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Decentralized Control with Information Structure Constraints

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Abstract. *In this paper, some problems related to the design of decentralized controllers are considered. Due to information structure constraints on the systems, we are interested in determine when a gain matrix corresponding to a control law can be designed having a required structure. To discuss this issue, we consider some generic classes of systems with different control strategies: optimal overlapping control, guaranteed cost control and* H_∞ *control. For each one of them, two scenarios are supposed: state feedback and output feedback controllers. In this line, some new contributions are offered.*

1 Introduction

The dynamic behavior of many physical processes is frequently complex. This situation naturally motivates the development of effective methods of control, taking into account particular features of these systems. Decentralized control can be a useful strategy to design controllers when the systems present complexities. Moreover, in many practical systems, specific structures of controllers are needed. The most obvious restrictions are those that are structural in nature. Thus, when *information-structure constraints* appear, the necessity of designing gain control matrices having preassigned structures arises. Different structures of the gain matrices are usually considered when information structure constraints occur [17], [19]. In the paper, some problems related to the design of gain matrices having predetermined structures are presented and discussed.

On the other hand, large-scale and complex systems are usually composed by subsystems sharing some components. These systems can be treated as interconnected but with overlapped parts. In this case, it is advantageous to utilize actuators sharing the information only among their neighbor subsystems but not with the overall subsystems, having "semidecentralized" feedback control structures. Overlapping information sets and the inclusion principle give a useful framework for such a design. A key point for this purpose is to obtain expanded systems with weak interconnections. Then, "virtual" decentralized controllers in the (non real) expanded system are designed, which are further transformed to be implemented into the real system as a unique controller [7], [16]. This method has been applied to different classes of overlapped systems and problems as illustrated for instance in [3], [4], [13], [15].

Our study is focussed on the obtention of gain matrices having a predetermined zero-nonzero structure, by considering state and output feedback control. We have focused the study on three kinds of control criteria: optimal control, guaranteed cost control and H_∞ control. Some new results are presented.

2 Overlapping quadratic optimal control

The inclusion principle provides conditions under which an initial system, with shared components, can be expanded to a higher dimensional space so that the overlapped subsystems appear as disjoint. The expanded space contains the essential information about the initial one in such a manner that a control methodology can be advantageously designed for this system and transformed (contracted) to have a final control law which is implementable into the initial system [7], [8], [9], [16]. The inclusion principle has been studied and applied satisfactory in different areas as mechanical systems [2], electric power systems [11], vehicles [12], [14] or control of structures [1].

Next, we summarize briefly the main ideas involved in the inclusion principle. Consider two linear time-invariant systems given by

$$\mathbf{S}: \ \dot{x}(t)=Ax(t)+Bu(t), \qquad \tilde{\mathbf{S}}: \ \dot{\tilde{x}}(t)=\tilde{A}\tilde{x}(t)+\tilde{B}\tilde{u}(t), \tag{1}$$

where $x(t)\in\mathbb{R}^{n}$, $u(t)\in\mathbb{R}^{m}$ are the states and the inputs for the system $\mathbf{S}$ at time $t\in\mathbb{R}^{+}$ and $\tilde{x}(t)\in\mathbb{R}^{\tilde{n}}$, $\tilde{u}(t)\in\mathbb{R}^{\tilde{m}}$ are the states and the inputs for $\tilde{\mathbf{S}}$. Matrices A, B and $\tilde{A}$, $\tilde{B}$ are constant of dimensions $n\times n$, $n\times m$ and $\tilde{n}\times\tilde{n}$, $\tilde{n}\times\tilde{m}$, respectively. Suppose that the dimensions of the state and input vectors $x(t)$, $u(t)$ of $\mathbf{S}$ are smaller than those of $\tilde{x}(t)$, $\tilde{u}(t)$ of $\tilde{\mathbf{S}}$. Denote $x(t)=x(t;x_0,u)$ the state behavior of $\mathbf{S}$ for a fixed input $u(t)$ and for an initial state $x(0)=x_0$. An analogous notation $\tilde{x}(t)=\tilde{x}(t;\tilde{x}_0,\tilde{u})$ is used for the state behavior of $\tilde{\mathbf{S}}$.

Consider the following transformations:

$$V:\mathbb{R}^{n}\longrightarrow\mathbb{R}^{\tilde{n}}, \qquad U:\mathbb{R}^{\tilde{n}}\longrightarrow\mathbb{R}^{n}, \qquad R:\mathbb{R}^{m}\longrightarrow\mathbb{R}^{\tilde{m}}, \qquad Q:\mathbb{R}^{\tilde{m}}\longrightarrow\mathbb{R}^{m}, \tag{2}$$

where V, U, R, Q are full-rank matrices such that $UV=I_n$ and $QR=I_m$, where I_n, I_m denote the identity matrices of indicated dimensions.

Definition 1 *(Inclusion Principle) A system $\tilde{\mathbf{S}}$ includes the system $\mathbf{S}$, denoted by $\tilde{\mathbf{S}}\supset\mathbf{S}$, if there exists a quadruplet of matrices (U,V,Q,R) satisfying $UV=I_n$, $QR=I_m$ such that for any initial state x_0 and any fixed input $u(t)$ of $\mathbf{S}$, the choice $\tilde{x}_0=Vx_0$ and $\tilde{u}(t)=Ru(t)$ implies $x(t;x_0,u)=U\tilde{x}(t;\tilde{x}_0,\tilde{u})$ for all $t\geqslant 0$. Figure 1 represents graphically this definition.*

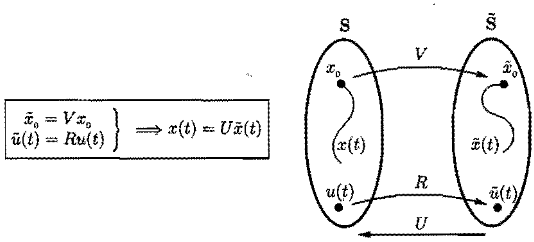

Fig.1 Inclusion principle

Associated with the systems $\mathbf{S}$ and $\tilde{\mathbf{S}}$ given in (1), we consider the quadratic cost functions

$$\begin{aligned} J(x_0,u(t)) &= \int_0^{\infty}\left[x^T(t)Q^*x(t)+u^T(t)R^*u(t)\right]dt, \\ \tilde{J}(\tilde{x}_0,\tilde{u}(t)) &= \int_0^{\infty}\left[\tilde{x}^T(t)\tilde{Q}^*\tilde{x}(t)+\tilde{u}^T(t)\tilde{R}^*\tilde{u}(t)\right]dt, \end{aligned} \tag{3}$$

where Q^*, $\tilde{Q}^*$ and R^*, $\tilde{R}^*$ are symmetric positive semidefinite and symmetric positive definite matrices, respectively.

Assume that the system S given in (1) has the following structure:

$$A=\left[\begin{array}{cc|c} A_{11} & A_{12} & A_{13} \\ \hline A_{21} & \multicolumn{1}{|c|}{A_{22}} & A_{23} \\ \hline A_{31} & \multicolumn{1}{|c}{A_{32}} & A_{33} \end{array}\right], \qquad B=\left[\begin{array}{cc|c} B_{11} & B_{12} & B_{13} \\ \hline B_{21} & \multicolumn{1}{|c|}{B_{22}} & B_{23} \\ \hline B_{31} & \multicolumn{1}{|c}{B_{32}} & B_{33} \end{array}\right], \tag{4}$$

where A_{ii}, B_{ij} for i=1,2,3, j=1,2,3 are $n_i \times n_i$, $n_i \times m_j$ dimensional matrices, respectively, and $\sum n_i = n$, $\sum m_j = m$. The expansion-contraction matrices are usually selected in the form

$$V=\begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & I_{n_2} & 0 \\ 0 & I_{n_2} & 0 \\ 0 & 0 & I_{n_3} \end{bmatrix}, \quad U=\begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{n_2} & \frac{1}{2}I_{n_2} & 0 \\ 0 & 0 & 0 & I_{n_3} \end{bmatrix}, \quad R=\begin{bmatrix} I_{m_1} & 0 & 0 \\ 0 & I_{m_2} & 0 \\ 0 & I_{m_2} & 0 \\ 0 & 0 & I_{m_3} \end{bmatrix}, \quad Q=\begin{bmatrix} I_{m_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{m_2} & \frac{1}{2}I_{m_2} & 0 \\ 0 & 0 & 0 & I_{m_3} \end{bmatrix}. \tag{5}$$

The expanded matrices $\tilde{A}$, $\tilde{B}$, $\tilde{Q}^*$ and $\tilde{R}^*$ of $\tilde{\mathbf{S}}$ can be expressed as

$$\tilde{A} = VAU + M, \quad \tilde{B} = VBQ + N, \quad \tilde{Q}^* = U^T Q^* U + M_{Q^*}, \quad \tilde{R}^* = Q^T R^* Q + N_{R^*}, \tag{6}$$

where M, N, M_{Q^*} and N_{R^*} are complementary matrices that can be chosen conveniently by the designer [3], [4], [5], [6]. The expanded matrices $\tilde{A}$=VAU and $\tilde{B}$=VBQ, without adding the complementary matrices M and N, have the form

$$\tilde{A}=\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}=\left[\begin{array}{cc|cc} A_{11} & \frac{1}{2}A_{12} & \frac{1}{2}A_{12} & A_{13} \\ A_{21} & \frac{1}{2}A_{22} & \frac{1}{2}A_{22} & A_{23} \\ \hline A_{21} & \frac{1}{2}A_{22} & \frac{1}{2}A_{22} & A_{23} \\ A_{31} & \frac{1}{2}A_{32} & \frac{1}{2}A_{32} & A_{33} \end{array}\right], \quad \tilde{B}=\begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix}=\left[\begin{array}{cc|cc} B_{11} & \frac{1}{2}B_{12} & \frac{1}{2}B_{12} & B_{13} \\ B_{21} & \frac{1}{2}B_{22} & \frac{1}{2}B_{22} & B_{23} \\ \hline B_{21} & \frac{1}{2}B_{22} & \frac{1}{2}B_{22} & B_{23} \\ B_{31} & \frac{1}{2}B_{32} & \frac{1}{2}B_{32} & B_{33} \end{array}\right]. \tag{7}$$

In this process, the basic idea is to achieve decoupled or weakly coupled expanded systems. For this reason, a proper choice of M and N is required [8], [9], [10]. In the expanded system $\tilde{\mathbf{S}}$, we can denote

$$\begin{aligned} \tilde{\mathbf{S}}_1 : \ \dot{\tilde{x}}_1(t) &= \tilde{A}_{11}\tilde{x}_1(t) + \tilde{B}_{11}\tilde{u}_1(t) + \tilde{A}_{12}\tilde{x}_2(t) + \tilde{B}_{12}\tilde{u}_2(t), \\ \tilde{\mathbf{S}}_2 : \ \dot{\tilde{x}}_2(t) &= \tilde{A}_{22}\tilde{x}_2(t) + \tilde{B}_{22}\tilde{u}_2(t) + \tilde{A}_{21}\tilde{x}_1(t) + \tilde{B}_{21}\tilde{u}_1(t), \end{aligned} \tag{8}$$

where $\tilde{A}_{ij}$, $\tilde{B}_{ij}$, i,j=1,2, $i \neq j$ are the interconnection matrices. The decoupled subsystems can be expressed in the following form:

$$\tilde{\mathbf{S}}_{\mathrm{D}}^1 : \ \dot{\tilde{x}}_1(t) = \tilde{A}_{11}\tilde{x}_1(t) + \tilde{B}_{11}\tilde{u}_1(t), \qquad \tilde{\mathbf{S}}_{\mathrm{D}}^2 : \ \dot{\tilde{x}}_2(t) = \tilde{A}_{22}\tilde{x}_2(t) + \tilde{B}_{22}\tilde{u}_2(t), \tag{9}$$

denoted by

$$\tilde{\mathbf{S}}_{\mathrm{D}} : \ \dot{\tilde{x}}(t) = \tilde{A}_D\tilde{x}(t) + \tilde{B}_D\tilde{u}(t) \tag{10}$$

in a more compact form, where $\tilde{A}_D$=diag$\{\tilde{A}_{11}, \tilde{A}_{22}\}$, $\tilde{B}_D$=diag$\{\tilde{B}_{11}; \tilde{B}_{22}\}$. With each subsystem given in (9) it is possible to associate local cost functions given by

$$\begin{aligned} \tilde{J}_D^1(\tilde{x}_{10}, \tilde{u}_1(t)) &= \int_0^\infty \left[\tilde{x}_1^T(t)\tilde{Q}_{11}^*\tilde{x}_1(t) + \tilde{u}_1^T(t)\tilde{R}_{11}^*\tilde{u}_1(t)\right] dt, \\ \tilde{J}_D^2(\tilde{x}_{20}, \tilde{u}_2(t)) &= \int_0^\infty \left[\tilde{x}_2^T(t)\tilde{Q}_{22}^*\tilde{x}_2(t) + \tilde{u}_2^T(t)\tilde{R}_{22}^*\tilde{u}_2(t)\right] dt, \end{aligned} \tag{11}$$

where $\tilde{x}_{10}$ and $\tilde{x}_{20}$ are the initial states of $\tilde{\mathbf{S}}_{\mathrm{D}}^1$ and $\tilde{\mathbf{S}}_{\mathrm{D}}^2$, respectively, and $\tilde{Q}_{11}^*$, $\tilde{Q}_{22}^*$, $\tilde{R}_{11}^*$ and $\tilde{R}_{22}^*$ are appropriate expanded matrices. The final cost function for the decoupled expanded system $\tilde{\mathbf{S}}_{\mathrm{D}}$ will be

$$\tilde{J}_D(\tilde{x}_0, \tilde{u}(t)) = \int_0^\infty \left[\tilde{x}^T(t)\tilde{Q}_D^*\tilde{x}(t) + \tilde{u}^T(t)\tilde{R}_D^*\tilde{u}(t)\right] dt, \tag{12}$$

where $\tilde{Q}_D^*$=diag$\{\tilde{Q}_{11}^*, \tilde{Q}_{22}^*\}$, $\tilde{R}_D^*$=diag$\{\tilde{R}_{11}^*, \tilde{R}_{22}^*\}$. The local control laws corresponding to the decoupled expanded subsystems $\tilde{\mathbf{S}}_D^1$ and $\tilde{\mathbf{S}}_D^2$ are given by

$$\tilde{u}_1(t) = \tilde{K}_{11}\tilde{x}_1(t), \qquad \tilde{u}_2(t) = \tilde{K}_{22}\tilde{x}_2(t). \tag{13}$$

This type of feedback control is called *overlapping control*. The problem of designing overlapping controllers can be formulated as a decentralized control problem in the expanded space $\tilde{\mathbf{S}}$.

The goal is to implement an overlapping controller in the system $\mathbf{S}$, denoted by $u_D(t)=K_D x(t)$, but as a contraction of the control law $u_D(t)=\tilde{K}_D\tilde{x}(t)$ designed in $\tilde{\mathbf{S}}$. Then, according to the previous structures, the gain matrices in the expanded and initial systems have the following form:

$$\tilde{K}_D = \left[\begin{array}{c|c} \tilde{K}_{11} & 0 \\ \hline 0 & \tilde{K}_{22} \end{array}\right] \xrightarrow{\text{contraction}} K_D = Q\tilde{K}_D V = \begin{bmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{bmatrix}. \tag{14}$$

Remark 1 *In this case, a quadratic optimal control has been applied to obtain the controllers $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$. However, other control criteria can be used following the proposed strategy of design. We can observe that $u_1(t)$ uses the information contained in $x_1(t)$ and $x_2(t)$ but does not use $x_3(t)$. Analogously, $u_3(t)$ only uses the information on $x_2(t)$ and $x_3(t)$ but does not use $x_1(t)$. Dealing with large-scale systems, where the number of variables can be notable, a decentralized control design may be a convenient approach.*

3 Guaranteed cost control

Consider a class of linear continuous-time uncertain systems described by the equations

$$\mathbf{S}: \begin{aligned} \dot{x}(t) &= [A+\Delta A(t)]x(t) + [B+\Delta B(t)]u(t), \\ y(t) &= C_y x(t), \end{aligned} \tag{15}$$

where $x(t)\in\mathbb{R}^n$ corresponds to the state, $u(t)\in\mathbb{R}^m$ is the input control and $y(t)\in\mathbb{R}^q$ is the measured output. A, B and C_y are known, real and constant matrices of appropriate dimensions. Norm-bounded time-varying uncertainties are supposed in the form

$$\Delta A(t) = H_A F_A(t) E_A, \qquad \Delta B(t) = H_B F_B(t) E_B, \tag{16}$$

where H_A, E_A, H_B and E_B are known real constant matrices of appropriate dimensions and F_A, F_B are unknown real time-varying matrices with Lebesgue measurable elements satisfying $F_i^T(t)F_i(t) \leqslant I$, for i=A, B.

Associated with the system (15) we consider the cost function

$$J = J(x_0, u(t)) = \int_0^\infty \left[x^T(t)Q^* x(t) + u^T(t)R^* u(t)\right] dt, \tag{17}$$

where Q^* and R^* are symmetric positive semidefinite and symmetric positive definite matrices, respectively.

The idea is to design robust controllers which make the resulting closed-loop systems not only asymptotically stable but also guaranteeing an adequate level of performance. The performance is measured with the standard quadratic cost function and an upper bound for the cost function is obtained.

Theorem 1 *Consider the system (15) satisfying (16) with an associated cost function (17). Suppose that there exist matrices $X>0$, Y, and scalars $\alpha_1>0$, $\alpha_2>0$ such that the following linear matrix inequality*

$$\begin{bmatrix} W_1 & X & XE_A^T & Y^TE_B^T & Y^T \\ X^T & -[Q^*]^{-1} & 0 & 0 & 0 \\ E_AX & 0 & -\alpha_1 I & 0 & 0 \\ E_BY & 0 & 0 & -\alpha_2 I & 0 \\ Y & 0 & 0 & 0 & -[R^*]^{-1} \end{bmatrix} < 0 \qquad (18)$$

is feasible, where $W_1 = AX + XA^T + BY + Y^TB^T + \alpha_1 H_AH_A^T + \alpha_2 H_BH_B^T$. Then, the output feedback controller $u(t)=Ky(t)$ is a quadratic guaranteed cost controller for the uncertain closed-loop system (15), where $KC_y=YX^{-1}$. Moreover, $J \leqslant x_0^TX^{-1}x_0$.

3.1 State feedback control

Consider $C_y=I$, the identity matrix, in the system (15). In this case, the control law $u(t)$ is a state feedback controller having the form $u(t)=Kx(t)$, where the gain matrix $K=YX^{-1}$ is obtained directly. When structural constraints appear in the model, it is necessary to consider some restrictions on the gain matrix K. Theorem 1 provides a gain matrix K, assuming that the LMI (18) is feasible, but without requirements on its structure. However, by using an LMI approach, we can impose some structural conditions on the variable matrices X and Y. Thus, if the matrix X has a diagonal form, the gain matrix K adopts the same structure that the matrix Y, which can be imposed a priori. For example,

$$K = \begin{bmatrix} * & * & 0 \\ 0 & * & * \end{bmatrix} = YX^{-1} = \begin{bmatrix} y_{11} & y_{12} & 0 \\ 0 & y_{22} & y_{23} \end{bmatrix} \begin{bmatrix} x_{11} & 0 & 0 \\ 0 & x_{22} & 0 \\ 0 & 0 & x_{33} \end{bmatrix}^{-1}. \qquad (19)$$

In general, this assertion is only true when the matrix X has a diagonal form. Moreover, the reduction in the number of variables in the LMI (18) can have a detrimental effect on the feasibility of the problem. Other factorizations of the gain matrix $K=YX^{-1}$ have been studied [17], [18], [19], [20], but not always with satisfactory results.

3.2 Output feedback control

From Theorem 1, when an output feedback controller is considered, the control gain matrix is given implicitly by the relation $KC_y=YX^{-1}$. Then, two issues appear simultaneously: (1) how to isolate the gain matrix K, and (2) how to impose some desired structure on K. To solve the problem (1), the following strategy can be used [18]:

Step 1) Select a full rank matrix Q of $n\times(n-q)$ dimension such that $C_yQ=0$.

Step 2) Solve the LMI given in (18) with

$$X = QX_qQ^T + C_y^T\left[C_yC_y^T\right]^{-1}C_y + C_y^TX_cC_y, \qquad Y = Y_cC_y, \qquad (20)$$

where X_q and X_c are unknown symmetric matrices of dimensions $(n-q)\times(n-q)$ and $q\times q$, respectively, and Y_c is an unknown $m\times q$ dimensional matrix.

Step 3) Supposing feasible the LMI (18), compute the gain matrix K as

$$K = Y_c\left[I - C_yX_0^{-1}C_y^TX_c\left[I + C_yX_0^{-1}C_y^TX_c\right]^{-1}\right], \qquad (21)$$

where $X_0=QX_qQ^T + C_y^T\left[C_yC_y^T\right]^{-1}C_y$. The procedure guarantees $KC_y=YX^{-1}$. This algorithm solves (1) but can not be used to obtain a preassigned structure on the matrix K.

Remark 2 *Other design strategies can be found in the literature to solve these problems. However, in many cases, the reduction of the number of variables in the LMI resulting from the change of variables leads to infeasibility. In other cases, the process depends on trial-error selection of some matrices and, in this case, the algorithm does not always assure feasibility of the problem.*

4 H_∞ control

Consider a class of linear continuous-time uncertain systems described by the equations

$$\begin{aligned}\mathbf{S}: \dot{x}(t) &= [A+\Delta A(t)]x(t)+[B+\Delta B(t)]u(t)+B_1w(t),\\ y(t) &= C_yx(t),\\ z(t) &= Cx(t)+Du(t),\end{aligned} \tag{22}$$

where $x(t)\in\mathbb{R}^n$ corresponds to the state, $u(t)\in\mathbb{R}^m$ is the input control, $w(t)\in L_2^p[0,\infty)$ the disturbance input, $y(t)\in\mathbb{R}^l$ is the measured output, and $z(t)\in\mathbb{R}^q$ is the controlled output. A, B, B_1, C_y, C, D are known, real and constant matrices of appropriate dimensions. Norm-bounded time-varying uncertainties satisfy (16).

The H_∞ control objective is to design controllers such that the closed-loop system is stable guaranteeing the disturbance attenuation of the closed-loop system from $w(t)$ to $z(t)$, i.e.

$$\|z(t)\|_2 \leqslant \gamma\|w(t)\|_2,\ \gamma>0, \tag{23}$$

for all non-zero $w(t)$, under zero initial conditions. In this paper, an LMI approach is used. With this idea in mind, consider an output feedback controller in the form $u(t)=Ky(t)$ for the system (22), where $K\in\mathbb{R}^{m\times l}$.

Theorem 2 *Consider a linear continuous-time uncertain system given in (22) with norm-bounded uncertainties (16) and a scalar $\gamma>0$. For given scalars $\beta_1>0$, $0<\beta_2<1$, suppose that there exists a symmetric positive-definite matrix X and a matrix W such that the following linear matrix inequality*

$$\begin{bmatrix} W_1 & XE_A^T & X & W^TE_B^T & W_2^T\\ E_AX & -I & 0 & 0 & 0\\ X & 0 & -I & 0 & 0\\ E_BW & 0 & 0 & -\beta_1 I & 0\\ W_2 & 0 & 0 & 0 & -\beta_2 I\end{bmatrix}<0 \tag{24}$$

holds, where

$$\begin{aligned}W_1 &= AX+XA^T+BW+[BW]^T+H_AH_A^T+(1+\beta_1)H_BH_B^T+\gamma^{-2}B_1B_1^T,\\ W_2 &= CX+DW.\end{aligned} \tag{25}$$

Then, there exists an output feedback controller in the form $u(t)=Ky(t)$ such that the resulting closed-loop system is asymptotically stable with H_∞ norm-bound γ. Moreover, the control gain matrix K is given implicitly as $KC_y=WX^{-1}$.

Remark 3 *Here, the problem is similar to the guaranteed quadratic output feedback control case. Now, we proposed a change of variables which allow to obtain explicitly the matrix K, taking into account a preassigned zero-nonzero structure on the matrix K.*

4.1 State feedback control

If we consider that $C_y=I_d$ in the system (22), then the gain matrix obtained from Theorem 2 has the form $K=WX^{-1}$. Then, due to the flexibility given by using an LMI approach, it is possible to impose the same structures as given in (19) on the variable matrices X and W. However, in general, it is not possible to achieve a particular form of the gain matrix K.

4.2 Output feedback control

4.2.1 Change of variables

Some strategies have been studied to obtain output feedback controllers [17], [19]. Here, the following change of variables, by adapting some previous ideas presented in [18], [20], are used:

$$X = \alpha X_0 + Q X_c Q^T, \qquad W = W_c C_y X_0, \tag{26}$$

where $\alpha>0$ is a scalar variable, X_0 is a constant symmetric matrix selected a priori, X_c and W_c are unknown $(n-l)\times(n-l)$ and $(m\times l)$ dimensional matrices, respectively, and Q is a constant $n\times(n-l)$ dimensional matrix such that rank $Q\leqslant(n-l)$ verifying $Q^T C_y^T=0$. Obviously, the selection of the matrix X_0 is not unique. It can be observed that

$$XC_y^T = \alpha X_0 C_y^T + Q X_c Q^T C_y^T = \alpha X_0 C_y^T \implies \alpha^{-1} C_y = C_y X_0 X^{-1}. \tag{27}$$

From (26) and (27), we have

$$WX^{-1} = W_c C_y X_0 X^{-1} = \alpha^{-1} W_c C_y. \tag{28}$$

Consider the Theorem 2 with the change of variables given in (26). If the LMI (24) is feasible for the new variables α, X_c and W_c, then $KC_y=WX^{-1}$ implies $KC_y=\alpha^{-1}W_c C_y$. As a result, a matrix K satisfying the previous equality can be chosen in the form

$$K = \alpha^{-1} W_c. \tag{29}$$

Remark 4 *By means of these changes of variables, two advantages are obtained: (1) the gain matrix K can be isolated, and (2) a zero-nonzero structure on the matrix K can be specified a priori, by imposing a desired structure on W_c in the corresponding LMI. The same idea could be applied to the output feedback guaranteed cost control.*

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5 Conclusions

Decentralized control strategies, when information structure constraints appear, have been discussed in the paper. Some generic classes of systems with different control criteria have been considered. Two kinds of control laws, state feedback and output feedback control laws, have been studied. In both cases, the possibility to obtain a preassigned gain matrix K has been commented.

References

[1] L. Bakule, F. Paulet-Crainiceanu, J. Rodellar, and J.M. Rossell. Overlapping reliable control for a cable-stayed bridge benchmark. *IEEE Transactions on Control Systems Technology*, 13(4):663–669, 2005.

[2] L. Bakule and J. Rodellar. Decentralized control and overlapping decomposition of mechanical systems. part 1: System decomposition. part 2: Decentralized stabilization. *International Journal of Control*, 61(3):559–587, 1995.

[3] L. Bakule, J. Rodellar, and J.M. Rossell. Generalized selection of complementary matrices in the inclusion principle. *IEEE Transactions on Automatic Control*, 45(6):1237–1243, 2000.

[4] L. Bakule, J. Rodellar, and J.M. Rossell. Structure of expansion-contraction matrices in the inclusion principle for dynamic systems. *SIAM Journal on Matrix Analysis and Applications*, 21(4):1136–1155, 2000.

[5] L. Bakule, J. Rodellar, and J.M. Rossell. Controllability-observability of expanded composite systems. *Linear Algebra and its Applications*, 332-334:381–400, 2001.

[6] L. Bakule, J. Rodellar, J.M. Rossell, and P. Rubió. Preservation of controllability-observability in expanded systems. *IEEE Transactions on Automatic Control*, 46(7):1155–1162, 2001.

[7] M. Ikeda and D.D. Šiljak. Overlapping decompositions, expansions and contractions of dynamic systems. *Large Scale Systems*, 1(1):29–38, 1980.

[8] M. Ikeda and D.D. Šiljak. Overlapping decentralized control with input, state, and output inclusion. *Control-Theory and Advanced Technology*, 2(2):155–172, 1986.

[9] M. Ikeda, D.D. Šiljak, and D.E. White. Decentralized control with overlapping information sets. *Journal of Optimization Theory and Applications*, 34(2):279–310, 1981.

[10] M. Ikeda, D.D. Šiljak, and D.E. White. An inclusion principle for dynamic systems. *IEEE Transactions on Automatic Control*, 29(3):244–249, 1984.

[11] S.S. Stanković, X.-B. Chen, and D.D. Šiljak. Stochastic inclusion principle applied to decentralized automatic generation control. *International Journal of Control*, 72(3):276–288, 1999.

[12] S.S. Stanković, M. Stanojević, and D.D. Šiljak. Decentralized overlapping control of a platoon of vehicles. *IEEE Transactions on Control Systems Technology*, 8(5):816–832, 2000.

[13] S.S. Stanković and D.D. Šiljak. Inclusion principle for linear time-varying systems. *SIAM Journal on Control and Optimization*, 42(1):321–341, 2003.

[14] D.M. Stipanović, G. Inalhan, R. Teo, and C.J. Tomlin. Decentralized overlapping control of a formation of unmanned aerial vehicles. *Automatica*, 40:1285–1296, 2004.

[15] A. Swarnakar, H.J. Marquez, and T. Chen. A design framework for overlapping controllers and its industrial application. *Control Engineering Practice*, 17:97–111, 2009.

[16] D.D. Šiljak. *Decentralized Control of Complex Systems*. Academic Press, New York, USA, 1991.

[17] D.D. Šiljak and A.I. Zečević. Control of large-scale systems: Beyond decentralized feedback. *Annual Reviews in Control*, 29:169–179, 2005.

[18] A.I. Zečević and D.D. Šiljak. Design of robust static output feedback for large-scale systems. *IEEE Transactions on Automatic Control*, 49(11):2040–2044, 2004.

[19] A.I. Zečević and D.D. Šiljak. A new approach to control design with overlapping information structure constraints. *Automatica*, 41:265–272, 2005.

[20] A.I. Zečević and D.D. Šiljak. Control design with arbitrary information structure constraints. *Automatica*, 44:2642–2647, 2008.