

AN EVALUATION OF A SIMPLE DYNAMICAL MODEL FOR IMPACTS BETWEEN RIGID OBJECTS

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Abstract – The main purpose of this work is to propose a dynamical model for simulating the response of different metallic objects when impacted by another rigid body. In addition, a methodology for estimating the model parameters is presented and discussed. Results from real experiments shows that by assuming certain characteristics on impacting objects, the dynamic model can reproduce the transient dynamics during contact time.

Keywords: Impacts, dynamic model, parameters estimation

1. INTRODUCTION

There are different works among literature that consider the study of collisions between bodies and propose different kind of models for analysing impacts [1-8]. Most of these investigations concentrate on estimating the post-impact conditions of the objects, or monitoring composite materials, rather than analyzing the behaviour of objects during the impact period. Breckenridge [1] presents an experimental study of different transient sources and their characteristic responses. Wildes [2] propose a dynamical model to study the characteristics of materials by “hearing” the sound produced by an impact, but does not consider the initial transient, and concentrates only on the steady state response. On [3], an interesting study on the dynamics of a one-degree-of-freedom system subjected to an impact is presented, focused from the point of view of control, for compensating the forces released during collision. In another kind of works, [4]-[5] present a different approach, they use collisions for evaluating the “health-state” of composite materials based on the “coin-tap” test method. [6]-[7] are interesting references for modelling and studying the effects of impacts on structures.

The motivation for developing this work arise from a previous investigation [9] where the authors presented the design of an inverse filter based on Neural Networks and a transient excitations. On that work, a model of impacts dynamics was required for training the network, and its improvement was left as a future development.

In this paper, we propose a mechanical model that describes the dynamics of two rigid objects during a collision. On Section 2, we begin by describing the model, proposing a methodology for measuring the parameters, and explaining the characteristics of the impacting device that

has been used for the experiments. On Section 3, we provide details of the experimental setup, and on final sections we present results and conclusions.

2. MODEL DESCRIPTION AND METHODOLOGY

The mechanical model we are proposing describes the collision between two rigid bodies, as shown in Fig. 1, where object 1 is a sensorised hammer that acts as the “impactor”, and object 2 is the “sample” that receives the impact, which for experimental purposes consist on a set of cylinders of different sizes and materials.

During impact, there is a small period of time, τ_p , where both objects are in contact and some amount of energy transfers from one body to the other. Modeling the dynamics of the “sample” during this period is not easy, mainly because the collision depends largely on three factors: the geometrical characteristics of the objects; the type of impact (direct impact, lateral, etc); and the velocities involved during the collision [3]. In order to simplify the statement of the model and equations, some assumptions are considered:

- Collisions are between very hard materials that have regular geometries, as cylinders.
- Objects will have a quasistatic behavior during impact: this means that all stress releases instantaneously to both objects.
- Velocities and forces during impact are low enough that they will produce an elastic impact: plastic deformations are negligible and kinetic energy is conserved.
- The impact develops in line with the center of mass of each body, and the movement of each object can be described by the displacement of their centre of mass.

These assumptions permit considering each object as a spring-mass-damper system, as shown in Fig. 2, where it is also possible to observe that only during impact both objects are in contact and act as a second-order spring-mass-damper system, while prior and after impact they behave as individual first-order single-degree-of-freedom systems. Equations (1) and (2) describe this process, where the first one illustrate the collision during contact time, τ_p , and the second specify the dynamics of the system once objects are no longer in contact. Initial conditions at $t = 0$, (3), consider that object 2 is at rest, while object 1 swings an angle $\Delta\theta$, during a time Δt , where, ω is the average angular-velocity of

the impactor, and L is the length of the supporting holder, as shown in Fig. 1.

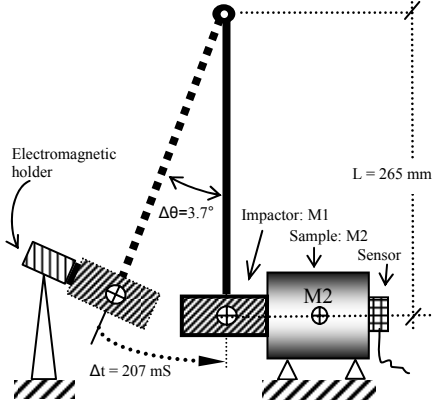


Fig. 1. Impacting device.

$$\left. \begin{aligned} m_1 \ddot{x}_1 &= -c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) \\ m_2 \ddot{x}_2 &= -F(t) + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) - c_2\dot{x}_2 - k_2x_2 \end{aligned} \right\} 0 \leq t \leq \tau_p \quad (1)$$

$$\left. \begin{aligned} m_1 \ddot{x}_1 &= -c_1\dot{x}_1 - k_1x_1 \\ m_2 \ddot{x}_2 &= -c_2\dot{x}_2 - k_2x_2 \end{aligned} \right\} t > \tau_p \quad (2)$$

$$\left. \begin{aligned} x_{1i} &= 0, \quad v_{1i} = \omega L \\ x_{2i} &= 0, \quad v_{2i} = 0 \end{aligned} \right\} t = 0 \quad (3)$$

Parameters, $c_{1,2}$, and, $k_{1,2}$, are the damping and stiffness of each body respectively, and are materials constants that depend directly on the type of material and geometrical properties of the impactor and the sample [7], while $m_{1,2}$ are the masses of each body.

An important variable in (1) is the impacting force, $F(t)$. Because impacts are transients that release a large amount of energy in a short period of time, this force is modeled as a single pulse of very short duration, that last only the time that both objects are in contact, τ_p . The duration and the shape of this pulse depends on the type of deformations produced by the collision, which are strongly relate to the speed and force level during impact, the type of materials

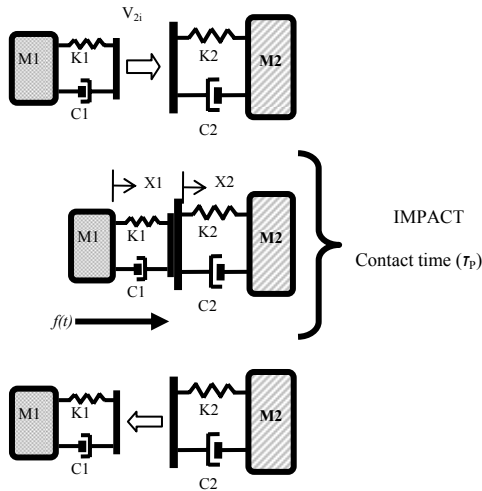


Fig. 2. Mechanical model of impact process.

(soft, hard), the geometry of bodies, and the location of the impact [8]. In order to describe the force $F(t)$, the impact has been divided in two parts, being the first the compression stage and the second the restitution stage; each having a characteristic shape that depends on how materials react during impact. In the case of an elastic collision, materials respond as linear springs, compressing and recovering their initial shape without deformations, with a symmetric pulse, similar to a versed-sine pulse [7], as in (4).

$$f(t) = \frac{F_0}{2} \left(1 - \cos \frac{2\pi t}{\tau_p}\right) = F_0 \sin^2 \left(\frac{\pi t}{\tau_p}\right), \quad 0 \leq t \leq \tau_p \quad (4)$$

$$f(t) = 0, \quad t > \tau_p$$

The period τ_p is defined in (5), where m_{eq} and k_{eq} are the relative mass [3] (6a), and relative stiffness [7] (6b), of the second-order system.

$$\tau_p = \pi \sqrt{\frac{m_{eq}}{k_{eq}}} \quad (5)$$

$$m_{eq} = \frac{m_1 \cdot m_2}{m_1 + m_2}, \quad k_{eq} = \frac{k_1 \cdot k_2}{k_1 + k_2} \quad (6a, b)$$

2.1 Proposed methodology for measuring model parameters

In order to calculate the parameters of the model proposed in (1) and (2), and to compare it with real data, the impacting device shown in Fig.1 was built and tested using 12 cylindrical samples of different materials and dimensions. The device consists on a sensorized impact hammer placed as a pendulum and embraced in a way that movement is possible only in one direction. The hammer is the ‘‘impactor’’, and the end that gets in contact with the samples has an aluminium tip located inline with the center of mass of the hammer and the samples. The hammer starts its swing from an angle $\Delta\theta$ from the vertical; to assure that the angle and the impacting velocities are constant on all the experiments, an electromagnetic holder supports the hammer on a fixed position, as shown in Fig. 1. Samples are located over hard foam, in a position that allows a direct impact to one of the flat surfaces of the cylindrical samples. To measure the response of the samples, another sensor, a piezoelectric accelerometer, is attached at the centre of the other flat surface, inline with the impact.

A. Measuring the pulse-duration: τ_p

From the point of view of an elastic collision and from (5), it is clear that the pulse-duration is a parameter related to the mass and the stiffness of the impacting bodies. This means that for every collision, between the impactor and each cylindrical sample, there will be a pulse of a particular length. In order to measure these pulses, 12 experiments are carried out (one for every sample): each sample is impacted a certain number of times with the hammer, and then the hammer and samples mean time-responses are calculated.

The measurement of the pulse-duration, τ_p , can be a difficult task because it is not easy to locate the actual starting and ending points on these types of pulses. As an alternative, it is convenient to use the hammers’ response as the reference signal, due its resemblance to a versed-sine like in (4), and to measure its average pulse-width, τ_{av} , by

simply measuring the pulse width at the half-amplitude of the mean time-response, and then calculate τ_p applying: $\tau_p = 2 \tau_{av}$ [7].

B. Measuring stiffness k_1 and k_2

The next step is to calculate the parameter k_1 (hammer's stiffness) by means of a "control" experiment. This requires that both bodies, the impactor and the sample, have similar characteristics: as to be made from the same material and to have similar dimensions and masses, $m_{Hammer} \approx m_{Sample}$. Considering this, we can assume that both objects have similar stiffness, $k = k_1 \approx k_2$, and from (6b) it is easy to observe that in this case, $k = 2 k_{eq}$. Once k_1 has been determined, the value of k_2 for each cylinder is calculated applying (5) and (6a-b).

C. Measuring damping constants c_1 and c_2

The damping is a parameter that has a larger effect on the system response once the impact excitation has finished and the system enters in a steady state. It is related to the amplitude decay of the oscillating response of the system. Its measurement is not always easy, particularly when damping ratio is equal or larger than 1, $\xi \geq 1$. For cases where the system is under-damped, $\xi \leq 1$, the ratio ξ can be estimated by measuring the exponential decay of the impulse response of the system, using (7a) and (7b), where U_n and U_{n+1} are consecutive peak values of the oscillating signal.

$$\Delta = \ln \frac{U_n}{U_{n+1}}, \quad \xi = \frac{\Delta}{\sqrt{4\pi^2 - \Delta^2}} \quad (7a, b)$$

$$\xi = \frac{1}{2} \frac{c}{\sqrt{km}} \quad (8)$$

We propose to measure the exponential decay, Δ , using U_1 as the maximum peak-acceleration produced by the impact, and use U_2 as the next peak found in the steady state oscillation. The relation between ξ and c is described in (8), where k and m are the particular parameters for each sample.

3. EXPERIMENTAL SET-UP

The 12 samples used for testing the model are cylinders made from four different materials: aluminium, steel, steel,

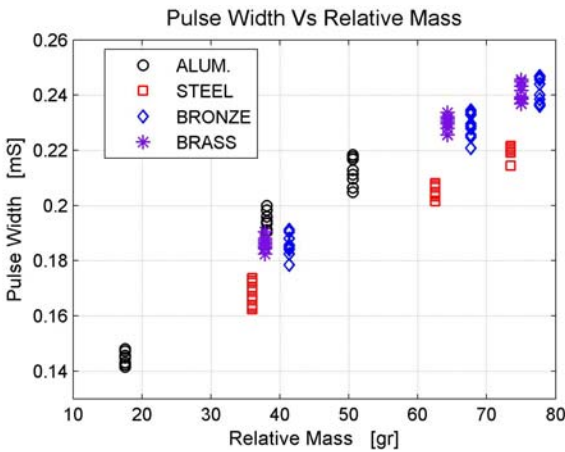


Fig. 3. Pulse-width Vs Relative mass.

bronze, and brass; in three sizes: all cylinders have a diameter of, $d_c = 30 \text{ mm}$, and the three lengths are: $L_1 = 10 \text{ mm}$, $L_2 = 30 \text{ mm}$, and $L_3 = 50 \text{ mm}$, designated as the small, medium and large samples, respectively. Table 1, shows the mass value of all samples. Each cylinder has in one of their flat surfaces a small holder for supporting a piezoelectric accelerometer (8309 from Brüel & Kjær) for recording the impacting time-response. Also, the impacting hammer (8206-002 from Brüel & Kjær) has an internal accelerometer that is used for measuring its time-response. Signals from both sensors were conditioned and then acquired with a 12-bit, National Instruments NI-PCI-6115, digitizer board. The sampling rate has been set to 2.5 MHz, and the recording time to 2 ms. Signals were filtered at 50 kHz, digitally, to reduce noise and high frequency components.

For the experiments, each sample was located in horizontal position on hard foam, as shown in Fig. 1. Using the electromagnetic holder for the hammer, each swing was reproduced under similar conditions. The hammer swings an angle of $\Delta\theta = 3.7^\circ$, in, $\Delta t = 207 \text{ ms}$, which gives an angular velocity of $\omega = 0.31 \text{ Rad/s}$. The length of the hammer, from the centre of mass to the supporting holder, is $L = 265 \text{ mm}$, which leads to a linear velocity of $v_{li} = 8.19 \text{ cm/s}$. This speed is constant for all impacts.

Table 1.

Material	Samples weight in [gr] Without accelerometer*		
	Small	Medium	Large
Aluminium	21.31	61.62	102.38
Steel	56.32	166.91	277.10
Bronze	70.48	209.53	348.13
Brass	60.86	180.14	299.60

Hammer effective seismic mass: $m_1 = 100 \text{ gr}$

*Accelerometer (8309) mass: 3 gr

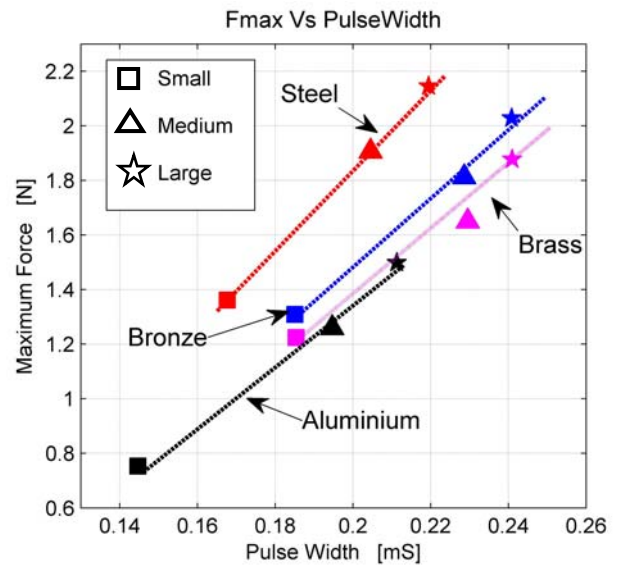


Fig. 4. Maximum amplitude Vs Pulse-width. Average values. Samples sizes: small (squares), medium (triangles), large (stars).

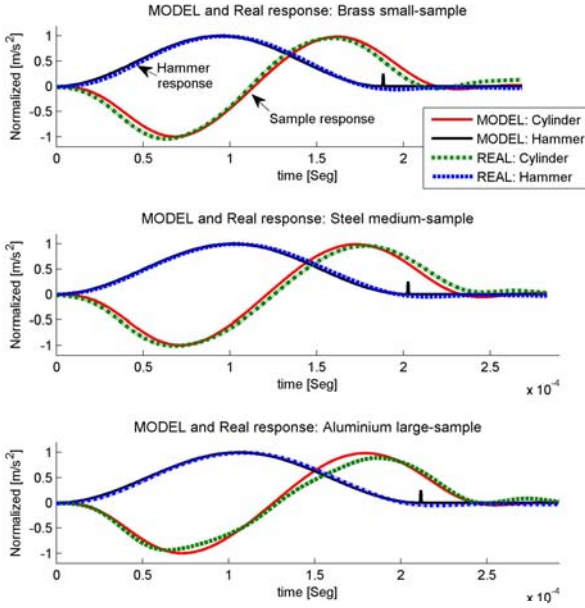


Fig. 5. Model and real cylinders responses.

The model described in (1) and (2), and all the equations used for calculating the parameters, were implemented using MATLAB.

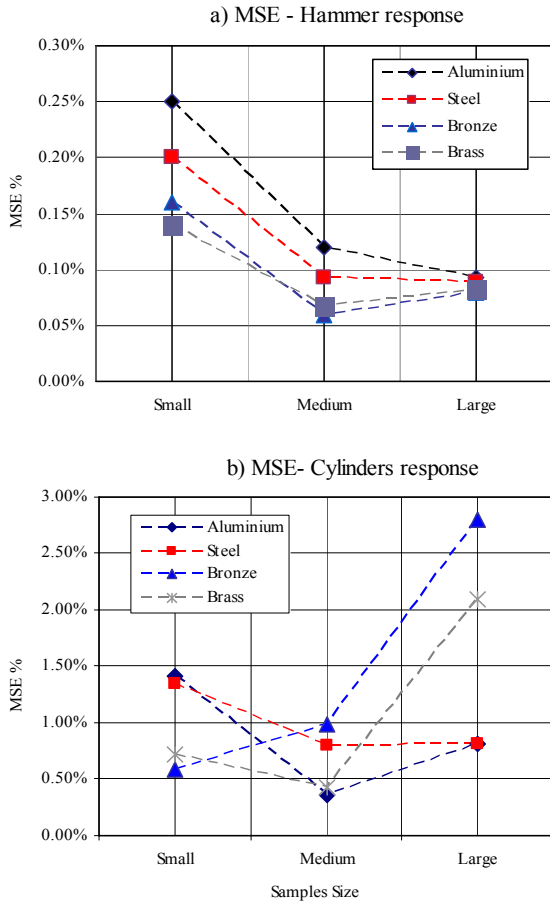


Fig. 6. Mean Square Error (MSE) between model and real data: a) for hammer response, b) for cylinders responses.

To measure the hammer stiffness k_1 , the large aluminium cylinder has been selected as the reference sample for the control experiment. From Table 1, it is possible to observe that both masses are similar, $m_{Hammer} \approx m_{L-alum}$; and that hammer's tip and cylinder are made from the same type of material (aluminium), and both have similar dimensions.

4. RESULTS

Model parameters were estimated following the procedure described in Section 2 and 3. For every sample, 10 impacts were produced and average time-responses were calculated for the hammer and the cylinders. At the end of the experiments, a group of 24 mean time-responses are available for analysis (12 from the hammer, and 12 from the cylindrical samples).

For measuring the pulse-duration, the average signals of the hammer were used, and results are shown in Table 2. As it can be observed, there is a relationship between the pulse-width and the mass of the samples. For every material, small samples have the shortest pulse-duration. This relation can be appreciated as well in Fig. 3, which shows the plot “pulse-width Vs relative-mass”, where impacts from all experiments are shown. It is possible to appreciate the variability of the pulse-width among different experiments considering individual samples. Also, it is clear that there is a relationship among pulse-duration, type of material and the relative mass.

Table 2.

Material	Samples pulse-duration, τ_p [μ S]		
	Small	Medium	Large
Aluminium	144.80	196.80	212.00
Steel	170.00	203.20	219.20
Bronze	188.00	226.40	239.20
Brass	189.00	288.10	244.00

Table 3 shows hammer and samples stiffness parameter values. Here, it is also possible to appreciate that stiffness changes according to the size of the samples. Another interesting plot is the “maximum amplitude Vs pulse-width”, shown in Fig. 4, where average values are shown, and again, it is possible to appreciate the relation between the acceleration level, the mass of the samples and the pulse-width.

Table 3.

Material	Samples Stiffness k_2 , [MN/m]		
	Small	Medium	Large
Aluminium	15.56	17.99	22.54
Steel	29.18	45.29	46.11
Bronze	24.89	31.26	33.22
Brass	20.71	26.99	27.87

Hammer stiffness: $k_1 = 22.54$ [MN/m]

Table 4 shows the values of the damping constants, and hammer estimated damping value. These were obtained using the logarithmic decay method assuming that all sample have and under-damped response.

Fig. 5 shows a comparison between the model simulated response and some real responses. For clarity, only three

cases are shown, those corresponding to the small brass-cylinder (*Top*), the medium steel-cylinder (*Middle*), and the large aluminium-cylinder (*Bottom*). Signals from the model and real experiments were normalized and shifted in time for comparison purposes. Solid lines correspond to the model response and the dashed lines are the real responses.

Table 4.

Material	Samples damping: c_2 , [N s/m]		
	Small	Medium	Large
Aluminium	206	531	802
Steel	483	1170	1670
Bronze	848	2028	2423
Brass	667	1070	2768

Hammer: $c_1 = 1909$ [N s/m]

The mean square error (MSE) was calculated to compare these results. Fig. 6a show the MSE, in percentage, between simulated and real hammer response, for every sample, and Fig. 6b shows the MSE between the simulated and the real cylinders response. Fig. 6a shows that the proposed versed-sine pulse describes properly the hammer's response, where the maximum errors correspond to the small samples, probably due to asymmetries in the pulse. In all cases, $MSE < 0.25\%$, which is a reasonable good result considering that the majority have similar MSE. Fig. 6b shows that the errors related to cylinders responses are a bit larger, being the worst ($\sim 2.8\%$) for the large bronze cylinder. All other errors are below 2%, which is also a reasonable good result, considering as well that the majority have similar MSEs.

4. CONCLUSIONS

This paper proposes a method for modelling the transient of an impact between two rigid bodies and for estimating the parameters of this model. An experimental impacting device has been designed for testing 12 different samples, which are cylinders made of aluminium, steel, bronze and brass; in three sizes, which allows testing the procedures with samples of different characteristics. To produce repeatable impacts, a electromagnetic holder was used to release an impacting hammer always from the same position.

Contact time, stiffness and damping parameters of each sample were calculated using the proposed method. These values were used in the proposed model to generate simulated results. The mean square error between the simulated responses and the real data shows that the model has a reasonably good behaviour during the transient of an impact. The MSE is less than 5% for samples simulated response, and less than 0.25% for the hammer simulated response.

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