

# Tunable Dual-Band filters based on capacitive-loaded stepped-impedance resonators

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**Abstract**— This paper proposes the design of tunable dual-band filters for multi-band multi-standard systems. The main objective is to provide frequency tunability in the second band while maintaining the first band fixed. To this end, the capacitive-loaded stepped-impedance resonator is used. The work is divided into two main parts. In the first part, a deep analysis of the capacitive-loaded stepped-impedance resonator structure is done, which provides analytical closed-form design equations which eases their design in front of the several approaches available in the literature up to now. The analysis is also particularized for the case of the capacitive-loaded uniform resonator. In the second part, resonators are integrated in two dual-band tunable filters.

## I. INTRODUCTION

The information society is rapidly approaching to a convergence between communication, multimedia and computing with the goal of a broad connection for all at any time and any place [1]. To operate in an effective and transparent way between all the different wireless access systems, terminals must be able to connect to different networks, driving the need for multi-radio platforms, front-ends and circuits. Moreover, the increasing demand in some wireless communication services requires unfolding the number of frequency bands assigned to a specific service. For instance, wireless local-area networks (WLAN) operate at 2.45 GHz but also at ISM 5 GHz bands.

In this context, dual-band filter structures are emerging for simultaneous operation in multiple frequency bands in wireless communication systems. A number of topologies for dual-band filters have been proposed so far and some of them use the stepped-impedance resonator (SIR) [2,3] to fix its spurious response by adjusting the line characteristic impedances [4,5]. In addition, within the framework of reconfigurable filters, several works have recently appeared proposing frequency-tunable dual-band filters based on stepped-impedance resonators, by shunt-connecting a variable capacitor at its center [6,7]; the concept of tunable stepped-impedance resonators was first proposed in [8], but applied to monoband filters.

Two are the objectives of this work. First, to propose a new analysis of the capacitive-loaded stepped-impedance resonator, leading to a closed-form analytical expression

which greatly eases the design of this structure, in comparison to the analyses available in the literature up to now. Second, to present two dual-band frequency-agile filters which permit to tune the second band while maintaining the first fixed, based on the capacitive-loaded stepped-impedance resonator in an open-loop form in order to miniaturize the whole structure.

## II. CAPACITIVE-LOADED STEPPED-IMPEDANCE RESONATOR

Fig. 1 shows the capacitive-loaded stepped-impedance resonator. The structure is based on a symmetrical stepped-impedance resonator (a resonator with lines of two different characteristic impedance,  $Z_1$  and  $Z_2$ ) with a capacitor  $C$  shunt-connected at the midpoint; here the case  $Z_1 > Z_2$  is considered, since this topology permits  $f_2 > 2f_1$  [2]. The input admittance of the resonator  $Y_{IN}$  is:

$$Y_{IN} = -jY_2 \frac{2(1+K)Z_3 \tan \theta - jZ_1(K - \tan^2 \theta)}{\left( \frac{(1+K)^2 \tan^2 \theta}{K - \tan^2 \theta} + (K \tan^2 \theta - 1) \right) Z_3 - jZ_1(1+K) \tan \theta} \quad (1)$$

where  $Y_2 = 1/Z_2$  is the characteristic admittance of the outer line,  $Y_1 = 1/Z_1$  is the characteristic admittance of the inner line and  $Z_3 = 1/j\omega C = -j|Z_3|$  is the impedance of the shunt-connected capacitor. The case  $\theta_1 = \beta_1 l_1 = \theta_2 = \beta_2 l_2 = \theta$  has been considered since it minimizes the total resonator length (being  $\beta_i$  the propagation constant at each line section and  $l_i$  their respective lengths). The parameter  $K$  is defined as the ratio between characteristic impedances  $K = Z_2/Z_1$ .

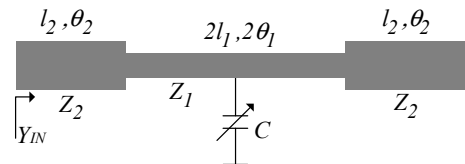


Fig. 1. Capacitive-loaded stepped-impedance resonator.

The resonance condition is  $|Y_{IN}|=0$ . From (1), the first resonance can be obtained with the condition:

$$K - \tan^2 \theta = 0 \quad (2)$$

Then, the fundamental resonance frequency is given by:

$$f_1 = \frac{v_1}{l_1} \frac{\tan^{-1}(\sqrt{K})}{2\pi} \quad (3)$$

where  $v_1$  is the propagation velocity in line 1. The second resonance is obtained from the numerator of (1), which after some re-ordering is:

$$Z_1 \tan^2 \theta - 2(1+K)Z_3 \tan \theta - KZ_1 = 0 \quad (4)$$

The solution to equation (4) will be:

$$\tan \theta = (1+K) \frac{|Z_3|}{Z_1} \pm \sqrt{(1+K)^2 \left( \frac{|Z_3|}{Z_1} \right)^2 + K} \quad (5)$$

Since (4) depends on  $Z_3$ , the second resonance will depend on the value of the capacitor  $C$ . If we consider  $C=0$  ( $|Z_3|=\infty$ ), we would have a stepped-impedance resonator, and the ratio between second ( $f_2$ ) and fundamental ( $f_1$ ) frequencies would depend only on the ratio between characteristic impedances  $K$ :

$$\frac{f_2}{f_1} = \frac{\pi}{2 \tan^{-1}(\sqrt{K})} \quad (6)$$

From now on, we will address the second resonance frequency that will be obtained (already considering  $C$ ) as  $f_r$ . Then,  $f_r=f_2$  for  $C=0$  (6) and  $f_r<f_2$  for  $C \neq 0$ . The ratio between resonance frequencies can be expressed as:

$$\frac{f_r}{f_1} = \frac{\theta_r}{\theta_1} = \frac{\theta_r}{2 \tan^{-1}(\sqrt{K})} \quad (7)$$

where  $\theta_r$  is the electrical length that corresponds to  $f_r$ . By using (7), (5) can be expressed as:

$$\frac{f_r}{f_1} = \frac{\tan^{-1} \left( \frac{1+K}{Z_1 \omega C} + \sqrt{\left( \frac{1+K}{Z_1 \omega C} \right)^2 + K} \right)}{\tan^{-1} \sqrt{K}} \quad (8)$$

Two resonators are considered along this work:  $K=1$  (uniform resonator, which is considered a particular case of the SIR) and  $K=0.56$  (SIR). For the case  $K=1$  the following parameters are used:  $f_1=2.45$  GHz,  $f_2=4.9$  GHz and  $Z_1=Z_2=50\Omega$ . For the case  $K=0.56$  the following parameters are considered:  $f_1=2.45$  GHz,  $f_2=6$  GHz,  $Z_1=60\Omega$  and  $Z_2=33.6\Omega$ . The reason for the case  $K=0.56$  is that it is a good example of  $K<1$  and will permit to use the resonators in filters suitable for WLAN applications (with  $f_1=2.45$  GHz and  $f_2$  tuned between 5.75 and 5.25 GHz). The dependence of  $f_r/f_1$  on  $Z_3$  (or  $C$ ) is obtained numerically solving (8) and is plotted in Fig. 2 for the two resonators.

It can be noticed in (2)-(3) that the first resonance frequency is independent of  $C$ , which only modifies  $f_2$ . Then, it can be considered for this analysis  $\omega=\omega_r=2\pi f_r$  and expression (4) can be rewritten as:

$$Z_1 \left( \tan^2 \left( \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right) - K \right) 2\pi f_r C - 2(1+K) \tan \left( \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right) = 0 \quad (9)$$

Only the positive solution is considered, since the negative leads to an unphysical solution ( $f_r<0$  or  $f_r>f_2$ ). Manipulating (9), the following expression is obtained:

$$\frac{f_r}{f_1} = \frac{1}{Z_1 \pi f_r C} \frac{\sin \left( 2 \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right)}{\left( \frac{1-K}{1+K} \right) - \cos \left( 2 \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right)} \quad (10)$$

Fig. 3 shows the graphical solution of (10) for the two resonators (Fig. 3.a for  $K=1$  and Fig. 3.b for  $K=0.56$ ). However, neither solving (8) nor (10) does not simplify the resonator design procedure; to this end, a closed-form analytical expression relating  $C$  and  $f_r/f_1$  is obtained next. Isolating the capacitance  $C$  in (10) and manipulating, expression (11) is obtained:

$$C = \frac{1}{Z_1 \pi f_r} \frac{\sin \left( 2 \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right)}{\left( \frac{1-K}{1+K} \right) - \cos \left( 2 \frac{f_r}{f_1} \tan^{-1} \sqrt{K} \right)} \quad (11)$$

Finally, developing expression (11) by using (6) and manipulating, expression (12) can be obtained which directly relates  $f_1, f_2, f_r, K$  ( $Z_1$  and  $Z_2$ ) and  $C$ :

$$C = \frac{1}{Z_1 \pi f_r} \frac{\sin \left( \frac{\pi f_r}{f_2} \right)}{\cos \left( \frac{\pi f_1}{f_2} \right) - \cos \left( \frac{\pi f_r}{f_2} \right)} \quad (12)$$

By using (12) it is very simple to design any capacitive-load stepped-impedance resonator, on the contrary to the works available up to now in the literature [7,8]. In [8] an analysis based on the ABCD parameters is provided, leading to complicated expressions which do not make its design straightforward. On the other hand, the authors provide in [7] a numerical approximation. Both works are far from the versatility and simplicity that the closed-form analytical expression (12) provides to the designer. A three-step simple design procedure can now be defined: 1/ Fix  $f_1$  and  $f_2$ , which correspond to the fundamental resonance and the maximum value for the second resonance (case  $C=0$ ), respectively. This determines  $K$ . 2/ Fix the values of  $Z_1$  and  $Z_2$ , according to line widths that can be physically fabricated. 3/ Fix the tuning range of the resonator; its limits determine two resonance

frequencies  $f_r$  ( $f_{r1}$  and  $f_{r2}$ ) which, in turn, determine the two extreme capacities  $C$  ( $C_1$  and  $C_2$ ).

In order to validate the proposed analysis, Fig.4 shows a comparison between the analytical expression (12) and an Agilent™ ADS-based design for the two resonators. A subplot with a detail of the region of maximum interest (small  $C$ ) is included. A good agreement is obtained.

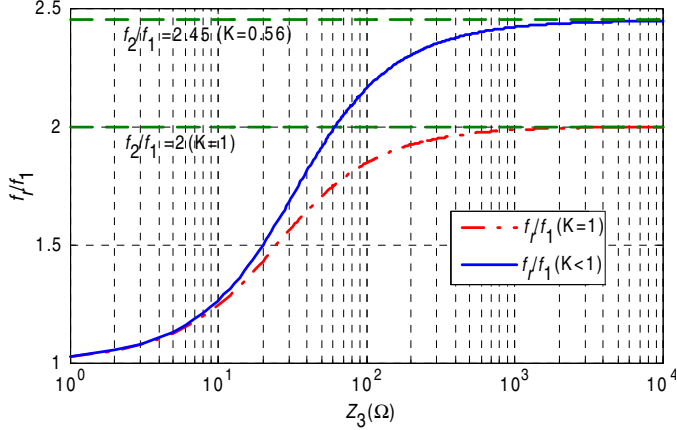


Fig. 2. Dependence of  $f_r/f_1$  on  $Z_3$ , obtained by numerically solving (8). Case  $K=1$  and  $K<1$  (0.56).

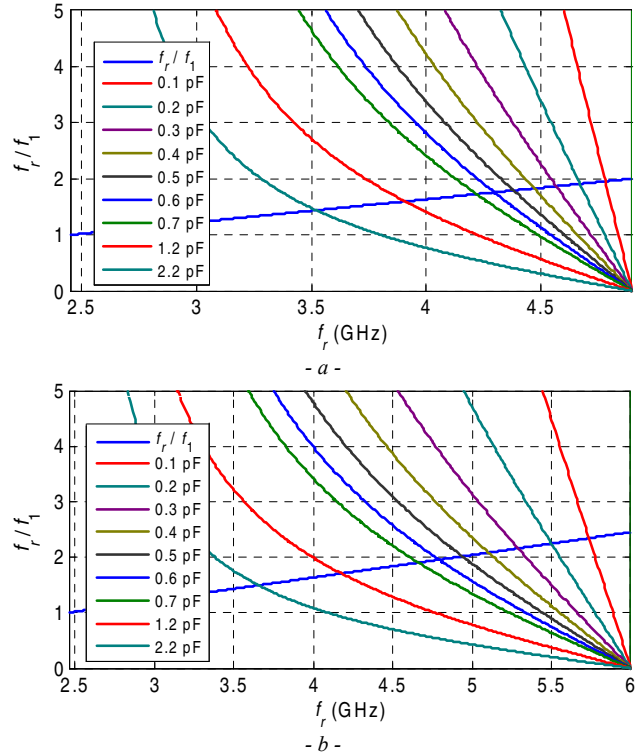


Fig.3. Graphical solution of (10) for the two resonators:  $K=1$  (a) and  $K<1$  (0.56) (b).

### III. TUNABLE DUAL-BAND FILTERS DESIGN

The capacitive-loaded stepped-impedance resonator has been integrated in two second-order Chebyshev filters, with  $K=1$  and  $K=0.56$ . In order to miniaturize both filters, the resonators have been designed in its open-loop form. They

have been manufactured on RO4003 substrate, with  $\epsilon_r=3.55$ , thickness  $t=0.813$  mm and loss tangent  $tg\delta=0.0027$ . Photographs of the manufactured filters are given in Fig. 5.

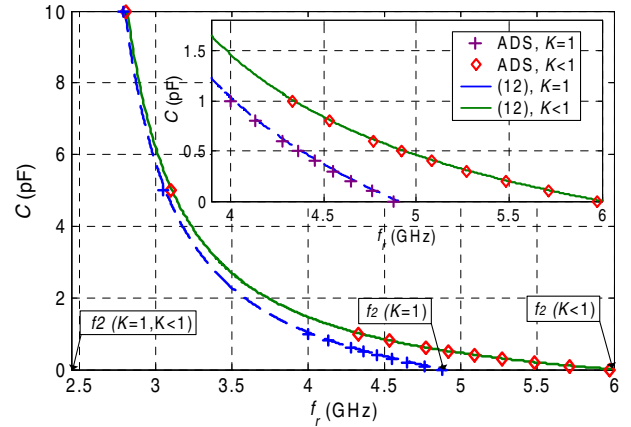


Fig. 4. Validation of the proposed closed-form analytical expression (12) for the capacitive-loaded stepped-impedance resonator for the cases  $K=1$  and  $K<1$  (0.56) by means of comparison to ADS simulation.

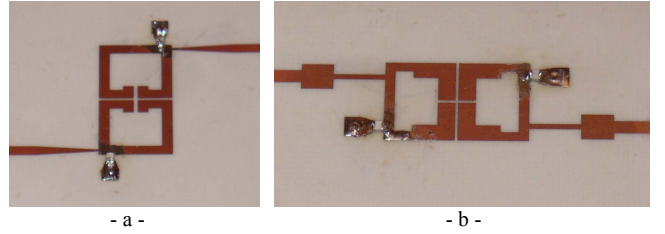


Fig. 5. Photographs of the manufactured filters. Tunable dual-band filter with  $K=1$  (a) and with  $K=0.56$  (b). (Not to scale).

The filter with  $K=1$  (see Fig. 5.a) has its first pass-band at 2.45 GHz and the second at 4.9 GHz. Fig. 6 shows the measured insertion and return loss compared to ADS/Momentum simulation.

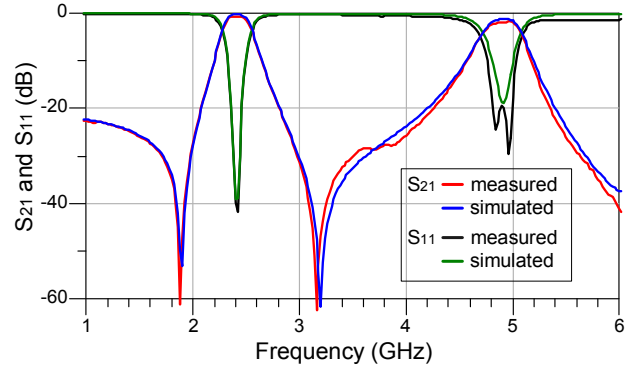


Fig. 6. Comparison between measurement and simulation of the tunable dual-band filter with  $K=1$  ( $C=0$ pF).

Fig. 7 shows the measured frequency tuning range of the filter by varying the shunt-connected capacitance from 0 to 0.5 pF. The maximum design frequency  $f_2$  for this filter is  $2f_1$  (when  $C=0$  pF). This limit can be pushed up by using the SIR resonator with  $K<1$ , as it has been demonstrated. A second filter has been designed with  $f_{r1}=2.45$ GHz and  $f_{r2}=6$ GHz (see Fig. 5.b). Since  $f_2$  is very different from  $2f_1$ , a dual-band

input/output matching network is designed based on using a stepped-impedance transmission line (nonsynchronous alternating-impedance transformer) [9,10]. The filter measured insertion and return loss are compared to simulation in Fig. 8. Fig. 9 shows its measured tuning range for  $C$  between 0 and 0.2 pF.

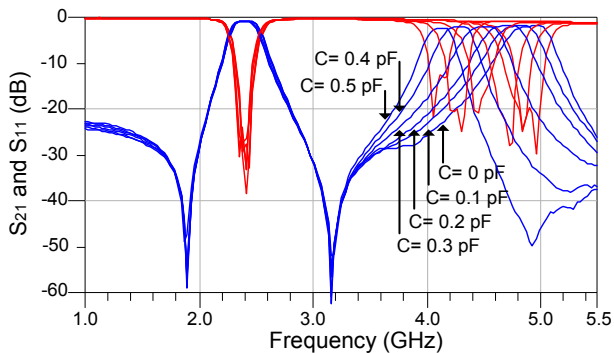


Fig. 7. Measurement of the tunable dual-band filter response with  $K=1$  for  $C$  between 0 and 0.5 pF, with a measured tuning range between 4.9 and 4.12 GHz, insertion loss 1.8 and 2.4 dB and relative bandwidth 8.1% and 6.3%, respectively. The measured first pass-band at 2.42 GHz has 0.63 dB insertion loss and 11.5% relative bandwidth.

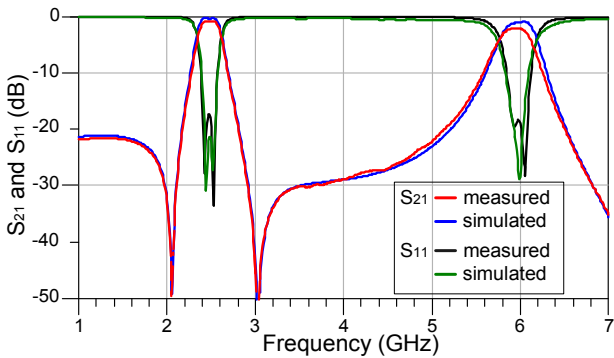


Fig. 8. Comparison between measurement and simulation of the dual-band filter with  $K=0.56$  ( $C=0$ pF).

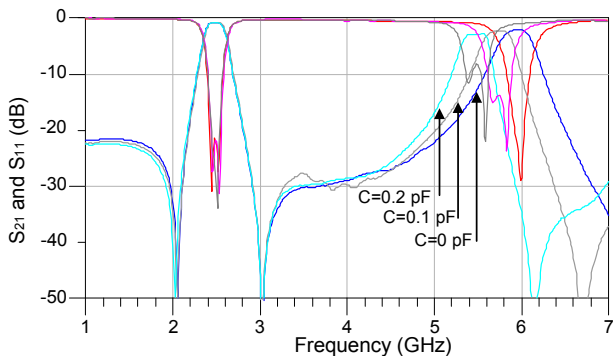


Fig. 9. Measurement of the tunable dual-band filter response with  $K=0.56$  for  $C$  between 0 and 0.2 pF, with a measured tuning range between 5.97 and 5.45 GHz, insertion loss 2.1 and 2.9 dB and relative bandwidth 7.2% and 6.1%, respectively. The measured first pass-band at 2.47 GHz has 0.83 dB insertion loss and 10.5% relative bandwidth.

A faster degradation in the filter performance has been experienced in the case  $K=0.56$ , since it has been proved that it is very sensitive to the precision in placing the variable capacitors. On the other hand, it has been demonstrated with

this filter that  $f_2$  can be placed beyond  $2f_1$ , which allows to design filters reconfigurable between the different frequency specifications of WLAN.

It is also important to note that when resonators are integrated into a filter the optimum position for the capacitor is no longer the physical midpoint and now this parameter is optimized by means of electromagnetic simulation. If the capacitor is not properly placed, a rapid degradation of the filter response in terms of ripple, losses and bandwidth is experienced while tuning the filter.

#### IV. CONCLUSIONS

This paper has proposed the design of tunable dual-band filters. They provide frequency tunability in the second band while maintaining the first band fixed. The capacitive-loaded stepped-impedance resonator is used. An analysis of the capacitive-loaded stepped-impedance resonator structure has been done, providing analytical closed-form design equations, which ease their design in front of the several approaches available up to now in the literature. The resonator has been integrated in two tunable dual-band filters which demonstrate good tuning range.

#### ACKNOWLEDGMENT

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