant when a user does not know how to precisely qualify a feature, and where the user can use a non-basic label in allowing the RS to capture and represent the ambiguity of a user's knowledge.

The RS proposed in this paper requires a measure of consensus defined in (Roselló et al. 2010) which is based on a definition of entropy for a qualitatively-described system. Absolute order-of-magnitude qualitative models are considered in this papers to describe users' preferences.

*This work has been partly funded by MEC (Spanish Ministry of Education and Science) VALORA Research Project (TIN2009-13902-C02-01).

Absolute Order-of Magnitude Models

Order-of-magnitude models (Dague 1993; Kalagnanam, Simon, and Iwasaki 1991; Struss 1988) aim to capture commonsense inferences (Travé-Massuyès, Dague, and Guerrin 1997) such as those commonly used in describing preferences and in recommendation processes. The absolute order-of-magnitude qualitative spaces (Travé-Massuyès and Dague 2003) are built from a set of ordered basic qualitative labels determined by a partition of the real line. A general algebraic structure, called Qualitative Algebra or Q-algebra, was defined based on this framework (Travé-Massuyès and Piera 1989), providing a mathematical structure to unify sign algebra and interval algebra through a continuum of qualitative structures built from the roughest to the finest partition of the real line. Q-algebras and their algebraic properties have been extensively studied (Missier, Piera, and Travé 1989; Travé-Massuyès and Dague 2003).

Let us consider a finite set of *basic* labels, $\mathbb{S}_* = \{B_1, \ldots, B_n\}$, which is totally ordered as a chain: $B_1 < \ldots < B_n$. Usually, each basic label corresponds to a linguistic term, for instance "extremely bad'" <"very bad" < "bad" < "acceptable" < "good" < "very good"<"extremely good".

The complete description universe for the Orders-of-Magnitude Space OM(n) with granularity n, is the set $\mathbb{S}_n$:

$$\mathbb{S}_n = \mathbb{S}_* \cup \{[B_i, B_j] \mid B_i, B_j \in \mathbb{S}_*, i < j\},$$

where the label $[B_i, B_j]$ with $i < j$ is defined as the set $\{B_i, B_{i+1}, \ldots, B_j\}$.

Consistent with the former example of linguistic labels, the label "moderately good" can be represented by ["acceptable", "good"], i.e., $[B_4, B_5]$. The label "don't know" is represented by ["extremely bad", "extremely good"], i.e., $[B_1, B_7]$. This least precise label is denoted by the symbol ?, i.e., $[B_1, B_n] \equiv$?.

There is a partial order relation $\leq_P$ in $\mathbb{S}_n$, "to be more precise than", given by $L_1 \leq_P L_2 \iff L_1 \subset L_2$. The structure $OM(n)$ permits working with all different levels of precision from the basic labels to the ? label.

The Algebraic Structure of the Set of Qualitative Descriptions Induced by the Users

Let $\Lambda = \{a_1, \dots, a_N\}$ be a set that represents a magnitude or a feature that is qualitatively described by means of the $\mathbb{S}_n$ labels. This qualitative description is carried out by each user and is represented by the function: $Q : \Lambda \to \mathbb{S}_n$,

Let $\mathcal{Q} = \{Q \mid Q : \Lambda \to \mathbb{S}_n\}$ be the set of qualitative descriptions of Λ over $\mathbb{S}_n$ given by a group of users. Given $Q, Q' \in \mathcal{Q}$, two different operations are defined between them.

Definition 1 *Given two qualitative descriptions $Q, Q' \in \mathcal{Q}$, the operation $Q \sqcup Q'$ leads to a new qualitative description function $Q \sqcup Q' : \Lambda \to \mathbb{S}_n$ such that, for any $a_t \in \Lambda$,*

$$(Q \sqcup Q')(a_t) = Q(a_t) \sqcup Q'(a_t),$$

where $\sqcup$ is the connex union of labels, i.e. the minimum label that contains $Q(a_t)$ and $Q'(a_t)$: $[B_i, B_j] \sqcup [B_h, B_k] = [B_{\min\{i,h\}}, B_{\max\{j,k\}}]$, using the convention $[B_i, B_i] = B_i$.

The concept of consensus between two qualitative descriptions, Q and Q', is required in order to introduce the common operation:

Definition 2 *Two qualitative descriptions Q, Q' are* in consensus*, $Q \rightleftarrows Q'$, iff*

$$Q(a_t) \cap Q'(a_t) \neq \emptyset, \quad \forall a_t \in \Lambda. \tag{1}$$

This last condition is equivalent to saying that $Q(a_t) \approx Q'(a_t), \forall a_t \in \Lambda$. [1]

It is clear that the relation $\rightleftarrows$ is symmetric and reflexive.

In general, a set $\{Q_i\}_{i \in I} \subset \mathcal{Q}$ of qualitative descriptions of Λ over $\mathbb{S}_n$ is *in consensus* iff $\cap_{i \in I} Q_i(a_t) \neq \emptyset \;\; \forall a_t \in \Lambda$. Note that, in this case, $Q \rightleftarrows Q'$ for all $Q, Q' \in \{Q_i\}_{i \in I}$.

Definition 3 *Given two qualitative descriptions Q and Q' where $Q \rightleftarrows Q'$, the* common *$Q \cap Q'$ operation produces a new qualitative description function $Q \cap Q' : \Lambda \to \mathbb{S}_n$ such that*

$$(Q \cap Q')(a_t) = Q(a_t) \cap Q'(a_t) \;\; \forall a_t \in \Lambda.$$

In general, if $\{Q_i\}_{i \in I} \subset \mathcal{Q}$ is in consensus, the operation *common* $\cap_{i \in I} Q_i$ produces a new qualitative description: $(\cap_{i \in I} Q_i)(a_t) = \cap_{i \in I} Q_i(a_t) \;\; \forall a_t \in \Lambda$.

The algebraic structure of the set $\mathcal{Q}$ and the $\sqcup$ and $\cap$ operations is given by the next proposition (the proof can be found in (Roselló et al. 2010)).

Proposition 1 *Let $\mathcal{Q}_L$ be a subset of $\mathcal{Q}$ which is in consensus. Then $(\mathcal{Q}_L, \sqcup, \cap)$ is a distributive lattice.*

[1] In the absolute order-of-magnitude theory, two labels $\mathcal{E}, \mathcal{F}$ are qualitatively equal, $\mathcal{E} \approx \mathcal{F}$, iff $\mathcal{E} \cap \mathcal{F} \neq \emptyset$.

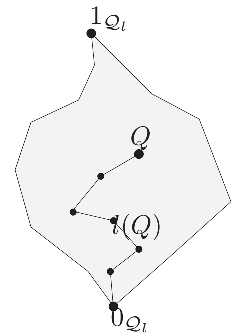

Figure 1: The null and universal elements, and the length of a qualitative description.

A Distance in Users' Sets in Consensus

This section is devoted to define a distance between two qualitative descriptions $Q, Q' \in \mathcal{Q}_L$.

A *chain* $[x, y]$ is a partially ordered set satisfying that $\forall x, y$ either $x \leq y$ or $y \leq x$, Equivalently, a chain is a subset of a poset totally ordered.

By *"x covers y"* it is meant that $y < x$ and that $y < z < x$ it is not satisfied by any z. A finite chain $x = a_1 < a_2 < \dots < a_n = y$ is a *maximal chain* if each a_{i+1} covers a_i for $i = 1, \dots, n-1$.

Definition 4 *In $(\mathcal{Q}_L, \sqcup, \cap)$ the* null element *$0_{\mathcal{Q}_L}$ is defined as*

$$0_{\mathcal{Q}_L} = \sqcup_{Q_i \in \mathcal{Q}_L} Q_i,$$

and the universal element *$1_{\mathcal{Q}_L}$ id defined as*

$$1_{\mathcal{Q}_L} = \cap_{Q_i \in \mathcal{Q}_L} Q_i.$$

The null element and the universal elements verify for all $Q \in \mathcal{Q}_L$ (see figure 1):

$$0_{\mathcal{Q}_L} \sqcup Q = 0_{\mathcal{Q}_L}, 0_{\mathcal{Q}_L} \cap Q = Q,$$
$$1_{\mathcal{Q}_L} \sqcup Q = Q, 1_{\mathcal{Q}_L} \cap Q = 1_{\mathcal{Q}_L},$$

and then $0_{\mathcal{Q}_L} \leq Q \leq 1_{\mathcal{Q}_L} \forall Q \in \mathcal{Q}_L$.

Let us assume that Λ is a finite set. Since $\mathbb{S}_n$ is also finite, hen all the chains in $(\mathcal{Q}_L, \sqcup, \cap)$ are finite.

Definition 5 *If $Q, Q' \in \mathcal{Q}_L$, the length of a chain $[Q, Q']$ is the cardinal of the maximal chain for $[Q, Q']$. The length of $Q \in \mathcal{Q}_L$, $l(Q)$, is the length of $[0_{\mathcal{Q}_L}, Q]$ (see figure 1).*

Let $(\mathcal{Q}_L, \sqcup, \cap)$ a distributive lattice in which all bounded chains are finite. Then

1. All finite connected chains between fixed end points have the same length (Jordan-Hölder theorem).
2. $\forall Q, Q' \in \mathcal{Q}_L$
$$l(Q) + l(Q') = l(Q \sqcup Q') + l(Q \cap Q') \tag{2}$$

Lemma 1 *Since in $\mathcal{Q}_L$ the operations $\sqcup$ and $\cap$ are the infimum and supremum respectively then:*

$$(Q \cap Q') \sqcup (Q' \cap Q'') \geq Q' \tag{3}$$
$$Q' \geq (Q \sqcup Q') \cap (Q' \sqcup Q''). \tag{4}$$

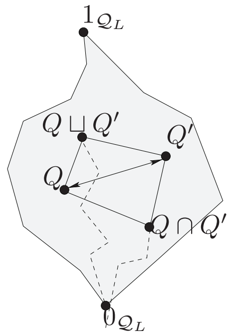

Figure 2: The weak partial lattice of the qualitative descriptions.

Proof: It is a simply exercise using the definition of $\leq$ and properties of $\sqcup$ and $\cap$. $\square$

The next theorem defines a distance in the lattice $(\mathcal{Q}_L, \sqcup, \cap)$:

Theorem 1 *In the lattice* $(\mathcal{Q}_L, \sqcup, \cap)$, *the function* $d : \mathcal{Q}_L \times \mathcal{Q}_L \to \mathbb{R}$ *defined as*

$$d(Q, Q') = l(Q \cap Q') - l(Q \sqcup Q'), \tag{5}$$

is a distance.

Proof:

1. Positive definiteness: From $Q \sqcup Q' \leq Q \cap Q'$ $\forall Q, Q'$ it is trivial to see that $l(Q \sqcup Q') \leq l(Q \cap Q')$, so $d(Q, Q') \geq 0$. If $Q = Q'$ then $d(Q, Q') = 0$. Conversely,
$$d(Q, Q') = 0 \Rightarrow l(Q \sqcup Q') = l(Q \cap Q'),$$
and this, together with the fact that $Q \sqcup Q' \leq Q \cap Q'$, and the Jordan-Hölder theorem leads to $Q \sqcup Q' = Q \cap Q'$. From the absorptive laws of lattices:
$$Q \cap (Q \sqcup Q') = Q \text{ and } Q \sqcup (Q \cap Q') = Q.$$
We have:
$$Q = Q \cap (Q \sqcup Q') = Q \cap (Q \cap Q') = Q \cap Q',$$
$$Q' = Q' \cap (Q \sqcup Q') = Q' \cap (Q \cap Q') = Q \cap Q',$$
therefore $Q = Q'$.
2. Symmetry: Since $\sqcup$ and $\cap$ are commutative, $d(Q, Q') = d(Q', Q)$.
3. Triangle inequality: For all $Q, Q', Q'' \in \mathcal{Q}_L$
$$d(Q, Q') \leq d(Q, Q'') + d(Q'', Q').$$
We have:
$$d(Q, Q'') + d(Q'', Q') = l(Q \cap Q'') + l(Q' \cap Q'') -$$
$$-(l(Q \sqcup Q'') + l(Q' \sqcup Q'')).$$
The two first summands can be expressed using the property (2):
$$l(Q \cap Q'') + l(Q' \cap Q'') = l((Q \cap Q'') \sqcup (Q' \cap Q''))+$$
$$+l((Q \cap Q'') \cap (Q' \cap Q'')),$$
and then, by (3):
$$l(Q \cap Q'') + l(Q' \cap Q'') \geq l(Q'') + l((Q \cap Q' \cap Q'')).$$
Similarly from (2)
$$l(Q \sqcup Q'') + l(Q' \sqcup Q'') = l((Q \sqcup Q'') \sqcup (Q' \sqcup Q''))+$$
$$+l((Q \sqcup Q'') \cap (Q'' \sqcup Q')),$$
and then, by (4):
$$l(Q \sqcup Q'') + l(Q' \sqcup Q'') \leq l(Q \sqcup Q' \sqcup Q'') + l(Q'').$$
So,
$$d(Q, Q'')+d(Q'', Q') \geq l(Q \cap Q' \cap Q'') - l(Q \sqcup Q' \sqcup Q'').$$
Now, using the fact that:
$$Q \cap Q' \cap Q'' \geq Q \cap Q' \Rightarrow l(Q \cap Q' \cap Q'') \geq l(Q \cap Q')$$
$$Q \sqcup Q' \sqcup Q'' \leq Q \sqcup Q' \Rightarrow l(Q \sqcup Q' \sqcup Q'') \leq l(Q \sqcup Q'),$$
we conclude that:
$$d(Q, Q'')+d(Q'', Q') \geq l(Q \cap Q') - l(Q \sqcup Q') = d(Q, Q').$$

$\square$

Consensus among Users

Measuring consensus has been tackled in the literature by several authors in different ways. The most studied approaches to measuring consensus use fuzzy linguistic information (Cabrerizo et al. ; Cabrerizo, Alonso, and Herrera-Viedma 2009; Mata, Martínez, and Herrera-Viedma 2009; Herrera-Viedma et al. 2007). In (Chiclana et al. 2008; Ngwenyama, Bryson, and Mobolurin 1996) the degree of consensus is computed through an average, and in (Eklund, Rusinowski, and De Swart 2007) it is performed by a distance.

Given a finite non empty set of features $\Lambda = \{a_1, \ldots, a_N\}$ and a group of users $\mathbb{E} = \{\alpha_1, \ldots, \alpha_M\}$, the *set of users* of Λ is considered as the pair $(\Lambda, \mathcal{Q}_{\mathbb{E}})$, where $\mathcal{Q}_{\mathbb{E}} = \{Q_i : \Lambda \to \mathbb{S}_n \mid i \in \{1, \cdots M\}\}$, and Q_i is the evaluation given by α_i.

When there is consensus among the set, i.e., $\cap_{i=1}^{M} Q_i(a_t) \neq \emptyset \ \ \forall a_t \in \Lambda$, the degree of consensus defined in (Roselló et al. 2010) can be computed. This degree is a number between 0 and 1; the closer it is to 1, the closer the set is to being unanimous in its preferences.

As said before, the necessary and sufficient condition for which there exists consensus is $\cap_{i=1}^{M} Q_i(a_t) \neq \emptyset, \forall a_t \in \Lambda$. If this situation does not hold then a process has to be initiated to obtain consensus. In (Chiclana et al. 2008; Eklund, Rusinowski, and De Swart 2007; Martínez and Montero 2007; Ngwenyama, Bryson, and Mobolurin 1996), different approaches to this problem are found framed within fuzzy sets theory and aggregation operators. The process considered in this paper is based on the algorithm introduced in (Roselló et al. 2010).

The Recommender System

Let us consider recommendation of a certain type of product (e.g. wines), for which there is a set of features Λ (sweetness, acidity, tannin,...) whose qualitative values can be used to describe customers' preferences. Each feature in Λ can be described by an element of space $\mathbb{S}_n$. Let $\mathbb{A}$ be the set of alternatives, i.e. products offered in the sales point. Let $\mathbb{E}$ be a set of customers that have already been recommended an alternative by an expert (sommelier). In order to recommend automatically an alternative to a new customer, the elements in $\mathbb{E}$, together with their recommended alternatives, are considered as the training set.

Each element in $\mathbb{E}$ is represented by a qualitative description $Q_i : \Lambda \longrightarrow \mathbb{S}_n$ that assign a label of $\mathbb{S}_n$to each feature. Let $\mathcal{Q}_{\mathbb{E}}$ be the set of these qualitative descriptions. The recommendations of the expert provide a function $f : \mathcal{Q}_{\mathbb{E}} \rightarrow \mathbb{A}$ that assigns each Q_i to an element of the set $\mathbb{A}$ of alternatives.

Example 1 *Let us consider a process for wines recommendation. The set Λ could be how much the user like the following characteristics:*

1. Wine Sweetness	***2. Acidity***
a. Bone dry	*a. Tart*
b. Dry	*b. Crisp*
c. Medium dry	*c. Soft*
d. Sweet	*d. Flabby*
e. Very sweet	

3. Tannin	***4. Balance and body***
a. Astringent	*a. Light bodied*
b. Firm	*b. Medium bodied*
c. Soft	*c. Full bodied*

So $\Lambda = \{1.a, 1.b, 1.c, \ldots, 4.c\}$. The set $\mathbb{A}$ is formed by a list of specific wines wines:

```
Healdsburg Unoaked Chardonnay 2007
Chateau La Freynelle Blanc
...
Siurana Clos de L'Obac Priorat 2004
```

The space $\mathbb{S}_5$ is formed by the labels: B_1 =`DISLIKE VERY MUCH`, B_2 =`DISLIKE`, B_3 =`NORMAL`, B_4 =`LIKE` and B_5 =`LIKE VERY MUCH`

One example of Q_i could be: $Q_i(1.a) = [B_1, B_2], Q_i(1.b) = B_2, Q_i(1.c) = [B_3, B_5], Q_i(1.d) = Q_i(1.e) = Q_i(1.f) = B_1$, and so on.

The function $f : \mathcal{Q}_{\mathbb{E}} \rightarrow \mathbb{A}$ maps each user to a wine depending on his or her preferences about features considered in Λ . This function is known in the training set and represents the sommelier's knowledge used to recommend the best wine to each customer.

The goal of the system is, for a new $Q' : \Lambda \rightarrow \mathbb{S}_n$, to assign to Q' an alternative $f(Q') \in \mathbb{A}$.

The main idea of the algorithm is that the best alternative for the new user with qualitative description Q' is the alternative of the user with nearest qualitative description Q. Notice that the considered distance is defined only in users'

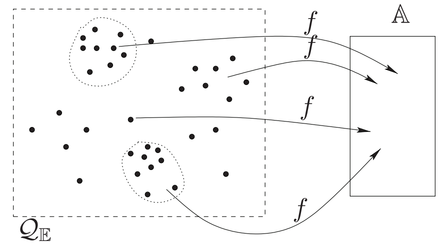

Figure 3: The training set of the recommender system.

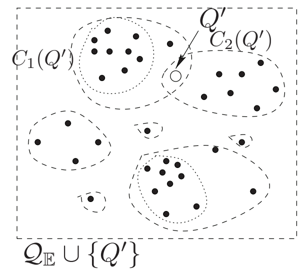

Figure 4: Here the automatic negotiation process has been applied one step. This has produced two subsets $C_1(Q')$ and $C_2(Q')$.
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sets in consensus. This fact reduces computing costs compared to other algorithms based on distances, such as k-NN algorithms.

This can be done in the following steps:

In figure 3 we can see the training set: each dot is an element $Q \in \mathcal{Q}_{\mathbb{E}}$ and the dotted closed lines represent that these two subsets are in consensus (of course each Q is in consensus with itself).

Let us denote by $C(Q')$ the set of the subsets of $\mathcal{Q}_{\mathbb{E}} \cup \{Q'\}$ that are in consensus and contain Q', and let $\mathcal{C}(Q')$ be the set of all $C_i(Q') \in C(Q')$ such that $C_i(Q') \neq \{Q'\}$:

$$\mathcal{C}(Q') = \{C_i(Q') \in C(Q') \mid |C_i(Q')| \geq 2\}, \qquad (6)$$

and let $i_{Q'}$ be its cardinal:

$$i_{Q'} = |\mathcal{C}(Q')|. \qquad (7)$$

First of all, the algorithm finds $\mathcal{C}(Q')$ and $i_{Q'}$. If $i_{Q'} \geq 1$ then the we choose the subset with highest degree of consensus (Roselló et al. 2010):

$$C(Q')^* = \arg \max_{C_i(Q') \in \mathcal{C}(Q')} \kappa(C_i(Q')).$$

The next step is to assign to Q' the alternative of the nearest Q in the subset $C(Q')^*$:

$$f(Q') = f(\arg \min_{Q \in C(Q')^*} d(Q, Q')),$$

where the distance is the expression in (5).

If in (7) $i_{Q'} = 0$, then we have to apply the automatic negotiation process (Roselló et al. 2010), in order to find at least one subset defined in (6) and get an $i_{Q'} \geq 1$ (see figure 4). Once it is found, the algorithms follows as before.

This algorithm is applied in the following example:

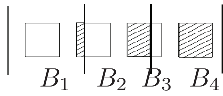
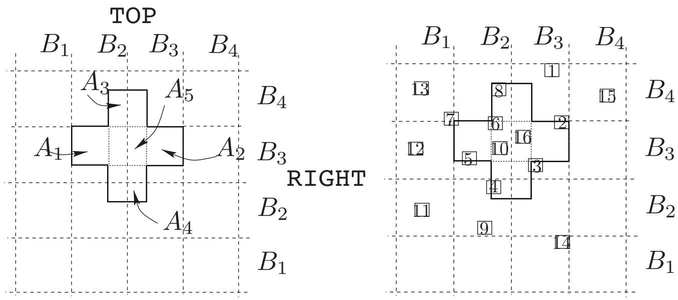


Figure 5: The intersections.



(a) The questionnaire

(b) The training set. For readability reasons, instead of Q_i it has been written i.

Figure 6: The recommendation system

Example 2 *The training set is composed by* $\mathbb{E} = \{\alpha_1, \ldots, \alpha_{16}\}$*, a set* $\Lambda = \{a_1, a_2, a_3\}$ *and the qualitative description is done over* $\mathbb{S}_4$*. The set of alternatives is* $\mathbb{A} = \{A_1, \ldots, A_6\}$*. The process of answer the questionnaire (qualitative description) and alternative assignment will be simulated by putting a little square on Figure 6(a). The little squares corresponding to the answers of* $\alpha_1, \ldots, \alpha_{16}$ *are represented in Figure 6(b) .*

The value of a_1 *and* a_2 *is how near is of* `RIGHT` *and* `TOP` *respectively. To simplify the example we will use only the basic labels* $\{B_1, \ldots, B_4\}$*. The third value* a_3 *is how much the square cuts the cross. This quantity is a label obtained by the connex union of the two labels that represent the intersection of the square in the horizontal and vertical lines of the cross, in Figure 5 we can see the case of the horizontal intersection.*

As an example, the qualitative descriptions corresponding to α_1 *and* α_2 *are:*

$$Q_1(\Lambda) = \{Q_1(a_1), Q_1(a_2), Q_1(a_3)\} = \{B_3, B_4, B_1\},$$
$$Q_2(\Lambda) = \{Q_2(a_1), Q_2(a_2), Q_2(a_3)\} = \{B_3, B_3, [B_2, B_3]\}.$$

The alternative assigned to each $Q_i \in \mathcal{Q}_{\mathbb{E}}$ *is related to the position of its corresponding little square with respect to the cross. There are five possible parts of the cross:* A_1, A_2, A_3, A_4 *and* A_5*, which are represented in Figure 6(a), and* $A_6 =$ `OUT_OF_CROSS`*. Then:*

$$f(Q_1) = A_6, f(Q_2) = A_2, f(Q_{10}) = A_5, etc.$$

The values of the training set are in the following table:

Q	$Q(a_1)$	$Q(a_2)$	$Q(a_3)$	$f(Q)$
Q_1	B_3	B_4	B_1	A_6
Q_2	B_3	B_3	$[B_3, B_4]$	A_2
Q_3	B_3	B_3	B_2	A_6
Q_4	B_2	B_2	$[B_3, B_4]$	A_4
Q_5	B_2	B_3	$[B_3, B_4]$	A_2
Q_6	B_2	B_3	$[B_3, B_4]$	A_5
Q_7	B_1	B_4	B_2	A_6
Q_8	B_2	B_4	B_4	A_3
Q_9	B_2	B_2	B_1	A_6
Q_{10}	B_2	B_3	B_4	A_5
Q_{11}	B_1	B_2	B_1	A_6
Q_{12}	B_1	B_3	B_1	A_6
Q_{13}	B_1	B_4	B_1	A_6
Q_{14}	B_3	B_1	B_1	A_4
Q_{15}	B_4	B_4	B_1	A_6
Q_{16}	B_3	B_3	B_4	A_5

In the initial configuration there are only two subsets in consensus with more than one element, see Figure 7(a) (each element is in consensus with itself):

$$\mathcal{C}_1 = \{Q_5, Q_6, Q_{10}\}, \text{ and } \mathcal{C}_2 = \{Q_2, Q_{16}\},$$

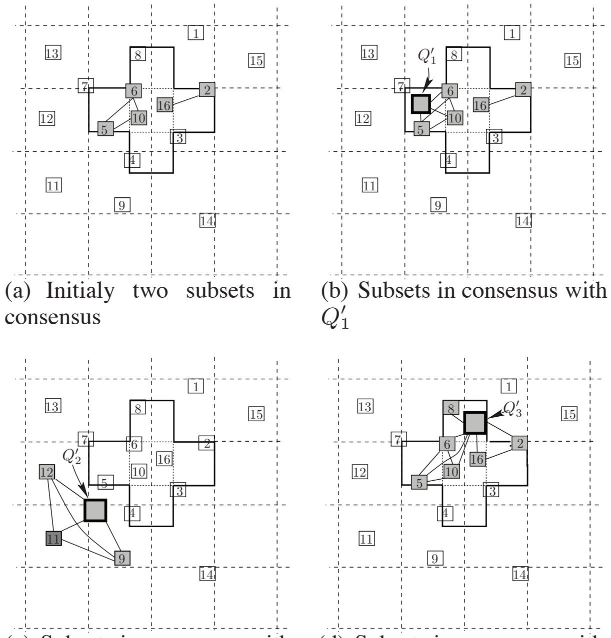
with degrees of consensus $\kappa(\mathcal{C}_1) = \kappa(\mathcal{C}_2) = 0.83$*. Let's suppose that the system receives a new qualitative description (see figure 7(b))* $Q'_1(\Lambda) = (B_2, B_3, B_4)$*. To assign* $f(Q'_1)$ *the algorithm finds the subsets of* $\{Q_1, \ldots, Q_{16}\}$ *that are in consensus with* Q'_1*. If there exists more than one of such subsets, we choose the one with highest consensus. Let's denote this subset as* $\mathcal{C}^*$*. Then it is necessary to find the* $Q^* \in \mathcal{C}^*$ *such that minimizes the distance* $d(Q'_1, Q^*)$ *(see expression 5). Then* $f(Q'_1) = f(Q^*)$*. In this case* Q'_1 *only is in consensus with the set* $\mathcal{C}_1$*, and* $Q^* = Q_{10}$ *so* $f(Q'_1) = A_5$*.*

If the system receives $Q'_2(\Lambda) = (B_2, B_3, B_1)$*, (see figure 7(b)) there is no subset in consensus with* Q'_2*, then it is necessary to start the automatic negotiation process (Roselló et al. 2010):*

$\phi \circ Q$	$\phi \circ Q(a_1)$	$\phi \circ Q(a_2)$	$\phi \circ Q(a_3)$
$\phi \circ Q_1$	$[B'_3, B'_4]$	$[B'_4, B'_5]$	$[B'_1, B'_2]$
$\phi \circ Q_2$	$[B'_3, B'_4]$	$[B'_3, B'_4]$	$[B'_3, B_5]$
$\phi \circ Q_3$	$[B'_3, B'_4]$	$[B'_3, B'_4]$	$[B'_2, B'_3]$
$\phi \circ Q_4$	$[B'_2, B'_3]$	$[B'_2, B'_3]$	$[B'_3, B'_5]$
$\phi \circ Q_5$	$[B'_2, B'_3]$	$[B'_3, B'_4]$	$[B'_3, B'_5]$
$\phi \circ Q_6$	$[B'_2, B'_3]$	$[B'_3, B'_4]$	$[B'_3, B'_4]$
$\phi \circ Q_7$	$[B'_1, B'_2]$	$[B'_4, B'_5]$	$[B'_2, B'_3]$
$\phi \circ Q_8$	$[B'_2, B'_3]$	$[B'_4, B'_5]$	$[B'_4, B'_5]$
$\phi \circ Q_9$	$[B'_2, B'_3]$	$[B'_2, B'_3]$	$[B'_1, B'_2]$
$\phi \circ Q_{10}$	$[B'_2, B'_3]$	$[B'_3, B'_4]$	$[B'_4, B'_5]$
$\phi \circ Q_{11}$	$[B'_1, B'_2]$	$[B'_2, B'_3]$	$[B'_1, B'_2]$
$\phi \circ Q_{12}$	$[B'_1, B'_2]$	$[B'_3, B'_4]$	$[B'_1, B'_2]$
$\phi \circ Q_{13}$	$[B'_1, B'_2]$	$[B'_4, B'_5]$	$[B'_1, B'_2]$
$\phi \circ Q_{14}$	$[B'_3, B'_4]$	$[B'_1, B'_2]$	$[B'_1, B'_2]$
$\phi \circ Q_{15}$	$[B'_4, B'_5]$	$[B'_4, B'_5]$	$[B'_1, B'_2]$
$\phi \circ Q_{16}$	$[B'_3, B'_4]$	$[B'_3, B'_4]$	$[B'_4, B'_5]$
$\phi \circ Q'_2$	$[B'_2, B'_3]$	$[B'_3, B'_4]$	$[B'_1, B'_2]$

It can be checked that the only subset where $\phi \circ Q'_2$ *is in consensus is* $\mathcal{C}^* = \{\phi \circ Q_9, \phi \circ Q_{11}, \phi \circ Q_{12}, \phi \circ Q'_2\}$*. Since* $Q^* = \phi \circ Q_9$*, then* $f(Q'_2) = A_6$*.*

The last case presented is a qualitative description Q'_3 *(see figure 7(d)) such that* $Q'_3(\Lambda) = (B_3, B_4, B_4)$*. Because* Q'_3 *is not in consensus with any subset, it is necessary the*



(a) Initialy two subsets in consensus

(b) Subsets in consensus with Q_1'

(c) Subsets in consensus with Q_2'

(d) Subsets in consensus with Q_3'

Figure 7: Subsets in consensus

automatic negotiation:

$$(\phi \circ Q_3')(\Lambda) = ([B_3', B_4'], [B_4', B_5'], [B_4', B_5']).$$

It can be checked that there are three subsets where $\phi \circ Q_3'$ *is in consensus:*

$\mathcal{C}_1 = \{\phi \circ Q_3', \phi \circ Q_2, \phi \circ Q_{16}\}$,
$\mathcal{C}_2 = \{\phi \circ Q_3', \phi \circ Q_5, \phi \circ Q_6, \phi \circ Q_{10}\}$,
$\mathcal{C}_3 = \{\phi \circ Q_3', \phi \circ Q_8\}$.

The consensus degrees are: $\kappa(\mathcal{C}_1) = 0.53$, $\kappa(\mathcal{C}_2) = 0.48$, $\kappa(\mathcal{C}_3) = 0.67$, *so now* $\mathcal{C}^* = \mathcal{C}_3$ *and* $Q^* = \phi \circ Q_8$, *then* $f(Q_3') = f(Q_8) = A_3$. *This example shows that the methodology presented assigns to each considered qualitative description its correct position with respect to the cross.*

Conclusions and Future Research

The paper presents on-going work, which provides a new strategy for recommendation systems. The RS presented takes into account the lack of precision of users' opinions. The proposed system, different from existing RSs, is based on the concept of entropy and allows the recommendation to be derived from the nearest neighbor within a group that is in consensus. The previous search of groups being in consensus with the user makes it easier to and with lower cost of calculating a minimum distance. Future work will be focussed in two directions. On the one hand, defining a simulated framework where evaluating the given algorithm and comparing it with existing recommender systems. On the other hand, the development of an automatic system to perform the recommendation process described will be implemented for validation in the perfume retailing industry.

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