Robust FDI/FTC using Set-membership Methods and Application to Real Case Studies

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Abstract: This paper reviews the use of set-membership methods in robust fault detection and isolation (FDI) and tolerant control (FTC). Set-membership methods use a deterministic unknown-but-bounded description of noise and parametric uncertainty (*interval models*). These methods aims to check the consistency between observed and predicted behavior by using simple sets to approximate the set of possible behaviors (in parameter or state space). When an inconsistency is detected a fault can be indicated, otherwise nothing can be stated. The same principle can be used to identify interval models for fault detection and to develop methods for fault tolerance evaluation. Finally, some real application of these methods will end the paper exemplifying the success of these methods in FDI/FTC.

Keywords: Fault detection, fault-tolerant control, robustness, interval models, set-membership

1. INTRODUCTION

Model-based fault detection of dynamic processes is based on the use of models to check the consistency of observed behaviours. However, when building a model of a dynamic process to monitor its behaviour, there is always some mismatch between the modelled and real behaviour due to the fact that some effects are neglected, some non-linearities are linearised in order to simplify the model, some parameters have tolerance when are compared between several units of the same component, some errors in parameters or in the structure of the model are introduced in the model calibration process, etc. These modelling errors introduce uncertainty in the model, but many times they could be bounded and included in the fault detection model. There are several ways of considering the uncertainty associated with the model (structured or non-structured). In FDI community a fault detection algorithm able to handle uncertainty is called *robust*. The robustness of fault detection algorithm is the degree of sensitivity to faults compared to the degree of sensitivity to uncertainty (Chen and Patton 1999). Research on robust fault detection methods has been very active in the FDI community these last years. One of the most developed families of approaches, called *active*, is based on generating residuals which are insensitive to uncertainty, while at the same time sensitive to faults. This approach has been extensively developed by several researchers using different techniques: unknown input observers, robust parity equations, H_∞, etc. In the book of Chen and Patton (1999), there is an excellent survey of this active approach. On the other hand, there is a second family of approaches, called *passive*, which enhance the robustness of the fault detection system at the decision-making stage. This approach is still under research. Several techniques have been used, but most of them are based on using an adaptive threshold at the decision-making stage.

In the present paper, passive robust fault detection when considering the nominal model plus the uncertainty on every parameter bounded by intervals is presented and reviewed. This type of uncertainty modelling provides a type of models known as an *interval model*. The use of interval models has received several names depending on the field of application (Jaulin *et al*., 2001): in circuit analysis it is known as *worst-case* (or *tolerance analysis*), in automatic control as *set-membership* (also known, in this field as *bounding* or *robust approach*) and in qualitative reasoning as *semi-quantitative*. In the automatic control literature, the set-membership (or robust) approach applied to parameter and state estimation has been treated extensively in the book of Milanese *et al.* (1996) while their application to control can be found in (Bhattacharyya, 1995; Ackermann, 2002). The worst-case analysis of circuits has been treated in the book of Kolev (1993) and in several research papers appearing in circuits journals and congresses. Finally, the semi-quantitative approach is treated in the book of Kuipers (1994) and in several papers appearing in artificial intelligence journals and congresses.

In FDI, the use of interval models for adaptive threshold generation started with the seminal work of Horak (1988). Since then several research groups have been working these last years to develop and apply this approach to FDI and FTC, see for example: (Armengol *et al*., 2000;2008; Hamelin *et al*, 2001; Sainz *et al*., 2002; Puig *et al*., 2002; 2008; Tornil *et al*, 2003; Fagarasan *et al*., 2004 and Ploix *et al*., 2006; Adrot *et al*, 2008). This approach has also been integrated with Qualitative Reasoning tools (from AI communities) giving a diagnostic tool known as CA~EN (Travé-Massuyes *et al*, 2001).

The paper also reviews the different approaches that can be used to identify interval models for fault detection. This research has started with the seminal work of (Ploix *et al.*, 1999). New fields of applications of set-membership methods to areas close to FDI as fault tolerant control are also presented. Finally, the paper presents several industrial applications where set-membership have been successfully used.

The remainder of the paper is organized as follows: in *Section 2*, interval models of dynamic systems for fault detection are introduced. In *Section 3* fault detection using the worst-case approach is recalled, while *Section 4* presents the fault detection using the set-membership approach. *Section 5* reviews the methods for identification of interval models using real data. *Section 6* presents the use of set-membership methods to the fault tolerance evaluation of control laws. *Section 7* presents several successful application of set-membership methods for fault detection based on interval models. Finally, the major conclusions are drawn in *Section 8*.

2. INTERVAL MODELS OF DYNAMIC SYSTEMS FOR FAULT DETECTION

2.1 Interval models of dynamic systems

Considering that the system to be monitored can be described by a MIMO linear uncertain dynamic model in discrete-time and state-space form as follows

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{x}(k)+\boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}(k)+\boldsymbol{w}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{x}(k)+\boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{u}(k)+\boldsymbol{v}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{y}(k)\in\Re^{ny}$, $\boldsymbol{u}(k)\in\Re^{nu}$, $\boldsymbol{x}(k)\in\Re^{nx}$ are the system output, input and the state-space vectors respectively; $\boldsymbol{A}(\boldsymbol{\theta})\in\Re^{nx\times nx}$, $\boldsymbol{B}(\boldsymbol{\theta})\in\Re^{nx\times nu}$, $\boldsymbol{C}(\boldsymbol{\theta})\in\Re^{ny\times nx}$ and $\boldsymbol{D}(\boldsymbol{\theta})\in\Re^{ny\times nu}$ are the state, the input, the output and the direct transmission matrices respectively; $\boldsymbol{\theta}\in\Re^{n\theta}$ is the vector of uncertain parameters where $\boldsymbol{\Theta}$ is a bounded set of box (interval) type $\boldsymbol{\theta}\in\boldsymbol{\Theta}$ such that each component $\theta_i\in\left[\underline{\theta}_i,\overline{\theta}_i\right]$ $i=1,..n_\theta$. This is why the resulting model is known as an *interval model*. The set $\boldsymbol{\Theta}$ contains all possible values of $\boldsymbol{\theta}$ when the system operates normally. Intervals for uncertain parameters can also be inferred from real data as will be discussed in *Section 5*.

The system in Eq. (1) can, alternatively, be expressed in input-output form using the shift operator q^{-1} and assuming zero initial conditions as follows:

$$\boldsymbol{y}(k)=\boldsymbol{M}(q^{-1},\boldsymbol{\theta})\boldsymbol{u}(k) \tag{2}$$

where $\boldsymbol{M}(q^{-1},\tilde{\boldsymbol{\theta}})$ is given by

$$\boldsymbol{M}(q^{-1},\boldsymbol{\theta})=\boldsymbol{C}(\boldsymbol{\theta})(q\boldsymbol{I}-\boldsymbol{A}(\boldsymbol{\theta}))^{-1}\boldsymbol{B}(\boldsymbol{\theta})+\boldsymbol{D}(\boldsymbol{\theta})$$

2.2 Interval models for fault detection

The principle of model-based fault detection is to test whether the measured input and output from the system is consistent with the behaviour described by a model of the faultless system. If the measurements are inconsistent with the model of the faultless system, the existence of a fault is proved. The *residual vector*, known also as *analytical redundant relation* (ARR), is usually used to check the consistency between the predicted, $\hat{\boldsymbol{y}}(k)$, and the real measured behaviour, $\boldsymbol{y}(k)$

$$\boldsymbol{r}(k)=\boldsymbol{y}(k)-\hat{\boldsymbol{y}}(k) \tag{3}$$

Ideally, the residuals should only be affected by the faults. However, the presence of disturbances, noise and modelling errors causes the residuals to become nonzero and thus interferes with the detection of faults. Therefore, the fault detection procedure must be *robust* against these undesired effects (Chen and Patton, 1999). In case of modelling a dynamic system using an interval model, the *worst-case* predicted output is described by a set that can be bounded at any iteration by an interval

$$\hat{y}_i(k)\in[\,\underline{\hat{y}}_i(k),\overline{\hat{y}}_i(k)\,] \tag{4}$$

in a non-faulty case. Such interval is computed independently for each output (neglecting couplings between outputs) as follows

$$\underline{\hat{y}}_i(k)=\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}}(\hat{y}_i(k,\boldsymbol{\theta}))\ \text{and}\ \overline{\hat{y}}_i(k)=\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}}(\hat{y}_i(k,\boldsymbol{\theta})) \tag{5}$$

Such interval can be computed using the algorithm based on numerical optimization presented in (Puig *et al.*, 2003). Then, the fault detection test is based either on propagating the parameter uncertainty to the residual, and checking if

$$\boldsymbol{y}(k)\in\left[\underline{\hat{\boldsymbol{y}}}(k)-\sigma,\overline{\hat{\boldsymbol{y}}}(k)+\sigma\right] \tag{6a}$$

or, equivalently

$$0\in\left[\underline{\boldsymbol{r}}(k),\overline{\boldsymbol{r}}(k)\right]=\boldsymbol{y}(k)-\left[\underline{\hat{\boldsymbol{y}}}(k)-\sigma,\overline{\hat{\boldsymbol{y}}}(k)+\sigma\right] \tag{6b}$$

holds or not. In case it does not hold a fault can be indicated. This test is named as *direct test*. Alternatively, the *inverse test* consists on checking if there exists a parameter values in the parameter uncertainty set $\boldsymbol{\Theta}$ such that model (2) is consistent with the system measurements. More formally, to check if

$$\exists\boldsymbol{\theta}\in\boldsymbol{\Theta}\mid\hat{\boldsymbol{y}}(k,\boldsymbol{\theta})\in\left[\boldsymbol{y}(k)-\sigma,\boldsymbol{y}(k)+\sigma\right] \tag{7}$$

In the case that is condition is not satisfied, a discrepancy between measurements and the model is detected and a fault should be indicated. This test can be implemented with the parameter estimation algorithms used in the *set-membership approach* (Milanese et al., 1996).

3. FAULT DETECTION USING THE WORST-CASE APPROACH

3.1 Fault detection using interval observers

The system described by Eq. (1) can be monitored using a linear observer with *Luenberger* structure. The resulting *interval observer* can be written as:

$$
\begin{aligned}
\hat{\boldsymbol{x}}(k+1,\boldsymbol{\theta}) &= (\boldsymbol{A}(\boldsymbol{\theta})-\boldsymbol{LC}(\boldsymbol{\theta}))\hat{\boldsymbol{x}}(k)+(\boldsymbol{B}(\boldsymbol{\theta})-\boldsymbol{LD}(\boldsymbol{\theta}))\boldsymbol{u}(k) \\
&\quad +\boldsymbol{Ly}(k) = \boldsymbol{A}_o(\boldsymbol{\theta})\hat{\boldsymbol{x}}(k)+\boldsymbol{B}_o(\boldsymbol{\theta})\boldsymbol{u}(k)+\boldsymbol{Ly}(k) \\
\hat{\boldsymbol{y}}(k,\boldsymbol{\theta}) &= \boldsymbol{C}(\boldsymbol{\theta})\hat{\boldsymbol{x}}(k)+\boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{u}(k)
\end{aligned} \tag{8}
$$

where ***u(k)*** is the measured system input vector, $\hat{\boldsymbol{x}}(k,\boldsymbol{\theta})$ is the estimated space-state vector and $\hat{\boldsymbol{y}}(k,\boldsymbol{\theta})$ is the estimated output vector for a given value of $\boldsymbol{\theta}\in\boldsymbol{\Theta}$. The observer gain matrix $\boldsymbol{L}\in\Re^{nx\times ny}$ is designed to stabilize the matrix $\boldsymbol{A}_o(\boldsymbol{\theta})$ and to guarantee a desired performance regarding fault detection for all $\boldsymbol{\theta}\in\boldsymbol{\Theta}$ (Chilali *et al*, 1996). Alternatively, the observer given by Eq. (8) can be expressed in input-output form using the *q*-transform and considering zero initial conditions as follows:

$$
\hat{\boldsymbol{y}}(k)=\boldsymbol{G}(q^{-1},\boldsymbol{\theta})\boldsymbol{u}(k)+\boldsymbol{H}(q^{-1},\boldsymbol{\theta})\boldsymbol{y}(k) \tag{9}
$$

where

$$
\boldsymbol{G}(q^{-1},\boldsymbol{\theta})=\boldsymbol{C}(\boldsymbol{\theta})\left(q\boldsymbol{I}-\boldsymbol{A}_o(\boldsymbol{\theta})\right)^{-1}\boldsymbol{B}_o(\boldsymbol{\theta})+\boldsymbol{D}(\boldsymbol{\theta})
$$

$$
\boldsymbol{H}(q^{-1},\boldsymbol{\theta})=\boldsymbol{C}(\boldsymbol{\theta})\left(q\boldsymbol{I}-\boldsymbol{A}_o(\boldsymbol{\theta})\right)^{-1}\boldsymbol{L}
$$

Interval observation requires solving the optimization problems introduced in Eq. (5) using Eq. (9). In order to preserve *uncertain parameter time-invariance* and to avoid the *wrapping effect*[1], the observer output prediction in Eq. (5) is substituted by

$$
\hat{\boldsymbol{y}}(k)=\boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{A}_0(\boldsymbol{\theta})^k\boldsymbol{x}_0 + \boldsymbol{C}(\boldsymbol{\theta})\sum_{j=0}^{k-1}\boldsymbol{A}_0(\boldsymbol{\theta})^{(k-1-j)}\boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{u}(j) \tag{10}
$$

When proceeding in this way, the optimization problems in Eq. (10) will not be convex since the non-linearity with respect to parameters. Therefore, the existence of a unique optimum is not guaranteed. In order to guarantee that the global optimum is reached, a global optimization algorithm must be used. In particular, a branch and bound interval arithmetic global optimization based on Hansen's algorithm (Hansen, 1992) can be used. An additional computational problem appears when using Eq. (10) since the degree of the polynomial in the objective function increases with time. This implies that the amount of computation needed is also increasing with time, being impossible to operate over a large time period. This problem can be solved if the interval system (1) is asymptotically stable (Puig et *al*., 2003). In this case, the predicted system output at time *k* depends, approximately, only on the inputs that occurred in a *time sliding window* with a length ℓ (whose value is of the order of the settling time) and the state at the beginning of such window. Then, Eq. (9) can be approximated by limiting the computation to a finite time horizon as it has been proposed in (Puig *et al*., 2003).

In case that uncertain parameters are considered *time-varying*, an iterative algorithm can be used that obtains the set of uncertain states at time *k*, $\mathbb{X}_k$ from the set $\mathbb{X}_{k-1}$ using the algorithm presented in Figure 1 (Bravo *et al*., 2008).

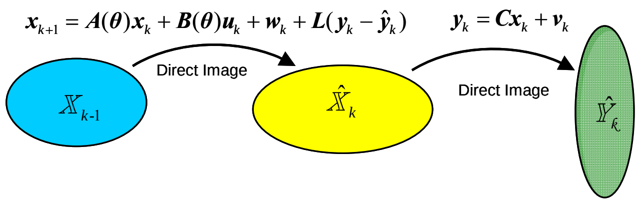

Fig. 1. Worst-case observer

To implement such algorithm the set of uncertain should be approximated since the exact set of estimated states would be difficult to compute. Several geometrical has been proposed in the literature ranging from parallelotopes or ellipsoids to zonotopes as proposed Alamo et al. (2005). A zonotope $\mathbb{X}$ of order *m* can be viewed as the Minkowski sum of *m* segments:

$$
\mathbb{X}=\boldsymbol{p}\oplus\mathbf{H}B^m=\left\{\boldsymbol{p}+\mathbf{Hz}:\mathbf{z}\in B^m\right\} \tag{11}
$$

where the segments are defined by the columns of matrix $\boldsymbol{H}$ and B^m is a unitary box composed of *m* unitary intervals. The order *m* is a measure for the geometrical complexity of the zonotopes (see Figure 2 for a zonotope of order 14).

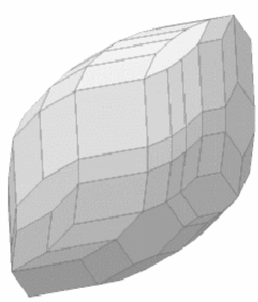

Fig. 2. Zonotope

Zonotope arithmetic possesses a set of operations (as sum, affine transformation, intersection) that can be very

[1] The problem of wrapping is related to the use of a crude approximation of set of states associated with the interval simulation. If at each iteration, the true solution set is wrapped into its interval hull, since the overestimation of the wrapped set is proportional to its radius, a spurious growth of the enclosures can result if the composition of wrapping and mapping is iterated.

efficiently implemented since they only involve operations with matrices

3.2 Interval ARMA parity equations

In case the observer gain in Eq. (7) is taken equal to zero (***L=0***), the observer becomes an *interval simulator*, since the output prediction is based only in the inputs and previous output predictions, and Eq. (9) becomes: $\hat{\boldsymbol{y}}(k)=\boldsymbol{M}(q^{-1},\boldsymbol{\theta})\boldsymbol{u}(k)$ while the residual is given by

$$\boldsymbol{r}(k)=\boldsymbol{y}(k)-\hat{\boldsymbol{y}}(k)=\boldsymbol{y}(k)-\boldsymbol{M}(q,\boldsymbol{\theta})\boldsymbol{u}(k) \qquad (12)$$

According to (Gertler, 1998), Eq. (12) corresponds with *ARMA primary parity equations* or *residuals*. This is an open-loop approach. Interval simulation requires solving the optimization problems following the same strategy as in the case of the interval observer but using the system matrices (1). In order to reduce computing complexity, as in observer case, a time window could be used. In this case this approach is known as ℓ order ARMA parity equation (Tornil *et al*., 2003).

3.3 Interval MA parity equations

On the other hand, in case observer gain in Eq. (7) is designed such that on the poles are at the origin (deadbeat observer), the observer becomes an *interval predictor*, since the output prediction is based only in measured inputs and outputs. The prediction equation (9) is moving average (MA) and follows a closed-loop approach. Thus, the corresponding residuals (11) are called *MA primary parity equations* or *residuals* (Gertler, 1998). The optimization problems (5) that must be solved now are linear with respect to parameters, and, therefore convex. This means that there exist very efficient algorithms to solve them (as the simplex algorithm). Because of the linearity, the existence of a unique optimum is guaranteed being located in one of the vertices of the parameter uncertainty intervals. Interval prediction is not affected by the problem of wrapping because the predicted output is based on the previous output measurements instead of the interval of the previous predicted outputs (Puig *et al*., 2003). Thus, interval prediction considers uncertain parameters as time varying. But, time invariance in uncertain is wanted to be preserved, a ℓ-order MA parity equation should be used (Tornil *et al*, 2003). Finally, Ploix *et al.* (2006) has, recently, proposed a method to obtain the interval parity equations directly from state-space using the Chow-Wilsky scheme.

3.3 Comparison

In Puig *et al*. (2008), the behaviour of the different interval fault detection approaches considered so far are studied and compared using the FDI benchmark proposed in DAMADICS project. Table 1 summarises the results of this comparison. This table can be used as a guideline to decide in which applications an approach is more suitable than the others. Prediction and simulation approaches have antagonist properties: prediction does not suffer from the wrapping effect, has low computational complexity, has low sensitivity to unmodeled dynamics but can suffer the following sensor fault effect and has high sensitivity to sensor noise. On the other side, the simulation approach has the opposite properties, presenting good performance to detect sensor faults in noisy systems. Finally, the observer approach is in the middle, with the advantage that since it has one more degree of freedom (the observer gain), it can be designed trying to minimize the bad effects and maximize the good effects.

Table 1. Interval-based fault detection approaches

Issue		Simulation	Observation	Prediction
Wrapping Effect		Yes	Yes	No
Computational Complexity		High	High	Low
Unmodeled Dynamics Sensitivity		High	Medium	Low
Initial Conditions Sensitivity		High	Medium	Low
Fault Sensitivity	actuator	Dynamic response	Dynamic response	Constant
	sensor	Constant	Pulse	Deadbeat
Noise Sensitivity	process	LP filter	LP filter	Gain
	sensor	Gain	HP filter	HP filter

4. FAULT DETECTION USING SET-MEMBERSHIP APPROACH

Alternatively to the worst-case approach presented in previous section, the set-membership (or consistency) based approach relies on checking whether the measured sequence of system inputs and outputs available at every time instant *k* could have been generated by the model (2) and parameter values in the parameter uncertainty set $\boldsymbol{\Theta}$ (Ocampo-Martínez *et al*., 2006a). This approach is related with the inverse test described in *Section 2*.

4.1 Fault detection test in the parameter space

The inverse test consists on checking if there exists a parameter in the parameter uncertainty set $\boldsymbol{\Theta}_k$ such that model (2) is consistent with the systems measurements. This test can be easily implemented using the set-membership parameter estimation procedure described in *Section 5*, that operates in the recursive form:

$$\boldsymbol{\Theta}_{k+1}=\boldsymbol{\Theta}_k \cap \mathbf{F}_k \qquad (13)$$

where: $\mathbf{F}_k=\left\{\boldsymbol{\theta}(\mathbf{p}_k)\in\mathbb{R}^{n_\theta} \mid y(k)-\sigma\leq\boldsymbol{\varphi}(k)\boldsymbol{\theta}(\mathbf{p}_k)\leq y(k)+\sigma\right\}$ is the strip of consistent parameters with the current measurement. In fault detection using the inverse test, the model is assumed invalidated and fault is indicated if $\mathbf{\Theta}_{k+1} = \varnothing$ (Ingimundarson *et al*., 2008)

Despite of outer approximation is the most used in fault detection due to it contains all the consistent models, the inner approximation, that contains only consistent parameters, can complement the use of outer approximation in order to improve fault detection behavior.

4.2 Fault detection test in the state space

A consistency-based state estimator assumes a priori boundson noise and uncertain parameters and constructs sets of estimated states that are consistent with the a priori bounds and current measurements. Several researchers as (Chisci *et al*., 1996)(Maksarov and Norton, 1996) (Shamma, 1997), (Calafiore, 2001) and (Kieffer *et al*., 2002), among others, have addressed this issue. Consider a system given by Eq. (1), an initial compact set $\mathbb{X}_o$ and a sequence of measured inputs and outputs, the uncertain state set at time k using the set-membership approach can computed using the algorithm presented in Figure 3. A fault is detected when $\mathbb{X}_k^e = \mathbb{X}_k^p \cap \mathbb{X}_k^y = \varnothing$ (Planchon and Lunze, 2006; Guerra *et al*., 2007).

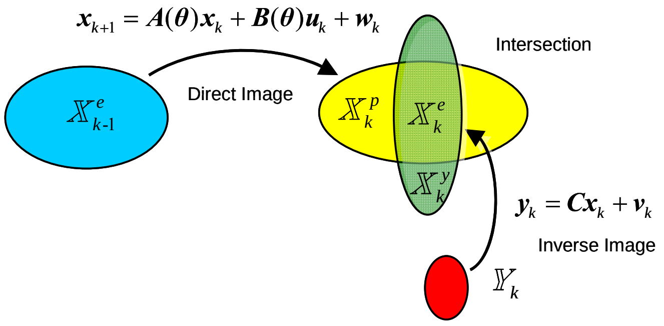

Fig. 3. Set-membership state estimation

5. IDENTIFICATION FOR ROBUST FAULT DETECTION

5.1 Model parametrisation

One of the key points in model based fault detection is how models are calibrated. Calibration would deliver a calibrated nominal model plus its modelling error in the form of interval parameters, that it will provide an interval of confidence for predicted behaviour. This type of models are known as "*interval models*". To this aim, several authors (Ploix *et al.*, 1999; Calafiore *et al.,* 2002; Campi *et al.,* 2009) have suggested an adaptation of classical system identifications methods to provide the nominal model plus the uncertainty intervals for parameters that guarantee that all registered data from the system in non-faulty scenarios will be included in the interval model. These algorithms are based on using classical identification methods (for example, least-squares) to provide the nominal estimate for system parameters. Then, the intervals of uncertainty for parameters are adjusted until all the measured data is covered by the model prediction interval. These algorithms proceed considering that the interval model (1) to be identified can be expressed in regressor form as follows

$$y(k) = \boldsymbol{\varphi}(k)\boldsymbol{\theta} + v(k) = \hat{y}(k) + v(k) \qquad (14)$$

where: $\boldsymbol{\varphi}(k)$ is the regressor vector of dimension $1 \times n_\theta$ which can contain any function of inputs $u(k)$ and outputs $y(k)$; $v(k)$ is additive noise bounded by a constant $|v(k)| \le \sigma$; $\boldsymbol{\theta} \in \mathbf{\Theta}_k$ is the parameter vector of dimension $n_\theta \times 1$ and $\mathbf{\Theta}_k$ is the set that bounds parameter values. described by a zonotope centered in the nominal model :

$$\mathbf{\Theta}_k = \boldsymbol{\theta}^0 \oplus \mathbf{H}B^n = \left\{ \boldsymbol{\theta}^0 + \mathbf{Hz} : \mathbf{z} \in B^n \right\} \qquad (15)$$

Notice that a particular case corresponds to the case the parameter set $\mathbf{\Theta}_k$ is bounded by an interval box:

$$[\theta_i] = \left[\theta_i^{\min}, \theta_i^{\max}\right] = \left[\theta_i^0 - \lambda_i, \theta_i^0 + \lambda_i\right]$$

with $i=1,\ldots,n_\theta$. This set can be viewed as a zonotope with $\mathbf{H}$ equal to a $n_\theta \times n_\theta$ diagonal matrix:

$$\mathbf{H} = diag(\lambda_1, \lambda_2, \ldots, \lambda_{n_\theta}) \qquad (16)$$

Given a sequence of M regressor vector values $\boldsymbol{\varphi}(k)$ in a fault free scenario and a model parameterised as in Eq. (14), the aim is to estimate model parameters and their uncertainty (model set) following either a *worst-case* or *set-membership parameter estimation* approach.

4.2 Worst-case parameter estimation

In this case, the set of uncertain parameters $\mathbf{\Theta}_k$ should be obtained in such a way that all measured data in a fault free scenario will be covered by the worst-case predicted output produced by using model (14) and the uncertainty parameter set ("***worst-case model***"), that is:

$$\bar{y}(k) \ge y(k) - \sigma \quad \text{and} \quad \underline{y}(k) \le y(k) + \sigma \qquad \forall k = 1,\ldots,M \qquad (17)$$

where:

$$\bar{y}(k) = \max\left(\boldsymbol{\varphi}(k)\boldsymbol{\theta}\right) \text{ with } \boldsymbol{\theta} \in \mathbf{\Theta}_k \qquad (18a)$$

$$\underline{y}(k) = \min\left(\boldsymbol{\varphi}(k)\boldsymbol{\theta}\right) \text{ with } \boldsymbol{\theta} \in \mathbf{\Theta}_k \qquad (18b)$$

This type of model identification was first suggested by Ploix *et al.* (1999) in the context of fault detection using a direct test and an interval LTI model in prediction.

Considering that the parameter set $\mathbf{\Theta}_k$ can be described as the zonotope (15) and proceeding as in Ploix *et al.* (1999), the maximum and minimum prediction provided by model (14) are given by

$$\bar{y}(k) = \hat{y}^0(k) + \left\|\boldsymbol{\varphi}(k)\mathbf{H}\right\|_1 \qquad (19a)$$

$$\underline{y}(k) = \hat{y}^0(k) - \left\|\boldsymbol{\varphi}(k)\mathbf{H}\right\|_1 \qquad (19b)$$

where $\hat{y}^0(k)$ is the model output prediction with nominal parameters: $\hat{y}^0(k) = \boldsymbol{\varphi}(k)\boldsymbol{\theta}^0(\mathbf{p}_k)$ where $\boldsymbol{\theta}^0 = (\theta_1^0, ..., \theta_{n_\theta}^n)$.

Notice that in the particular case of interval parameters:

$$\|\boldsymbol{\varphi}(k)\mathbf{H}\|_1 = \sum_{i=1}^{n} \lambda_i |\boldsymbol{\varphi}_i(k)|$$

Replacing equations (19a) and (19b) in inclusion conditions (17), the optimal zonotope that fulfills the ***"worst-case condition"*** can be computed using the following algorithm.

Algorithm 1 **"Worst-case Parameter Estimation " (general case)**

$$\min_{\mathbf{H}} f(\boldsymbol{\Theta}_\mathbf{k}(\mathbf{H}))$$

subject to: $\|\boldsymbol{\varphi}(k)\mathbf{H}\|_1 \geq |y(k) - \hat{y}^0(k)| - \sigma \quad \forall k = 1, ..., M$

where the cost function *f* in *"worst-case approach"* is usually the interval prediction thickness that can be calculated as

$$\sum_{k=1}^{N} (\bar{y}(k) - \underline{y}(k)) = 2\sum_{k=1}^{N} \|\boldsymbol{\varphi}(k)\mathbf{H}\|_1 \tag{20}$$

In order to reduce the complexity of *Algorithm 1*, the zonotope that bounds $\boldsymbol{\Theta}_\mathbf{k}$ can be parameterised such that $\mathbf{H} = \lambda\mathbf{H}_0$, that corresponds with a zonotope with predefined shape (determined by $\mathbf{H}_0$) and a scalar λ. Then, in this case interval prediction thickness (20) is given by

$$\sum_{k=1}^{N} (\bar{y}(k) - \underline{y}(k)) = 2|\lambda| \sum_{k=1}^{N} \|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1 = f(|\lambda|) \tag{21}$$

and restrictions of *Algorithm* 1 can be expressed as follows:

$$\lambda\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1 \geq |y(k) - \hat{y}^0(k)| - \sigma \Rightarrow \lambda \geq \frac{|y(k) - \hat{y}^0(k)| - \sigma}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \tag{22}$$

such that *Algorithm* 1 can be rewritten as

Algorithm 2 "Worst-case Parameter Estimation" (particular case)

$$\min_{\lambda} 2|\lambda| \sum_{k=1}^{N} \|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1$$

subject to: $\lambda \geq \dfrac{|y(k) - \hat{y}^0(k)| - \sigma}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \quad \forall k = 1, ..., M$

The optimal solution provided by such algorithm is:

$$\lambda = \sup_{k \in \{1,...,M\}} \left(\frac{|y(k) - \hat{y}^0(k)| - \sigma}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \right) \tag{23}$$

4.3 Set-membership parameter estimation

On the other hand, the set of uncertain parameters $\boldsymbol{\Theta}_k$ using a set-membership parameter estimation approach is obtained in such a way that the predicted behavior is consistent with all the measured data in a fault-free scenario. In this case the model is called a "***consistent model***" since the predicted behavior is always inside the interval of possible measurements. That is:

$$\hat{y}(k) - \sigma \leq y(k) \leq \hat{y}(k) + \sigma \qquad \forall k = 1, ..., M \tag{24}$$

where: $\hat{y}(k) = \boldsymbol{\varphi}(k)\boldsymbol{\theta}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}_k$.

Algorithms for identifying such kind of model are also known as "***set-membership parameter estimation***" algorithms. In Milanese *et al.* (1996) there is a survey of such methods.

Using this approach, the parameter set $\boldsymbol{\Theta}_k$ that contains all models consistent with data, known as *Feasible Parameter Set* (**FPS**), is defined as follows:

$$\mathbf{FPS} = \{\boldsymbol{\theta} \in \boldsymbol{\Theta}_k \mid y(k) - \sigma \leq \boldsymbol{\varphi}(k)\boldsymbol{\theta} \leq y(k) + \sigma, k = 1, \cdots, M\} \tag{25}$$

The exact description of **FPS** is in general not simple, and existing algorithms usually approximate the **FPS** using an inner/outer simpler shapes as boxes, ellipsoids or zonotopes (Milanese *et al.* 1996). The approximation set is called approximated feasible parameter set (**AFPS**). In this paper, algorithms that provided inner/outer **AFPS** using zonotopes in case of using the model parameterised as in (14) are presented.

Outer approximations

Outer approximation algorithms find the parameter set $\boldsymbol{\Theta}_k$ of minimum volume such that $\mathbf{FPS} \subseteq \boldsymbol{\Theta}_k$. This kind of algorithms usually implies an excessive computational cost and recursive forms have been proposed as the one described in Bravo *et al.* (2006). This recursive approach is based in computing iteratively the **AFPS** using zonotopes and related operations as follows:

$$\mathbf{AFPS}_{k+1} = \mathbf{AFPS}_k \cap \mathbf{F}_k \tag{26}$$

where $\mathbf{F}_k = \{\boldsymbol{\theta}(\mathbf{p}_k) \in \mathbb{R}^{n_\theta} \mid y(k) - \sigma \leq \boldsymbol{\varphi}(k)\boldsymbol{\theta}(\mathbf{p}_k) \leq y(k) + \sigma\}$

Inner approximations

Inner approximation algorithms find the parameter set $\boldsymbol{\Theta}_k$ of maximum volume such that $\boldsymbol{\Theta}_k \subseteq \mathbf{FPS}$.

A set-membership inner approximation using zonotopes parameterised as in Eq. (15) for models expressed as in (14) can be obtained in a similar way as proposed in *Algorithm 2* for the worst-case zonotope. The inner approximation algorithm comes from fact the FPS conditions (25) can be bounded by:

$$y(k) - \sigma \leq \underline{y}(k) \leq \boldsymbol{\varphi}(k)\boldsymbol{\theta}(\mathbf{p}_k) \leq \bar{y}(k) \leq y(k) + \sigma$$

where $\underline{y}(k)$ and $\bar{y}(k)$ are defined as in (18) and, in the case of $\boldsymbol{\Theta}_k$ is a zonotope, calculated as in (19). Then, the maximum inner zonotope, centered in $\boldsymbol{\theta}^0$, with consistent parameters can be computed using the following algorithm

Algorithm 3 **"Inner Set-membership Zonotope" (general case)**

$$\max_{\mathbf{H}} f(\mathbf{\Theta_k}(\mathbf{H}))$$

subject to: $\|\boldsymbol{\varphi}(k)\mathbf{H}\|_1 \le \sigma - |y(k) - \hat{y}^0(k)| \quad \forall k = 1,\dots,M$

where the cost function *f* in the *"set-membership approach"* is usually the volume of the zonotope defined by (15). This volume only depends of matrix **H** and of B^n with a volume equal to 2^n. In the particular case, **H** is a square matrix ($n_\theta = n$): $vol(\mathbf{\Theta_k}) = 2^n |\det(\mathbf{H})|$. See (Montgomery, 1989) for more details.

As in *Algorithm 1*, it will be considered the particular case $\mathbf{H} = \lambda \mathbf{H}_0$. Then, if $\mathbf{H_0}$ is a square matrix $vol(\mathbf{\Theta_k}) = |2\lambda|^n |\det(\mathbf{H}_0)|$ and restrictions of *Algorithm 3* can be expressed as:

$$\lambda \|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1 \le \sigma - |y(k) - \hat{y}^0(k)| \Rightarrow \lambda \le \frac{\sigma - |y(k) - \hat{y}^0(k)|}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \qquad (27)$$

such that it can be rewritten as follows

Algorithm 4 **"Inner set-membership zonotope" (particular case)**

$$\max_{\lambda} vol(\mathbf{\Theta}_k) = f(|\lambda|)$$

subject to: $\lambda \le \dfrac{\sigma - |y(k) - \hat{y}^0(k)|}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \qquad \forall k = 1,\dots,M$

The optimal solution provided by such algorithm is:

$$\lambda = \inf_{k \in \{1,\dots,M\}} \left(\frac{\sigma - |y(k) - \hat{y}^0(k)|}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \right) \qquad (28)$$

6. FAULT TOLERANCE EVALUATION USING SET-MEMBERSHIP APPROACHES

6.1 Motivation

The objective of this section is to assess the tolerance of a certain actuator fault configuration considering a linear predictive/optimal control law with constraints. This problem has been already treated in the literature for the case of LQR problem without constraints (Staroswiecki, 2003), thanks to the existence of analytical solution. However, constraints (on states and control signals) are always present in real industrial control problems and could be easily handled using linear constrained Model Predictive Control (MPC). In general, an analytical solution for these kind of control laws does not exist, which makes difficult to do this type of analysis. The method proposed in this section is not of analytical but of computational nature.

It follows the idea proposed by (Lydoire, 2004) in which the calculation of the control law for a predictive/optimal controller with constraints can be divided in two steps: First, the calculation of solutions set that satisfies the constraints (feasible solutions) and then, the optimal solution determination.

Faults in actuators will cause changes in the set of feasible solutions since constraints on the control signals have varied. This causes that the set of admissible solutions for the control objective could be empty. Therefore, the admissibility of the control law facing the actuator faults can be determined knowing the feasible solutions set. This section provides a method to compute this set and then evaluate the admissibility of the control law.

To find the feasible solutions set for the problem of MPC, a constraints satisfaction problem could be formulated (Ocampo-Martínez *et al*., 2006b). However, this problem is computationally demanding and should be solved approximately in a iterative way in time, bounding it by its interval hull. Moreover, when proceeding in this way, an interval simulation problem is implicitly solved appearing typical difficulties associated with it (as wrapping effect, among others) (Puig *et al.*, 2003). In order to avoid such problems, the region of possible states should be approximated using more complex domains than intervals. In this section, a zonotope-based method to evaluate the admissibility of fault actuator configurations is proposed and discussed.

6.2 Admissibility of the control law

The solution of a control problem consists on finding a control law in a given set of control laws $\mathbb{U}$ such that the controlled system a*chieves the control objectives* $\mathbb{O}$ while its behavior satisfies a set of constraints $\mathbb{C}$. The solution of the problem is completely defined by the triple: $\langle \mathbb{V}, \mathbb{O}, \mathbb{C} \rangle$. In the case of a linear constrained predictive control law:

$$\mathbb{O} : \min_{\tilde{u}} J(\tilde{x}, \tilde{u})$$

subject to:

$$\mathbb{C} : \begin{cases} x_{k+1} = Ax_k + Bu_k \\ u_k \in \mathbb{U} \quad k = 1,\cdots,N-1 \\ x_k \in \mathbb{X} \quad k = 0,\cdots,N \end{cases}$$

where:

$$\mathbb{U} = \left\{ u_k \in \mathbb{R}^m \middle| u_{min} \le u_k \le u_{min} \right\}$$

$$\mathbb{X} = \left\{ x_k \in \mathbb{R}^n \middle| x_{min} \le x_k \le x_{min} \right\}$$

and

$$\tilde{u}_k = (u_j)_0^{k-1} = (u_0, u_1, \cdots, u_{k-1})$$

$$\tilde{x}_k = (x_j)_0^{k-1} = (x_0, x_1, \cdots, x_k)$$

The *feasible solution* set is given by

$$\Omega = \left\{ \tilde{x}, \tilde{u} \middle| \left(x_{k+1} = Ax_k + Bu_k \right)_0^{N-1} \right\}$$

and gives the input and state sets compatible with system constraints which originate the set of predictive states.

The *feasible control objectives set* is given by

$$J_{\Omega} = \{ J(\tilde{x},\tilde{u}) | (\tilde{x},\tilde{u}) \in \Omega \}$$

and corresponds to the set of all values of $J(\tilde{x},\tilde{u})$ obtained from feasible solutions.

The *admissible solution set* is given by

$$\mathbb{A} = \{ (\tilde{x},\tilde{u}) \in \Omega_f \,|\, J(\tilde{x},\tilde{u}) \in \mathcal{J}_{\mathbb{A}} \}$$

where Ω_f corresponds to the feasible solution set of a actuator fault configuration and $\mathcal{J}_{\mathbb{A}}$ defined as the admissible control objective set.

The admissibility evaluation using a set computation approach starts obtaining the feasible solution set Ω given a set of initial states $\mathbb{X}_o$, the system dynamic and the system operating constraints over N using the algorithm presented in Figure 4.

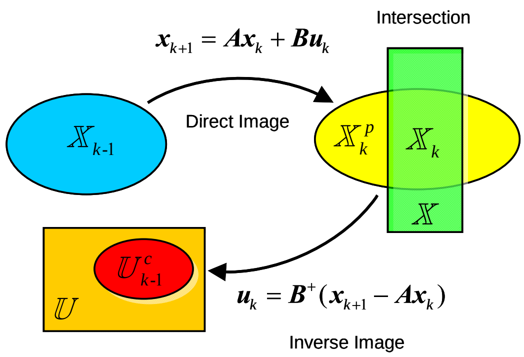

Fig 4. Feasible solution set computation

At the same time that the feasible solution set is computed Ω, the feasible control objectives set J_{Ω} at time *k* can be obtained using the algorithm presented in Figure 5.

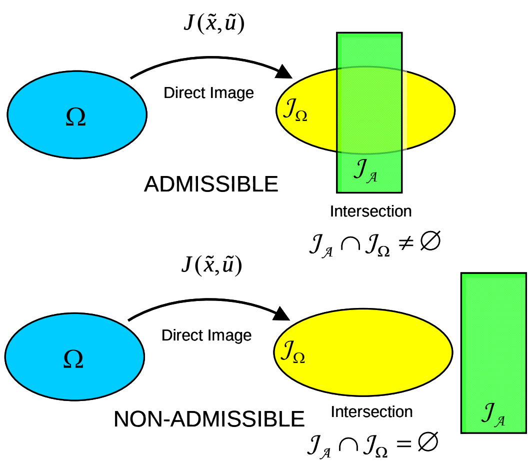

Fig. 5. Admissibility evaluation

7. REAL APPLICATIONS

The set-membership methods described in this paper have been used in some real applications where the SAC-UPC research group have been deeply involved.

7.1 TIGER-SHEBA

The TIGER and TIGER SHEBA European Projects were both leaded by Intelligent Applications Ltd. The consortium included end users and a research team composed of LAAS-CNRS and SAC-UPC. TIGER, full name "Real-Time Assessement of Dynamic, Hard to Measure Systems", ran within the European ESPRIT program from 1992 to 1995. This project set the basis of the TIGER™ tool, commercialized by IA Ltd. TIGER-SHEBA, full name "TIGER with Model Based Diagnosis", succeeded to TIGER in the ESPRIT TRIAL Applications European program starting in 1998 and ending in 2000 (Project n°27548). TIGER SHEBA enhanced TIGER™ by integrating the model-based diagnosis system CA~EN. The design of TIGER™, the CA~EN software and its integration in TIGER-SHEBA have led to an innovative product for gas turbine monitoring and diagnosis that has been commercialized all over the world, and to the dissemination of scientific ideas in recognized scientific journals and conferences. The focus of the development performed within TIGER SHEBA was to use the CA~EN software. The CA~EN system was used on critical sub-systems of several gas turbines such as the gas and liquid fuel systems. Extensive testing of the TIGER /CA~EN system was carried out. Numerous test runs were conducted varying from 2 weeks to 2 months at a time covering key periods of operation for the considered gas turbines. In addition, the combined system runs continuously on-line on the Frame 6 gas turbine of the National Power's cogeneration plant at Aylesford (UK). Details about the application of set-membership methods in TIGER and TIGER-SHEBA projects can be found in the following publications (Travé *et al.* 1996;1997; 2001;2006) (Milne *et al.*, 1995)(Escobet *et al.*, 2001).

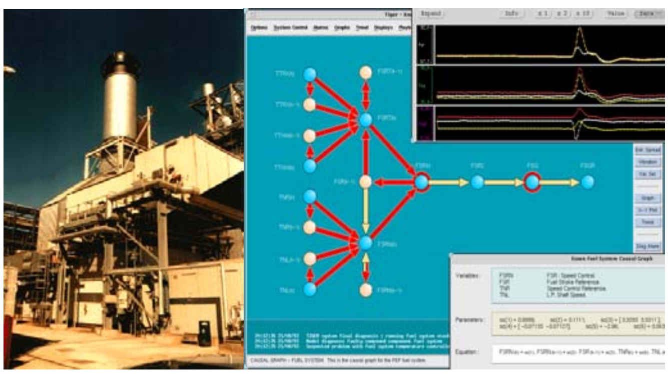

Fig 6. Fault diagnosis system used in TIGER project

7.2 DAMADICS

DAMADICS was a Research Trained Network funded by the European Commission under Framework V. It started in 2000 and ends in 2003. The objectives were providing training and mobility in the synthesis and development of methods and on-line diagnostic tools for applications in power, food processing and chemical industries. During this Network, it was developed a diagnosis benchmark case study based on an industrial smart actuator used in the evaporation station of a sugar factory in Poland (Bartys and de las Heras, 2003). The smart actuator consists of a control valve, a pneumatic servomotor and a smart positioner. It allowed testing and comparing the methods presented in *Section 3*. The obtained results are published in (Puig *et al*. 2006) and (Puig, *et al*. 2008), among others. The main conclusions of this comparative study were simulation/observation approach gives a very time consuming due to the complexity of optimisation problems must be solved, but, it provides a persistent fault indication when a fault appears in the control valve. This has lead to the research of new methods not based on optimization that try to alleviate such computational complexity (see *Section 5.1*).

Fig 7. Fault diagnosis benchmark used in DAMADICS

7.3 Barcelona Sewer Network

Sewer networks are complex large-scale systems which in turn require highly sophisticated supervisory-control systems to ensure that high performance can be achieved and maintained under adverse conditions. They are geographically distributed and decentralized with a hierarchical structure. Each sub-system (catchment) is composed of a large number of elements with time-varying behavior, exhibiting numerous operating modes and subject to changes due to external conditions (weather) and operational constraints. Most cities around the world have sewage systems that combine sanitary and storm water flows within the same network. This is why these networks are known as *Combined Sewage Systems* (CSS). During rain storms, wastewater flows can easily overload these CSS, thereby causing operators to dump the excess of water into the nearest receiver environment (rivers, streams or sea). This discharge to the environment, known as *Combined Sewage Overflow* (CSO), contains biological and chemical contaminants creating a major environmental and public health hazard. Environmental protection agencies have started forcing municipalities to find solutions in order to avoid those CSO events. A possible solution to the CSO problem would be to enhance existing sewer infrastructure by increasing the capacity of the wastewater treatment plants (WWTP) and by building new underground retention tanks (see Figure 1). But in order to take profit of these expensive infrastructures, it is also necessary a highly sophisticated *real-time control* (RTC) scheme which ensures that high performance can be achieved and maintained under adverse meteorological conditions (Schütze, 2004) (Marinacki, 2005). The advantage of RTC applied to sewer networks has been demonstrated by an important number of researchers during the last decades. Comprehensive reviews that include a discussion of some existing implementations are given by (Schilling, 1996) (Schütze, 2004) and cited references therein, while practical issues are discussed by (Schütze, 2002), among other. The RTC scheme in sewage systems might be *local* or *global*. When local control is applied, flow regulation devices use only measurements taken at their specific locations. While this control structure is applicable in many simple cases, in a big city, with a strongly interconnected sewer network and a complex infrastructure of sensors and actuators, it may not be the most efficient alternative. Conversely, a global control strategy, which computes control actions taking into account real-time measurements all through the network, is likely the best way to use the infrastructure capacity and all the available sensor information. Global RTC deals with the problem of generating control strategies for the control elements in a sewer network, ahead of time, based on a predictive dynamic model of the system, and readings of the telemetry system, in order to avoid street flooding, prevent CSO discharges to the environment, minimize the pollution, homogenise the utilization of sewage system storage capacity and, in most of cases, minimize the operating costs (Marinacki 2005). The multivariable and large-scale nature of sewer networks have lead to the use of some variants of Model Predictive Control (MPC), as global control strategy (Ocampo-Martínez, 2008).

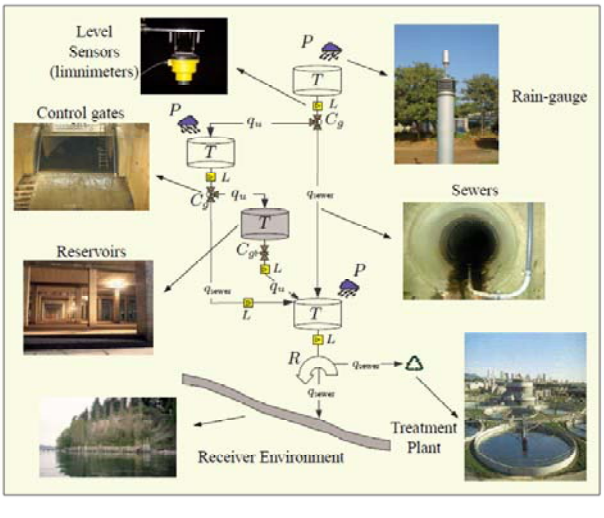

Fig 8. Elements of a sewer network

The global RTC need of operating in adverse meteorological conditions involve, with a high probability, sensor and actuator malfunctions (faults). This problem calls for the use of an on-line FDI system able to detect such faults and correct them (if possible) by activating fault tolerance mechanisms, as the use of soft sensors or using the embedded tolerance of the MPC controller, that avoid that the global RTC control should be stopped every time that a fault appears.

The problem FDI in rain gauges and limnimeters used for the RTC of a sewer network is addressed in (Puig, 2009). The proposed FDI strategy is based on building an interval linear model for every instrument. Then, each instrument reading is compared with the prediction provided by its interval model. While, the real measurement of instrument is inside the interval of predicted behaviour (or *envelope*) generated using its interval model, no fault can be indicated. However, when the measurement is outside its envelope, a fault can be indicated using the approach presented in detail in (Puig, 2008). Once the fault has been detected, a fault isolation procedure is initiated in order to isolate the faulty instrument. The proposed fault isolation algorithm is based on the combined use of several fault signature matrices that considers additional information to the typical binary one. More precisely, fault signature matrices containing information about residual fault sensitivity and time/order of residual activation are used. To exemplify the FDI problem in sewer networks and the proposed FDI methodology, the Barcelona network is used as the case study. Such network has a telemetry system containing 22 rain gauges and more than 100 limnimeters used for the RTC system. The problem of reconstructing measurements from faulty sensors will also be addressed as once the FDI system has located them.

Ocampo-Martínez *et al*. (2006; 2009) address the problem of faults in actuators by using the embedded fault tolerance in the MPC controller used for the global control of the network will be provided. In Ocampo-Martínez et al. (2007), the use of set-membership methods tolerance evaluation is presented in detail.

8. CONCLUSIONS

This paper has reviewed the use of set-membership methods in robust fault detection and isolation (FDI) and tolerant control (FTC). Alternatively to the statistical methods, set-membership methods use a deterministic unknown-but-bounded description of noise and parametric uncertainty (interval models). Using approximating sets to approximate the set of possible behaviours (in parameter or state space), these methods allows to check the consistency between observed and predicted behaviour. When an inconsistency is detected a fault can be indicated, otherwise nothing can be stated. The same principle has been used to estimate interval models for fault detection and to develop methods for fault tolerance evaluation. Finally, same real application of these methods has been used to exemplify the successful uses in FDI/FTC.

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