SLIDING MODE CONTROL OF STRUCTURES WITH UNCERTAIN COUPLED SUBSYSTEMS AND ACTUATOR DYNAMICS

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Abstract

This paper deals with the problem of stabilizing a class of structures subject to an uncertain excitation due to the temporary coupling of the main system with another uncertain dynamical subsystem. A sliding mode control scheme is proposed to attenuate the structural vibration. In the control design, the actuator dynamics is taken into account. The control scheme is implemented by using only feedback information of the main system. The effectiveness of the control scheme is shown for a bridge platform with crossing vehicle.

1 Introduction

Vibrations in dynamical flexible structures, as those encountered in civil engineering, are often caused by environmental (seismic or wind) excitations and human made (traffic or heavy machinery) excitations. One way for attenuating the structural vibrations is to use the active control systems so that the safety of the structure and comfortability of the human beings are improved [1]. Different active control methods have been used to account for uncertainties in the structural models and the lack of knowledge of the excitations [2]−[6]. This paper considers a class of structures whose excitation comes through the uncertain coupling with another dynamical system during a certain time. One prototype of this class of systems is illustrated by considering a bridge platform with an unknown moving vehicle as a coupled exciting subsystem. A sliding mode control scheme is proposed to reduce the vibration of bridge induced by the crossing vehicle. In the control design, only the feedback information from the controlled structure (bridge) is used. Numerical simulation is done to show the effectiveness of the proposed active control scheme for an elastically suspended bridge when a truck crosses it.

2 Problem formulation

Consider the problem of active control of an elastically suspended bridge with crossing vehicles as shown in Figure 1. The bridge section consists of a rigid platform with elastic mounts on the left-hand and right-hand sides [7]. The main variables to be measured are the vertical deviation z of the center of mass of the bridge and the inclination Θ with respect to the horizon of the bridge platform. Vibration of the bridge is produced when a truck crosses the bridge with velocity $v(t)$ within a time interval $[t_0, t_f]$. Without the loss of generality, t_0 is set to zero and t_f denotes the final time of interaction between the structure and the truck. The truck is modelled by a mass m with an elastic suspension of damping c and stiffness k. Additional variables ξ, η and ζ are chosen according to Figure 1. The mass of the platform is given by M, and the moment of inertia with respect to C by the parameter J.

The active control is implemented by two actuators located between the ground and the bridge at the left and the right ends respectively. The actuators A_1 and A_2 supply vertical control forces Mu_1 and Mu_2 which complement the resistant passive forces F_1 and F_2 given by the elastic supports. u_1 and u_2 are the control variables. The objective is to attenuate the vibration of the bridge induced by the crossing vehicle by using active forces Mu_1 and Mu_2.

Equations of motion of the truck:

When the truck is not in the bridge (for $t < 0$ and $t > t_f$), the equation of motion of the truck is $m\,\ddot{\eta} = k\,\eta_0 - m\,g$, where η_0 is the position of relaxed suspension. When $t \in [0, t_f]$, the truck is crossing the bridge. Assume that the declination angle Θ is small, then the dynamic motion of the truck is described ny the following equation

$$\left\{ \begin{array}{lcl} m\ddot{\eta} & = & F - mg \\ F: & = & k[\eta_0 - (\eta + \zeta)] - c(\dot{\eta} + \dot{\zeta}) \\ \zeta: & = & z + (\xi - a)\Theta \end{array} \right. \tag{1}$$

Equations of motion of the bridge:

For $t < 0$ the bridge is in a steady state. For $t \in [0, t_f]$, the dynamic behavior of the bridge is described by the following equations of motion:

$$\left\{ \begin{array}{lcl} M\,\ddot{z} & = & M\,g + F - F_1 - F_2 - Mu_1 - Mu_2 \\ J\,\ddot{\Theta} & = & (\xi - a)F + aF_1 - bF_2 + aMu_1 - bMu_2 \\ F: & = & k[\eta_0 - (\eta + \zeta)] - c(\dot{\eta} + \dot{\zeta}) \\ F_1 & = & k_1(-z_{1,0} + z - a\Theta) + c_1(\dot{z} - a\dot{\Theta}) \\ F_2 & = & k_2(-z_{2,0} + z + b\Theta) + c_2(\dot{z} + b\dot{\Theta}) \end{array} \right. \tag{2}$$

where $z_{1,0}$ and $z_{2,0}$ represent the vertical positions of relaxed left-hand and right-hand suspension, respectively.

We consider the bridge as the main system and the truck as the attached uncertain subsystem. The space state variables are split into the measurable ones, $\mathbf{x} := (z, \Theta, \dot z, \dot\Theta)^T$, and the unmeasurable ones $\mathbf{y} := (\eta, \dot\eta)^T$. $\mathbf{u} := (u_1, u_2)^T$ are control signals. The uncertain coupling between the bridge and the truck is due to the scalar force F. When the truck has left the bridge for $t > t_f$, the two systems are obviously decoupled with $F = 0$ and then the equations of motion of the bridge are

$$\left\{\begin{array}{rcl} M\,\ddot z &=& M\,g - F_1 - F_2 - Mu_1 - Mu_2, \\ J\,\ddot\Theta &=& aF_1 - bF_2 + aMu_1 - bMu_2. \end{array}\right. \tag{3}$$

In the above models, consider that the structural parameters of the bridge (M, J, c_1, c_2, k_1, k_2) are known, while the parameters related to the truck (m, c, k, η_0, ξ, v) are assumed to be uncertain but bounded; i.e.,

$$\begin{array}{rcl} \dfrac{k}{m} &=& \omega_0 + \Delta\omega, \quad \text{with } |\Delta\omega| \le \bar\omega, \\ \dfrac{c}{m} &=& \sigma_0 + \Delta\sigma, \quad \text{with } |\Delta\sigma| \le \bar\sigma, \\ \dfrac{k}{M} &=& \Omega, \quad \text{with } \Omega \le \bar\Omega, \\ \dfrac{c}{M} &=& \Upsilon, \quad \text{with } \Upsilon \le \bar\Upsilon, \\ |\eta_0| &\le& \bar\eta_0, \qquad |v(t)| \le \bar v \end{array} \tag{4}$$

where ω_0 and σ_0 are known nominal values and $\bar\omega$, $\bar\sigma$, $\bar\Omega$, $\bar\Upsilon$, $\bar\eta_0$ and $\bar v$ are known bounds. Finally the equations of motion (1) and (2) can be rewritten into the following form:

$$\left\{\begin{array}{rcl} \dot{\mathbf{x}} &=& \mathbf{A_c x} + \mathbf{B_c u} + \mathbf{g_c}(\mathbf{x}, \mathbf{y}, t), \\ \dot{\mathbf{y}} &=& \mathbf{A_r y} + \mathbf{g_r}(\mathbf{x}, \mathbf{y}, t) \end{array}\right. \tag{5}$$

where the parameters of the matrices $\mathbf{A_c}$, $\mathbf{B_c}$ and $\mathbf{A_r}$ are known. The functions $\mathbf{g_c}$ and $\mathbf{g_r}$ include the uncertain coupling effects.

$$\mathbf{A_c} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1+k_2}{M} & \dfrac{ak_1 - bk_2}{M} & -\dfrac{c_1+c_2}{M} & \dfrac{ac_1 - bc_2}{M} \\ \dfrac{ak_1 - bk_2}{J} & -\dfrac{a^2k_1 + b^2k_2}{J} & \dfrac{ac_1 - bc_2}{J} & -\dfrac{a^2c_1 + b^2c_2}{J} \end{pmatrix} \tag{6}$$

$$\mathbf{B_c} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & -1 \\ \dfrac{aM}{J} & -\dfrac{bM}{J} \end{pmatrix}, \quad \text{and} \quad \tilde{\mathbf{g}}_\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ g_{c,3} \\ g_{c,4} \end{pmatrix} \tag{7}$$

Here for $t \in [0, t_f]$:

$$\begin{array}{rcl} g_{c,3}(\mathbf{x}, \mathbf{y}, t) &:=& -\dfrac{k}{M}z - \dfrac{1}{M}[k(\xi(t) - a) + cv]\Theta - \dfrac{c}{M}\dot z \\ && -\dfrac{c}{M}(\xi(t) - a)\dot\Theta - \dfrac{k}{M}\eta - \dfrac{c}{M}\dot\eta + \dfrac{k}{M}\eta_0 \\ && +\dfrac{k_1}{M}z_{1,0} + \dfrac{k_2}{M}z_{2,0} + g \end{array} \tag{8}$$

$$\begin{array}{rcl} g_{c,4}(\mathbf{x}, \mathbf{y}, t) &:=& -\dfrac{k}{J}(\xi(t) - a)z - \dfrac{1}{J}[k(\xi(t) - a)^2 + cv(\xi(t) \\ && -a)]\Theta - \dfrac{c}{J}(\xi(t) - a)\dot z - \dfrac{c}{J}(\xi(t) - a)^2\dot\Theta \\ && -\dfrac{k}{J}(\xi(t) - a)\eta - \dfrac{c}{J}(\xi(t) - a)\dot\eta + \dfrac{k}{J}(\xi(t) \\ && -a)\eta_0 - \dfrac{ak_1}{J}z_{1,0} + \dfrac{bk_2}{J}z_{2,0} \end{array} \tag{9}$$

while, for $t > t_f$,

$$g_{c,3} := \frac{k_1}{M}z_{1,0} + \frac{k_2}{M}z_{2,0} + g \tag{10}$$

$$g_{c,4} := -\frac{ak_1}{J}z_{1,0} + \frac{bk_2}{J}z_{2,0} \tag{11}$$

$$\mathbf{A_r} = \begin{pmatrix} 0 & 1 \\ -\omega_0 & -\sigma_0 \end{pmatrix} \tag{12}$$

$$\mathbf{g_r}(\mathbf{x}, \mathbf{y}, t) = \begin{pmatrix} 0 \\ g_{r,2} \end{pmatrix} \tag{13}$$

For $t \in [0, t_f]$,

$$\begin{array}{rcl} g_{r,2} &=& -\dfrac{k}{m}z - \dfrac{1}{m}[k(\xi(t) - a) + cv]\Theta - \dfrac{c}{m}\dot z - \dfrac{c}{m}(\xi(t) \\ && -a)\dot\Theta - \Delta\omega\,\eta - \Delta\sigma\,\dot\eta + \dfrac{k}{m}\eta_0 - g \end{array} \tag{14}$$

and for $t > t_f$,

$$g_{r,2} = -\Delta\omega\,\eta - \Delta\sigma\,\dot\eta + \frac{k}{m}\eta_0 - g \tag{15}$$

Denote $\mathbf{e} = (e_1, e_2)^T$

$$\begin{array}{rcl} e_i(\mathbf{x}, \mathbf{y}, t) &=& e_{i,1}(t)\,z + e_{i,2}(t)\,\Theta + e_{i,3}(t)\,\dot z + e_{i,4}(t)\,\dot\Theta \\ && + e_{i,5}(t)\,\eta + e_{i,6}(t)\,\dot\eta + e_{i,7}(t). \end{array} \tag{16}$$

Now, it can be verified that $\mathbf{A_c}$ and $\mathbf{A_r}$ are stable matrices and the function $\mathbf{e}(\mathbf{x}, \mathbf{y}, \cdot)$ is continuous for all t except a set $\{0, t_f\}$ and there exist known non-negative scalars α_c^c, α_c^r, δ_c, such that, for all $\mathbf{x}$, $\mathbf{y}$ and t, one has

$$\mathbf{g_c} = \mathbf{B_c\,e}, \quad \text{and} \quad \|\mathbf{e}(\mathbf{x}, \mathbf{y}, t)\| \le \alpha_c^c\|\mathbf{x}\| + \alpha_c^r\|\mathbf{y}\| + \delta_c \tag{17}$$

with

$$\alpha_c^c = \sqrt{2}\,\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2}, \tag{18}$$

$$\alpha_c^r = \sqrt{2}\,\sqrt{\alpha_5^2 + \alpha_6^2}, \tag{19}$$

$$\delta_c = \sqrt{2}\,\alpha_7. \tag{20}$$

where

$$\alpha_1 = \bar{\Omega} \tag{21}$$

$$\alpha_2 = \begin{cases} \frac{1}{(a+b)}\left(\bar{\Omega}a^2 + (a\bar{\Omega} + \bar{\Upsilon}\bar{v})a + a\bar{\Upsilon}\bar{v}\right), & if\ a \geq b \\ \frac{1}{(a+b)}\left(\bar{\Omega}b^2 + (b\bar{\Omega} + \bar{\Upsilon}\bar{v})b + b\bar{\Upsilon}\bar{v}\right), & if\ a < b \end{cases}$$

$$\alpha_3 = \bar{\Upsilon} \tag{22}$$

$$\alpha_4 = \begin{cases} \frac{2a^2}{(a+b)}\bar{\Upsilon}, & \text{if } a \geq b \\ \frac{2b^2}{(a+b)}\bar{\Upsilon}, & \text{if } a < b \end{cases} \tag{23}$$

$$\alpha_5 = \bar{\Omega} \tag{24}$$

$$\alpha_6 = \bar{\Upsilon} \tag{25}$$

$$\alpha_7 = max\left\{\frac{1}{(a+b)}\left[\bar{\Omega}(a+b)\bar{\eta}_0 + \frac{(a+b)k_1 z_{1,0} + g}{M}\right], \frac{1}{(a+b)}\left[\bar{\Omega}(a+b)\bar{\eta}_0 + \frac{(a+b)k_2 z_{2,0} + ag}{M}\right]\right\} \tag{26}$$

Indeed, solving the linear system $\mathbf{g_c} = \mathbf{B_c}\,\mathbf{e}$, it is easy to get that $\mathbf{e} = (e_1, e_2)^T$, where

$$e_1 = \frac{-bMg_{c,3} + Jg_{c,4}}{(a+b)M} \quad ; \quad e_2 = -\frac{aMg_{c,3} + Jg_{c,4}}{(a+b)M}. \tag{27}$$

3 Controller Design

The objective of active control is to attenuate the vibration of the bridge induced by a crossing truck through the uncertain coupling between the dynamics of the bridge and the truck. The controller design will be based on the principle of sliding mode controller [8]−[9], in which only the feedback information of the bridge (not the truck) is used. Define a sliding function as follows

$$\boldsymbol{\sigma}(t) = \boldsymbol{D}\boldsymbol{x}(t) \quad with \quad \sigma_i(t) = \boldsymbol{d}_i^T\boldsymbol{x}(t) \quad (i = 1, 2) \tag{28}$$

where $\boldsymbol{D} = [\boldsymbol{d}_1, \boldsymbol{d}_2]^T \in \boldsymbol{R}^{2\times 4}$ is a matrix to be chosen by the designer in order to guarantee the asymptotic stability of the closed-loop system in sliding mode

$$\dot{\boldsymbol{x}}(t) = \left[\boldsymbol{I}_4 - \boldsymbol{B}_c(\boldsymbol{D}\boldsymbol{B}_c)^{-1}\boldsymbol{D}\right]\boldsymbol{A}_c\boldsymbol{x}(t) \tag{29}$$

For the system (5), a simple choice for $\boldsymbol{D}$ is

$$\boldsymbol{D} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \tag{30}$$

Consequently, the following two sliding functions are defined

$$\sigma_1(t) = \dot{z}(t) + z(t) \quad ; \quad \sigma_2(t) = \dot{\Theta}(t) + \Theta(t) \tag{31}$$

When the system (5) is in sliding mode, $\sigma_i(t) = 0$, $(i = 1, 2)$, one has

$$z(t) = z(t_s)e^{-(t-t_s)} \quad ; \quad \Theta(t) = \Theta(t_s)e^{-(t-t_s)} \tag{32}$$

where t_s is the time instant when sliding motion is generated in the system. Thus, the closed-loop control system in sliding mode is exponentially stable.

In order to design the sliding mode controller, define a Lyapunov function candidate:

$$V(\boldsymbol{\sigma}) = \frac{1}{2}\boldsymbol{\sigma}^T(t)\boldsymbol{\sigma}(t) \tag{33}$$

The derivative of the Lyapunov function is obtained as follows:

$$\begin{aligned} \dot{V}(\boldsymbol{\sigma}) &= \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^T\left[\boldsymbol{D}\boldsymbol{A}_c\boldsymbol{x} + \boldsymbol{D}\boldsymbol{B}_c\boldsymbol{u} + \boldsymbol{D}\boldsymbol{B}_c\boldsymbol{e}(\boldsymbol{x}, \boldsymbol{y}, t)\right] \\ &\leq H(\boldsymbol{x}, \boldsymbol{u}) + H(\boldsymbol{y}) \end{aligned} \tag{34}$$

where

$$\begin{aligned} H(\boldsymbol{x}, \boldsymbol{u}) &=: \boldsymbol{\sigma}^T\{\boldsymbol{D}\boldsymbol{A}_c\boldsymbol{x} + \boldsymbol{D}\boldsymbol{B}_c\boldsymbol{u}\} + ||\boldsymbol{\sigma}^T|| \cdot ||\boldsymbol{D}\boldsymbol{B}_c|| \cdot \\ &\quad \cdot\{\delta_c + \alpha_c^c||\boldsymbol{x}||\} \end{aligned} \tag{35}$$

$$H(\boldsymbol{y}) =: \alpha_c^r||\boldsymbol{\sigma}^T|| \cdot ||\boldsymbol{D}\boldsymbol{B}_c|| \cdot ||\boldsymbol{y}|| \tag{36}$$

Since the state variable $\boldsymbol{y}(t)$ of the coupled uncertain subsystem (the truck) is usually not measurable, the objective of the sliding mode control is to minimize the $\dot{V}(\boldsymbol{\sigma})$ by making the $H(\boldsymbol{x}, \boldsymbol{u}) < 0$. If we denote $\boldsymbol{u}_d(t)$ as the "desired" control signal (without taking into account the actuator dynamics), then the following "desired " sliding mode control law will be used for the generation of sliding motion:

$$\begin{aligned} \boldsymbol{u}_d &= -\boldsymbol{k}_c\boldsymbol{x} - (\boldsymbol{D}\boldsymbol{B}_c)^{-1}\{\psi_0 + \psi_1|z| + \psi_2|\Theta| + \psi_3|\dot{z}| \\ &\quad + \psi_4|\dot{\Theta}|\}\left[sgn(\sigma_1), sgn(\sigma_2)\right]^T \end{aligned} \tag{37}$$

where

$$\boldsymbol{k}_c = \frac{1}{4}\boldsymbol{D}\boldsymbol{B}_c\boldsymbol{D} + (\boldsymbol{D}\boldsymbol{B}_c)^{-1}\boldsymbol{D}\boldsymbol{A}_c \tag{38}$$

$$\begin{cases} \psi_0 > \sqrt{2}\alpha_7||\boldsymbol{D}\boldsymbol{B}_c|| \\ \psi_1 \geq \sqrt{2}\alpha_1||\boldsymbol{D}\boldsymbol{B}_c|| \\ \psi_2 \geq \sqrt{2}\alpha_2||\boldsymbol{D}\boldsymbol{B}_c|| \\ \psi_3 \geq \sqrt{2}\alpha_3||\boldsymbol{D}\boldsymbol{B}_c|| \\ \psi_4 \geq \sqrt{2}\alpha_4||\boldsymbol{D}\boldsymbol{B}_c|| \end{cases} \tag{39}$$

It is easy to verify that if the controller gains are chosen to accomplish the relationships eqns.(38)-(39) then $H(\boldsymbol{x}, \boldsymbol{u}) < 0$. In practice, the continuous approximation is used for the control law (37) to attenuate the high-frequency chattering

$$sgn(\cdot) \Longrightarrow \frac{(\cdot)}{|(\cdot)| + \gamma} \tag{40}$$

where γ is a positive small constant. Thus, the corresponding continuous "desired" sliding mode control law is

$$\begin{aligned} \boldsymbol{u}_d &= -\boldsymbol{k}_c\boldsymbol{x} - (\boldsymbol{D}\boldsymbol{B}_c)^{-1}\{\psi_0 + \psi_1|z| + \psi_2|\Theta| + \psi_3|\dot{z}| \\ &\quad + \psi_4|\dot{\Theta}|\}\left[\frac{\sigma_1}{|\sigma_1| + \gamma_1}, \frac{\sigma_2}{|\sigma_2| + \gamma_2}\right]^T \end{aligned} \tag{41}$$

Now, assume that a hydraulic actuator is used for the implementation the control action generated by the "desired" sliding mode controllers (37) or (41). The dynamic behavior of the hydraulic actuator is described by the following equation [10]:

$$v_i(t) = P_v \dot{u}_i(t) + P_l u_i(t) + P_a \dot{z}(t), \qquad i = 1, 2 \tag{42}$$

where

$$P_v = \frac{C_v}{4\beta P_a}, \qquad P_l = \frac{C_l}{P_a}, \qquad P_a > 0 \tag{43}$$

The equation (42) represents the internal dynamics of a hydraulic actuator's chamber, with $u_i(t)$ being the average output actuator force, $v_i(t)$ the total fluid flow rate of the actuator's chamber, P_a the actuator effective piston's area, C_v the chamber's volume, β the bulk modulus of the hydraulic fluid, C_l the coefficient of leakage and $\dot{x}_i(t)$ the velocity of the piston. Denote $\tilde{\boldsymbol{u}}(t)$ as the tracking error between the "real" control action $\boldsymbol{u}(t)$ and the "desired" control action $\boldsymbol{u}_d(t)$; i.e.,

$$\tilde{\boldsymbol{u}}(t) = \boldsymbol{u}(t) - \boldsymbol{u}_d(t) \tag{44}$$

Apply the control force $\boldsymbol{v}(t) = [v_1(t), v_2(t)]^T$ in (42) to the bridge platform and define a new Lyapunov function candidate $V(\boldsymbol{\sigma}, \tilde{\boldsymbol{u}})$

$$V(\boldsymbol{\sigma}, \tilde{\boldsymbol{u}}) = V_1(\boldsymbol{\sigma}) + V_2(\tilde{\boldsymbol{u}}) \tag{45}$$

$$V_1 = \tfrac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma} \qquad V_2(\tilde{\boldsymbol{u}}) = \tfrac{1}{2}P_l^{-1}P_v \tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \tag{46}$$

The controller design is made through the minimization of the derivative of a Lyapunov function candidate. From eqn.(44)

$$\boldsymbol{u}(t) = \boldsymbol{u}_d(t) + \tilde{\boldsymbol{u}}(t) \tag{47}$$

Then, the derivative of $V(\boldsymbol{\sigma}, \tilde{\boldsymbol{u}})$ is obtained as follows by using eqns. (37), (42) and (44):

$$\dot{V}(\boldsymbol{\sigma}, \tilde{\boldsymbol{u}}) \leq H(\boldsymbol{x}, \boldsymbol{u}, \tilde{\boldsymbol{u}}) + H(\boldsymbol{y}) \tag{48}$$

where

$$\begin{aligned} H(\boldsymbol{x}, \boldsymbol{u}, \tilde{\boldsymbol{u}}) \quad &=: \quad H(\boldsymbol{x}, \boldsymbol{u}) - \frac{1}{4}\boldsymbol{\sigma}^T\boldsymbol{B}_c^T\boldsymbol{D}^T\boldsymbol{D}\boldsymbol{B}_c\boldsymbol{\sigma} + \boldsymbol{\sigma}^T\boldsymbol{D}\boldsymbol{B}_c\tilde{\boldsymbol{u}} \\ &\qquad -\tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \\ &= \quad H(\boldsymbol{x}, \boldsymbol{u}) - (\tilde{\boldsymbol{u}} - \frac{1}{2}\boldsymbol{D}\boldsymbol{B}_c\boldsymbol{\sigma})^T(\tilde{\boldsymbol{u}} - \frac{1}{2}\boldsymbol{D}\boldsymbol{B}_c\boldsymbol{\sigma}) \\ &\leq H(\boldsymbol{x}, \boldsymbol{u}) < 0 \end{aligned} \tag{49}$$

Therefore, the "real" control action $\boldsymbol{u}(t)$ (taking into account the actuator dynamics) can minimize the derivative of Lyapunov function $\dot{V}(\boldsymbol{\sigma}, \tilde{\boldsymbol{u}})$ by making $H(\boldsymbol{x}, \boldsymbol{u}, \tilde{\boldsymbol{u}}) < 0$, which is similar to the case when a "desired" control action $\boldsymbol{u}_d(t)$ (without taking into account the actuator dynamics) is applied to the bridge platform.

4 Numerical Simulation Results

In the numerical simulation, an actively suspended bridge platform is considered as the main system and the excitation is induced by a truck when it crosses the bridge. The following parameters are used for the controller design and numerical simulation:

Nominal parameters and bounds for uncertainties: $\bar{\eta}_0 = 1$ [m], $\omega_0 = 40$ [N/(m kg)], $\bar{\omega} = 20$ [N/(m kg)], $\sigma_0 = 1$ [Ns/(m Kg)], $\bar{\sigma} = 5$ [Ns/(m Kg)], $\bar{\Omega} = 5$ [N/(m kg)], $\bar{\Upsilon} = 0.5$ [Ns/(m Kg)], $\bar{v} = 8.33$ [m/s] ($\bar{v} = 30$ [km/h]), $k_0 = 4 \cdot 10^5$ [N/m], $c_0 = 10^4$ [Ns/m].

Bridge: $M = 10^5$ Kg, $J = 2 \cdot 10^7$ Kg m^2, $a = b = 25$ m, $k_i = 4 \cdot 10^6$ N/m and $c_i = 4 \cdot 10^4$ N s/m for each $i = 1, 2$. $z_{1,0} = z_{2,0} = -0.125$ m, which correspond to the equilibrium position for the platform without truck and no control.

The parameters of the truck, which are unknown for the controller design, are the following:

Truck: $m = 10^4$ Kg, $v = 8.33$ m/s (= 30 Km/h), $k = 4 \cdot 10^5$ N/m, $c = 10^4$ N s/m, $\eta_0 = 0.75$m.
The parameters of the hydraulic actuator are the following: $P_a = 2.4 \times 10^{-2}$, $P_v = 3.57 \times 10^3$ and $P_l = 1.99 \times 10^{-5}$

With the above parameters, we obtain $\boldsymbol{\alpha} = (5, 129.165, 0.5, 12.5, 5, 0.5, 500.0025)^T$. In the controller design The matrix $\boldsymbol{D}$ is chosen as

$$\boldsymbol{D} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

in order to make a reasonable trade-off between performance and control effort and the controller gains are chosen as follows according to eqns.(38) and (39):

$$\boldsymbol{\Psi} = (\psi_0, \psi_1, \psi_2, \psi_3, \psi_4)^T = (1 \times 10^{-3}, 10, 258.33, 1, 25)^T$$

The platform is excited by the crossing of the truck for time $t \in [0, 6]$ seconds, and after $t = 6$ seconds no excitation is evolved with between the platform and the truck. The time history of structural vibration of the bridge platform for the uncontrolled case (dash line) and the controlled case (solid line) are shown in Figures 2 and 3. Concretely, Figure 2 shows the main effect of the control, which is to add damping to the bridge platform. Without control, the platform has very low damping, thus exhibiting a highly oscillatory response. The damping coefficients of the two end supports are $c_1 = c_2 = 4 \times 10^4$ N s/m, which corresponds to a damping factor of 4.5% approximately. The control modifies this behavior, forcing a practically overdamped response. It is seen how the vertical deflection z of the center of mass of the platform evolves slowly but smoothly towards its equilibrium position with the truck ($z = 0.125m$). After $t = 6$ seconds the excitation disappears and the platform deflection evolves to recover the initial equilibrium position. Figure 3 shows that the inclination Θ of the bridge has been significantly improved. Figures 4 and 5 display the control signals u_1 and u_2, which are feasible for practical actuators.

5 Conclusions

An active sliding mode control scheme has been proposed in this paper to attenuate the vibrations of a main system excited by an temporarily coupled uncertain subsystem. Only the feedback information of the main system has been used in the control design, without measuring the response of the coupled uncertain subsystem. It has been shown that the active controller also works well when the actuator dynamics is taken into account. The results of numerical simulation have illustrated the effectiveness of the proposed control scheme for an active controlled suspended bridge platform with crossing vehicles.

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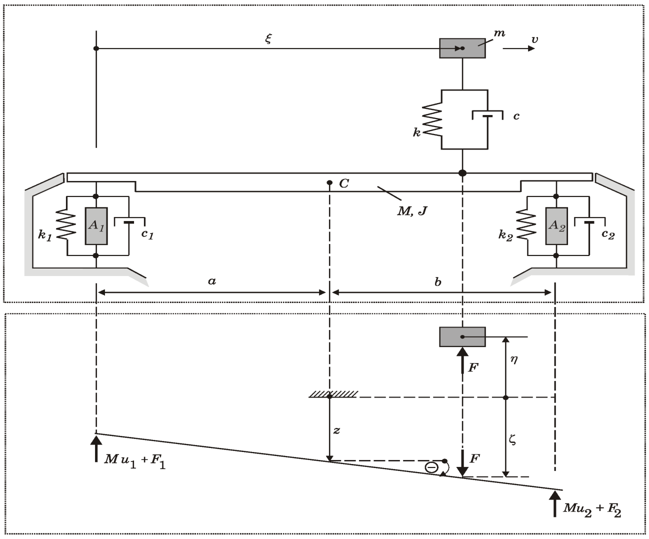

Figure 1: Actively controlled bridge platform with crossing vehicle

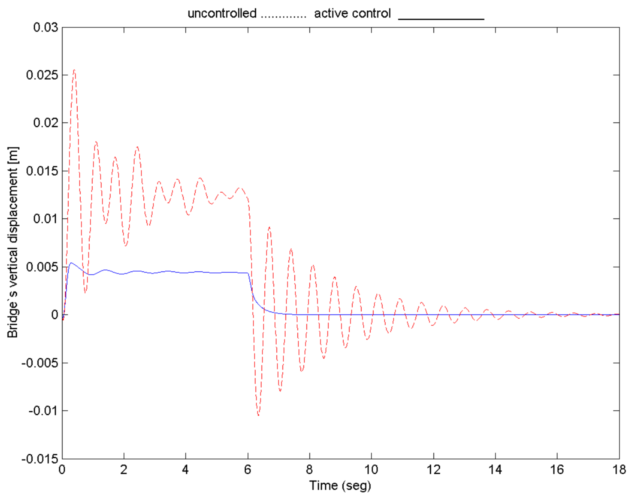

Figure 2: Vertical vibration of the bridge

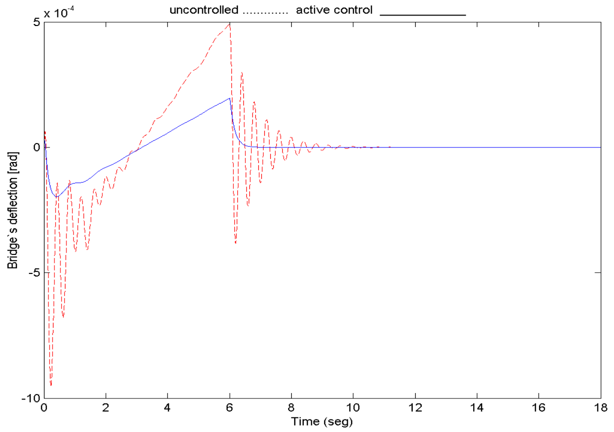

Figure 3: Inclination of the bridge

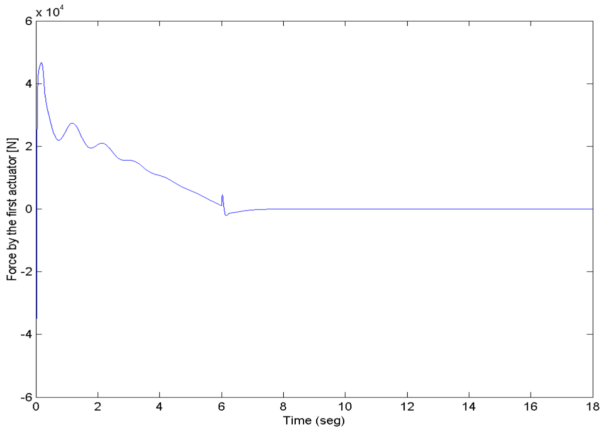

Figure 4: Control force of the 1st actuator

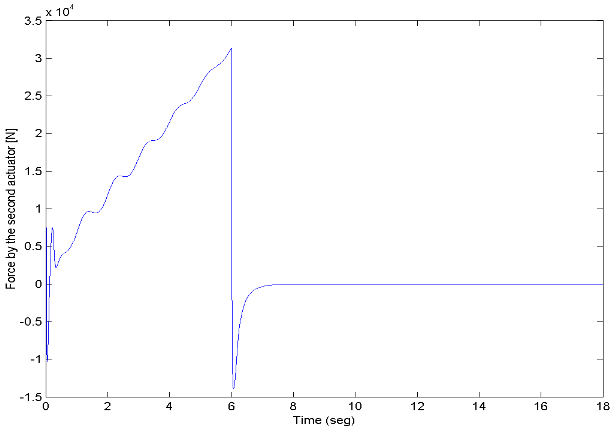

Figure 5: Control force of the 2nd actuator