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Development of frequency-domain source models for railway vibration impact assessment

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ABSTRACT

Many studies have been focused on the development of vibration source models, which comprise both the force applied on the track top and the vibration behaviour of the fixation system. Numerous researchers model the interaction between wheel and rail as a constant load moving along the track. Other authors consider a combination of harmonic and non-harmonic moving axle loads. Whereas most of these models are intended to validate time-domain vibration results, they become useless for predicting the frequency-domain vibration impact far from the track, data which is required in order to assess the fulfilment implied by most of the national regulations.

In this work, two source models are presented and described. The first model comprises a series of empirical-statistical models, based on vibration measurements carried out in rail tracks. These models allow predicting the mean frequency-dependent applied force by high-speed, conventional and underground rolling stock. The second model consists in an analytic-deterministic approach based on the theoretical model of the wheel-rail deformation. This deformation is used to obtain the wheel-rail contact force through the Hertz's Theory of mechanical contacts. The model includes the superstructure motion, considering the rail as a Bernoulli-Euler beam, the sleepers as a punctual mass, and the pad, ballast and ground impedances.

These source models will be included in the prediction tool for evaluating the vibration impact for new railway infrastructures, which is being developed within the CATdBTrén project. This project has been awarded a R&D funding from the Catalanian Government.

1. INTRODUCTION

The CATdBTrén project is aimed to develop a prediction tool allowing the evaluation of the vibration impact for new railway infrastructures. Moreover, new types of fastening systems with high vibration isolation properties will be designed. This software will include models of contact forces caused by high-speed, conventional and underground rolling stock. It will be able to reproduce infrastructure vibration transmission behaviour, ground vibration propagation, terrain-foundation coupling and building vibration behaviour. Thus, the CATdBTrén prediction tool will estimate the influence of the rolling stock, rail and wheel roughness, fastening system, substructure, soil propagation properties and building characteristics, all in the final vibration impact. This tool is intended to be user-friendly and to produce results with average accuracy,

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so detailed studies of problematic areas will still be required. All the elements in the prediction chain are shown in Figure 1:

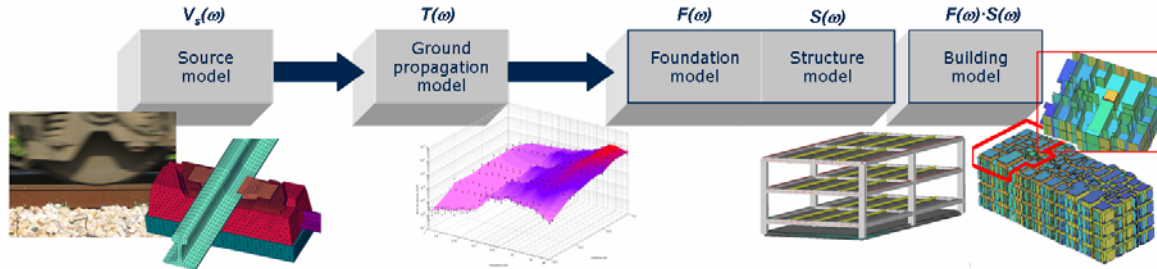


Figure 1: Calculation flow for the complete prediction model

Consequently the one-third octave band vibration level inside the building $V_r(\omega)$ will be calculated using the ground vibration level at track point $V_s(\omega)$ and the transfer functions of propagation through the terrain $T(\omega)$, through the building foundations $F(\omega)$ and through the building structure $S(\omega)$, according to the expression:

$$V_r(\omega) = V_s(\omega) \cdot T(\omega) \cdot F(\omega) \cdot S(\omega) \quad (1)$$

The source model will provide the ground vibration level at track point $V_s(\omega)$. Two calculation methodologies will be available. The first one comprises a collection of empirical-statistical models both for rolling stock and fastening and superstructure systems. These models are based on vibration and extensimetric measurements carried out in in-situ and in-service rail tracks. The second one is an analytic-deterministic model based on the theoretical model of the wheel-rail deformation and the superstructure motion. This second (deterministic) model requires a large amount of data but it allows assessing the variations due to modifications in the parameters. In case a limited scope of data is available, it is preferable to use the first (statistical) model which also gives the engineer the information regarding the expected variability in the prediction.

2. ANALYTICAL SOURCE MODEL

A. Wheel-rail contact model

An analytical model, based on the Hertz's Theory of Elastic Contacts^{1,2,3}, is presented here. This analytical approach allows to calculate the contact force $F(t)$ by means of the wheel-rail deformation $\delta(t)$; where this deformation is function of the vertical motion and the irregularities of both rail track and the wheel. The Hertz's Theory is based on the expression below:

$$F(t) = k_{Hertz} \delta(t)^{3/2} \quad (2)$$

where k_{Hertz} can be calculated from the geometry of the solids in contact, the rail and the wheel. Likewise, the wheel-rail deformation time history $\delta(t)$ can be evaluated from the motion of the superstructure and the rolling stock. Then, the expression (3) shows the same contact force, now in terms of wheel vertical displacement $y_w(t)$, rail vertical displacement $y_r(t)$ and irregularities of both solids $\xi(t)$.

$$F(t) = k_{Hertz} (y_r(t) - y_w(t) - \xi(t))^{3/2} \quad (3)$$

B. Superstructure model

It is undoubtedly important to define a superstructure model that allows to calculate the rail track vertical motion. Furthermore, to know the vibration level at near ground due to railway transit, the dynamics of superstructure is the most important factor.

Basically, there are two types of models in literature: continuous^{4,5,6,7} and discrete^{8,9} models. The difference between these two approaches resides in the treatment of sleepers, considering those as a continuous or as a discrete foundation, respectively. Several references show that both systems have only two significant natural frequencies. This fact facilitates the simplification of the model, becoming a two-degrees-of-freedom system, with its equivalent masses, stiffnesses and dampings.

Therefore, the proposed analytical model is a simplification of the typical discrete model, where the rail track will be taken as a punctual mass. The stiffness and damping coefficients below the rail track of this equivalent system will be calculated by means of the application of a force of known amplitude and phase (see Figure 2).

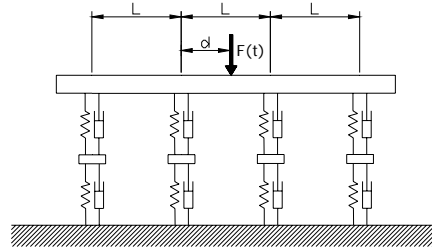


Figure 2: Discrete parameters model

The equation of motion of the entire rail, assuming it as an Euler-Bernoulli beam with a vertical moving load and with a discrete sleeper foundation, has the following form:

$$EJ \frac{\partial^4 y_r}{\partial x^4} + \mu \dot{y}_r + k_{pad} (y_r - y_s) + c_{pad} (\dot{y}_r - \dot{y}_s) = -F(t) \cdot \delta(x - c_{train}t) \quad (4)$$

where E is the rail's Young modulus, J is the inertia moment of the rail, μ is the rail mass per unit length, k_{pad} and c_{pad} are the impedance of the fixation system and $\delta(x - c_{train}t)$ is the delta Dirac function that describes the contact force movement along the rail. On the other hand the equation of motion of any discrete sleeper is:

$$k_{pad} (y_r - y_s) + c_{pad} (\dot{y}_r - \dot{y}_s) - k_{B-G} y_s - c_{B-G} \dot{y}_s = m_t \ddot{y}_s \quad (5)$$

where k_{B-G} and c_{B-G} are the impedance of both ballast and "sub-ballast", m_t is the mass of a sleeper and $y_s(t)$ is the sleeper vertical displacement. Figure 3 represents the resulting simplified model. It considers the effects of ballast and "sub-ballast" independently. The system "sub-ballast" includes all layers under ballast: the ground and the superstructure foundation.

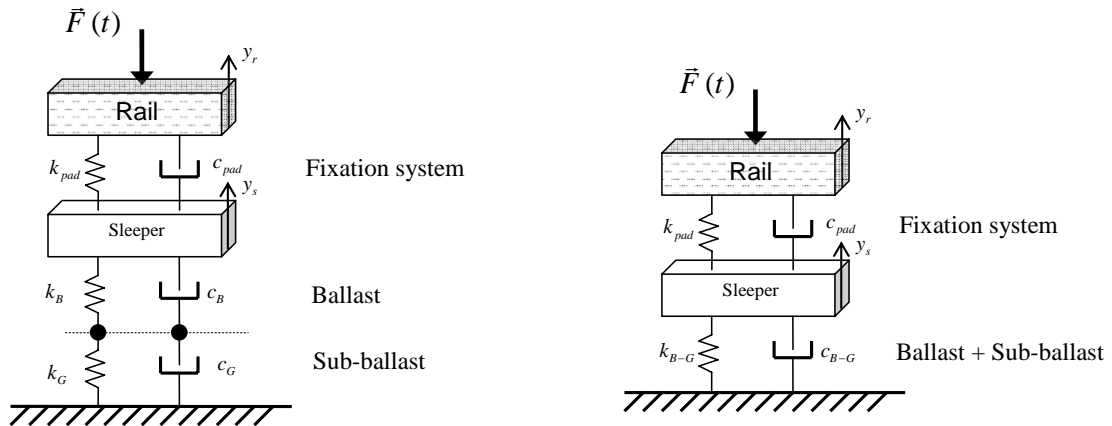


Figure 3: The simplified 2DOF model

Finally and to obtain an accurate enough model, it is necessary to adjust the frequency response curves, mainly the natural frequencies, of complete and simplified models.

C. Rolling stock model

Over the rail track, the model is completed with the rail-wheel contact force, the wheel (another punctual mass), the bogie's primary suspension (a damped spring) and the static force (weight) of the train. The sprung mass is considered as a fixed solid: its natural frequencies are much lower than the excitation frequencies of the wheel-rail contact force. Figures 4 and 5 graphically represent this model.

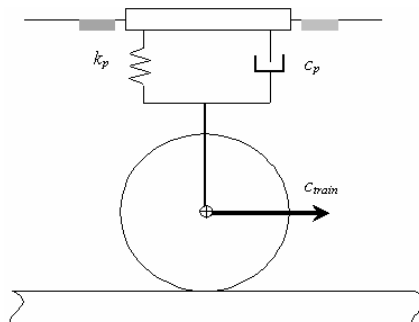


Figure 4: Rolling stock model including the wheel and the primary suspension

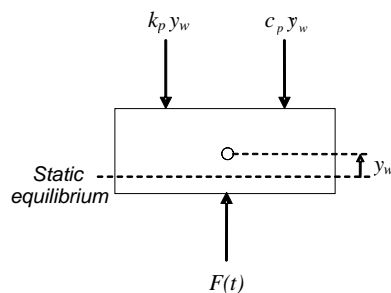


Figure 5: Free solid diagram of the wheel

As figure 4 shows, the secondary suspension is not taken into account. The equation of motion of the wheel can be written as follows:

$$m_w \ddot{y}_w + c_p \dot{y}_w + k_p y_w = F(t) \quad (6)$$

where k_p and c_p are the impedance primary suspension, m_w is the wheel mass and $y_w(t)$ is the wheel vertical displacement.

D. Roughness description

The irregularities of any surface, like a rail or wheel contact profiles, are random functions. Therefore, they have been often described by means of their power spectral density in the spatial domain.

Several authors show that these roughness profiles of rail and wheel could be considered a zero mean Gaussian isotropic random field in the spatial domain, and a normal stationary ergodic random process in the time domain. So, the roughness can be described by its power spectral density in the wavelength domain^{10,11} $S_\xi(k)$, and it could be transformed into the frequency domain $S_\xi(\omega)$ by means of the train velocity c_{train} :

$$S_\xi(k) = c_{train} \cdot S_\xi(\omega) \quad (7)$$

Once the PSD is known, a time history of roughness $\xi(t)$ can be rebuilt. This process uses a random phase cosines summation, each one of them with its amplitude, which can be computed from the PSD in the frequency domain. The next expressions describe the process with more accuracy.

$$\xi(t) = \sum_{k=1}^M A_k \cos(\omega_k t + \varphi_k) \quad (8)$$

$$\omega_k = \omega_1 + \left(k - \frac{1}{2}\right) \Delta\omega; \quad \Delta\omega = (\omega_m - \omega_1) / M; \quad (9)$$

$$A_k = \sqrt{2S_\xi(\omega_k) \Delta\omega} \quad (10)$$

where A_k is the amplitude of each cosine function, ω_k is the frequency of each cosine function, ω_1 and ω_m are the lower and upper limits of the frequency range, respectively, $\Delta\omega$ is the frequency resolution, M is the resolution of the system and φ_k is a random phase. This process generates a roughness description that briefs the roughness profile data in a very useful way. This description allows classifying all the variety of temporal histories in a few typologies of roughness, which depend on the maintenance, the steel mechanical properties and others.

E. Complete model

It is clear from the foregoing that the contact force model and the superstructure and rolling stock models are coupled to each other, and they form a single model that is a 4-DOF system, which includes the vertical motion of the wheel, the rail, the sleeper and the ground.

3. STATISTICAL SOURCE MODEL

A. Train model

As seen before, the vibration generation phenomenon depends on the characteristics of both rolling stock and fastening system as well as the coupled interaction between them. On the other hand, the statistical source model is intended to comprise independent models for rolling stock and fastening system in order to allow the user to select any combination of train and fixation. So it becomes necessary to determine at what frequency this coupling can be considered negligible and independent models can be defined.

In order to characterize the energy input due to passing-by trains, the rail vertical acceleration between two sleepers was chosen. At medium and high frequencies, this location is not supposed to depend strongly on the fixation system or the terrain stiffness. Several measurement campaigns were performed in order to characterize each rolling stock typology. It was necessary to carry out measurements in numerous locations in order to assess the frequency-dependent influence of the superstructure typology. Figure 6 shows the pass-by of a train (left hand) and several measurement locations in rail and sleeper (right hand).



Figure 6: Train pass-by (left hand) and accelerometer locations in rail and sleeper (right hand)

Additionally, each rolling stock unit has a unique roughness profile that depends mostly on the date of the last maintenance operation and such other factors as the number and intensity of braking manoeuvres^{12,13}. Hence, as it can be seen experimentally, each unit produces a unique vibration spectrum. Figure 7 shows the rail vibration levels measured for four regional trains (left hand) and for six freight trains (right hand) in two tested locations.

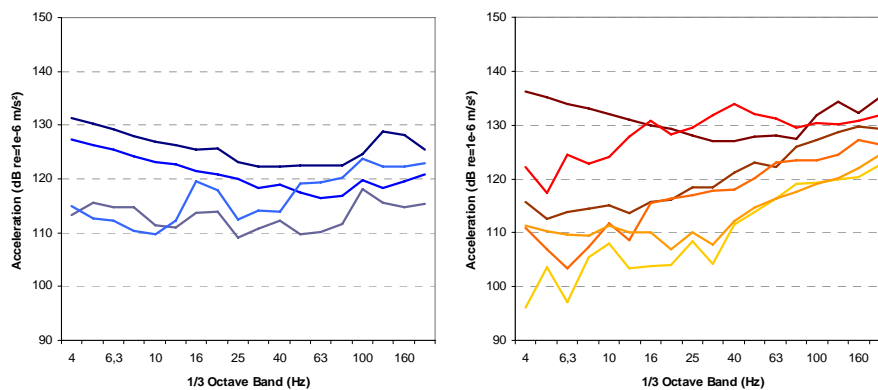


Figure 7: Rail vibration levels measured for four regional trains (left hand) and for six freight trains (right hand) each group passing by with approximately the same speed

In this case, variations up to 18 dB and 51 dB are observed (regional and freight train, respectively) which suggest the existence of a strong statistical behaviour. Figure 8 shows the standard deviation due to the pass of trains measured in different locations with several fixation systems.

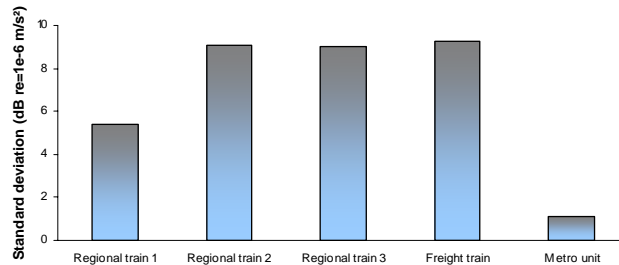


Figure 8: Standard deviation for several rolling stock typologies including regional, freight and metro trains

As it can be seen, measurements of the regional train typology 1 pass-by, which is the most modern regional train in this set, have the smallest standard deviation. On the other hand, the metro unit was unique in all the concerned measurements, so the standard deviation is related in this case only to the experimental repeatability. If the experimental campaign was broad enough, it would be possible to define the statistical source models including mean and variance data for each typology of rolling stock. Figure 9 shows the mean and the expected vibration values in 95% confidence range.

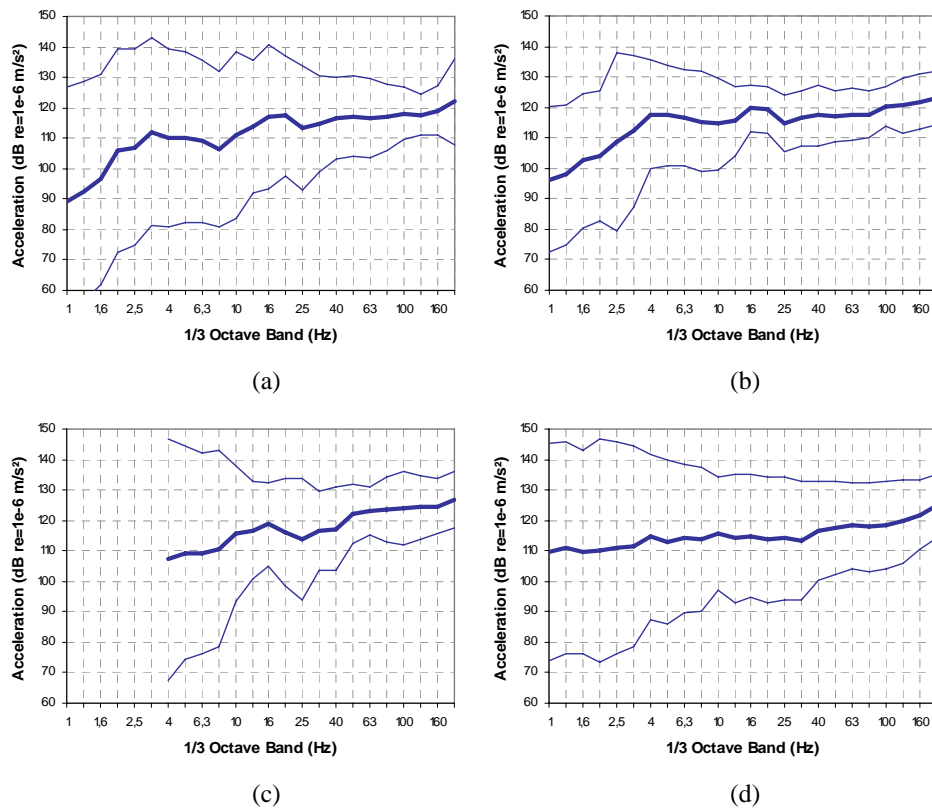


Figure 9: Rail pass-by measured acceleration levels for regional trains (a, b, c) and freight trains (d). Mean level in thick line, 95% confidence range in thin line

These source models allow not only to predict the input vibration levels but also to understand the variability of those as well as the real uncertainty when dealing with this kind of predictions. Finally, these data will be used as input for the superstructure model, which is described below.

B. Superstructure model

As it was mentioned, the superstructure models consist in the transfer function between the rail vertical vibration level and the ground vibration level ($V_s(\omega)$). Once again obtaining significant data requires a large quantity of measurements. In this case, several fastening systems were tested: Ballast, ATD, Rheda 2000 and Rheda Dywidag¹⁴. Figure 10 shows the transfer functions obtained for ballast and ATD fastening systems. Comparison between the mean data for ballast and all the fastening systems is shown in Figure 11.

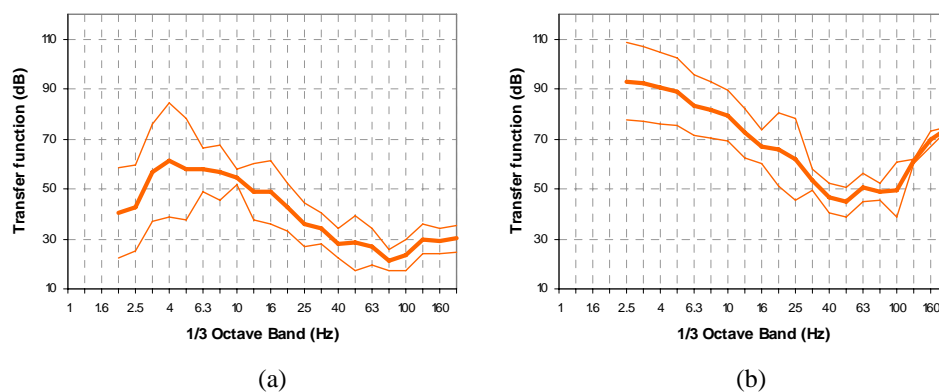


Figure 10: Measured transfer functions for Ballast (a) and ATD fastening system (b) (differences between rail and ground vertical vibration levels). Mean level in thick line, 95% confidence range in thin line

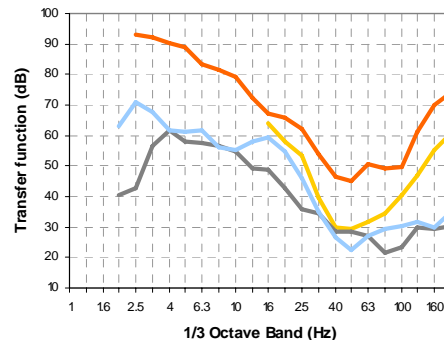


Figure 11: Mean transfer functions (differences between rail and ground vertical vibration levels) for ballast (grey line), ATD (orange line), Rheda 2000 (yellow line) and Rheda Dywidag (blue line)

As it can be seen, these mean values allow to assess the vibration isolation efficiency for each superstructure system. And as it was expected¹⁵, ballast produces the worst isolation results in almost all the frequency bands. Figure 12 shows the standard variation measured for each superstructure typology. In this case, considering the influence of the passing-by rolling stock as negligible, these variations are related mostly to the experimental repeatability. Here standard variations are not close to zero but around 5 dB, which implies a strong stochastic behaviour. This conclusion points out the convenience of considering the statistical point of view and the uncertainty, always intrinsically associated with this kind of predictions.

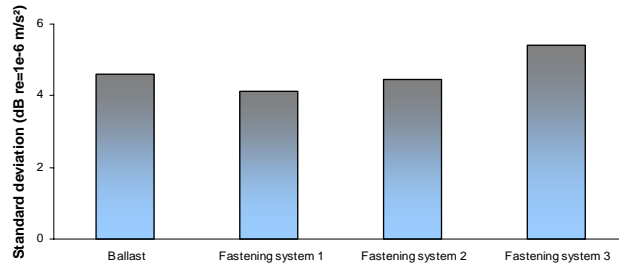


Figure 12: Standard deviation for ballast and for other three assessed fastening systems

C. Complete statistical source model

Assembling both the train and superstructure models in a complete train-superstructure vibrational source model requires operating as below (in decibels):

$$m = m_r + m_s \quad (11)$$

$$\sigma = \sqrt{\sigma_r^2 + \sigma_s^2} \quad (12)$$

Where m is the mean ground vibration level $\langle V_s(\omega) \rangle$ and m_r and m_s are the mean rail vertical acceleration and the mean fastening system transfer function, respectively. σ is the standard deviation for the ground vibration level and σ_s and σ_r are the standard deviation for the rail vertical acceleration and for the fastening-system transfer function, respectively.

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