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A MILP model for the Visibility Windows Assembly Line Balancing Problem (VWALBP): the case of the Müller-Hannemann & Weihe problem[†]

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ABSTRACT

This note introduces the Visibility Windows Assembly Line Balancing Problem (VWALBP), in which only a portion of the workpieces can be reached from each station, and proposes a MILP model for the specific case described by Hannemann & Weihe in a previous paper.

1. Introduction

Assembly lines are an important element of many production systems in which a wide variety of objects can be produced.

Usually, in the literature regarding assembly line problems, it is assumed implicitly that, at any cycle, there is exactly one workpiece in each station and that all the workpiece is visible from any station at any cycle. However, the *visibility windows* of the workstations may not coincide with the dimensions of the workpieces and this gives rise to the problem that we name Visibility Windows Assembly Line Balancing Problem (VWALBP).

In particular, in some assembly lines, the size of the workpiece is large relative to the dimensions of the workstations and, because of this, only a specific limited portion of the unit can be reached from each station at any moment.

In many actual assembly lines a cycle decomposes into stationary stages in which the workpieces stood motionless. Between stationary stages, the assembly line, with all the workpieces on it, moves forward. The size of the forward steps may be equal or different and usually it is a multiple of a given quantity that depends on the technology of the assembly line. The operations can only be performed during the stationary stages. At any stage, therefore, only a subset of the tasks that make up the production process are within reach of each station and are the only tasks that can be performed during the stage.

Therefore, during a given stationary stage, one workpiece may be processed by several stations and one workstation may process several workpieces.

Müller-Hannemann &Weihe (2006) deals with a specific case of the VWALBP, in which the workpieces are large relating to the size of the visibility windows, whose main assumptions are: (i) the workpieces have only two relevant dimensions (in the real

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case considered in Müller-Hannemann &Weihe's paper they are PC boards); (ii) all workpieces are identical; (iii) the number of stations is fixed; (iv) each task corresponds to a defined position in the workpiece and this position is defined by a single coordinate, since the visibility windows of the workstations are limited by two different values corresponding to the, say, the horizontal axe of coordinates and do not have any practical limit concerning the vertical axe; (v) each task can be performed only in one specific station (i.e., the tasks are assigned a priori to the workstations); (vi) there are no precedence relationships between the tasks; (vii) the distance between the left borders of two consecutive workpieces is constant; (viii) the visibility windows of the workstations do not overlap; (ix) all the forward movements must be a multiple of a given elementary step. The authors formalise the problem and propose a heuristic algorithm to solve it.

The purpose of the present paper is to introduce a Mixed Integer Linear Programming (MILP) model for the problem stated in Müller-Hannemann & Weihe (2006). Next section contains the notation, adapted from that used in Müller-Hannemann &Weihe (2006) and the model.

2. A MILP model for the Müller-Hannemann &Weihe problem

Data

N	number of jobs (tasks)
m	number of machines (stations)
$\left[L_i, R_i\right]$	visibility region $\left(i=1,...,m\right)$, where: $L_1 = 0$ $L_i < R_i \quad i=1,...,m$ $R_i < L_{i+1} \quad i=1,...,m-1$
A_0	size of the workpiece (which we will cal length) along the horizontal axe (assuming that the direction of the movement of the pieces coincides with that of this axe),
A	distance between the left borders of two successive workpieces on the assembly line; therefore, equal to the workpiece length, A_0, plus the gap between two consecutive pieces.
T	setup time (depends on the time for accelerating and slowing down the workpieces plus the time for resetting the robot arms between two consecutive stationary stages); the total time of the cycle is equal to the sum of (i) the times corresponding to the stationary stages constituting a cycle plus (ii) the setup time multiplied by the number of stationary stages plus (iii) the time for moving the pieces forward along the cycle at the maximal speed (since this time is constant it can be disregarded as long as the optimisation is concerned).
Δ	length of an elementary step (all the forward steps of the line must be a multiple of Δ; therefore, $\gcd\left(A,\Delta\right)=\Delta$).
J_0	set of jobs (tasks); $\left\|J_0\right\| = N$
J_i	set of jobs to be performed on machine i $\left(i=1,...,m\right)$, where: $\bigcup_{i=1,...,m} J_i = J_0$ and $J_i \cap J_k = \varnothing \quad \forall i,k$

p_j processing time of job j $\left(j = 1,\ldots,N\right)$

a_j $\left(0 \le a_j \le A_0\right)$ distance to the right border of the workpiece corresponding to the job j $\left(j = 1,\ldots,N\right)$

S upper bound on the number of forward steps in a cycle $\left(S \le \frac{A}{\Delta}\right)$. $S = \frac{A}{\Delta}$ *iff* all the forward steps consists in one single elementary step; specific considerations may lead to a lower value of this upper bound.

Variables

$x \ge 0$ initial shift of the right border of the workpiece with respect to the left limit of station 1 (recall that $L_1 = 0$) : $0 \le x \le R_1 + a_1^{\min}$, where $a_1^{\min} = \min_{j \in J_1} a_j$.

$\delta_s \in Z^+$ number of elementary steps of the forward step s $\left(s = 1,\ldots,S\right)$; $\delta_s \le \frac{A}{\Delta}$

$\eta_s \in \{0,1\}$ $\eta_s = 1$ iff the forward step s $\left(s = 1,\ldots,S\right)$ exists (i.e., has a positive number of elementary steps).

$y_{js} \in \{0,1\}$ $y_{js} = 1$ iff the job j is performed during the stationary stage following forward step $s-1$ $\left(j = 1,\ldots,N; s = 1,\ldots,S\right)$

$k_j \in Z^+$ number of workpieces on the line that precede a workpiece when job j is performed on the latter $\left(j = 1,\ldots,N\right)$; $\left\lceil \frac{L_i + a_j - R_1 - a_1^{\min} - A + \Delta}{A} \right\rceil \le k_j \le \left\lfloor \frac{R_i + a_j}{A} \right\rfloor$, where $i \mid j \in J_i$.

C_s completion time, for the whole line, corresponding to the stationary stage following forward step $s-1$ $\left(s = 1,\ldots,S\right)$

Model

$$\left[MIN\right] z = T \cdot \sum_{s=1}^{S} \eta_s + \sum_{s=1}^{S} C_s \tag{1}$$

$$\sum_{s=1}^{S} \delta_s = \frac{A}{\Delta} \tag{2}$$

$$\delta_s \le \frac{A}{\Delta} \cdot \eta_s \qquad s = 1,\ldots,S \tag{3}$$

$$\eta_{s+1} \le \eta_s \qquad s = 1,\ldots,S-1 \tag{4}$$

$$A \cdot k_j - a_j + x + \Delta \cdot \sum_{l=1}^{s-1} \delta_l \ge L_i - M_{js} \cdot \left(1 - y_{js}\right) \quad s = 1,\ldots,S;\; j = 1,\ldots,N;\; i \middle| j \in J_i \tag{5}$$

$$A\cdot k_j - a_j + x + \Delta\cdot\sum_{l=1}^{s-1}\delta_l \le R_i + M'_{js}\cdot\left(1-y_{js}\right) \quad s=1,\ldots,S\,;\; j=1,\ldots,N\,;\; i\,\middle|\, j\in J_i \tag{6}$$

$$\sum_{s=1}^{S} y_{js} = 1 \qquad j=1,\ldots,N \tag{7}$$

$$\sum_{j\in J_i} p_j\cdot y_{js} \le C_s \qquad i=1,\ldots,m\,;\; s=1,\ldots,S \tag{8}$$

$$C_s \le \max_{i=1,\ldots,m}\left(\sum_{j\in J_i} p_j\right)\cdot\eta_s \qquad s=1,\ldots,S \tag{9}$$

$$\sum_{j\in J_i} y_{js} \le \left|J_i\right|\cdot\eta_s \qquad i=1,\ldots,m; s=1,\ldots,S \tag{9'}$$

$$y_{js} \le \eta_s \qquad j=1,\ldots,N; s=1,\ldots,S \tag{9''}$$

where:

$$M_{js} \ge L_i - A\cdot k_j + a_j - x - \Delta\cdot\sum_{l=1}^{s-1}\delta_l$$

$$M'_{js} \ge A\cdot k_j - a_j + x + \Delta\cdot\sum_{l=1}^{s-1}\delta_l - R_i$$

and, then ($i \mid j\in J_i$):

$$M_{js} = L_i - A\cdot\left\lceil \frac{L_i + a_j - R_1 - a_1^{\min} - A + \Delta}{A}\right\rceil + a_j - \Delta\cdot\left(s-1\right)$$

$$M'_{js} = A\cdot\left\lfloor \frac{R_i + a_j}{A}\right\rfloor - a_j + R_1 + a_1^{\min} + A - \Delta - R_i =$$

$$= A\cdot\left(\left\lfloor \frac{R_i + a_j}{A}\right\rfloor + 1\right) + R_1 - R_i - \Delta - a_j + a_1^{\min}$$

The objective (1) is to minimise the cycle time. Constraint (2) imposes that the number of elementary steps in a cycle corresponds to the distance between the left borders of two consecutive workpieces; (3) enforce that the forward step *s* exists if the number of the corresponding elementary steps is positive; constraints (4) eliminate symmetries, assuring that the forward step *s* exists only if forward step *s*-1 exists; (5) and (6) guarantee, for each task, that it is accessible, from the only station that is able to perform the task, during the stationary stage in which the task will be carry out; (7) impose that each task has to be assigned to one, and only one, stationary stage; (8), that the time corresponding to the stationary stages is not less than the processing time at any station; finally, (9), (9’) and (9”) are alternative ways to force the existence of a stationary stage when at least one task is assigned to it.

Reference

Müller-Hannemann, M.; Weihe, K. (2006) “Moving policies in cyclic assembly line scheduling”. *Theoretical Computer Science*, 351, 425-436.