

Performance of Coherent Lidar Receivers Using Atmospheric Compensation Techniques

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Abstract

We present recent studies on the impact of phase and amplitude fluctuations on Doppler lidars using coherent detection. As there is an emphasis on elucidating those implications of the atmospheric propagation problem that bear on the design and reliability of optical coherent lidar systems, we consider in a unified framework the effects of atmospheric turbulence and diffuse target speckle on the performance of coherent receivers utilizing atmospheric compensation techniques. As the effects ascribed to turbulence and speckle are random, and subsequently must be described in a statistical sense, we define a mathematical model for the probability density function of the received coherent signal after propagation through the atmosphere. In our model, the parameters describing the signal statistics depend on the turbulence conditions and the degree of modal compensation applied in the receiver. We provide analytical expressions and use them to study the effect of various parameters on performance, including turbulence level, signal strength, receive aperture size, speckle effective area, and the extent of compensation.

1. Introduction

Evaluating the performance of a heterodyne or homodyne lidar receiver in the presence of atmospheric turbulence and target speckle is generally difficult because of the complex ways turbulence and speckle affect the coherence of the received signal that is to be mixed with the local oscillator. The downconverted heterodyne or homodyne power is maximized when the spatial field of the received signal matches that of the local oscillator. Any mismatch of the amplitudes and phases of the two fields will result in a loss in downconverted power. Adaptive compensation of atmospheric wave-front phase distortion to improve the performance of atmospheric systems has been an important field of study for many years. In particular, modal compensation method involves correction of several modes of an expansion of the total phase distortion in a set of basis functions.

Here, we study in a unified framework the effects of both wavefront distortion and amplitude scintillation on the performance of heterodyne receivers utilizing atmospheric compensation techniques. The effects ascribed to turbulence and speckle are random and subsequently must be described in a statistical sense. Early works quantified turbulence-induced fading through its mean and variance [1][2], although these are not adequate to fully characterize system performance. Recently, a full-wave simulation of beam propagation has been used to examine the uncertainty inherent to the process of optical power measurement with a practical heterodyne receiver because of the presence of refractive turbulence [3].

In this study we define a mathematical model for the received signal scattered by the atmospheric target after propagation through the atmosphere. In synchronous optical receivers on free-space optical communication links, by noting that the downconverted signal current affected by turbulence can be characterized as the sum of many contributions from different coherent regions within the aperture, we show that the probability density function (PDF) of this current can be well-approximated by a modified Rice distribution [4]. Here, being concerned with the basic problem of optical heterodyne detection, in this study we need to consider the case of back-scattered light coming from remote atmospheric targets. In remote sensing lidar systems power fluctuations could result from a number of physical mechanisms other than refractive turbulence, mainly speckle [5], so these mechanisms must also be the focus of this analysis. In general, fluctuations induced by turbulence are not as intense as those due to speckle in optical remote sensing systems but, although their normalized variance is smaller, they still need to be considered to properly describe the performance of any practical coherent lidar. This specific situation requires some additional considerations. Now, the speckle process is driven by the turbulence random process and the problem must be analyzed by the application of conditional speckle statistics. There is a compounding of the statistics for the downconverted signal current affected by speckle, where we can regard the speckle current distribution to be a conditional PDF, conditioned on knowledge of the mean value as described by the modified Rice distribution characterizing the current affected by turbulence. We suggest a continuous PDF from the family of K distributions, the

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gamma transform of a non-central chi-squared distribution, to model the smearing due to speckle of the signal-to-noise ratio (SNR) in a heterodyne lidar receiver affected by atmospheric turbulence. In our model, the parameters describing the PDF depend on the turbulence conditions and the degree of modal compensation applied in the receiver. We provide analytical expressions for the statistical moments of coherent lidar signals, and use them to study the effect of various parameters on performance, including turbulence level, signal strength, receive aperture size, speckle effective area, and the extent of compensation. We consider phase compensation and both spatial and temporal (averaging) forms of signal diversity combining.

2. First-order statistics

In a coherent receiver, downconversion from the optical domain to the electrical domain can be achieved using heterodyne methods. In order to assess the impact of turbulence, both log-amplitude and phase fluctuations should be considered. We have already modeled the impact of atmospheric turbulence-induced phase and amplitude fluctuations on free-space optical links using synchronous detection and found that the SNR γ is described by a noncentral chi-square distribution with two degrees of freedom [4]:

$$p_\gamma(\gamma) = \frac{1+r}{\bar{\gamma}} \exp(-r) \exp\left[-\frac{(1+r)\gamma}{\bar{\gamma}}\right] I_0\left[2\sqrt{\frac{(1+r)r\gamma}{\bar{\gamma}}}\right], \quad (1)$$

where the average SNR $\bar{\gamma}$ (detected photocounts) and the parameter $1/r$ consider turbulence effects. In this model, the signal is characterized as the sum of a constant (coherent) term and a random (incoherent) residual halo. The contrast parameter $1/r$ is a measure of the strength of the residual halo to the coherent component. The parameter r ranges between 0 and ∞ . It can be shown that when the constant term is very weak ($r \rightarrow 0$), turbulence fading makes the SNR to become negative-exponential-distributed, just as in a speckle pattern. Likewise, when the dominant term is very strong ($r \rightarrow \infty$), the density function becomes highly peaked around the mean value $\bar{\gamma}$, and there is no fading to be considered. Both $\bar{\gamma}$ and $1/r$ are described in terms of log-normal amplitude fluctuations and Gaussian phase fluctuations as characterized by their respective statistical variances, σ_χ^2 and σ_ϕ^2 ,

$$\sigma_\chi^2 = \log_e(1 + \sigma_\beta^2) \quad \sigma_\phi^2 = C_J \left(\frac{D}{r_0}\right)^{5/3} \quad (2)$$

The intensity variance σ_β^2 is often referred to as the scintillation index [6]. The coefficient C_J depends on the number J of Zernike terms corrected by a receiver employing active modal compensation [7]. Phase-compensated receivers offer the potential for overcoming atmospheric limitations by adaptive tracking of the beam wave-front and consequent correction of atmospherically-induced aberrations. The modal compensation method is a correction of several modes of an expansion of the total phase distortion in a set of basics functions. Here, we have considered a model for a modal compensation system, a hypothetical device whose response functions are components of some expansion basis. Different sets of functions can be used for the expansion although most often they are Zernike polynomials, a set of orthonormal basis modes defined on a unit circle and that are related to the classical Seidel aberrations [8]. The modes are a product of angular functions and radial polynomials when polar coordinates are considered. We will assume that the modal compensation system has infinite spatial resolution in the correction of phase distortions. In Eq. (2), the receiver aperture diameter D is normalized by the wavefront coherence diameter r_0 , which describes the spatial correlation of phase fluctuations in the receiver plane [1].

Normally, for a single data point, the probability density function of the SNR γ affected by speckle can be considered to be a gamma distribution,

$$p_\gamma(\gamma) = \left(\frac{NM}{\bar{\gamma}}\right)^N \frac{\gamma^{N-1}}{\Gamma(N)} \exp\left[-\frac{NM}{\bar{\gamma}}\gamma\right] \quad (3)$$

where we have considered a general case where the signal is not defined by a single speckle pattern, but instead by the sum of N independent speckle patterns as might be generated when several equal-strength independent laser shots are averaged. Here, $\bar{\gamma}_s/M$ is the effective detected photocount, or number of photons detected from a single shot in a correlation area of the speckle field pattern on the aperture plane. The number of spatial speckle modes M depends on the width of a speckle

and the area of integration of the detector. The PDF Eq. (1) modeling the impact of atmospheric turbulence on coherent receivers can be compounded with the target speckle statistics Eq. (3) to obtain the corresponding distribution function for heterodyne lidar signals. The unconditional probability function of the SNR γ is found by averaging the speckle density function with respect to the statistics of the conditional mean SNR affected by turbulence. This density function can be shown to be an example from the family of K distributions.

3. Performance of heterodyne lidars

The moments about the origin of the atmospheric SNR γ can be calculated

$$\overline{\gamma^n} = \int_0^\infty d\gamma \gamma^n p_\gamma(\gamma) = \frac{\Gamma(n+N)}{\Gamma(N)} \frac{\Gamma(n+1)}{(1+r)^n} L_n(-r) \left(\frac{1}{N} \frac{\bar{\gamma}}{M} \right)^n \quad (4)$$

in terms of the simple Laguerre polynomials L_n for the parameter r describing the ratio of the coherent component to the residual halo in the atmospheric coherent signal collected by the receiving aperture. In Eq. (4), Γ is the complete gamma function. Of special interest is the ratio of the SNR standard deviation to its mean describing the uncertainty in the measurement process. Note that, as the heterodyne lidar SNR is the consequence of the compounding of two sources of fluctuations, atmospheric turbulence and target speckle, this magnitude could be greater than unity.

Figures 1 and 2 show the effect of atmospheric turbulence and speckle on the performance of Doppler lidars modal-compensated heterodyne receivers and signal diversity. We study the mean and the ratio of the standard deviation to its mean as a function of several parameters: the average turbulence-free SNR γ_0 per symbol, the receiver aperture diameter D , the number of spatial modes J removed by the compensation system, and the strength of atmospheric turbulence. Turbulence is quantified by two parameters: the phase coherence length r_0 and the scintillation index σ_β^2 . The value of the scintillation index $\sigma_\beta^2 = 1$ corresponds to strong scintillation, but still below the saturation regime. When the turbulence reaches the saturation regime, wavefront distortion becomes so severe that it would be unrealistic to consider phase compensation. When we assume no scintillation, $\sigma_\beta^2=0$, the effect of turbulence is simply to reduce the coherence length r_0 . For a fixed aperture diameter D , as r_0 is reduced, the normalized aperture diameter D/r_0 increases, and turbulence reduces the heterodyne downconversion efficiency.

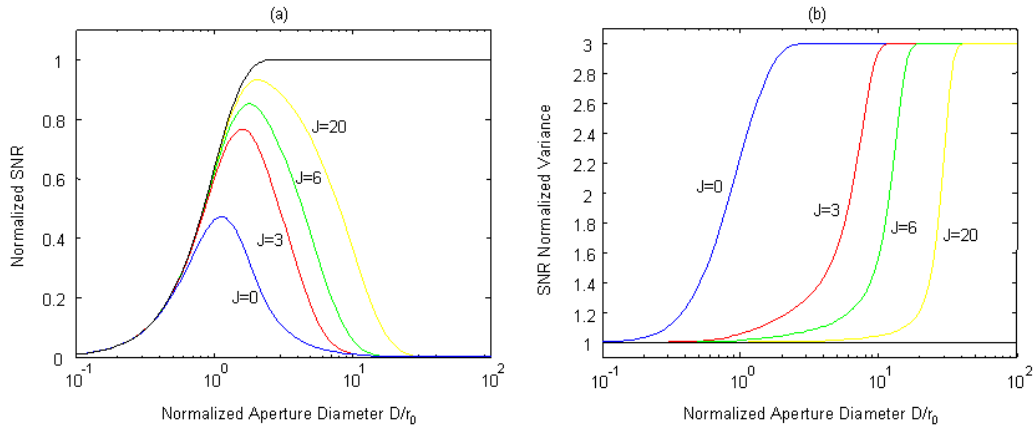


Fig. 1. Performance is shown for different values of the normalized receiver aperture diameter D/r_0 and the number of modes J removed by phase compensation. Turbulence is characterized by a fixed phase coherence diameter r_0 . Amplitude fluctuations are neglected by assuming $\sigma_\beta^2=0$. The compensating phases are expansions up to tilt ($J=3$), astigmatism ($J=6$), and 5th-order aberrations ($J=20$). The no-correction case ($J=0$) is also considered. The black line corresponds to the measurement uncertainty associated with one-pulse speckle measurement. Without loss of generality, in all cases considered in these plots the correlation diameter of the speckle field is equated to the turbulence coherence diameter.

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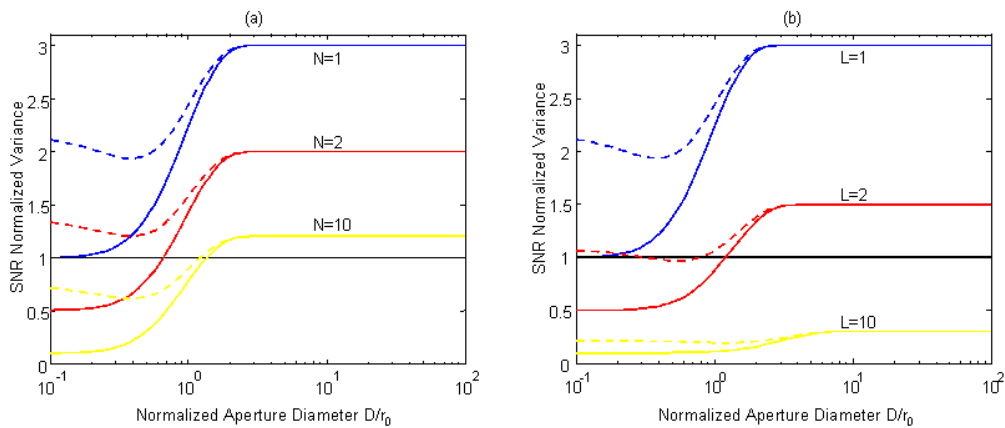


Fig. 2. SNR normalized variance as a function the normalized receiver aperture diameter D/r_0 . Turbulence is characterized by a fixed phase coherence diameter r_0 . In (a), performance is shown for different values of the averaged pulses N . In (b), performance is studied for different values of the total diversity branches L . Amplitude fluctuations are neglected (solid lines) by assuming $\sigma_\beta^2=0$. When scintillation is considered (dashed lines), the scintillation index is fixed at $\sigma_\beta^2=1$. The black line corresponds to the error associated with one-pulse, one-branch speckle measurement where, assuming that all the power samples are independent, the variance is expected to decrease as $1/(N \times L)$. Once again, and without loss of generality, all plots consider the correlation diameter of the speckle field equal to the turbulence coherence diameter r_0 .

More detailed results and comments on our analysis will be presented at the meeting. This study was partially funded by the Spanish Department of Science and Technology MCYT Grant No. TEC 2006-12722. The research of Aniceto Belmonte was supported by a Spain MEC Secretary of State for Universities and Research Grant Fellowship.

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