MIMO Channel Metrics

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Abstract*— *Space diversity processing when suitable either at the transmitter side or at the receiver side provides a reduction on the transmitted power for the same received signal quality. The paper discusses the importance of the channel entropy in order to design, derived directly from the channel sounder, first, a system able to decide whether it is worth that the transmitter knows the channel or not (CSIT or Channel State Information at the Transmitter). Second, a channel metric is proposed in order to classify the channel regardless of the Tx-Rx space processing to be used. Among other aspects, this channel classifier would help in designing a vector quantifier to feed-back the channel state information to the transmitter when CSIT is required.

I. INTRODUCTION

Transmitting and receiving data through multiple channels remains, in many senses, as an open problem from the signal processing point of view, and a risky investment from the technology or the engineering side. Regardless of acoustic or electromagnetic propagation and no matter the existing differences between space diversity and traditional diversities like frequency time and code, the multi-channel scenario is difficult to characterize in a single label, such that it tell us up to what degree it is a good or bad channel [1]. This label has to be as independent as possible of the processing strategies at the transmitting and at the receiving sites. The existence of this label is crucial to cope with proper classification of multi-channel scenarios and to fully develop concepts like software defined radio and adaptive modulation.

The purpose of this paper is to extend the work of [2] by studying more in detail the channel capacity of a Rayleigh fading MIMO channel and how it may help to make decisions on different transmission and reception strategies. In the taxonomy of the signal processing techniques that are of interest for transmitting information through a multi-channel scenario we consider: i) Use or not of CSIT (channel state information at the transmitter); ii) Single user or multi-user link; iii) BER (Bit Error Rate) or Rate criteria; iv) Long time range or short time range. Next, Section II provides a summary of signal processing strategies arranged in terms of this taxonomy. Special attention is given to the rate criterion and Section III finds out channel features with direct impact on the achievable rate or capacity. The aim is to build a channel sounder that would help to take decisions concerning the use or not of CSIT. Finally, Section V reports distance criteria which provide a formal support to the choice made for the goodness label and help to classify the MIMO channel. Among others, this classification is a very useful tool when the receiver has to report CSI to the transmitter.

Since all signal processing strategies rely on the multi-channel matrix $\mathbf{H}$ and on the noise covariance matrix $\mathbf{R}_0$, the performance depends only on matrix $\mathbf{R_H}$, as it is defined in Eq. 1, the paper concentrates on finding which attribute of this matrix labels it better.

$$\mathbf{R}_{\mathrm{H}} = \mathbf{H}^{H}\mathbf{R}_{\mathrm{o}}^{-1}\mathbf{H} \qquad (1)$$

II. BER AND MEAN SQUARE ERROR (MSE) STRATEGIES

Space diversity processing when suitable either at the transmitter side (Tx) or at the receiver side (Rx) provides a reduction on the transmitted power for the same received signal quality. When used simultaneously at both sides the multi-channel system provides significant gains in capacity and quality (i.e. BER and MSE). The problem with this radio multi-channel is that, in order to obtain its best, both the transmitter and the receiver require information about the so-called MIMO channel, formed as the set of physical radio channels provided by multiple antennas deployed at Tx and Rx. Due to the fast time varying behaviour of the channel, the channel state information at Tx (CSIT) represents a difficult task at the engineering level without a clear revenue in terms of rate and or quality.

Clearly CSI strategy, also present in single channel, adds complexity when determining the global quality of a multi-channel scenario since it may seriously impact on the achievable performance. Furthermore, depending on the major objective in the communication systems design, either capacity or BER, the impact of CSI may be different also. In addition, short time range and long (average) time range objective will exhibit, in general, different performance.

Finally, when passing from the traditional mode, namely point to point though the multi-channel (i.e. single symbol transmission through MIMO wireless) to the distributed mode, namely multiple streams belonging to different information streams, a new set of problems open up due to the mismatch between global quality criteria and distributed quality criteria. In other words, it may be that the multi-channel scenario is a good channel for a single user, but it behaves as a bad channel for a multi-user link. Note that for distributed quality criteria the minimum mean square error per stream MSE(q), index q denotes the information stream, does not indicates the actual SINR, neither the BER, unless the MSE matrix is diagonal. Furthermore, in the distributed quality case, per stream MSE, SINR and BER do not get profit of the channel on the same manner; in consequence, again the concept of good and bad channel may become quite difficult and fuzzy.

In the quality study presented next, we distinguish between single stream and multiple stream strategies. The aim is to summarize those results presented in [2] that reinforce the use of the geometric mean of the channel eigenvalues not only for the rate driven systems presented in Section III but also for the general MIMO channel classifier of Section IV.

A. 2.1. Single Stream Strategies

Also called delayed decision architectures, single stream strategies transmit one symbol per channel access. When *instantaneous CSIT* is available, maximizing the instantaneous BER with the constraint of global energy at transmission arises to a performance that depends on:

- $l_{\max}$ in the case of CSIT
- l_{arith} in the case of no CSIT, where

$$l_{\text{arith}} = \frac{1}{\mathrm{n_o}} \sum_{q=1}^{\mathrm{n_o}} l_q \qquad (2)$$

where $\mathrm{n_o} = \min\left(\mathrm{n_T}, \mathrm{n_R}\right)$.

We observe that both performance will be the same when all the eigenvalues are the same; that is, when $l_{\text{geom}} \approx l_{\text{arith}}$, with

$$l_{\text{geom}} = \prod_{i=1}^{\mathrm{n_o}} l_i^{1/\mathrm{n_o}} \qquad (3)$$

where the logarithm of the geometric mean is the entropy of the channel. Note that entropy for constrained energy reveals the flatness of the eigenvalues set as well as how big is their spread. Therefore, for a given arithmetic mean, the less is the entropy the better is the channel for increasing performance from the availability of CSIT.

B. Multiple Stream Strategies

When *instantaneous CSIT* is available, Singular Value Decomposition techniques preserve optimality, regardless of the different signal processing techniques that can be applied at transmission and at reception. Without loss of generality it can also be considered that the number of streams to be transmitted is equal to the minimum size of antennas at the transmitter or receiver, otherwise a unitary transform is required previous to the power allocation stage. In [2] it is shown that:

- High $?_{\text{geom,}}$ that is flatness or high entropy, implies good channel in fair strategies (Tx).
- Low $?_{\text{geom}}$, or low entropy, implies that the channel is suitable for low fairness strategies (Tx) in multiple streams.

All the previous derivations have been done for the instantaneous multi-channel, if average BER is considered then *average* or *statistical CSIT* is required. In [2] and references therein, it is shown that:

- The geometric mean of the eigenvalues of covariance matrices provides also the basic gain for no CSIT.
- The gain for CSIT depends basically on the harmonic mean of the activated eigenmodes.

In conclusion, maximum entropy is the relevant feature for non CSIT scenarios. Also maximum entropy increases the goodness on fairness criteria for the multiplexing case. Just CSIT scenarios with low fairness objectives, or single stream, benefit of the minimum entropy of the selected eigen-channels.

It is worthwhile to remark that the gain due to CSIT implies that some modes are not activated, due to an inherent water-filling to the optimization procedure and that the number of elements in the CSIT gain summation can be hardly reduced when low quality receiving antennas are used. The same comment applies for the geometric mean that is severely reduced when low values are considered.

III. CAPACITY

To obtain capacity, given the multi-channel matrix, it is also necessary that the Tx/Rx perform SVD of the channel matrix, reducing the problem to the proper power allocation.

$$\max_{\mathbf{Q}, Tr(\mathbf{Q}) = \mathrm{E_T}} \mathrm{Ln}\left|\mathbf{I} + \mathbf{Q}\mathbf{R}_{\mathrm{H}}\right| = \left|\begin{matrix}\mathbf{R}_{\mathrm{H}} = \mathbf{ULU}^{H} \\ \mathbf{Q} = \mathbf{UZU}^{H}\end{matrix}\right| \qquad (4)$$
$$= \max_{z_i, \sum_i z_i = \mathrm{E_T}} \sum \mathrm{Ln}\left(1 + z_i l_i\right)$$

The optimum power allocation is traditional water-filling,

$$z_i = \left[m - \frac{1}{l_i} \right]^+ , \quad \text{with} \quad \sum_{i=1}^{n_0} z_i = E_T \tag{5}$$

The capacity expression is then

$$C_{CSIT} = k \log\left[l_G \left(\frac{E_T}{k} + l_H^{-1} \right) \right] \tag{6}$$

where

$$l_G = \prod_{i=1}^{k} l_i^{1/k} \qquad l_H^{-1} = \frac{1}{k}\sum_{i=1}^{k} \frac{1}{l_i} \tag{7}$$

where k is the number of activated channel modes in the waterfilling. Note that in fact k, the geometric mean and the harmonic mean depend on E_T, thus, $k(E_T)$, ?$_G(E_T)$, ?$_H(E_T)$,. In order to simplify the formulation we have dropped out this dependence. When no CSIT is available, uniform power allocation or UPA in Eq. 8 is the best choice at the transmitter

$$C_{UPA} = \sum_{i=1}^{n_0} \log\left(1 + \frac{E_T}{n_T} l_i \right) \tag{8}$$

By inspecting Eq. 6 and Eq. 8, note that for the high SNR (Signal to Noise Ratio) regime, both capacities are proportional to the logarithm of the geometric mean of the channel, which is the entropy of the channel (i.e. all the eigenvalues or channel modes are taken into account). Therefore, a good channel is that with maximum entropy and, as we have concluded after reviewing different quality schemes in the previous section, maximum entropy is the relevant feature for non CSIT scenarios. However, for a given ?$_{\text{arith}}$, as ?$_{\text{geom}}$ decreases, then CSIT should be considered. In other words, concerning the relevance of CSIT for capacity, improvement increases when the entropy decreases, that is, when the directivity of the impinging energy increases.

Next, in order to design a system able to decide on transmitting with or without CSIT we should give more details on the SNR range where the entropy is the most representative metric and which are the other interesting metrics by default.

A. *CSIT decision system*

The goal is to build a Decision System that, as shown in Fig. 1, based on 6 inputs should be able to decide on transmitting with or without CSIT.

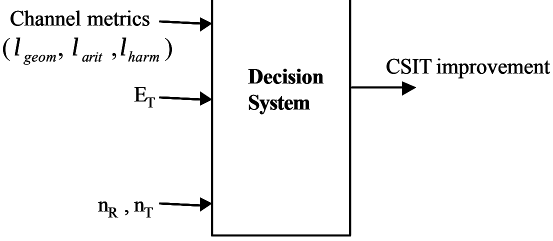

Fig. 1. Decision System

When evaluating the Capacity with UPA in Eq. 6 for extreme SNR values we get

$$C_{UPA}^{\text{low SNR}} = \log\left(1 + \frac{n_o}{n_T} E_T \, l_{arith} \right) \qquad C_{UPA}^{\text{high SNR}} = n_o \log\left(\frac{E_T}{n_T} l_{geom} \right) \tag{9}$$

Without loss of generality we consider MIMO channels with energy normalized to $n_T n_R$,

$$\text{SNR} = \frac{E_T . Tr\left(E\{\mathbf{R}_H\} \right)}{n_T n_R} \tag{10}$$

Therefore, SNR depends not only on the transmitted energy but also on the channel variance. Fig. 2 shows the importance of three different channel metrics depending on the SNR. More specifically, the figure plots the percentage of the Mean Square error between the actual UPA capacity and the one approximated by three different metrics. Note that the plot depends on the number of transmitting and receiving antennas.

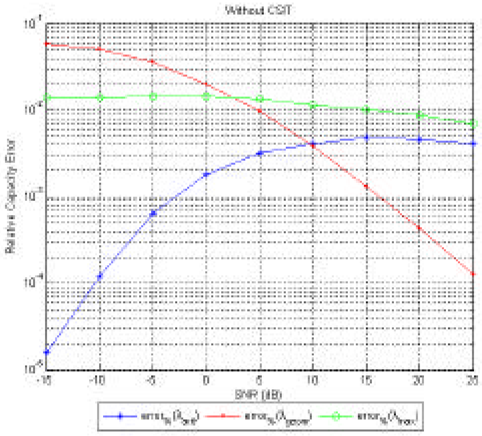

Fig. 2. Relative C_{UPA} error for a 4x4 MIMO system

Therefore, we can represent the importance of the metric in order to sort capacity without CSIT as shown in Fig. 3. Two fuzzy sets have been defined, one where capacity is given by ?$_{\text{arith}}$ and the other where is given by ?$_{\text{geom}}$. The vertical axis is the degree of membership of the SNR to each of the sets. In this way we can incorporate the fact of having a range of SNR where neither ?$_{\text{arith}}$ nor ?$_{\text{geom}}$ uniquely describe the capacity without CSIT. The crossing point between the two sets moves to the right the more antennas we have.

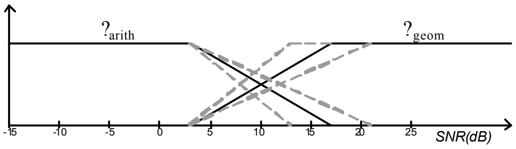

Fig. 3. Fuzzy sets to describe the importance of the metric in order to sort capacity without CSIT (in 2x2, 4x4 and 8x8 case)

When evaluating the *Capacity with CSIT* in Eq. 6 for different SNR values we obtain Fig. 4 for different channel metrics, where all the channel eigenvalues have been taken into account.

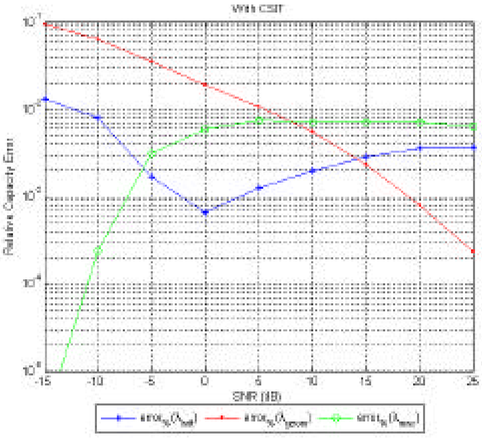

Fig. 4. Relative C_{CSIT} error for a 4x4 MIMO system

As we have done for the UPA case, the importance of the different metrics can be represented as Fig. 5 shows for a 4x4 MIMO system.

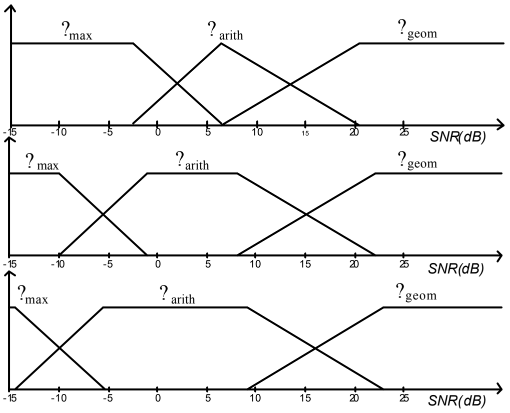

Fig. 5. Fuzzy sets to describe the importance of the metric in order to sort capacity with CSIT (in 2x2, 4x4 and 8x8 case)

Once we know the SNR ranges we can infer the following decision rules in order to know whether CSIT is useful or not for a given number of n_T, n_R. Let us assume a 4x4 MIMO system then two rules can be inferred for the extreme SNR values:

IF SNR>+20 dB THEN no CSIT
IF SNR<-10 dB THEN CSIT

The improvement in the worst SNR regime is

$$\log\left(\frac{1+E_T\, l_{max}}{1+E_T\, l_{arit}}\right) \qquad (11)$$

For the intermediate values of –10dB to +20dB, we observe that different number of eigenmodes are activated by the waterfilling and a CSIT strategy adds complexity to the decision process. Note that the additional input to the system of the number of activated eigenmodes would be desirable. Fig. 6 plots the number of activated eigenmodes for the capacity waterfilling for different values of transmitted energy.

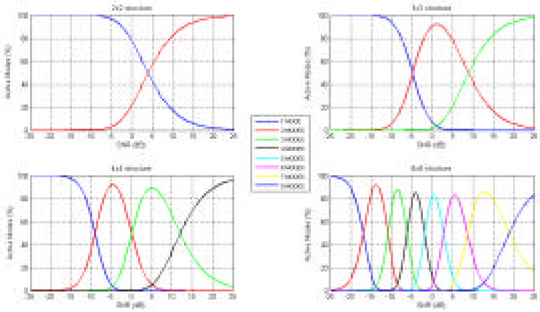

Fig. 6. Number of activated eigenmodes (% of the total simulated channels) of 2,3,4 and 8 antenna structures.

However, the decision process would then depend on the waterfilling strategy. For this reason, we would like to keep on working on the whole set of channel eigenmodes. Fig. 7 shows that in a 4x4 system when SNR>5 dB, $?_{harm}$ gives the improvement in performance by using CSIT (that is Eq.6 in front of Eq.8). Note that its more representative than the condition number, which is, to the authors knowledge, the only metric reported for that purpose so far. In other antenna structures the results also agree. Finally, for the range -10dB to +5dB, nothing can be said based on $?_{harm}$ and the improvement has to be computed directly from Eq. 6 and Eq. 8. The two last rules are finally

IF +5dB<SNR<+20 dB THEN CSIT
(the improvement is sorted by $1/\ ?_{harm}$)
IF -10dB<SNR<+5 dB THEN CSIT
(the improvement is Eq.6/Eq.8)

Due to the high uncertainty within the -10dB to +5 dB range, one possible alternative is to use transmission techniques as in [3] that are robust to these bad quality channels.

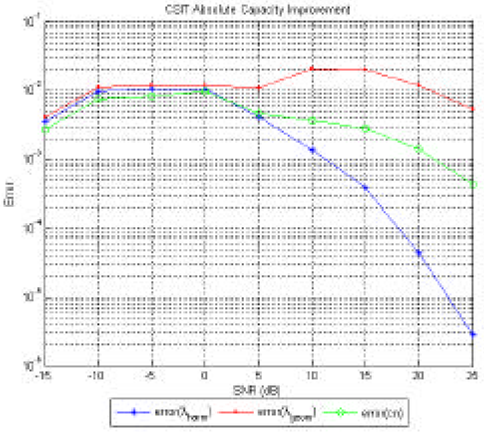

Fig. 7. Error when sorting the capacity improvement following harmonic mean, geometric mean and condition number (4x4 case). A set of 10000 channels has been generated.

Once decided the use of CSIT, a MIMO channel classification would be useful in order to perform a quantization for the feedback. Although the geometric mean has proved to be a representative metric for SNR > 5 dB and that within the same range the gain of CSIT is associated to the harmonic mean, note that in Eq. 6 the gain summation can be hardly reduced when low quality channel modes are used. The same comment applies for the geometric mean that is severely reduced when very low values are considered. To overpass these problems a good measure of the comparative goodness of the channel has to be done in terms of weak logarithm-majorization of the corresponding eigenvalues. Given two multi-channels with matrices R_{H1} and R_{H2} respectively, channel H1 is better than H2 when the eigenvalues of H2 are weakly logarithm-majorized (WLM) by the eigenvalues of H1. In other words, any set of eigenmodes activated in the first channel have more entropy than the alternative channel. Let us say, we compare the fractional entropy of the channel

$$\underline{l}_1 \succ_x^w \underline{l}_2$$
$$\sum_{q=1}^{n} Ln\left(l_{1q}\right) \geq \sum_{q=1}^{n} Ln\left(l_{2q}\right) \quad \forall n = 1,\ldots,n_T \qquad (12)$$

IV. A DISTANCE CRITERIA

In addition to the channel measures used in the previous section, most of the times results quite useful to find out a MIMO channel classifier that could make CSIT feedback easy or that simply could indicate up to what degree two channels are close enough to decide that a given space-time processing is or not suitable for both of them. Take adaptive modulation or link adaptation as an example. We propose a classifier based on the idea of WLM.

The results are successful as the classifier is, as desired, independent of SNR conditions, and also follows the Capacity ordering typologies already described. In high SNR the system works perfectly in accordance to the geometric mean and for low SNR in accordance to the arithmetic mean. The classifier separates channels into clusters, and inside each one the mean value of the metrics are different depending on the cluster. Also the metric variance inside each cluster is very limited. In all the plots only the capacity without CSI is represented but similar ordering is obtained for capacity with CSI.

A. *SNR=30dB, 4x4.*

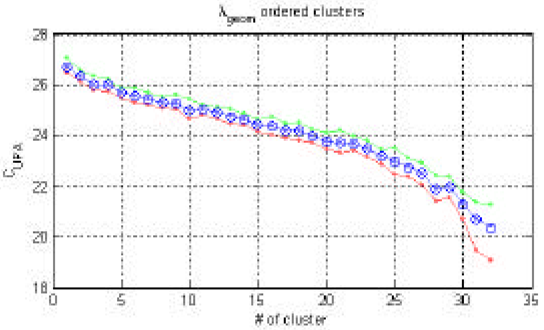

Fig. 8. Capacity value of each cluster. Observe that variance is minimum. 32 clusters, 5000 channels.

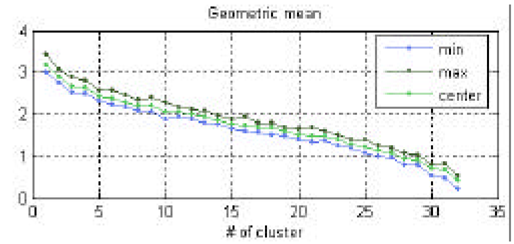

Fig. 9. Capacity value with geometric mean. The grey scale is a gradient between the first cluster (black) and the last one (light grey). 32 clusters, 5000 channels.

B. *SNR=-10dB, 4x4.*

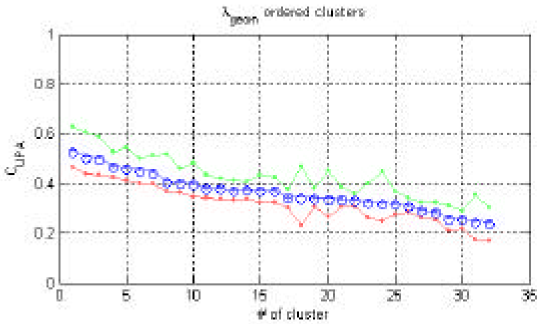

Fig. 10. Capacity value of each cluster. Observe that variance is minimum. 32 clusters, 5000 channels.

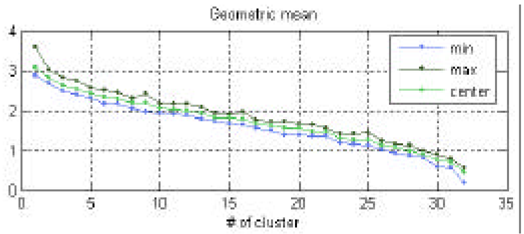

Fig. 11. Capacity value with geometric mean. The grey scale is a gradient between the first cluster (black) and the last one (light grey). 32 clusters, 5000 channels.

C. *Capacity and geometric mean regions*

The proposed clustering is able to group channels with similar geometric mean and capacity, in all SNR ranges. Fig. 12. presents the results for SNR=0dB. Note the way in which the classifier groups channels. Clusters are in grey colour, the darker correspond to the first cluster and the ligher to the last one.

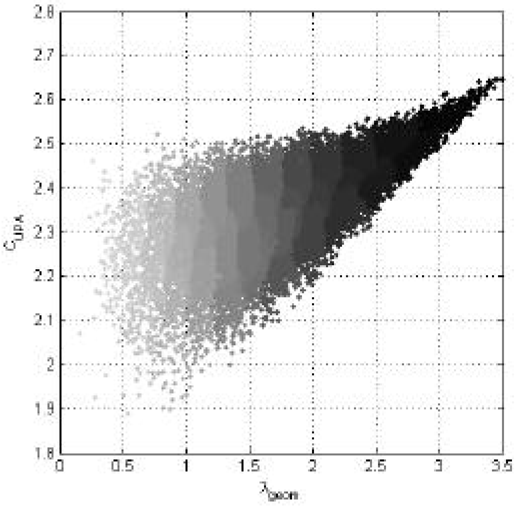

Fig. 12. Capacity versus geometric mean. 32 clusters, 20000 channels.

Similar results have been obtained changing the number of transmitting and receiving antennas.

V. CONCLUSIONS

Independently of the specific architecture and signal processing techniques currently available for MIMO transmission, this paper proposes, first, a decision system on the usefulness of CSIT, second a channel classifier. Further research is carried out in order to validate the results with quality measures (BER and MSE). The promising low intra-cluster deviations of the significant means ensures a robustness of low CSI schemes like adaptive modulation where only the cluster index is sent through the feedback channel. Also opportunistic schemes may benefit of the proposed clustering.

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