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Adaptive backstepping control of some uncertain nonlinear oscillators

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***Abstract*— A backstepping-based adaptive controller is designed for a class of uncertain second orded nonlinear systems under the strict-feedback form. It is shown that the closed loop is globally uniformly ultimately bounded and we give explicit bounds on both the asymptotic and transient performance. The control strategy is applied to a system typically found in base isolation schemes for seismic active protection of building structures. This system exhibits a hysteretic nonlinear behavior which is described analytically by the so-called Bouc–Wen model. Unlike other control schemes, the developed backstepping control does not require an exact knowledge of the model parameters. They are only defined within known intervals. The practical effectiveness of the controller is illustrated by numerical simulations.**

I. INTRODUCTION

Backstepping-based control has been proposed in recent years as a powerful method for stabilizing nonlinear systems both for tracking and regulation purposes [1]. The main advantage of these designs is the *systematic* construction of a Lyapunov function for the closed loop, allowing the analysis of its stability properties. The adaptive version of these designs, especially the tuning functions design, offers the possibility to synthetize in a systematic way controllers for a wide class of nonlinear systems (those under the strict-feedback form) whose structure is known but with *unknown* parameters [1]. They also offer the possibility to analyze the transient behavior of the closed loop *in the absence of uncertainties*. Despite the fact that the robustness of the tuning functions design has been studied extensively in the case of linear systems [2], [3], [4], [5], [6], [7], much more is to be done in the case of nonlinear systems [8], [9]. In [10] a robust adaptive scheme for nonlinear systems with globally exponentially stable unmodeled dynamics has been developed for the *regulation* case. For the class of nonlinearities studied in [10] the unmodeled dynamics enter to the system state equations as functions which can be unbounded with respect to the state, but bounded with respect to the time. Despite the fact that the scheme in [10] ensures arbitrary asymptotic performance, it does not allow the quantification of the transient performance as an *explicit* function of the design parameters.

In this paper, we propose a simple backstepping-based adaptive scheme for a class of strict-feedback nonlinear systems for the *tracking* problem. The systems studied in the present paper arise from a class of nonlinear second order oscillators, which are common in structural engineering models of base isolation devices for seismic protection of buildings [11].

The proposed adaptive scheme uses the switching σ-modification [2], [4] and new terms that incorporate part of the information on the uncertainties. The adaptive algorithm allows the quantification of both transient and asymptotic performance as explicit functions of the design parameters. The uncertain nonlinear part of the open loop is written as the sum of the scalar product of -possibly- unknown coefficients with known functions, plus a residual which may be unbounded with respect to the state, but is bounded with respect to the time. This representation has the practical advantage of giving an estimation of the uncertain part by an open loop identification. For structural systems, which are stable in open loop, this is often possible. The reduction of the size of the uncertainty often results in a reduction of the amplitude of the control signal since the nonlinear terms which counteract the effect of the uncertainty are smaller.

In order to test the practical potential of the proposed control scheme, it is applied to design an active controller for a seismic base isolation scheme which has a nonlinear hysteretic behavior. This behavior is described by the so-called Bouc–Wen model [12], which is well accepted in the context of structural mechanics for its ability to describe analytically a wide spectrum of hysteretic loops [13].

Hysteresis is encountered in a wide variety of processes in which the input-output dynamic relations between variables involve memory effects. Examples are found in biology, optics, electronics, ferroelectricity, magnetism, mechanics, structures, among other areas. This paper is primarily concerned with hysteresis in mechanical and structural systems. In these systems, hysteresis appears as a natural mechanism of materials to supply restoring forces against movements and dissipate energy [14]. This mechanism has been exploited in recent years in building damping devices and vibration isolation schemes [15], [16]. In a near context, mechanical and structural hysteresis is also encountered when using new “smart” materials and actuators for vibration control, as the cases of shape memory alloys [17] and electro/magnetorheological fluids [18].

While there is an extensive literature about physical characterization and mathematical modelling of hysteretic systems in different areas, only a few references are found reporting feedback controllers in the general literature on control systems [19], [20], [21], [22]. In structural systems, feedback controllers in the presence of hysteretic components have been primarily encountered when dealing with smart actuators and base isolation schemes. A passivity based control strategy has been presented in [23] along with a hysteretic Preisach model. In base isolated structures, feedback control problems arise when hysteretic isolators are coupled with active feedback controllers. In this case, the Bouc-Wen model [12] has been extensively used to describe the hysteretic behavior. In [24] stochastic linearization of the model is used in conjunction with a linear optimal control. A robust sliding mode control strategy has been proposed in [25] considering that the output of the hysteresis model can be bounded by an uncertain function with linear bounds.

A contribution of this paper to the problem of controlling base isolation schemes is in the use of the Bouc-Wen model with uncertain parameters without relying on any linearization. We consider that all the model parameters are defined within known intervals, without the need of knowing the exact values of the parameters. In practical problems, these intervals can be obtained through identification of real structures [26]. The effectiveness of the controller is shown by means of numerical simulations.

II. PROBLEM STATEMENT

The aim is to control the second order oscillator

$$m\ddot{x} + c\dot{x} + \Phi(x,t) = f(t) + u(t), \qquad (1)$$

where m and c are real parameters, which can be thought as the mass and the viscous damping coefficient, respectively, of a mechanical system (a base isolator-device, for instance). $f(t)$ represents an external excitation (like an earthquake force) and Φ represents a nonlinear restoring force. We write the nonlinear part as

$$\Phi(x,t) = \phi_1 \psi_1\left(\frac{x}{a},t\right) + \ldots + \phi_n \psi_n\left(\frac{x}{a},t\right) + R(x,t), \qquad (2)$$

where $\psi_1, \ldots, \psi_n$ are known (possibly unbounded) locally Lipschitz functions with respect to x, piecewise continuous and bounded with respect to the time. The known constant a is a positive scaling factor which has the same dimension as the displacement x. As we shall see later, we can take

$$a \triangleq \sqrt{\frac{1}{T_0}\int_0^{T_0} x_{ol}^2(t)dt},$$

that is the root mean-square of the open loop displacement response x_{ol} to some "standard" excitation $f(t)$ during some given period of time T_0. The constant uncertain parameters $\phi_1, \ldots, \phi_n$ have the same physical dimension (that of a force). The nonlinear restoring force may not be available for on-line measurement. We assume the following:

Assumption 1: There exists a known (not necessarily bounded) function $r(x,t)$ which is locally Lipschitz with respect to x, piecewise continuous and bounded with respect to t, such that $|R(x,t)| \le r(x,t)$.

Assumption 2: The unknown constant vector $\theta_\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T$ lies inside a known sphere. That is, we know a positive constant M_ϕ such that $\|\theta_\phi\| \le M_\phi$.

Assumption 3: The uncertain parameters m and c lie in known intervals, that is there exist known positive constants $m_{\max}$ and $c_{\max}$ such that $0 < m \le m_{\max}$ and $0 \le c \le c_{\max}$.

Assumption 4: A known bound F on the unknown disturbance $f(t)$ is available. That is $|f(t)| \le F$ for all $t \ge 0$.

Assumption 5: The displacement x and velocity $\dot{x}$ are available for on-line measurement.

The objective is to design a backstepping-based adaptive control law such that the closed loop is globally uniformly ultimately bounded and such that the tracking error can be made arbitrarily small both in the transient and asymptotically by an explicit choice of the design parameters.

III. CONTROLLER DESIGN

We first rewrite equation (1)-(2) in the state space following form:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{1}{m}\Big(-cv\frac{x_2}{v} - \phi_1\Psi_1(\frac{x_1}{a},t) - \ldots - \phi_n\Psi_n(\frac{x_1}{a},t) \\
&\qquad -R(x_1,t) + f(t) + u(t)\Big) \\
&= \frac{1}{m}\left(\theta^T\varphi\left(\frac{x_1}{a},\frac{x_2}{v},t\right) - R(x_1,t) + f(t) + u(t)\right),
\end{aligned} \qquad (3)$$

where $x_1 = x$, $x_2 = \dot{x}$, $\theta = (cv, \phi_1, \ldots, \phi_n)^T$ is the (constant) vector of uncertain parameters and

$$\varphi = \left(-\frac{x_2}{v}, -\Psi_1(\frac{x_1}{a},t), \ldots, -\Psi_n(\frac{x_1}{a},t)\right)^T.$$

The known positive constant v is introduced to have dimensionless components in the regression vector φ and (force) dimension-like terms in the vector of parameters θ. As before, we take

$$v \triangleq \sqrt{\frac{1}{T_0}\int_0^{T_0} \dot{x}_{ol}^2(t)dt},$$

which is the root mean-square of the open loop velocity response $\dot{x}_{ol}$ to the "standard" excitation $f(t)$ during the period of time T_0.

From Assumptions 2 and 3 it follows that

$$\|\theta\| \le \sqrt{(c_{\max}v)^2 + M_\phi^2} \triangleq M_\theta.$$

It is worth noting that in equation (3) the control $u(t)$ is multiplied by an unknown term. Thus we need to construct an estimator $\hat{m}(t)$ of the parameter m.

Consider now the standard variables:

$$\begin{aligned}
z_1 &= x_1 - y_r \qquad \text{(tracking error)}, \\
\alpha_1 &= -c_1\frac{v}{a}z_1, \\
z_2 &= x_2 - \dot{y}_r - \alpha_1,
\end{aligned}$$

where $y_r(t)$ is a known bounded reference signal such that $\dot{y}_r(t)$ and $\ddot{y}_r(t)$ are bounded and piecewise continuous.

The control law and parameters update laws are given in equations (4) and (5) below:

$$\begin{aligned} u(t) = &-\hat{\theta}^T\varphi - c_1\frac{v}{a}(x_2-\dot{y}_r)\hat{m} - \frac{v^2}{a^2}\hat{m}z_1 + \hat{m}\ddot{y}_r \\ &-\frac{a}{v^3 m_{\max}}d_2 z_2 r^2 - \frac{v m_{\max}}{a}c_2 z_2 \\ &-\mathrm{sg}\left(\frac{z_2}{v}\right)\mathrm{cf}\left(\frac{|z_2|}{v}\right)gF, \end{aligned} \tag{4}$$

and

$$\begin{cases} \dot{\hat{\theta}} = \dfrac{M_\theta^2}{m_{\max}v^2}\Gamma\varphi z_2 - \dfrac{v}{a}\Gamma\sigma_\theta\left(\dfrac{\|\hat{\theta}\|}{M_\theta}\right)\hat{\theta}, \\ \dot{\hat{m}} = \gamma m_{\max}\left(\dfrac{c_1}{av}x_2 + \dfrac{1}{a^2}z_1 - \dfrac{1}{v^2}\ddot{y}_r - \dfrac{c_1}{av}\dot{y}_r\right)z_2 \\ \qquad -\gamma\dfrac{v}{a}\sigma_m\left(\dfrac{|\hat{m}|}{m_{\max}}\right)\hat{m}. \end{cases} \tag{5}$$

In the above expressions c_1, c_2, d_2 are dimensionless positive design parameters and $0 \le g \le 1$ adjusts the part of the information on the perturbation f to be included in the control law; Γ is a (dimensionless) positive definite design matrix, $\sigma_\theta(y) = \bar{\sigma}_\theta\sigma(y)$, $\sigma_m(y) = \bar{\sigma}_m\sigma(y)$, and $\mathrm{cf}(y) = \sigma(y/\varepsilon_1)$ where $\sigma(y) = \{0$ if $y \le 1$, $y-1$ if $y \in [1,2]$, 1 if $y \ge 2\}$. In the above expression $\bar{\sigma}_\theta$, $\bar{\sigma}_m$ and ε_1 are (dimensionless) positive design parameters. The function sg is defined as follows: $\mathrm{sg}(y) = \{-1$ if $y \le -\varepsilon_2$, $(1/\varepsilon_2)y$ if $y \in [-\varepsilon_2, \varepsilon_2]$, 1 if $y \ge \varepsilon_2\}$, where ε_2 is a (dimensionless) positive design parameter.

IV. MAIN RESULTS

In this section we state stability and performance results concerning the above control scheme. The results are proven in [27]. The tracking error both of the closed loop displacement and velocity will be measured by the root mean-square norm defined as

$$\|y\|_{[0,T]} \triangleq \sqrt{\frac{1}{T}\int_0^T y(t)^2 dt},$$

for some time interval $[0,T]$.

Theorem 1: The closed loop consisting of the system (3) subject to Assumptions 1-5, along with the control law given by (4) and (5), is globally uniformly ultimately bounded. Moreover, the control signal is bounded.

Theorem 2: Consider system (3) subject to Assumptions 1-5 along with the control law given by (4) and (5), then the following statements hold:

(a) The transient displacement tracking error performance is given by

$$\begin{aligned} \left(\frac{\|z_1\|_{[0,T]}}{\|x_{ol}\|_{[0,T_0]}}\right)^2 \le &\left(\frac{m}{m_{\max}} + \frac{m_{\max}}{m}\right)\left(\frac{1}{\gamma}\left(\frac{\tilde{m}(0)}{m_{\max}}\right)^2\right. \\ &+\left\|\frac{\tilde{\theta}(0)}{M_\theta}\right\|^2_{\Gamma^{-1}} + \frac{\bar{\sigma}_m}{2c_1} + \frac{\bar{\sigma}_\theta}{2c_1}\frac{\|\theta\|^2}{M_\theta^2} + \frac{1}{2c_1 d_2} \\ &\left.+\frac{(1-g)^2}{4c_1c_2}\frac{\|f\|^2_{[0,T]}}{f_{av}^2}\right) + \frac{g(4\varepsilon_1+2\varepsilon_2)}{c_1}\frac{F}{f_{av}}, \end{aligned}$$

for all $T \ge 0$, where $\|X\|_P \triangleq \sqrt{X^TPX}$ for any vector X and positive definite matrix P and $f_{av} \triangleq \frac{\frac{1}{2}mv^2}{a}$.

(b) The asymptotic displacement tracking error performance is given by

$$\begin{aligned} \left(\frac{\|z_1\|_{[t_0,\infty]}}{\|x_{ol}\|_{[0,T_0]}}\right)^2 \le &\left(\frac{m}{m_{\max}} + \frac{m_{\max}}{m}\right)\left(\frac{1}{4c_1d_2}\right. \\ &\left.+\frac{(1-g)^2}{8c_1c_2}\frac{\|f\|^2_{[t_0,\infty]}}{f_{av}^2}\right) + \frac{1}{c_1}g(2\varepsilon_1+\varepsilon_2)\frac{F}{f_{av}}, \end{aligned}$$

for all $t_0 \ge 0$.

(c) The transient velocity tracking error performance is given by

$$\begin{aligned} \left(\frac{\|\dot{x}-\dot{y}_r\|_{[0,T]}}{\|\dot{x}_{ol}\|_{[0,T_0]}}\right)^2 \le &2\left(\frac{m}{m_{\max}} + \frac{m_{\max}}{m}\right) \\ &\left(\frac{1+c_1^2}{\gamma}\left(\frac{\tilde{m}(0)}{m_{\max}}\right)^2 + (1+c_1^2)\left\|\frac{\tilde{\theta}(0)}{M_\theta}\right\|^2_{\Gamma^{-1}} +\right. \\ &+\bar{\sigma}_m\left(\frac{c_1}{2} + \frac{m}{c_2 m_{\max}}\right) + \bar{\sigma}_\theta\cdot\frac{\|\theta\|^2}{M_\theta^2}\left(\frac{c_1}{2}+\frac{1}{c_2}\right) + \\ &\left.+\frac{1}{d_2}\left(\frac{1}{c_2}+\frac{c_1}{2}\right)\left(\frac{c_1}{c_2}+\frac{m}{m_{\max}c_2^2}\right)(1-g)^2\frac{\|f\|^2_{[0,T]}}{f_{av}^2}\right) \\ &+g\left(\frac{2}{c_2}+c_1\right)\left(1+\frac{m}{m_{\max}}\right)(8\varepsilon_1+4\varepsilon_2)\frac{F}{f_{av}}, \end{aligned}$$

for all $T \ge 0$.

(d) The asymptotic velocity tracking error performance is given by

$$\begin{aligned} \left(\frac{\|\dot{x}-\dot{y}_r\|_{[t_0,\infty]}}{\|\dot{x}_{ol}\|_{[0,T_0]}}\right)^2 \le &2\left(\frac{m}{m_{\max}} + \frac{m_{\max}}{m}\right)\cdot \\ &\cdot\left(c_1+\frac{2}{c_2}\right)\left(\frac{1}{4d_2} + \frac{(1-g)^2}{8c_2}\cdot\frac{\|f\|^2_{[t_0,\infty]}}{f_{av}^2}\right) \\ &+g\left(c_1+\frac{2}{c_2}\right)(4\varepsilon_1+2\varepsilon_2)\left(1+\frac{m}{m_{\max}}\right)\frac{F}{f_{av}}, \end{aligned}$$

for all $t_0 \ge 0$.

Remarks: (1) The transient performance is improved as the initial estimation errors $\tilde{m}(0)$ and $\tilde{\theta}(0)$ are improved. (2) We may decrease the effect of the error estimates by increasing the gains γ and Γ. This increase has no effect on the asymptotic tracking performance. (3) Over–estimating the mass leads to a poor transient and asymptotic performance. (4) To improve the displacement tracking performance we may also increase the gains c_1, c_2, d_2 or decrease ε_1, ε_2, $\bar{\sigma}_\theta$ and $\bar{\sigma}_m$. However, increasing the gain c_1 increases also the root mean-square norm of the velocity tracking error. Improving the closed loop displacement behavior may be done at the expense of an increase in the control signal amplitude. (5) Fixing the gain c_1, increasing c_2, d_2 and decreasing ε_1, ε_2 we can achieve a velocity tracking mean-square error as small as desired both in the transient and asymptotically. (6) The gain g may be used for a trade-off between the desired tracking performance and an acceptable control amplitude. (7) The displacement and velocity tracking performance bounds depend *explicitly* on the design parameters.

V. APPLICATION: CONTROL OF THE BOUC–WEN HYSTERETIC OSCILLATOR

In this section, we consider a system within the class considered in Section II, which is part of a base isolation scheme installed to supply passive and active resistance to structures against seismic excitations. In this case the nonlinear restoring force Φ comes from a hysteretic behavior of the isolator materials, which is described by the Bouc–Wen model, widely used in structural mechanics [12]:

$$\begin{aligned}\Phi(x,t) &= \alpha k x(t) + (1-\alpha) D k z(t), \\ \dot{z} &= D^{-1}\left[A\dot{x} - \beta|\dot{x}|\,|z|^{n-1}z - \lambda\dot{x}|z|^n\right].\end{aligned} \tag{6}$$

This model considers the restoring force $\Phi(x,t)$ as the superposition of an elastic component αkx and a hysteretic component $(1-\alpha)Dkz$, in which $D>0$ is the yield constant displacement and $\alpha \in [0,1]$ is the post to pre-yielding stiffness ratio. The hysteretic part involves an auxiliary variable z obtained by solving the above nonlinear differential equation, in which A, β and γ are nondimensional parameters which control the shape and the size of the hysteretic loop, and n is an integer that governs the smoothness of the transition from elastic to plastic response.

With the above model, the state space system under consideration is

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = m^{-1}\left[-cx_2 - \alpha k x_1 - (1-\alpha)kDz + f(t) + u(t)\right], \\ \dot{z} = D^{-1}\left[Ax_2 - \beta|x_2|\,|z|^{n-1}z - \lambda x_2|z|^n\right]. \end{cases} \tag{7}$$

Now we consider system (7) with the following values of the parameters: $m_e = 156\cdot10^3 Kg$, $k_e = 6\cdot10^6 N/m$, $c_e = 2\cdot10^4 Ns/m$, $\alpha_e = 0.6$, $D_e = 0.6m$, $A_e = 1$, $\beta_e = 0.1$, $\lambda_e = 0.5$, $n_e = 3$, where the index e refers to the exact value of the parameter. In fact, we do not need to know these values to implement the controller, only their range is needed. That is, for each of these parameters p we assume that some identification process led to the knowledge of $p_{\min}$ and $p_{\max}$ such that $p_{\min} \le p \le p_{\max}$. We denote $p^* = (p_{\min} + p_{max})/2$. This is always possible because although the hysteretic force may not be available on-line, an identification off-line is possible [26].

With these notations, we write $\Phi(x,t)$ as

$$\begin{aligned}\Phi(x,t) &= (\alpha k - \delta)x + (1-\alpha)Dkz + \delta x \\ &= \phi_1 \frac{x}{a} + (1-\alpha)Dkz + \delta x,\end{aligned} \tag{8}$$

where $\delta = \alpha^* k^*$. Equation (8) is under the form (2) with $\phi_1 = a(\alpha k - \delta)$, $\psi_1(x) = \frac{x}{a}$ and $R(x,t) = (1-\alpha)Dkz$. Since the term δx is known, it will be incorporated into the control u. The residual term R is bounded as follows: $|R(x,t)| \le (1-\alpha_{\min}) D_{\max} k_{\max} \max_{t\ge0}|z(t)| \triangleq r$. The bound $\max_{t\ge0}|z(t)|$ can be determined from the analytical and numerical analysis of the Bouc–Wen model given in [27]. A bound on ϕ_1 may be determined as follows : $|\phi_1| \le a\max(\alpha_{\max}k_{\max} - \alpha^*k^*, \alpha^*k^* - \alpha_{\min}k_{\min}) \triangleq M_\phi$.

The control law is obtained from equation (4):

$$\begin{aligned}u(t) = &-\hat{\theta}^T\varphi - c_1\frac{v}{a}(x_2 - \dot{y}_r)\hat{m} - \frac{v^2}{a^2}\hat{m}z_1 + \hat{m}\ddot{y}_r \\ &- \frac{a}{v^3 m_{\max}} d_2 z_2 r^2 - \frac{v m_{\max}}{a} c_2 z_2 \\ &- \mathrm{sg}\left(\frac{z_2}{v}\right)\mathrm{cf}\left(\frac{|z_2|}{v}\right) gF + \delta x_1,\end{aligned}$$

where the known term δx_1 has been incorporated to the control law.

The excitation on the system is due to an earthquake, whose horizontal ground acceleration is $a_e(t)$. In this way, the excitation force takes the form $f(t) = -ma_e(t)$. As a prototype, we consider the Taft's earthquake, whose acceleration is plotted in Figure 1. An upper bound of the exciting force for the control law design is chosen as $F = 1.2m_e$, considering a larger allowable excitation than the prototype case. To choose the scaling factors a and v, we determine the open loop response of the hysteretic system under the Taft's earthquake excitation, and then we take a and v as the root mean-square of the open loop displacement and velocity respectively during the time period $T_0 = 20$ seconds. This gives $a = 0.0121$ and $v = 0.0758$.

We take the following design parameters: $\gamma = 20$, $\Gamma = 1000\times I_2$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $\bar{\sigma}_\theta = 0.1$, $\bar{\sigma}_m = 0.1$, $c_1 = 1$, $c_2 = 0.02$, $d_2 = 0.007$, and $g = 0.333$. We set $\hat{\theta}(0) = (c_{\max}v, M_\phi)^T$ and $\hat{m}(0) = m_{\max}$.

We choose the following reference signal:

$$y_r(s) = \frac{\omega_r^2}{s^2 + 2\xi_r\omega_r s + \omega_r^2} r_e(s),$$

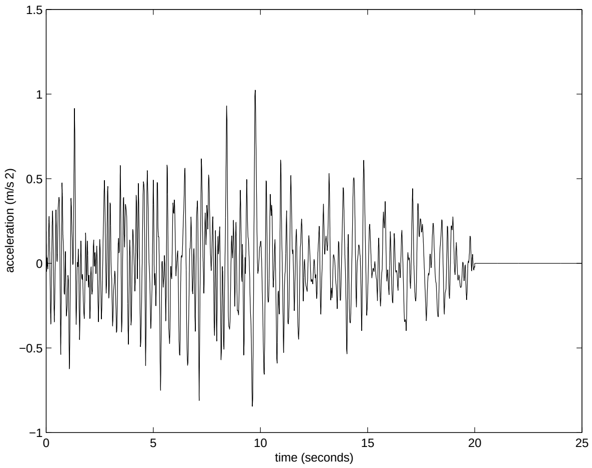

Fig. 1. Earthquake acceleration.

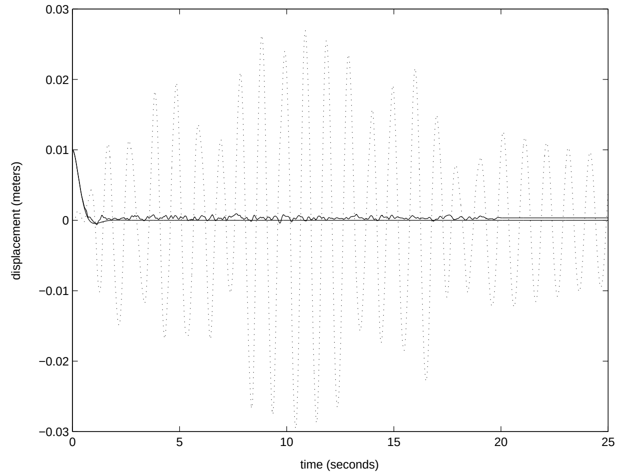

Fig. 2. Controlled (solid) and uncontrolled (dashed) displacement.

where

$$\xi_r = 0.7, \quad \omega_r = 4 rad/s$$

The input reference $r_e(s)$ is set to zero, so that the reference signal is excited only by the initial conditions of the process.

Figures 2 and 3 show the time history of the state variables x_1 (displacement) and x_2 (velocity). A significant reduction in both displacement and velocity can be observed. Also it can be seen that the transient performance of the system has been improved as a result of the control action. The static error in Figure 2 can be reduced as desired by adjusting the design parameters (this may increase the amplitude of the control). Note that, since an internal model of the disturbances is not available, it is not possible to achieve exact asymptotic tracking. Figure 4 displays the control acceleration signal, that is $u(t)/m_e$. The magnitude and shape of this control signal resembles the seismic excitation acceleration in Figure 1, what seems reasonable.

Since, according to (7), the equilibrium of the closed loop is characterized by $u(\infty) = \alpha k x_1(\infty) + (1-\alpha)kDz(\infty)$ and $x_1(\infty) \neq 0$, it is not possible to guarantee theoretically that $u(\infty) = 0$. In practice, an additional criterion can be implemented to cut the control action after the excitation has dissapeared.

VI. CONCLUSION

This paper has presented an application of the adaptive backstepping control technique to an hysteretic second order mechanical system which is common in base-isolation devices for seismic protection of structures. The control scheme gives explicit bounds on the tracking error both asymptotically and during the transient. The practical efficiency to substantially reduce the response of the system has been tested by means of numerical simulations.

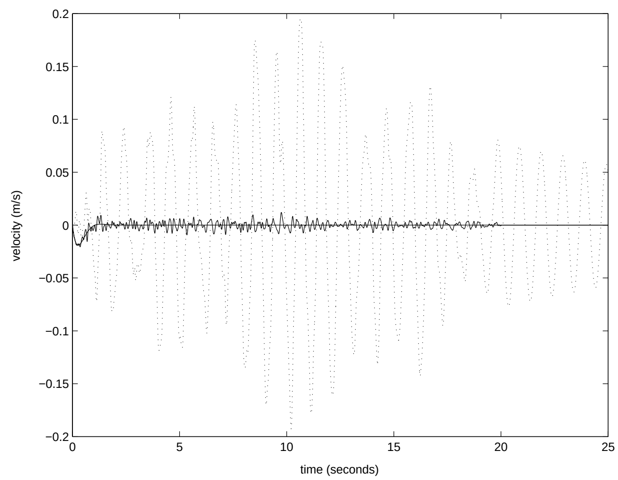

Fig. 3. Controlled (solid) and uncontrolled (dashed) velocity.

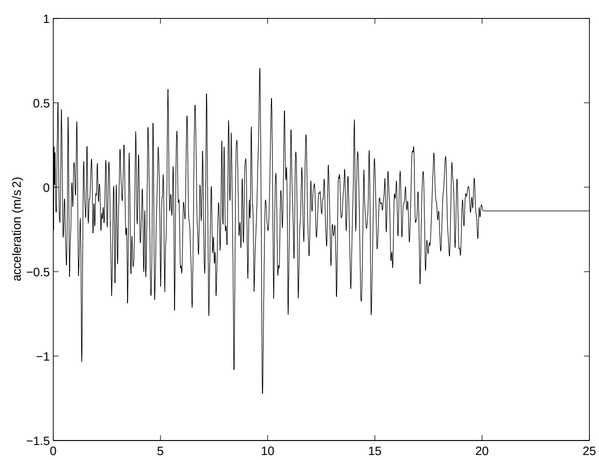

Fig. 4. Control signal.

VII. ACKNOWLEDGMENTS

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