When Almost Distance-Regularity Attains Distance-Regularity *

C. Dalfó†, E.R. van Dam‡, M.A. Fiol†, and E. Garriga†

†Universitat Politècnica de Catalunya, Dept. de Matemàtica Aplicada IV
Barcelona, Catalonia (e-mails: {cdalfo,fiol,egarriga}@ma4.upc.edu)
‡Tilburg University, Dept. Econometrics and O.R.
Tilburg, The Netherlands (e-mail: edwin.vandam@uvt.nl)

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Abstract

Generally speaking, 'almost distance-regular graphs' are graphs which share some, but not necessarily all, regularity properties that characterize distance-regular graphs. In this paper we first propose four basic different (but closely related) concepts of almost distance-regularity. In some cases, they coincide with concepts introduced before by other authors, such as walk-regular graphs and partially distance-regular graphs. Here it is always assumed that the diameter D of the graph attains its maximum possible value allowed by its number $d+1$ of different eigenvalues; that is, $D = d$, as happens in every distance-regular graph. Our study focuses on finding out when almost distance-regularity leads to distance-regularity. In other words, some 'economic' (in the sense of minimizing the number of conditions) old and new characterizations of distance-regularity are discussed. For instance, if $\boldsymbol{A}_0, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_D$ and $\boldsymbol{E}_0, \boldsymbol{E}_1, \ldots, \boldsymbol{E}_d$ denote, respectively, the distance matrices and the idempotents of the graph; and $\mathcal{D}$ and $\mathcal{A}$ stand for their respective linear spans, any of the two following 'dual' conditions suffice: (*a*) $\boldsymbol{A}_0, \boldsymbol{A}_1, \boldsymbol{A}_D \in \mathcal{A}$; (*b*) $\boldsymbol{E}_0, \boldsymbol{E}_1, \boldsymbol{E}_d \in \mathcal{D}$. Moreover, other characterizations based on the preintersection parameters, the average intersection numbers and the recurrence coefficients are obtained. In some cases, our results can be also seen as a generalization of the so-called spectral excess theorem for distance-regular graphs.

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