

DISTURBANCE DECOUPLING FOR SINGULAR SYSTEMS BY PROPORTIONAL AND DERIVATIVE FEEDBACK AND PROPORTIONAL AND DERIVATIVE OUTPUT INJECTION

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Abstract

We study the disturbance decoupling problem for linear time invariant singular systems. We give necessary and sufficient conditions for the existence of a solution to the disturbance decoupling problem with or without stability via a proportional and derivative feedback and proportional and derivative output injection that also makes the resulting closed-loop system regular and/or of index at most one. All results are based on canonical reduced forms that can be computed using a complete system of invariants that can be implemented in a numerically stable way.

Key words

Singular Systems, equivalence relation, disturbance decoupling.

1 Introduction

We consider linear and time-invariant continuous singular systems of the form

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + Gg(t), & x(t_0) = x_0, & t \geq 0 \\ y(t) = Cx(t), \end{cases} \quad (1)$$

(1) where $E, A \in M_n(C)$, $B \in M_{n \times m}(C)$, $C \in M_{p \times n}(C)$, $G \in M_{n \times q}(C)$ and $\dot{x} = dx/dt$. The term $g(t)$, $t \geq 0$, represents a disturbance, which may represent modeling or measuring errors, noise, or higher order terms in linearization. Singular systems arise naturally in circuits design, mechanical multibody systems and a large variety of the applications (see [5] and [6], for example), and they have been studied under different points of view. The problem of constructing feedbacks and/or output injections that suppress this disturbance in the sense that $g(t)$ does not affect the input-output behavior of the system is analyzed. In the case of standard state space systems the disturbance decoupling problem has been largely studied (see [1],[7],[8]

for example), This problem for singular systems has also been studied (see [2], [4] for example). In this paper we study the disturbance decoupling problem for singular systems that can be stated as follows: Find necessary and sufficient conditions under which we can choose state and derivative feedback as well state and derivative output injection such that, the matrix pencil $(E + BF_E^B + F_E^C C, A + BF_A^B + F_A^C C)$ is regular of index at most one and

$$C(s(E + BF_E^B + F_E^C C) - (A + BF_A^B + F_A^C C))^{-1} G = 0.$$

We assume without loss of generality that matrices B , G are full column rank and C is full row rank, i.e., $\text{rank } B = m$, $\text{rank } G = q$, $\text{rank } C = p$. If this is not the case, then this can be easily achieved, by removing the nullspaces and appropriate renaming of variables.

2 Notations

In the sequel we will use the following notations.

- I_n denotes the n -order identity matrix,
- N denotes a nilpotent matrix in its reduced form $N = \text{diag}(N_1, \dots, N_t)$, $N_i = \begin{pmatrix} 0 & I_{n_i-1} \\ 0 & 0 \end{pmatrix} \in M_{n_i}(C)$,
- J denotes the Jordan matrix $J = \text{diag}(J_1, \dots, J_t)$, $J_i = \text{diag}(J_{i_1}, \dots, J_{i_s})$, $J_{i_j} = \lambda_i I_{i_j} + N$,
- L denotes the diagonal matrix $L = \text{diag}(L_1, \dots, L_q)$, where $L_j = \begin{pmatrix} I_{n_j} & 0 \\ 0 & 0 \end{pmatrix} \in M_{n_j \times (n_j+1)}(C)$,
- R denotes the diagonal matrix $R = \text{diag}(R_1, \dots, R_p)$, where $R_j = \begin{pmatrix} 0 & I_{n_j} \\ 0 & 0 \end{pmatrix} \in M_{n_j \times (n_j+1)}(C)$.

We represent systems of the form (1) as quadruples of matrices (E, A, B, C) in the case of disturbance do not appear or it is not considered and a quintuples of matrices (E, A, B, C, G) otherwise.

- iii) $C_1(sI_{n_3} - N_4)^{-1}G_3 = 0$ if and only if $\text{rank} \begin{pmatrix} sI_{n_3} - N_4 & G_3 \\ C_2 & 0 \end{pmatrix} = n_3$
- iv) $(sI_{n_4} - J)^{-1}G_4 = 0$ if and only if $G_4 = 0$
- v) $(sN_1 - I_{n_5})^{-1}G_5 = 0$ if and only if $G_5 = 0$

As a consequence we have.

Corollary 1. Let (E, A, B, C, G) a quintuple of matrices in its reduced form, and we assume $\bar{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ according to the decomposition of the system. If $G_2 = 0$, $G_4 = 0$, $G_5 = 0$, $\text{rank} \begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} = n_1$ and $\text{rank} \begin{pmatrix} sI_{n_3} - N_4 & G_3 \\ C_1 & 0 \end{pmatrix} = n_3$, then the given system is trivially disturbance decoupled.

The disturbance decoupling problem is called with stability if one imposes the additional constraint that the close-loop $(E + BF_E^B + F_E^C C)\dot{x}(t) = (A + BF_A^B + F_A^C C)x(t) + Bu(t) + Gg(t)$, $y(t) = Cx(t)$ system is stable. Remember that a singular system is stable if and only if the spectrum of the system lies in C^{-1} .

Proposition 4. Given a singular system (E, A, B, C) . There exist a proportional and derivative feedback as well a proportional and derivative output injection such that the close-loop system $(E + BF_E^B + F_E^C C, A + BF_A^B + F_A^C C, B, C)$ is stable (and we call stable under proportional and derivative feedback and proportional and derivative output injection) if and only if $\text{rank} \begin{pmatrix} sE - A & B \\ C & 0 \end{pmatrix} = n$, $\forall s \in C^+$.

Proof. The spectrum of a system coincides with the spectrum of the associate pencil, and the spectrum is invariant under equivalence relation.

As a consequence we have.

Corollary 2. Let (E, A, B, C, G) a quintuple of matrices in its reduced form, and we assume $\bar{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ according to the decomposition of the system. If $G_2 = 0$, $G_4 = 0$, $G_5 = 0$, $\text{rank} \begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} = n_1$, $\text{rank} \begin{pmatrix} sI_{n_3} - N_4 & G_3 \\ C_1 & 0 \end{pmatrix} = n_3$ and $\sigma(J) \subset C^{-1}$. Then the given system is trivially disturbance decoupled with stability.

5 Conclusions

In this paper a qualitative description of the disturbance decoupling problem is considered and a necessary and sufficient condition for the existence of a proportional and derivative feedback as well a proportional and derivative output injection such that the close-loop

system is regular with index at most one and for systems in its reduced form a condition for decoupling is presented.

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