

TRANSIENT ANALYSIS OF RF CAVITIES UNDER BEAM LOADING

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Abstract

The conventional electrical model analogy of a RF cavity is a shunt RLC circuit supplied by two current sources representing the RF amplifier and the beam. In the literature, the impedance of the cavity is often calculated in the Fourier domain. This type of cavity modelling has two drawbacks: First, it assumes a perfect matching between the cavity and the amplifier therefore it neglects the reflected voltage. And, second, it does not provide any information about the cavity transient response, for example at start-up or upon beam arrival, while this information can be very important for the design of the regulation loops. In this work we will remove these drawbacks by calculating the cavity impedance in Laplace domain taking the reflected voltage into account. We will then modify our model so that it also includes the influence of the beam on the cavity. For transient RF simulations, though, a typical problem is the long simulation time due to the relatively slow transient response compared to the RF period. To overcome this problem, finally, we will use a mathematical method to map the cavity frequency response from RF to baseband to reduce the simulation time significantly.

CAVITY IMPEDANCE

The conventional electrical model of RF cavities is shown in Fig. 1 [1]:

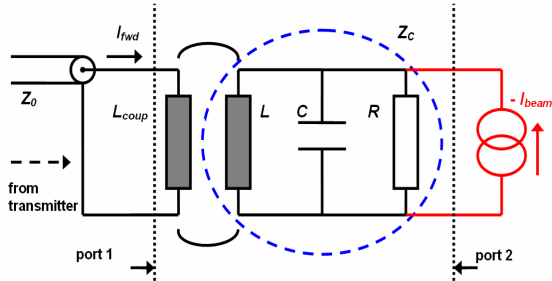


Figure 1: Conventional cavity model with beam.

In this model the RLC circuit and the transformer represent the cavity and its input power coupler respectively.

Cavity Impedance Without Beam

Based on the model shown in Fig. 1, the impedance of the cavity without beam looking into the port 1 can be calculated as the following:

$$Z_{amp}(s) = \frac{V_{cav}(s)}{I_{fwd}(s)} = \frac{2\beta Z_0 L s}{RLC.s^2 + L(\beta + 1).s + R} \quad (1)$$

In this equation $V_{cav}(s)$ is the cavity voltage (the sum of

forward and reflected voltages), $I_{fwd}(s)$ is the cavity forward current (on the transformer primary side), β is the coupling factor and Z_0 is the characteristic impedance of the transmission line feeding the cavity.

Cavity Impedance with Beam

Looking into the port 2 in Fig. 1 the cavity impedance seen by the beam can be calculated as the following:

$$Z'_{beam}(s) = \frac{V'_{cav}(s)}{-I'_{beam}(s)} = \frac{RL.s}{RLC.s^2 + L(\beta + 1).s + R} \quad (2)$$

where the negative sign in front of the beam current $I'_{beam}(s)$ is due to the negative charge of the circulating particles (electrons) and primes are used when the corresponding variable is on the secondary side. Using the Superposition Theorem the total cavity voltage due to the amplifier and the beam can be calculated as the following:

$$V_{total}(s) = I_{fwd}(s).Z_{amp}(s) - I_{beam}(s).Z_{beam}(s) \quad (3)$$

Despite their compact form, Equations (1-3) represent the dynamics of the cavity as seen by the beam and amplifier currents taking the coupler effect into account. Furthermore, as the impedances are calculated in the Laplace domain, they can be used to study the transient response of the cavity. This would be particularly important for the design of the LLRF regulations loops for example when the aim is to analyse the evolution of the cavity voltage (amplitude and phase) with time.

Table 1: ALBA RF Parameters

Parameter	Value	Unit
Energy E	3	GeV
Revolution frequency f_r	1.115	MHz
DC Beam current I_{DC}	400	mA
Beam power per cavity	90	kW
Energy loss per turn U_0	1.02	MeV
Momentum compaction factor α_p	8.8e-4	
Number of cavities	6	
RF frequency f_0	500	MHz
Unloaded shunt impedance R	3.5	M Ω
RF power per cavity P_{cav}	150	kW
Peak RF voltage per cavity	600	kV _p
Unloaded quality factor Q_0	29000	
Cavity coupling factor β	2.67	
Synchronous phase φ_s	159	Deg.
Harmonic number h	448	
Max IOT output power (per IOT)	80	kW

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Figure 2 shows the simulated response of the ALBA cavity with the parameters shown in Table 1. In these simulations the amplifier current starts at $t = 0$ sec and the beam ($I_{DC} = 400$ mA, $\phi_s = 159^\circ$) arrives at $t = 100 \mu\text{s}$ with the cavity detuned by $\Delta f = f - f_0 = 37.5$ kHz and the coupling factor β set to 2.67:

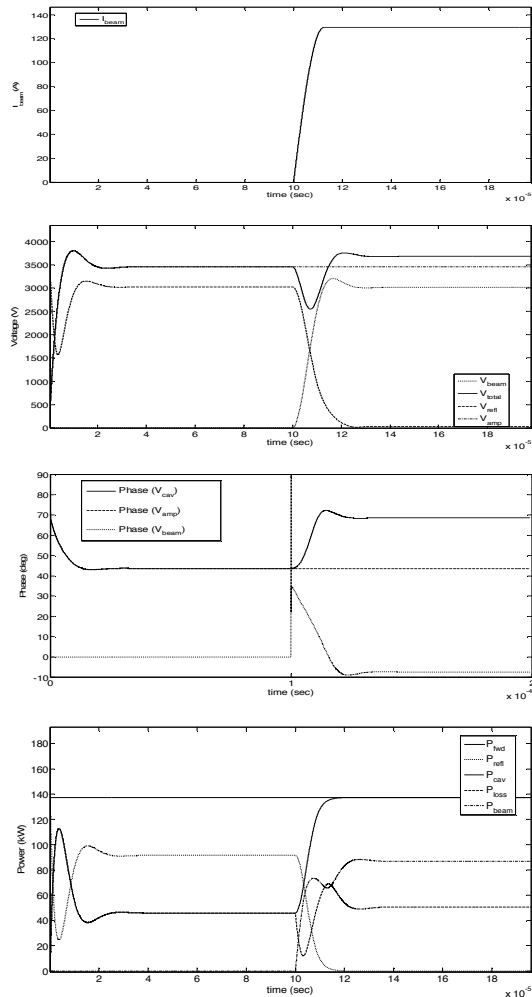


Figure 2: Open-loop simulation results of the cavity with the amplifier and the beam (all voltages are referred to the primary side).

As can be seen in Fig. 2, after the beam arrival, the phase of the cavity voltage settles at its desired value (i.e. $159^\circ - 90^\circ = 69^\circ$). This is due to the right choice of the cavity coupling factor and detuning corresponding to the magnitude and phase of the beam. In this condition, the steady state reflected voltage from the cavity is zero as the cavity behaves like a pure resistive load for the amplifier.

It should be noted however that practically, the beam does not enter the storage ring all at once. In real conditions, a fraction of each bucket around the ring will be filled with each injection shot from the ALBA booster. Therefore, the variations of the voltage imposed on the cavity upon each injection shot will be much smaller than what is shown in the figures. Nevertheless, these simulations give a good insight as show the cavity

dynamics seen by the beam and amplifier currents. It also worth's mentioning that although in these simulations the beam was modelled by a sinusoidal current source running at the RF frequency, Equation (3), in its general form, also gives the possibility to analyse a more realistic beam current (ex. pulsed). This would be useful for example when the aim is to study the influence of the filling pattern on the cavity transient voltage.

IQ-BASED CAVITY FIELD REGULATION

Figure 5 shows a simplified schematic of the ALBA ALLRF (Analog Low Level Radio Frequency) for amplitude and phase regulation [3].

Baseband-Equivalent Model of the IQ Loops

Here, we simulate the control system of Fig. 5 in MATLAB / Simulink. It should be noted, however, that as the complete loop consists of an RF part (i.e. the cavity and the transmitter) and a baseband part (i.e. the LLRF), a conventional simulation would be very slow. This is linked to the comparably slow variations of the baseband signals due to the very high quality factor of the cavity. Here, we apply a mathematical technique based on rotating reference frame [2] to our cavity model to map the RF frequencies into baseband leading to the baseband-equivalent model of the cavity with significantly larger simulation speed but with the same precision as the conventional model. Figure 3 compares the block diagrams of the baseband-equivalent model with the conventional model.

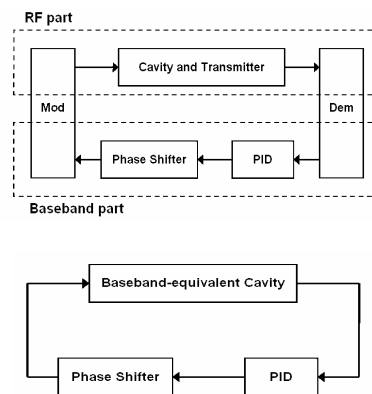


Figure 3: Comparison between the conventional simulation model (above) and the baseband-equivalent model (below) of the regulation loops.

As in the baseband-equivalent model all the signals run at baseband frequency, the simulation time is much shorter. Details on the derivation of the baseband-equivalent model are presented in another article which is currently under preparation by the same authors.

Figure 4 shows the simulation results of the IQ loops regulating the voltage of the matched cavity at $V_{total} = 3705$ V, 69° with the beam ($I_{beam} = 800$ mA, 0°) being imposed at $t = 100 \mu\text{s}$. It should be noted that in these simulations the simulation time was about 1000 times shorter than the conventional model.

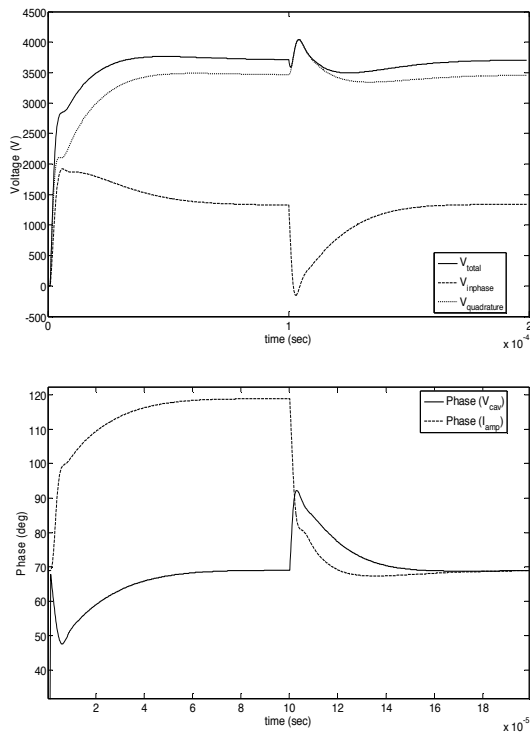


Figure 4: Closed-loop simulations of the baseband-equivalent cavity model with beam.

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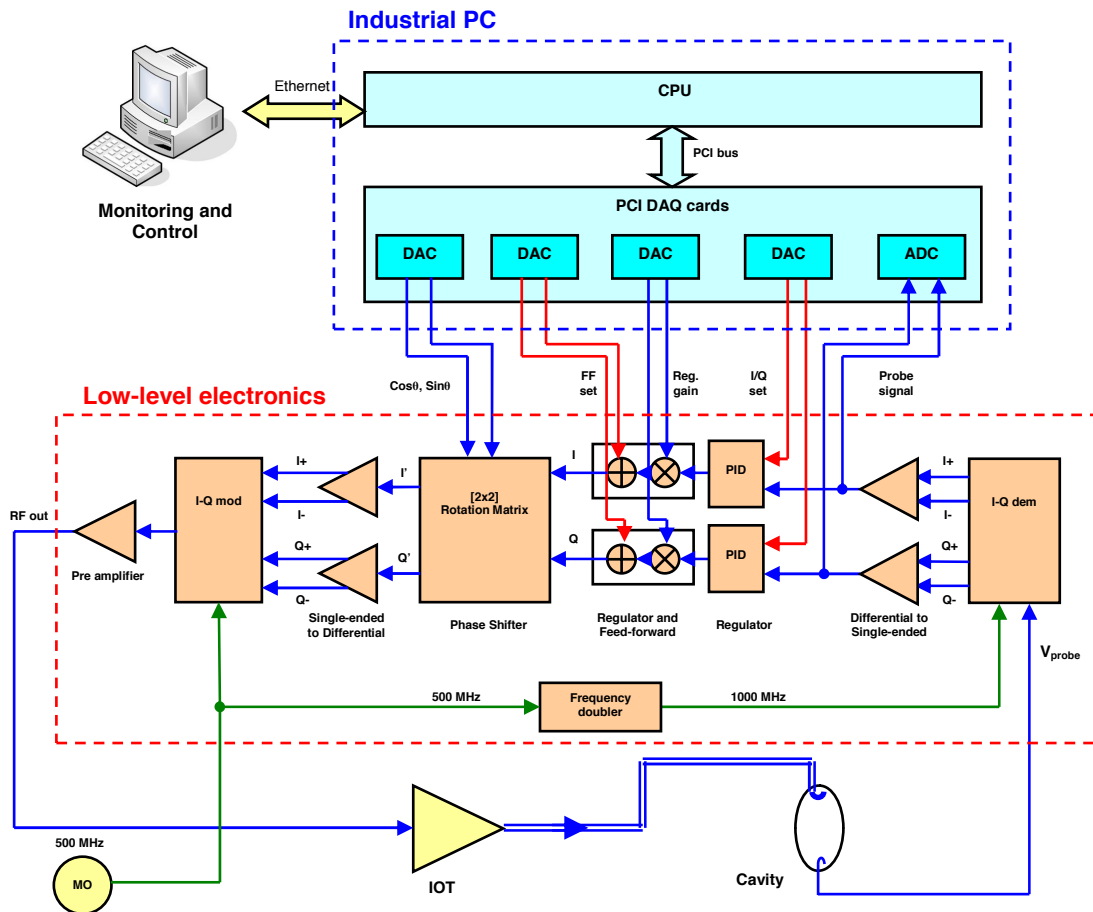


Figure 5: Schematics of the real implementation of the IQ-based amplitude and phase loops of the ALBA ALLRF.